

DEVELOPMENT OF AN IMPROVED ERGODIC CAPACITY OF UNDERLAY COGNITIVE RADIO WITH IMPERFECT CHANNEL STATE INFORMATION

BY

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NIGERIA

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NIGERIA

JULY, 2017

DECLARATION

Ideclare that the work in thisDissertationentitled“Development of an Improved Ergodic Capacity of Underlay Cognitive Radio with Imperfect Channel State Information”has been carried out by me in the Department of Electrical and Computer Engineering. The information derived from the literature has been duly acknowledged in the text and a list of references provided. No part of this dissertation was previously presented for another degree or diploma at this or any other Institution.

Aminu Muhammad ABBA _____ _____

(Student) Signature Date

CERTIFICATION

This dissertationentitled **"DEVELOPMENT OF AN IMPROVED ERGODIC CAPACITY OF UNDERLAY COGNITIVE RADIO WITH IMPERFECT CHANNEL STATE INFORMATION"**by Aminu Muhammad ABBA meets the regulations governing the award of the degree ofMaster of Science (MSc) in Telecommunications Engineering of theAhmadu Bello University, and it is approved for its contribution to knowledge and literary presentation.

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(Dean, School of Postgraduate Studies)Signature Date

DEDICATION

This research work is dedicated to Almighty God.

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ABSTRACT

This research work presentsthedevelopment of an improved ergodic capacity of underlay Cognitive Radio (CR) with imperfect Channel State Information(CSI). An underlay CR network under a Peak Interference Power (PIP) constraint imposed by a Primary User (PU) was considered.Aclosed formergodic capacity expression of the Secondary User (SU)under PIP constraintwas derivedin order to determine the ergodic capacity under different fading types using Nakagami-m distributions.The m-parameter of the Nakagami-m fading channel measures the ratio of the Line-of-Sight (LoS) signal power to that of the multipath component. The improved ergodic capacity expression developed was validated with the existing ergodic capacity expression at m=1. The results showed that the ergodic capacity of the SU could be extended to different fading types in the CR path by adjusting the m-parameter.The impact of channel estimation errors, σ^2 and channelcorrelation coefficient, ρ on the SUergodic capacity was studiedunder different fading types to provide an insight on the capacity behavior of CR network. The resultsobtained atdifferent values of m parametersshowedthe ergodic capacity degraded as a result of increasing σ^2 and it increases with increasing ρ .It was observed that at m=1/2,1,2 and 3, a significant capacity gains of 21.65%, 19.71%, 17.43%, and 15.82% were achieved, when 3% interference outage $\left(P_{out}\right)$ was considered for all the m values from $\sigma^2 = 1$ to $\sigma^2 = 0$. However, a capacity gains of 23.59%, 22.31%, 21.02%, and 20.26% were also achieved respectively at 1% P_{out} . It was also found that at m=1/2,1,2 and 3, a spectral efficiency in (bits/S/Hz) of 0.5007, 0.2888,0.2660, and 0.2596 were achieved when 3% P_{out} was considered from $\rho = 0$ to $\rho = 1$, while a spectral efficiency

of 0.2548, 0.1771, 0.1211, 0.0931 were also achieved at 1% P_{out} for the respective m values.

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LIST OF ABBREVIATIONS

Acronyms	Definition
AIP	Average Interference Power
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
CDF	Cumulative Density Function
CR	Cognitive Radio
CRN	Cognitive Radio Network
CSI	Channel State Information
GSC	Generalized Selective Combining
ISM	Industrial, Scientific and Medical
LoS	Line of Sight
MAC	Message Authentication Code
MMSE	Minimum Mean Square Error
MRC	Maximal ratio combining
NBC	Nigerian Broadcasting Commission
NCC	Nigerian Communication Commission
PDF	Probability Density Function
PIP	Peak Interference Power Constraint
PRx	Primary Receiver

PTx Primary Transmitter

PU Primary User

QoSQuality of Service

RFRadio Frequency

SEP Symbol Error Probability

SNRSignal to Noise Ratio

SUSecondary User

SRxSecondary Receiver

STxSecondary Transmitter

TVTelevision

UHFUltra High Frequency

U-NII Unlicensed National Information Infrastructure

VHFVery High Frequency

CHAPTER ONE

INTRODUCTION

1.1 Background To the Study

Wireless communications offers a wide range of services like mobile internet, data exchange, location tracking, space communications, etc. In recent years there has been a dramatic increase in the demand for radio spectrum. However, the radio spectrum is a limited natural resource. Access to it is regulated by the government agencies such as the Federal Communications Commission (FCC) in the United Sates, Nigerian Communication Commission (NCC) and Nigerian Broadcasting Commission (NBC).

The limited radio spectrum led to the development of the cognitive radio (CR) with a view to exploiting the available spectrum efficiently. This has led to the emergence of a spectrum sharing technique in whichunlicensed/ secondary users (SU's) can share the spectrum of licensed/ primary users (PU's) without harming primary communications (Sboui, 2013).

The cognitive radio has been studied widely as it provide ways to improve the spectrum efficiency by allowing SU to concurrently access the spectrum band licensed to the PU while causing limited interference to the PU (Haykins, 2005).

1.2 Problem Statement

Capacity analysis is useful in investigating the ultimate performance limits of a CR network. A CR user may coexist with the incumbent PUs on either a non-interfering basis or on an interfering-tolerant basis. The interference-tolerant case works such that the CR users are allowed to operate on the frequency band assigned for the PUs as long

as the interference received at the primary user remains below a certain threshold. The channel state information (CSI) between the secondary transmitter (STx) and the primary receiver (PRx) is used by the STx to calculate the appropriate transmit power to limit the interference on the PRx. However, in practical communication systems, the CSI is frequently imperfect for spectrum sharing systems, it is difficult for the SU to perfectly acquire the CSI in the STx to the PRx channel due to channel estimation errors and feedback delay. The cognitive radio channel may also undergoesdifferent fading types, which affects the ergodic capacity. The existing ergodic capacity under imperfect CSI by Xu*et al.*, (2013b) is restricted to study the performance behavior of CR in a Rayleigh fading environment. This work presents the development of an improvedergodic capacity of underlay CRsystem with imperfect CSI under different fading conditions using a Nakagami-m distribution.

1.3 Significance of Research

A number of studies have been conducted on spectrum sharing environments, especially on the secondary link capacity based on using power control to satisfy interference constraints. In most of the works (Ghasemi& Sousa, 2007; Rezki&Alouini, 2012a), the capacity of the secondary link was analyzed under the assumption that the secondary user knows the CSI perfectly between the STx and the PRx. However, the obtained CSI is frequently outdated in practical systems due to channel estimation errors and feedback delay. In a spectrum sharing system the problems caused by this outdated CSI are severe (Kim *et al*, 2012). The channel estimation errors and feedback delay exist simultaneously, and neglecting either one of them leads to violation of the interference

power constraint at the PU. Motivated by the results of Xu*et al*. (2013b), there is a need to extend the work in order to consider the fading conditions in the cognitive radio channel, this is because the cognitive radio path may experience different fading types. The performance analysis on the behavior CR on the ergodic capacity of the SU under different fading conditions can be carried out when this fading conditions are considered.

1.4 Aim and Objectives

The aim of this research is to developan improved ergodic capacity of underlay cognitive radio with an imperfect channel state information. The objectives which was employed to achieve this aim is as follows:

1. To determine the ergodic capacity under the peak interference power constraint
2. To examine the impact of channel estimation errors using the estimation errors variance, σ^2 and the channel correlation coefficientusing the correlation coefficient, ρ on the ergodic capacity
3. To validate the results obtained using the work of Xu*et al*., (2013b)

1.5 Scope of the Research

The scope of this dissertation is to study the performance of CR under different fading conditions. The study is based on developing an improved ergodic capacity expression with imperfect CSI using Nakagami-m fading distribution in the cognitive radio path, by adjusting the m parameter of theNakagami-m fading distribution, the behavior of CR under different fading conditions can be studied.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This section presents the review of the fundamental concepts and the review of similar works which are relevant to this research.Under fundamental concepts, discussion covered cognitive radio networks, principle model equations and techniques available to use in order to justify the suitable one for this work. On the other hand, the review of similar researches established the extent of research in the subject area which led to a different approach used to achieve the desired significant contributions.

2.2 Review of Fundamental Concepts

In this section, concepts that are fundamental toCognitive radio, Cognitive radio paradigm, Cognitive radio networks, Wireless channel, Wireless channel characteristics, Rayleigh Distribution, Nakagami-m Distribution,Channel state information (CSI), Channel capacity and Ergodic capacity are presented:

2.2.1 Cognitive Radio

The term “cognitive radio” was coined by Joseph Mitola III(Mitola&Manguire, 1999). Mitola described how a cognitive radio could enhance the flexibility of personal communication services through a new language called the radio knowledge representation language.

The potential for CR is a novel efficient methodology, extension of software-defined radio, to transmit and receive information over various wireless communication devices. Being aware of the environment, cognitive radio chooses the best available option based on performance for each application. The different measurement parameters include frequency, power, antenna, transmitter bandwidth, modulation scheme, etc. This means the said radio has to deal with different RF spectrum baseband varieties at the same time, thus requiring a more robust, efficient and reconfigurable hardware architecture (Hayar*et al.*, 2007).

CR can therefore be defined as an intelligent wireless communication system that is aware of its surrounding environment (i.e., outside world), and uses the methodology of understanding-by-building to learn from the environment and adapt its internal states to statistical variations in the incoming RF stimuli by making corresponding changes in certain operating parameters (e.g., transmit-power, carrier frequency, and modulation strategy) in real-time with two primary objectives in mind (Haykins, 2005):

1. Highly reliable communication whenever and wherever needed
2. Efficient utilization of the radio spectrum

2.2.2 Cognitive Tasks

The cognitive process starts with passive sensing of radio frequency (RF) stimuli and culminates with action. Haykins,(2005) in his work focused on three cognitive tasks.

1. Radio-scene analysis, which encompasses the following:
    i) Estimation of interference temperature of the radio environment;
    ii) Detection of spectrum holes

2. Channel identification, which encompasses the following:

    i) Estimation of channel-state information (CSI);

    ii) Prediction of channel capacity for use by the transmitter

3. Transmit-power control and dynamic spectrum management

The tasks in 1 and 2 are usually carried out in the receiver while task 3 is carried out in the transmitter. Through interaction with the RF environment, these three tasks form a cognitive cycle, which is illustrated in Figure 2.1:

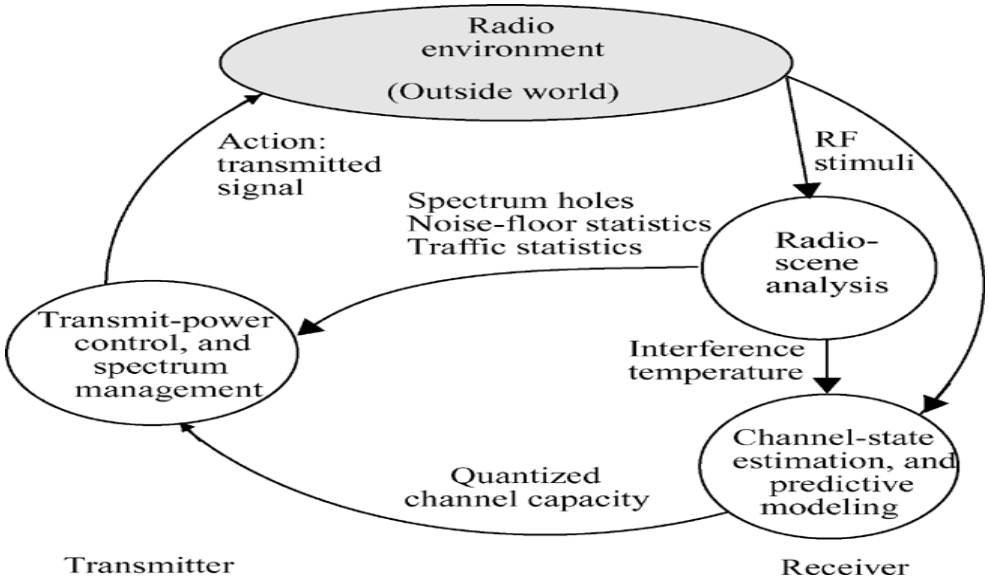


Figure 2.1: Cognitive Cycle (Haykins, 2005)

From Figure 2.1, it has become necessary that the cognitive module in the transmitter must work in a harmonious manner with the cognitive module in the receiver. In order to maintain this harmony between the CR transmitter and receiver at all times, there is a need for a feedback channel connecting the receiver to the transmitter. Through the

feedback channel, the receiver is enabled to convey information on the performance of the forward link to the transmitter (Haykins, 2005).

2.2.3 Cognitive Radio Paradigm

In a CR network, cognitive users (or secondary users (SUs)), in general unlicensed users, share the same wireless network with licensed primary users (PUs). This operation could have one of the three following modes (Sboui, 2013):

2.2.3.1 *Underlay*

In CR underlay mode, the SU is used, at the same time, the same frequency of the PU with previous knowledge of the interference caused by its transmitter to receiver. Consequently, cognitive communication is not performed unless this interference is below a previously defined threshold. Meanwhile, the SU user is optimizing power consumption with adequate power allocation.

2.2.3.2*Overlay*

As in underlay mode, in the overlay, the SU shares the spectrum with PU, knowing PUs messages and codebook. Then, the SU uses advanced signal processing and coding in order to relay primary communication when transmitting its own signal. In the case of multi-SUs, cooperative communications can be adopted to relay primary communication and exploit spectrum.

2.2.3.3*Interleave*

The interleave is different from previous modes because, in this mode, the SU is opportunistically exploiting spectral holes (frequency, time, space) in order to perform its communication with minimal interference to PU communication.

2.2.4 Cognitive Radio Networks

The general CR network architecture, its functional framework and its mode of operation is presented as follows (Lassila&Pentilen, 2008):

2.2.4.1 Cognitive radio architecture

The CR architecture consists of network components, spectrum heterogeneity and network heterogeneity.

1. **Network Components:** The components of the CR network architecture, as shown in Figure 2.2, can be classified in two groups: the primary network (or licensed network) and the CR network. The primary network is referred to as an existing network, where the PU activities are controlled through primary base stations. Due to their priority in spectrum access, the operations of the primary should not be affected by the SUs. The CR network does not have a license to operate in a desired band. Hence, additional functionality is required for CR users to share the licensed spectrum band. The CR network may also include spectrum brokers that play a role in distributing the spectrum resources among different CR networks.

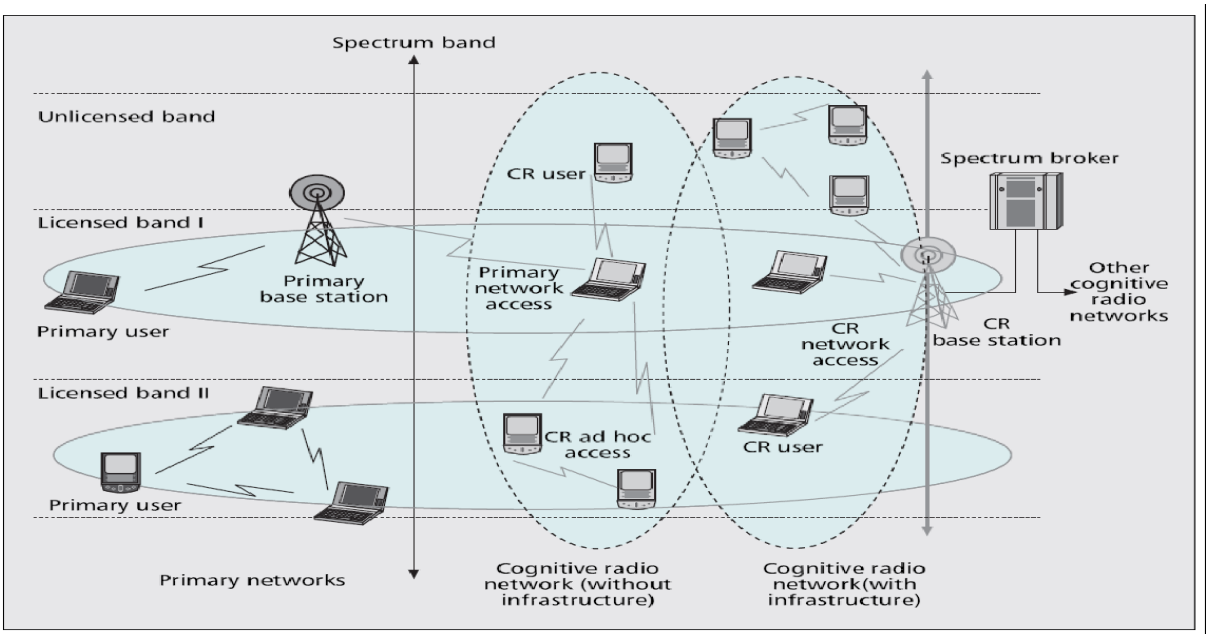


Figure 2.2: Network Architecture for CR Networks (Lassila&Pentilen, 2008)

2. **Spectrum Heterogeneity:** CR users can access both the licensed portion of the spectrum used by PU and the unlicensed portions of the spectrum through the use of wideband access technology. The operation types for CR network can be classified as licensed band operation and unlicensed band operation. The licensed band is primarily used by the primary network, the CR focused mainly on the detection of PUs in this case. In the absence of PU, the SU have the same right to access the spectrum. Hence, spectrum sharing methods are required for CR users to compete for unlicensed band.
3. **Network heterogeneity:** The CR user have the opportunity to perform three different access types:
    I. **Cognitive Radio Network Access:**SUs can access their own cognitive base station, on both licensed and unlicensed spectrum bands.

II. **Cognitive radio Ad- hoc Access:**SUs can communicate with other SUs through an ad hoc connection on both licensed and unlicensed spectrum bands.

III. **Primary Network Access:**SUs can also access the primary base station through the licensed band. The SUs require an adaptive message authentication code (MAC) protocol, which enables roaming over multiple primary networks with different access technologies.

2.2.4.2 Functional framework

Cognitive radio networks impose unique challenges due to their coexistence with primary network as well as diverse QoS requirements. Thus, there is a need for spectrum management functions with the following critical design challenges: CR network should avoid interference with primary networks, QoS awareness to support different traffic types and seamless communication regardless of the appearance of PU. To address these challenges, different spectrum management functionalities in CR networks are required:

1. **Spectrum Sensing:** the SU monitors the available spectrum bands, capture their information and then detect spectrum holes, which are the unused portion of the spectrum for allocation.
2. **Spectrum Decision:** the allocation of spectrum is not only dependent on spectrum availability, but it is also based on internal and (possibly external policies)
3. **Spectrum Sharing:** The CR network access should be coordinated to prevent multiple users colliding in overlapping portions of the spectrum.

4. **Spectrum Mobility:** If the specific portion of the spectrum in use is required by a PU, the communication must be continued in another vacant portion of the spectrum.

2.2.5 Wireless Channel

A wireless channel contains multipath components from direct line of sight (LoS) component and various reflected and refracted components. It is made by the constructive and destructive addition of different multipath components introduced by the channel. The channel characteristics depend upon the number of scattered objects, number of multipath, the size of the objects and the amount of absorption by the surrounding environment such as wall and roofs, as illustrated in Figure 2.3.(Garg, 2007). Another characteristic of the wireless channel is its time-varying nature. This variation in time arises because either the transmitter or the receiver is moving (Sharma & Mishra, 2015).

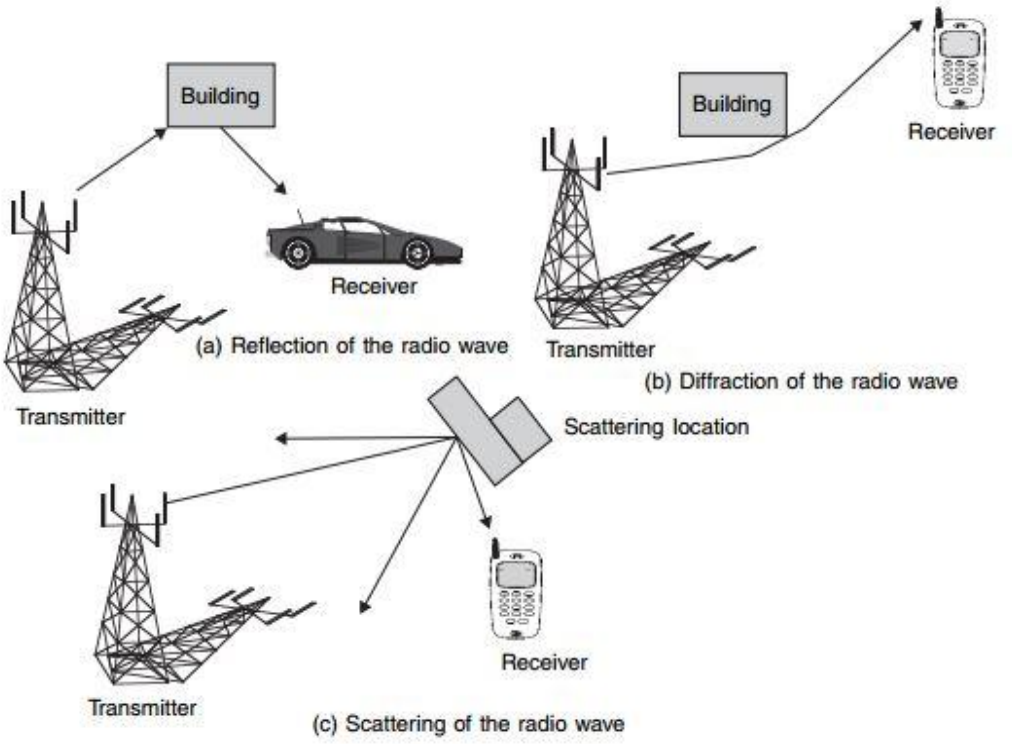


Figure 2.3: Reflection, Diffraction and Scattering of Radio Wave (Garg, 2007)

2.2.5.1Characteristics of wireless channel

The wireless channel is different and much more unpredictable than the wired line channel because of factors such as (Sboui, 2013):

1. **Path loss:** as signal propagates, the path loss describes the signal attenuation depending mainly on the distance and the environment (rural, urban etc.). Path loss is defined as the ratio of the received power to transmit power. In free space the path loss is inverse to the square of the distance.
2. **Shadowing:** accrues when transmitted signals are blocked by huge obstacles (mountains, hills, buildings, etc.) causing a permanent decrease of the signal power level in some regions.

3. **Multipath (or fast fading):** occurs when the signal is received from many reflections or paths (such as reflection with buildings) as illustrated in Figure 2.4 Fast fading affects, especially high frequencies (radio frequencies), where the wavelength is small and the signal could increase or decrease dramatically within very reduced distance. One of the most adopted models for multipath is the Rayleigh fading statistical model (Sharma& Mishra, 2015).

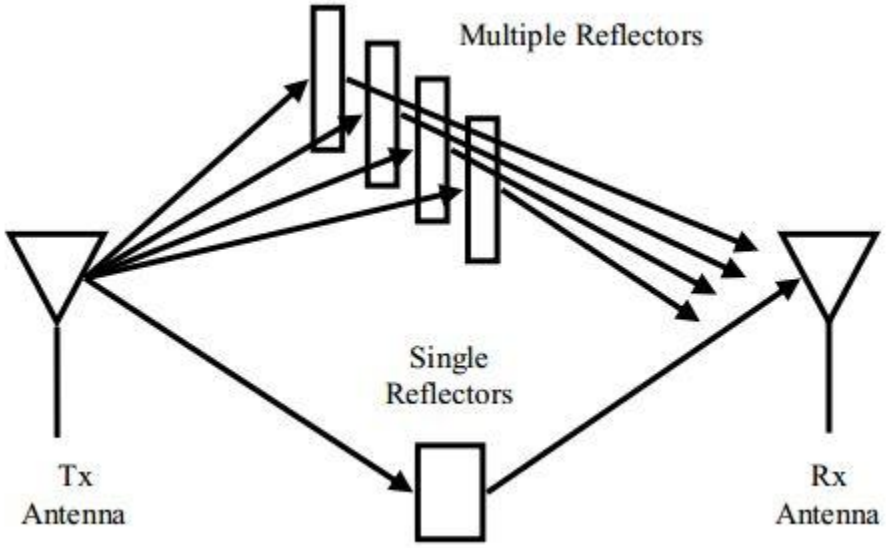


Figure 2.4: Multipath due to Single and Multiple Reflectors (Sharma & Mishra, 2015)

4. **Interference:** is caused by the use of the same frequency by more than one user. In mobile networks, interference is, mainly caused by the reuse of frequency in different cells. However, new techniques such as spectrum sharing, allow a secondary communication within the same frequency but under a tolerated interference threshold (Sboui, 2013).

5. **Doppler Shift:** is a phenomena related to user mobility and causes a shift in the frequency noted f_D , the Doppler frequency shift can be expressed as (Sboui, 2013):

$$f_D = \frac{v \cos\theta}{\lambda} \quad (2.1)$$

where v refers to the velocity of the mobile user, θ is the angle between when the direction of the signal is linear to the direction of the mobile user and λ is the wavelength of the signal. Hence, the Doppler shift, f_D is often called the fading bandwidth or fading fate of the channel, it enlarges the bandwidth of the signal and cause the signal deterioration over a period of time equal $\frac{1}{f_D}$

All these effects as illustrated in Figure 2.5 should be taken into consideration when designing new systems. As the channel gain is random, preliminary calculations are needed in order to satisfy the wireless system specification and prevent system outage (Sboui, 2013).



Figure 2.5: Various Fading Effects on Signal Power Level (Sboui, 2013)

2.2.5.2 Rayleigh distribution

The Rayleigh distribution is used to model multipath fading with no line of sight (LoS) path. The channel fading amplitude, α for a Rayleigh distribution is distributed according to (Simon &Alouini, 2000):

$$p_\alpha(\alpha) = \frac{2\alpha}{\Omega} \mathrm{e}^{-\left(\frac{\alpha^2}{\Omega}\right)}, \alpha \geq 0 \quad (2.2)$$

The instantaneous signal-to-noise power ratio (SNR) per symbol, $\gamma = \alpha^2 E_s / N_0$, and the average SNR per symbol by $\bar{\gamma} = \Omega E_s / N_0$, where E_s is the energy per symbol and Ω is the mean square value of α and N_0 is the power spectral density. The probability density function, PDF of γ is obtained by introducing a change of variables in the expression for the PDF of $p_\alpha(\alpha)$ of α yielding

$$p_\gamma(\gamma) = \frac{p_\alpha\left(\sqrt{\Omega_\gamma / \bar{\gamma}}\right)}{2\sqrt{\gamma \bar{\gamma} / \Omega}} \quad (2.3)$$

Introducing change of variables to the probability density function (PDF), $p_\alpha(\alpha)$, it becomes $p_\gamma(\gamma)$, hence following equation (2.3), the PDF in Rayleigh channel is distributed according to an exponential distribution given by (Simon &Alouini, 2000):

$$p_\gamma(\gamma) = \frac{1}{\bar{\gamma}} e^{-\left(\frac{\gamma}{\bar{\gamma}}\right)}, \quad \gamma \geq 0 \quad (2.4)$$

2.2.5.3 Nakagami-m distribution

The Nakagami-m distribution is a more versatile fading model which better fits a wide range of empirical data by adjusting a single parameter. This parameter, m, measures the ratio of the line-of-sight (LoS) signal power to that of the multi-path component (Ghasemi& Sousa, 2007).

The channel response of a wireless channel is a complex quantity and for this reason, the Nakagami-m fading channel Z , can be represented as follows (Sharma & Mishra, 2015):

$$Z = X + jY = R\left(e^{j\theta}\right) \quad (2.5)$$

where X, Y, R, θ represents the in-phase component, the quadrature-phase component, the envelope and the phase component respectively. The PDF of a Nakagami-m distributed fading envelop R can be expressed as (Sharma & Mishra, 2015):

$$f_R(r) = \frac{2m^m r^{2m-1}}{\Gamma(m)\Omega^m} \mathrm{e}^{-\left(\frac{mr^2}{\Omega}\right)}, 0 \le r < \infty \quad (2.6)$$

where $\Omega = E\left[R^2\right]$ is the expected value of the average power and m is shaping parameter which controls the shape of the distribution. When m is an integer, R is the square root of the sum of the amplitude square of m-dimensional independent and identically distributed (m.i.i.d) complex Gaussian random variables (Sharma & Mishra, 2015).

$$R = \sqrt{\left(\left|X_1^2\right| + \left|X_2^2\right| + \left|X_3^2\right| + \ldots\ldots\ldots\ldots \left|X_m^2\right|\right)} \quad (2.7)$$

where $X_i = 1, 2, 3, \ldots\ldots\ldots\ldots\ldots\ldots, \mathrm{m}$ is a complex Gaussian random variable (RV) with zero mean and variance Ω/m.

The inverse of the normalized variance of R^2, m is calculated from the expression (Sharma & Mishra, 2015):

$$m = \frac{\left(E\left[R^2\right]\right)^2}{Var\left(R^2\right)} = \frac{\Omega^2}{Var\left(R^2\right)} \quad (2.8)$$

where $Var\left(R^2\right)$ is the variance of R^2.The value of m ranges between $\frac{1}{2}$ and ∞. When $m \to \infty$, the channel converges to a static channel, i.e. it no longer remains variant channel. In special cases, the Nakagami-m becomes a one sided Gaussian distribution for $m = \frac{1}{2}$ and Rayleigh when $m = 1$. For the values of $m > 1$, the Nakagami-m distribution closely approximates to the Racian distribution.

The Nakagami-m phase envelope joint distribution is given by (Sharma & Mishra, 2015):

$$f_{R\Theta}\left(\theta\right) = \frac{m^m \left|\sin\left(2\theta\right)\right|^{m-1} r^{2m-1}}{2^{m-1}\Omega^m \Gamma^2\left(\frac{m}{2}\right)} e^{\left(\frac{mr^2}{\Omega}\right)}, m \geq 0.5 \quad (2.9)$$

The envelope pdf $f_R\left(r\right)$ is given by equation and the phase envelope is given by (Sharma & Mishra, 2015):

$$f_\Theta\left(\theta\right) = \frac{\Gamma\left(m\right)\left|\sin\left(2\theta\right)\right|^{m-1}}{2^m \Gamma^2\left(\frac{m}{2}\right)}, -\pi \leq \theta < \pi \quad (2.10)$$

For a Nakagami fading channel, channel power gain g is distributed according to the following gamma distribution (Simon &Alouini, 2005)

$$f_g\left(x\right) = \frac{m^m x^{m-1}}{\Gamma(m)\Omega^m} e^{-\left(\frac{mx}{\Omega}\right)}, \mathrm{m} \geq \frac{1}{2} \quad (2.11)$$

where $\Gamma(.)$ is the gamma function, defined as $\Gamma\left(x\right) = \int_0^\infty t^{x-1} e^{-t} dt$.

2.2.6 Channel State Information

The channel state information (CSI) refers to the available knowledge about the channel and propagation environment (path loss, shadowing, fading etc.). Knowing the CSI allows the transmitter to adapt signal transmission depending on real-time channel condition. CSI is determined by the level of knowledge of the channel coefficient h, Depending on this knowledge, the CSI could be presented in one of these forms (Sboui, 2013).

1. **Instantaneous CSI (ICSI):** the channel conditions are perfectly known. Hence the transmission could be perfectly adapted in this case.
2. **Imperfect CSI:** the channel conditions are partially known. It may be due to a combination of channel estimation errors and feedback delay (Xu*et al*., 2013b) as illustrated in Figure 2.6. The time varying characteristics of the fading channel cause estimation errors (Kim & Lee, 2003). If the STx or the PRx has mobility, the STx obtained an outdated channel information due to feedback delay (Kim *et al.,* 2012).Basically, the imperfect CSI can be described by using one of two models: the channel estimation errors model and the correlation model.

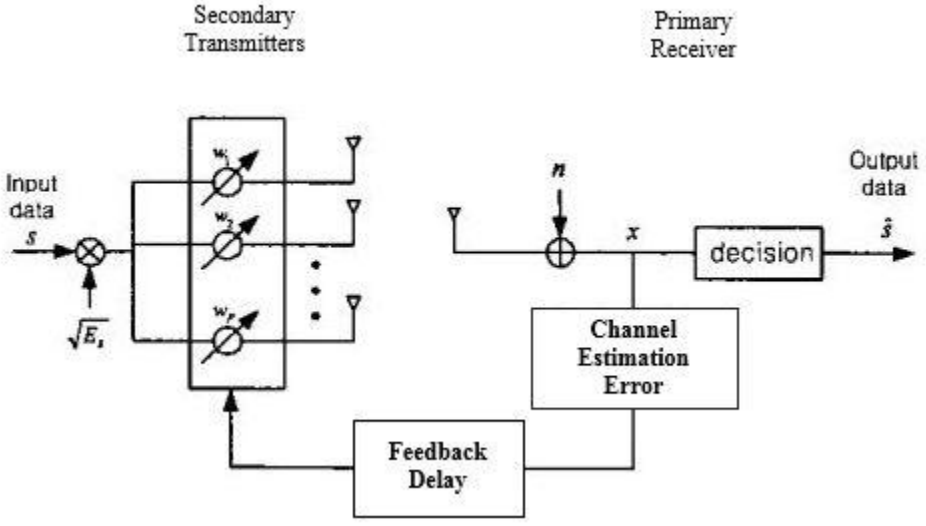


Figure 2.6: A Block Diagram of a Spectrum Sharing System (Kim & Lee, 2003)

2.2.7 Channel Capacity

When designing a wireless communication system, understanding achievable limits of data transmission is a preliminary and important step. Consequently, the notion of channel capacity was developed in order to derive these limits. The channel capacity is defined as the maximum data rate that could be supported by the channel without error (even with a noisy channel). The channel capacity is used as a basic performance tool for the analysis and design of an efficient technique to improve spectrum efficiency. Based on information theory, the capacity of an AWGN channel was given by (Shannon, 1949):

$$C_{awgn} = B\log_2(1+SNR) \quad (2.12)$$

where B is the bandwidth, and SNR is the signal-to-noise ratio. The capacity is expressed in bits per second (bit/s) if it is expressed with $\log_2$

From equation (2.12), it can be seen that the channel capacity of AWGN can be increased by either increasing the transmission bandwidth of the transmitted signal or by improving the SNR ratio of the channel.

Meanwhile, the capacity could also be normalized with unit bandwidth $(B=1Hz)$ and expressed in bit/Hz or bits per channel use $(bpcu)$, commonly known as spectral efficiency (Sboui, 2013).

However, in some cases, the Neperian logarithm log is used in order to simplify calculations and avoid having the constant $\log_e 2$ on the results.

2.2.7.1 Fading channel capacity

Through the expression given in equation (2.12), the capacity depends mainly on the SNR and the bandwidth, in fading channels the SNR is given by (Sboui, 2013):

$$SNR = \frac{P}{N_0 B}|h|^2 \quad (2.13)$$

where P is the signal power, N_0 is the noise variance and h is the channel coefficient. Hence the instantaneous capacity of the channel, C_0 with channel coefficient equal to h_0 is given by:

$$C_0 = \log_2\left(1 + \frac{P}{N_0 B}|h_0|^2\right) \quad (2.14)$$

However, h_0 is a random variable and its variation is very fast in fast-fading channel. Therefore, the ergodic capacity is introduced.

2.2.8 Ergodic Capacity

The ergodic capacity is defined as the maximum of the long-term average achievable rate with arbitrarily small probability of error, subject to constraints on the power (Leila &Aissa, 2009). Figure 2.7, shows a spectrum sharing model with a single primary user (PU) and a single secondary user (SU) sharing the same network frequency. The ergodic capacity is expressed as (Kabiri, 2015):

$$\frac{C}{B} = \max_{\mathrm{E}[g_0 p(g_0, g_1) \le Q]} \mathrm{E}\left[\log_2\left(1 + \frac{g_1 p(g_0, g_1)}{N_0 B}\right)\right] \quad (2.15)$$

where C is the capacity of the channel,

$\mathbb{E}$ is the statistical expectation,

g_0 is the channel gain between the Secondary user transmitter and the Primary User receiver

g_1 is the channel gain between the Secondary user transmitter and the Secondary user receiver

$P(g_0, g_1)$ denotes the secondary user transmit power as a function of channel power gains g_0 and g_1

Q is the interference power constraint

From equation (2.15), the ergodic capacity is an optimization problem.The interference power constraint is imposed on either the average or the peak power received at the primary receiver. The ergodic capacity can be computed under the average power received at the primary receiver, known as the average interference power constraint or under the peak power received at the primary receiver, known as the peak interference power constraint.

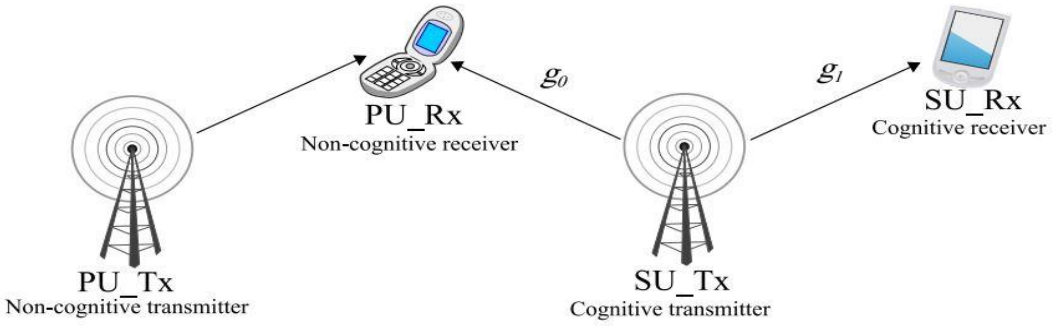


Figure 2.7: A CR Network Model (Sboui*et al.*, 2013b)

2.2.8.1 Impact of correlation coefficient on the ergodic capacity

The secondary user can obtain the CSI from the band manager mediating between the PUs and SUs or from the PRx directly (Suraweera*et al.*, 2010). If the STx or PRx has mobility, the STx-PRx link channel is time varying by Doppler Effect, and the STx

obtains an imperfect CSI due to feedback delay. The transmission power of the SU is allocated using this imperfect CSI, hence, the interference power to the PRx does not satisfy the interference power constraint in a spectrum sharing environment making it impossible for the secondary system to share the spectrum of the primary system. However, the imperfect channel and the real value can actually be correlated as a result of the time varying channel using a channel correlation based model. The correlation means a reliability of CSI at the STx, and it has to be considered in order to achieve capacity enhancement for the SU (Kim *et al*., 2012).

In a correlation based model, an estimate of channel coefficient h can be written as (Suraweera*et al*., 2010):

$$h = \rho\tilde{h} + \left(\sqrt{1-\rho^2}\right)\in_1 \quad (2.16)$$

where $\tilde{h}$ is the estimated CSI at the STx, h is the perfect channel coefficient of the STx-PRx link, ρ is the channel correlation coefficient, $0 \le \rho \le 1$ is a constant that determines the average quality of the channel estimates over all channel states of h. $\in_1$ is $CN\left(0,\lambda\right)$ an is uncorrelated with $\tilde{h}$. ρ is the correlation coefficient of the channels and is related to the Doppler shift and the feedback delay.

The channel gain g is calculated as:

$$g = |h|^2 \quad (2.17)$$

The expression of ρ depends on the propagation environment. For land mobile environment, Jakes modelled ρ as (Jakes & Cox, 1994):

$$\rho = J_0\left(2\pi f_D \tau\right) = J_0\left(\omega_d \tau\right) \quad (2.18)$$

where, $J_0(.)$ is the zeroth-order Bessel function of the first kind,

f_D is the Doppler shift and τ is the feedback delay

Figure 2.8 shows a plot of ρ against the τ using equation (2.18). However, in literatures(Simon &Aloini, 2000;Kim *et al*., 2012; Xu*et al*., 2013a), it is assumed that once ρ is zero, ρ remains zero at all higher τ and it has a range of $0 \le \rho \le 1$. Based on this assumption, a higher τ corresponds to lower ρ. Therefore, the impact of τ on the ergodic capacity of the SU can be studied by investigating the impact of ρ. Considering the correlation coefficient, the ergodic capacity of the Secondary user is a function of $\rho, \breve{g}_0, g_1, N_0$ and B.

The ergodic capacity of the SU under the average interference power (AIP) constraint is given by (Kim *et al.*, 2012):

$$C_{er} = \max_{P\left(\rho, \breve{g}_0, g_1\right) \ge 0} \int\int B \log\left(1 + \frac{g_1 P\left(\rho, \breve{g}_0, g_1\right)}{N_0 B}\right)$$

$$\times f_{g_1}(g_1)\, f_{\breve{g}_0}\left(\breve{g}_0\right) dg_1 d\breve{g}_0 \qquad (2.19)$$

$$\text{subjected to } \mathrm{E}\left[\breve{g}_0 P\left(\breve{g}_0, g_1, \rho\right)\right] \le Q_{avr} \qquad (2.20)$$

The ergodic capacity under the peak interference power (PIP) constraint is given by (Kim *et al*., 2012):

$$C_{pk} = \int_{g_1/\breve{g}_0} \mathrm{B} \log\left(1 + \frac{g_1 P\left(\rho, \breve{g}_0, g_1\right)}{N_0 B}\right) f_{\frac{g_1}{\breve{g}_0}}(x)\, dx \qquad (2.21)$$

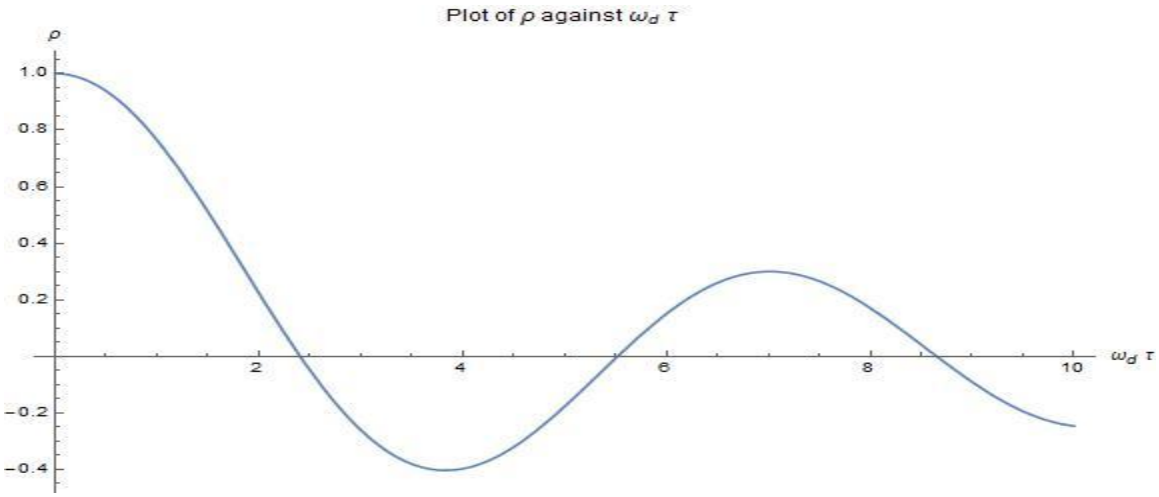


Figure 2.8: A Plot of ρ against $\omega_d \tau$ (Jakes & Cox, 1994)

2.2.8.2 Impact of estimation errors σ^2 on the ergodic capacity

In the channel estimation errors model, the outdated property of the channel is expressed by the variance of the error term (Kim *et al.*, 2012). This model is effective for analyzing practical channel estimation. The secondary user is assumed to perform minimum mean square errors (MMSE) estimation of $\tilde{h}$ (Sboui*et al.*, 2011, Kim *et al.*, 2012, Xu*et al.*, 2013b).

$$\tilde{h} = \hat{h} + \in_2 \quad (2.22)$$

where $\hat{h}, \hat{h} \sim CN\left(0,(1-\sigma^2)\lambda\right)$ is the outdated channel impulse response with channel estimation errors that is available at the STx, $\in_2$ is the channel estimation error that is uncorrelated with $\tilde{h}$ and σ^2, where $\in_2, \in_2 \sim CN\left(0,\sigma^2\lambda\right)$ and $\sigma^2\left(0 \le \sigma^2 \le 1\right)$ is a measure of the accuracy of the channel estimation errors. λ is the mean channel gain, as modelled by the propagation path model: $\lambda = -A - B\log_2(d)$ in dB (Gao*et al.*, 2007).

where A and B are constants depending on propagation environments, and d is the distance.

Using the channel errors estimation model, the ergodic capacity is expressed as a function of $\sigma^2, \breve{g}_0, g_1, N_0$ and B.

The capacity under the AIP constraint, Q_{av} is given by (Xu*et al.*, 2013b):

$$C_{er} = \max_{P\left(\rho,\breve{g}_0,g_1\right)\geq 0} \int\int B\log\left(1+\frac{g_1 P\left(\sigma^2,\breve{g}_0,g_1\right)}{N_0 B}\right)$$

$$\times f_{g_1}(g_1)\, f_{\breve{g}_0}\left(\breve{g}_0\right) dg_1 d\breve{g}_0 \qquad (2.23)$$

$$\text{subjected to } \mathrm{E}\left[\breve{g}_0 P\left(\breve{g}_0, g_1, \sigma^2\right)\right] \leq Q_{avr} \qquad (2.24)$$

The ergodic capacity under the peak interference power (PIP) constraint is given by (Xu*et al.*, 2013):

$$C_{pk} = \int_{g_1/\breve{g}_0} \mathrm{B}\log\left(1+\frac{g_1 P\left(\sigma^2,\breve{g}_0,g_1\right)}{N_0 B}\right) f_{\frac{g_1}{\breve{g}_0}}(x)\, dx \qquad (2.25)$$

2.2.8.3 Impact ofestimation errors and correlation coefficient onthe ergodic capacity

In previous literatures (Suraweera*et al.*, 2010;Rezki &Aloini, 2012;Kim *et al.*, 2012;Sboui *et al.*, 2013a), the channel property of the channel is expressed either by correlation based model or the MMSE based model. In practice the CSI available at the STx contains both channel estimation errors as well as error due to feedback delay.

The imperfect channel model addressing errors due to estimation error as well as due to feedback delay can be written as (Xu*et al.*, 2013b):

$$h = \rho \hat{h} + \in \quad (2.26)$$

where $\hat{h}$ is the imperfect CSI available at the transmitter of the SU with $\hat{h} \sim CN\left(0,(1-\sigma^2)\lambda\right)$, $\in \sim CN\left(0,\alpha\lambda\right)$ is uncorrelated with $\hat{h}$, and $\alpha = 1-\left(1-\sigma^2\right)\rho^2$

The capacity under the AIP constraint, Q_{av} is given by (Xu*et al.*, 2013b):

$$C_{er} = \max_{P\left(\rho,\breve{g}_0,g_1\right)\geq 0} \int\int B\log\left(1+\frac{g_1 P\left(\breve{g}_0,g_1,\rho,\sigma^2\right)}{N_0 B}\right)$$

$$\times f_{g_1}(g_1)\, f_{\breve{g}_0}\left(\breve{g}_0\right) dg_1 d\breve{g}_0 \quad (2.27)$$

subjected to $\mathrm{E}\left[\breve{g}_0 P\left(\breve{g}_0,g_1,\rho,\sigma^2\right)\right] \leq Q_{avr}$ (2.28)

2.3 Review of Similar Works

The following present a critical review of similar research works:

Suraweeraet al., (2010) investigated the impact of imperfect channel knowledge of the primary-secondary link on the SU mean capacity under a peak power constraint at the primary receiver. The impact of channel estimation errors on the gain of the secondary transmitter to the primary receiver link was considered in an AWGN channel. The impact of quantizing the imperfect CSI was also studied to propose an approximate correlation coefficient to mimic the quantization of the CSI. The accuracy of the proposed correlation coefficient was confirmed from simulations. However, the expression for the

transmit power of the SU to satisfy the interference power outage probability at the PU was not provided.

Sbouiet al., (2011)studied the ergodic capacity of secondary user channel in a spectrum sharing scenario in which the secondary transmitter is instantaneously aware of estimated versions of the cross link (between the secondary transmitter and the primary receiver) and the secondary link CSI. The ergodic capacity was determined under the average interference constraint (AIP) and the PIP constraint.Simulation results presented showed that in a Rayleigh channel,the capacity of the SU was affected by the estimated crosslink CSI with no influence in the case of estimated secondary link CSI at high power regime. At low power regime, the capacity depends on the estimated secondary link CSI. However, the impact of channel correlation coefficient was not considered which could have significant effect on the ergodic capacity of the SU.

Rezki&Alouni, (2012a) investigated the capacity of an independent identically distributed (i.i.d) flat Nakagami-m fading channels with perfect CSI at thePRx and at the STx. The ergodic capacity of a single input single output (SISO) Nakagami-m fading channel was investigated in order to derive the closed form expression for different antenna configurations and also provide the asymptotically low Signal to Noise (low-SNR) characterization of the capacity.Numerical results showed that for a channel mean-square $\Omega = 1$, the Nakagami-m fading channel figure depicts that of i.i.d Rayleigh fading channel in nats per channel use (npcu) at m=1. Intuitively as m approaches infinity, the channel tended to be an AGWN channel and the capacity scales only proportionally with SNR. The work gives the channel of the secondary user under a perfect CSI conditions, however, in practical communications system, the obtained CSI was frequently imperfect

due to the impact of channel estimation errors and feedback delay errors, which could also have an effect on the ergodic capacity of the secondary user.

Rezki&Aloini, (2012b) investigated a communication system where the PU and the SU share the same spectrum under interference constraint with an imperfect CSI. The SU was aware of the instantaneous CSI of the secondary link, but knows only the statistics and an estimated version of the secondary transmitter-primary receiver link. The optimum power profile and the ergodic capacity of the secondary link for general fading channels with a continuous probability density function (PDF) under the average and peak transmit power constraints and with respect to two different interference constraint : the outage interference constraint and a signal-to-interference outage constraint were derived. Numerical results showed that for a Rayleigh fading channel, the capacity does not increase with the power, implying that the interference constraint was harmful at high power regime, whereas at low power regime, it has a marginal impact and no-interference performance, which also corresponds to the ergodic capacity under average or peak transmit power constraint. However, the impact of correlation coefficient on the capacity of the channel was not considered which could also affect the SU capacity.

Kim et al., (2012) investigated the impact of outdated channel information on the capacity of secondary user in spectrum sharing environments. A spectrum sharing system in which the SU shares the spectrum of the primary user within the predetermined interference power constraint was considered to derive the ergodic capacity of the SU according to the interference from the PTx under the AIP constraint and that under the PIP constraint using a channel correlation model.Both numerical and simulated results demonstrated that the ergodic capacity of the secondary user was more robust to the

outdated channel environment than the PIPconstraint. However, the channel estimation errors was not considered which could also have a significant effect on the ergodic capacity.

Sbouiet al., (2013a) studied a spectrum sharing communication scenario in which a primary and secondary users are communicating, simultaneously, with their respective destinations using the same frequency carrier. The SU was assumed to have a noisy version of the cross link and the secondary link CSI. The optimal power profile and ergodic capacity were derived for fading channels, under an average transmit power and an instantaneous interference constraint. An asymptotic capacity analysis at high and low signal-to-noise ratio (SNR) was also provided. Numerical results applied for independent Rayleigh fading channels, showed that at low SNR regime, only the secondary channel estimation matters, with no effect on the cross link on the capacity; whereas at high SNR regime, the capacity was rather driven by the cross link CSI.However, the impact of channel correlation coefficient was not considered which could have effect on the SU capacity.

Sbouiet al.,(2013b) studied the capacity of spectrum sharing cognitive radio systems over Nakagami-m fading channels at low SNR. A setting where the CR user was solely constraint by a low interference threshold was considered. The primary user was licensed to freely exploit the spectrum; whereas the CR user was allowed to share the spectrum with the PU without affecting the primary communication. The channel capacity of the CR user was determined under the PIP constraint and also the AIP constraint for a perfect CSI at the cross link and also in the case when there is no CSI. The impact of different fading parameters on the channel capacity was also highlighted. However, in practical

communications system, the CSI is imperfect due to feedback delay errors and channel estimation errors, these errors can have a significant effect on the SU capacity.

Xuet al., (2013)investigated the impact of channel estimation errors and feedback delay on the ergodic capacity for spectrum sharing cognitive radio network. An imperfect channel state information (CSI) was assumed in the channel between the secondary transmitter (STx) and the primary receiver (PRx), which was due to channel estimation errors and feedback delay. It was also assumed that the STx has a perfect knowledge of the secondary receicver (SRx).The ergodic capacities under the imperfect CSI was derived for an AWGN fading environment. Numerical results showed that the ergodic capacity of the SU was insensitive to the channel estimation errors and the feedback delay under high feedback delay. It was also showed that decreasing the distance between the STx and the SRx or increasing the distance between the STx and the PRx can mitigate the impacts of the channel estimation errors and feedback delay thereby significantly increasing the ergodic capacity of the secondary user. However, there was need to derive a unified ergodic capacity expression to analyze the impact of channel estimation errors and channel correlation coefficient for different channel fading environments, this was due the fact that theSTx-SRx path could experience different fading types due to factors like different path length or shadowing.

Ho-Van et al., (2014)investigated an underlay cooperative CR network with imperfect CSIunder a strict transmit power constraint. The interference cumulative distribution function of licensed users, and outage probability of unlicensed users was analyzed in realistic operating conditions such as the imperfect fading channel information and maximum transmit power constraint, over Nakagami-m fading channels. Simulation

results showed that: the channel information imperfection degrades considerably the performance of both the unlicensed network in terms of outage probability and licensed network in terms of interference levels; underlay cooperative CR experience the outage saturation phenomenon; the probability that the interference power constraint was satisfied was relatively low and depends significantly on the corresponding fading severity conditions as well as the channel estimation quality; there exist a critical performance trade-off between licensed and unlicensed networks. However, the expression for the transmit power of the SU to satisfy the interference power outage probability at the primary user (PU) was not provided.

Deng et al., (2015) proposed a cognitive spectrum sharing with generalized selection combining (GSC) at the SU in the presence of multiple PTx with outdated CSI. The impact of GSC and outdated channel information on the outage probability of CR sharing subject to two physical power constraints; the maximum transmit power at the STx and peak interference temperature at the PRx were determined. The closed-form expressions for the exact and asymptotic outage probability in Rayleigh fading were derived. Simulation results confirmed that the diversity order of GSC was entirely dependent of the secondary network and it was equal to the available number of receiver antennas at the SU. This result was consistent with those of maximal-ratio combining (MRC) and selection combining (SC) in CR spectrum sharing. More, importantly, the results showed that the outage probability decreases with an increase in the correlation coefficient of the outdated channel. However, the expression for the transmit power of the SU to satisfy the interference power outage probability at the PU was not provided.

Krishnan et al., (2015) proposed a new joint optimization of subcarrier and power allocation in a Multi User orthogonal frequency division multiplexing (MU-OFDM) based CR systems. The optimization was performed by modifying the water filling algorithm by using the convex optimization theory in order to maximize capacity. The convex optimization theory is a non-uniform distributed power control algorithm based on the solution of Langrange equation. A protection for the PUs against interference caused by the SUs and the power allocation range was determined by the threshold power and QoS of the SU which make it possible for both PU and SU to coexist. Mainly two goals are satisfied here. i.e the system and the user goals. The system goal was mainly concerned with the total capacity and coverage, whereas the user goal was concerned with the minimization of power consumption and with the provision of proper QoS.Simulation results showed that injected interference of the PU channel has decreased and obtained capacity of SU has been improved. However, there was a need to analyze imperfect spectrum sensing conditions and consider other different modulation schemes in order to reduce the interference caused by SU.

Kunduet al., (2015) studied the performance of spectrum sharing networks with imperfect CSI. A system model with a primary link that consists of aPTx and a PRx, with the secondary link consists of aSTx and aSRx was considered. The performance of the SU was analyzed in terms of outage probability and symbol error probability. Simulation results showed that higher imperfection in the CSI degrades the outage performance of the PU and as well the SU. Power control has to be done depending on the imperfection in CSI so as to maintain the primary Qos. Higher interference from the PU increases outage and symbol error probability (SEP) of the SU, while its outage improves with

increase in tolerable interference threshold of PU. However, the expression for the transmit power of the SU to satisfy the interference power outage probability at the primary user (PU) was not provided.

Xiangqun and Qingxin, (2015)investigated the capacity gains achievable under a given delay quality-of-service (QoS) constraint in asymmetric fading channels. A spectrum sharing cognitive radio system in which CR users may access the spectrum allocated to primary users PUs. The channel between the STx andPRx and the channel between the STx and the SRx may undergo different types and arbitrary link power gains. A closed form expression for the effective capacity under an AIP power constrained was developed.

Analytical and simulated results showed that the effective capacity was sensitive to the fading types and link power gains. The fading parameters of the interference channel play a vital role in the effective capacity for the looser delay constraints. However, the capacity expressions of the ergodic capacity was not provided.

Arankumar and Kumaran, (2016) carried out cooperative relaying and distributed beam forming for spectrum sharing systems with the objective of improving the performance of the secondary system while respecting the interference constraint imposed by the PU. A study of the outage performance of cognitive opportunistic decode-and-forward relaying operating in the secondary network with multiple PU receivers and in the presence of interference from a PU transmitter was done. A closed form of the expression was derived for the outage probability with all system links following Rayleigh distribution. Simulation results showed that with more number of relays used for the secondary usersQoS was greatly improved. The simulation was

validated using Monte Carlo simulation. However, there was a need to analyze the primary users Bit Error Rate (BER) performance over a number of relays used for SUs to further improve the system performance and to moderate the moderation level used for PU to maximize the SUs performance with maximum possible relays.

Rosas et al., (2016) proposed an optimal power allocation technique using genetic algorithm for maximization of ergodic capacity under different fading channel approaches in CR network. The technique studies how to maximize SUErgodic capacity considering some constraints such as maximum transmit power of SU, interference threshold at PRx and interference on STx. A CR network with single PU and SU was assumed and a study of the maximum SU capacity was done under different channel environment. Genetic Algorithm (GA) was used to solve the optimization power allocation strategies to enhance power, performance and maximize ergodic capacity of underlay cognitive radio network under different fading channels. The GA optimized result was validated with that of Langrange optimization technique. However, there was a need to study the ergodic capacity of the SU under imperfect channel condition.

From the reviewed literature, it is evident that much work has been done to determine theergodic capacity of underlay cognitive radio. The underlying assumptions are mostly based on the availability of the CSI to determine the SU capacity under the PIP constraint or the AIP constraint that the PU can tolerate in an underlay CR systems. However, in practical communications system, only a partial channel state information is obtained due to the effect of feedback delay and the channel estimation errors. The impact of channel estimation errors and correlation coefficient has been investigated in a Rayleigh fadingenvironment by Xu*et al.,* (2013b). However, to the best of our knowledge, no

work has been carried out to investigate the impact ofchannel estimation errors and correlation coefficient so asto improve the ergodic capacity of underlay CRnetwork under different fading condition usinga Nakagami-m fading distribution.

CHAPTER THREE

MATERIALS AND METHODS

3.1 Introduction

This chapter describes the detailed procedures carried out in developing a closed form expression of the ergodic capacity of an underlay CR under the PIP constraint.The methodology for carrying out the work in order to achieve the research aim is discussed in this chapter.

3.2 Materials

The materials used in carrying out this research work include: Laptop, Wolfram Mathematica: a computational program, powerful in symbolic computation, Matlab: a numerical based computing environment based on Matrix manipulation used for plotting illustration results in this research, Office Visio: For drawing block diagrams.

3.3 Methodology

The following methodology was adopted in carrying out this research work:

1. Adopting and modifying the cognitive radio model from the work of Xu et al., (2013b)
2. Determining the peak interference power constraint
3. Deriving the closed form ergodic capacity expression of the Secondary User (SU)
4. Examining the impact of estimation errors variance, σ^2 on the SU capacity
5. Examining the impact of correlation coefficient, ρ on the SU capacity

6. Comparison of the spectrum sharing capacity in (4) and (5) with the work of Xu et al., (2013b).

3.4 System and Channel Models

An underlay CR system in which the primary channel consists of aPTx and aPRx, and the cognitive radio channel consists ofaSTx and aSRx was considered. The additive noises at the primary link and the secondary link were all assumed to be independent AWGN, which have zero mean and power spectral density by N_0 with a received signal bandwidth, B .The interference channel between the PRx and STx is assumed to be a Rayleigh fading distribution and that of the cognitive radio channel between the STx and SRx is assumed to be a Nakagami-m Fading channel as shown in Table 3.1. The Rayleigh fading channel can be obtained as a special case of Nakagami-m (m=1). The instantaneous complex channel impulse responses of the STx-PRx link and the STx-SRx link are $h_0[n]$ and $h_1[n]$ respectively as shown in Figure.3.1, where n denotes the time index. $h_0[n]$ and $h_1[n]$ are independent and are zero mean circularly symmetric complex gaussian (ZMCSCG) random variables with variances λ_0 and λ_1, respectively, that is, $h_0[n] \sim CN(0, \lambda_0)$ and $h_1[n] \sim CN(0, \lambda_1)$.

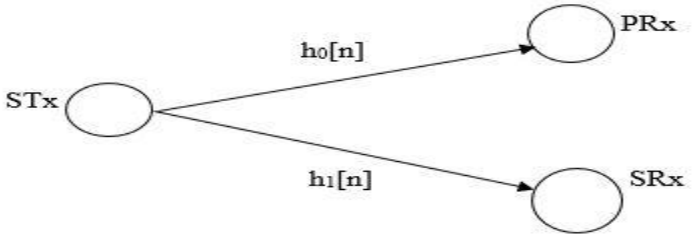


Figure 3.1: Spectrum Sharing Model (Xu*et al.*, 2013b)

Table 3.1: Interference and Cognitive Radio Channel Model

Scenario	**Channel Model**	
	Interference Channel Model	**Cognitive Radio Channel Model** (m)
1	Rayleigh	Nakagami-m (m=1/2)
2	Rayleigh	Nakagami-m (m=1)
3	Rayleigh	Nakagami-m (m=2)
4	Rayleigh	Nakagami-m (m=3)

The perfect knowledge of $h_1[n]$ was assumed to be available at the STx. However, $h_0[n]$ was not perfectly known at the STx. On one hand, the channel correlation coefficient was used to model the outdated property of the channel due to feedback delay (Xu*et al.*, 2013b):

$$h_0[n] = \rho \hat{h}_0[n] + \sqrt{1-\rho^2}\varepsilon_1 \quad (3.1)$$

where ρ $(0 \le \rho \le 1)$ is the channel correlation coefficient, $\hat{h}_0[n]$ is the outdated channel impulse response with no channel estimation errors $(\hat{h}_0[n] \square CN(0, \lambda_0))$, ε_1 is $CN(0, \lambda_0)$

and ε_1 is uncorrelated with $\hat{h}_0[n]$. Due to the time varying property of the channel, ρ reflects the correlation of the channels and is related to the Doppler shift and feedback delay. For a land mobile environment, $\rho = J_0(2\pi f_d \tau)$, where $J_0(.)$ is the zeroth-order Bessel function of the first kind, f_d is the Doppler shift and τ is the feedback delay. On the other hand, the SU is assumed to perform minimum mean square errors (MMSE) estimation of $\hat{h}_0$ and $\hat{h}_0 = \breve{h}_0 + \in_2$, where $\breve{h}_0 \square CN(0,(1-\sigma^2)\lambda_0)$ is the outdated channel impulse response with channel estimation errors that is uncorrelated with $\breve{h}_0$, and σ^2 $(0 \le \sigma^2 \le 1)$ is a measure of the accuracy of the channel estimation. We denote the channel gains as g_1 and $\breve{g}_0$, where $g = |h|^2$. A unit variance noise, N_0 and a unit bandwidth, B was considered for spectral efficiency. The ergodic capacity is presented in bits/S/Hz. Figure 3.2 is the block diagramshowing the step by step procedure fordeveloping the improvedergodic capacity under the peak interference power (PIP) constraint.

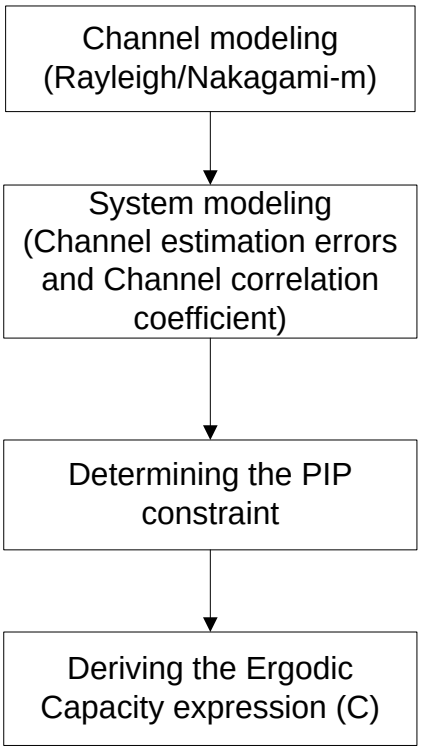


Figure 3.2: Block diagram for the ImprovedErgodic Capacity Expression

3.5 Ergodic Capacity

In the section, a study of the ergodic capacity of the SU with the imperfect CSI was done. In order to gain more insight into the fundamental limits of spectrum sharing in fading environments under imperfect channel condition, the ergodic capacity subject to non-interfering constraint was evaluated. Two types of primary communication protection can be adopted to avoid the interference caused by the secondary user (Ghasemi& Sousa,2007). They are the Average interference power constraint (AIP) and the Peak Interference power constraint (PIP). In this research, the PIP constraint was considered. The PIP constraint considers the instantaneous signal-to-noise (SNR) at the primary receiver which renders it appropriate in ensuring that the Primary user QoS is satisfied.

3.3.1 Ergodic Capacity under the PIP constraint

In determining the ergodic capacity under the PIP constraint, the STx has to guarantee that the PIP at the PRx is below predefined limit as (Xu*et al.*, 2013b),

$$g_0 P\left(g_1, \breve{g}_0, \rho, \sigma^2\right) \leq Q_0 \qquad (3.2)$$

where Q_0 is the PIP limit, g_0 is the channel gain between the STx and PRx, $P\left(g_1, g_0, \rho, \sigma^2\right)$ is the secondary transmit power. However, it is impossible for the STx to satisfy the constraint in equation (3.2), since the STx only knows an imperfect estimation of the channel on STx-PRx link, $\breve{g}_0$. Under this strict PIP constraint, when the secondary user has an imperfect channel information for the interfering link $\breve{g}_0$. The STx has to cease transmission at all times and hence results in zero ergodic capacity. For this reason, a constraint based on stochastic concept is considered. The primary user should allow a certain percentage of outage, which is referred to as interference outage. The term interference outage means that the interference power at the PRx exceeds the predetermined threshold for a certain percentage of time (Musavian&Aïssa, 2009). Based on this, the PU informs the SU to use a reduced value of Q_0. i.e. $\breve{Q}_0$, so that a certain percentage of outage, P_0, is allowed, which is given as follows as (Xu*et al.*, 2013b):

$$\Pr\left(g_0 P\left(g_1, \breve{g}_0, \rho, \sigma^2\right) \geq Q_0\right) \leq P_0 \quad (3.3)$$

where $\breve{g}_0$ is the PDF of the imperfect channel in the STx-PRxlink is define as,

$$\breve{g}_0 = \frac{m_o^{m_0} \breve{g}_0^{m_0 - 1}}{\Gamma(m_0)\left(1-\sigma^2\right)\lambda_0} e^{-\frac{m_0 \breve{g}_0}{\left(1-\sigma^2\right)\lambda_0}} \quad (3.4)$$

Therefore, the new PIP constraint is written as $P\left(g_1, \breve{g}_0, \rho, \sigma^2 \right) \le \frac{\breve{Q}_0}{\breve{g}_0}$, and the value of $\breve{Q}_0$ needs to be determined so that the constraint (3.3) is satisfied.

3.3.1.1 Determining the PIP constraint

In order to obtain $\breve{Q}_0$, $P\left(g_1, \breve{g}_0, \rho, \sigma^2 \right) \le \frac{\breve{Q}_0}{\breve{g}_0}$ is substituted into the constraint in equation (3.3) at equality as follows:

$$P_o = \Pr\left(\frac{g_0}{\breve{g}_0} > \frac{Q_0}{\breve{Q}_0} \right)$$

$$= 1 - F_{\frac{g_o}{\breve{g}_0}}\left(\frac{Q_0}{\breve{Q}_0} \right) \quad (3.5)$$

where $F_x(.)$ denotes the cumulative density function (CDF) of x. The joint PDF of g_0 and $\breve{g}_0$ is given as (Simon &Alouini, 2000):

$$f_{g_0, \breve{g}_0}\left(g_0, \breve{g}_0 \right) = \frac{1}{\left(1-\sigma^2\right)\alpha\lambda_0{}^2} e^{-m_0\left(\frac{\left(1-\sigma^2\right)g_0 + \breve{g}_0}{\left(1-\sigma^2\right)\alpha\lambda_0} \right)} I_{m_0-1}\left(\frac{2m_0\rho\sqrt{g_0\,\breve{g}_0}}{\alpha\lambda_0} \right) \quad (3.6)$$

where $I_{m_0-1}(.)$ is the (M-1)th- order modified Bessel function of the first kind. Then, the PDF of $\frac{Q_0}{\breve{Q}_0}$ is calculated as (Xu*et al*., 2013b):

$$f_{\underset{\cup}{g_0}}(y)=\int_0^{\infty} z f_{\underset{\cup}{g_0},g_0}(yz,z)dz$$

$$=\int_0^{\infty}\frac{z}{\left(1-\sigma^2\right)\alpha{\lambda_0}^2}e^{-m_0\frac{\left(1-\sigma^2\right)yz+z}{\left(1-\sigma^2\right)\alpha\lambda_0}}J_{m_0-1}\left(\frac{2m_0\rho\sqrt{y}zi}{\alpha\lambda_0}\right)dz \quad (3.7)$$

Equation (3.6) can be simplified as follow:

$$f_{\underset{\cup}{g_0}}(y)=\frac{2\alpha m_0\left(\left(1-\sigma^2\right)y+1\right)\left(\frac{4m_0\rho\sqrt{y}}{\alpha}\right)^{m_0-1}\Gamma\left(m_0+\frac{1}{2}\right)}{\pi\left(1-\sigma^2\right)\left[{m_0}^2\left(y^2+\left(\frac{2}{\left(1-\sigma^2\right)}+4\rho\right)y+\frac{1}{\left(1-\sigma^2\right)^2}\right)\right]^{m_0+\frac{1}{2}}} \quad (3.8)$$

For a Rayleigh fading channel $m_0=1$

$$f_{\underset{\cup}{g_0}}(y)=\frac{\left(2\alpha\left(1-\sigma^2\right)y+1\right)\Gamma\left(\frac{3}{2}\right)}{\pi\left(1-\sigma^2\right)\left[y^2+\left(\frac{2}{\left(1-\sigma^2\right)}+4\rho\right)y+\frac{1}{\left(1-\sigma^2\right)^2}\right]^{\frac{3}{2}}}$$

Then the CDF of $\frac{\overset{\cup}{Q_0}}{Q_0}$ is given as (see appendix A):

$$F_{\underset{\cup}{g_0}}(y)=\int_0^{y} f_{\underset{\cup}{g_0}}(z)dz$$

$$=\frac{1}{2}-\frac{1-\left(1-\sigma^2\right)y}{2\left(1-\sigma^2\right)\sqrt{\left(y+\frac{1}{\left(1-\sigma^2\right)}\right)^2-4\rho^2y}} \quad (3.9)$$

substituting equation (3.9) into equation (3.5) yields

$$P_0 = 1 - \left(\frac{1}{2} - \frac{1 - \left(1-\sigma^2\right)\frac{Q_0}{\breve{Q}_0}}{2\left(1-\sigma^2\right)\sqrt{\left(\frac{Q_0}{\breve{Q}_0} + \frac{1}{\left(1-\sigma^2\right)}\right)^2 - 4\rho^2\frac{Q_0}{\breve{Q}_0}}} \right) \quad (3.10)$$

From equation (3.10) for a given value of P_0, the value of $\breve{Q}_0$ is derived as:

$$\breve{Q}_0 = \left(\frac{\alpha - \left(1-2P_0\right)\sqrt{\alpha\left(1-\left(1-\sigma^2\right)\left(1-2P_0\right)^2\rho^2\right)}}{2P_0\left(1-P_0\right)} \right) \times \left(1-\sigma^2\right) Q_0 \quad (3.11)$$

It is seen that the value of $\breve{Q}_0$ is Q_0 multiplied by a constant factor that is related to ρ, σ^2 and P_0. Then, based on equation (3.11), the STx transmits power according to $P\left(g_1, \breve{g}_o, \rho, \sigma^2\right) = \frac{\breve{Q}_0}{Q_0}$ which satisfies the constraint in equation (3.3)

3.3.1.2 Ergodic capacity for a given PIP constraint

The ergodic capacity of the SU can be derived as (Xu*et al.*, 2013b):

$$C_{er} = \int_0^\infty \int_0^\infty \mathrm{B}\log_2\left(\frac{\breve{Q}_0 g_1}{N_0 B \breve{g}_0}\right) f_{g_1}(g_1) f_{\breve{g}_0}(\breve{g}_0)\,\mathrm{d}g_1\, d\breve{g}_0 = \int_0^\infty \mathrm{B}\log_2\left(\frac{\breve{Q}_0 x}{N_0 B}\right) f_{\frac{g_1}{\breve{g}_0}}(x)\,\mathrm{d}x \quad (3.12)$$

where, g_1 is the PDF of the channel between the STx-SRx link which is a Nakagami-m fading channel, its PDF is given as:

$$g_1 = \frac{m^m g_1^{m-1}}{\Gamma(m)\lambda_1} e^{-\frac{mg_1}{\lambda_1}} \quad (3.13)$$

$f_{\frac{g_1}{g_0}}(x)$ is a joint pdf of $x = \frac{g_1}{g_0}$, the corresponding joint PDF is given by(Ghasemi& Sousa, 2007):

$$f_{\frac{g_1}{g_0}}(x) = \left(\frac{\frac{m_0}{(1-\sigma^2)\lambda_0}}{\frac{m}{\lambda_1}} \right)^{m_0} \frac{x^{m-1}}{Beta(m, m_0)\left(x + \frac{\frac{m_0}{(1-\sigma^2)\lambda_0}}{\frac{m}{\lambda_1}} \right)^{m_0+m}} \quad (3.14)$$

where $Beta(m, m_0)$ is the beta function of m and m_0

But $m_0 = 1$

$$f_{\frac{g_1}{g_0}}(x) = \left(\frac{\frac{1}{(1-\sigma^2)\lambda_0}}{\frac{m}{\lambda_1}} \right) \frac{x^{m-1}}{Beta(m,1)\left(x + \frac{\frac{1}{(1-\sigma^2)\lambda_0}}{\frac{m}{\lambda_1}} \right)^{1+m}} \quad (3.15)$$

substituting equation (3.15) into equation (3.12),

$$C_{er} = \int_0^{\infty} \mathrm{B}\log_2\left(\frac{\breve{Q}_0\, x}{N_0 B}\right)\left(\frac{\dfrac{1}{\left(1-\sigma^2\right)\lambda_0}}{\dfrac{m}{\lambda_1}}\right)\frac{x^{m-1}}{Beta\left(m,1\right)\left(x+\dfrac{\dfrac{1}{\left(1-\sigma^2\right)\lambda_0}}{\dfrac{m}{\lambda_1}}\right)^{1+m}}dx$$

$$\frac{C_{er}}{B} = \frac{\breve{Q}_0}{\breve{Q}_0 - m\left(1-\sigma^2\right)\beta N_0 B}\left(\log_2\left(\frac{\breve{Q}_o}{m\left(1-\sigma^2\right)\beta N_0 B}\right)+\gamma+\varphi\left(m\right)\right) \quad (3.16)$$

where γ is Euler's constant known as Euler Gamma, and β is the mean channel gain ratio. $\beta = \frac{\lambda_0}{\lambda_1}$, $\varphi\left(z\right)$ is a digamma function. λ_0 is mean link channel gain of the distance between STx-PRx link, λ_1 mean link channel gain of the distance between the STx-SRx link and B is the bandwidth. The developed ergodic capacity expression under the PIP constraint in equation (3.16) was compared to the result in equation (23) in the work ofXu*et al*., (2013b). It can be seen that, the improved closed form ergodic capacity expression developed in equation (3.16), at m=1 yields the result of equation (23) in Xu*et al*., (2013b).

The ergodic capacity under the PIP constraint in equation (23) in the work ofXu*et al*., (2013b) is expressed as:

$$\frac{C_{er}}{B} = \frac{\eta}{\eta - 1}\log_2 \eta \quad (3.17)$$

where $\eta = \frac{\breve{Q}_{pk}}{\left(1-\sigma^2\right)\beta N_0 B}$

However, equation (3.17) is restricted to the scenario at m=1 only, but the developed expression in equation (3.16) can be extended to other fading conditions by adjusting the m-parameter.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Introduction

In this section, the results obtained on the study of the impact of channel estimation errors and correlation coefficient on the ergodic capacity under the PIP constraint was presented. The goal is to investigate and analyze the performance limit of CR with imperfect channel condition under different fading conditions.

4.2 Impact of σ^2 on the SU capacity under different m values

In Figure 4.1, the plot of the ergodic capacity of the SU capacity under the PIP constraint against the channel estimation errors variance, σ^2 at different values ofm-parameter was presentedusing the Matlab codein Appendix B2. The interference path experience a

Rayleigh channel, and the Cognitive Radio path experiences a Nakagami-m fading at m=1/2,1, 2 and 3. The value for, $\beta = 1, \rho = 0.5$ and $Q_0 = 0$ dB. It was observedthat the ergodic capacity decreases with increasing σ^2. This indicates that the ergodic capacity degrades as a result of channel estimation errors. This reveals an important fact that there exist a fundamental tradeoff between ergodic capacity and channel estimation errors. It was also shown that a small amount of interference-outage permitted by the primary receiver, e.g., $P_0 = 0.03$ results in a significant gain in the ergodic capacity over different fading conditions compared to a system with a stricter peak interference constraint at $P_0 = 0.01$. It was also observed that the ergodic capacity of the SU at m=3 is lower than when m=1/2. This indicates that the fading of the cognitive path is harmful to the capacity gain regardless of σ^2. At m=1, the cognitive radio path becomes a Rayleigh channel, the values obtained agree with that of the work inXu*et al.*, (2013b).

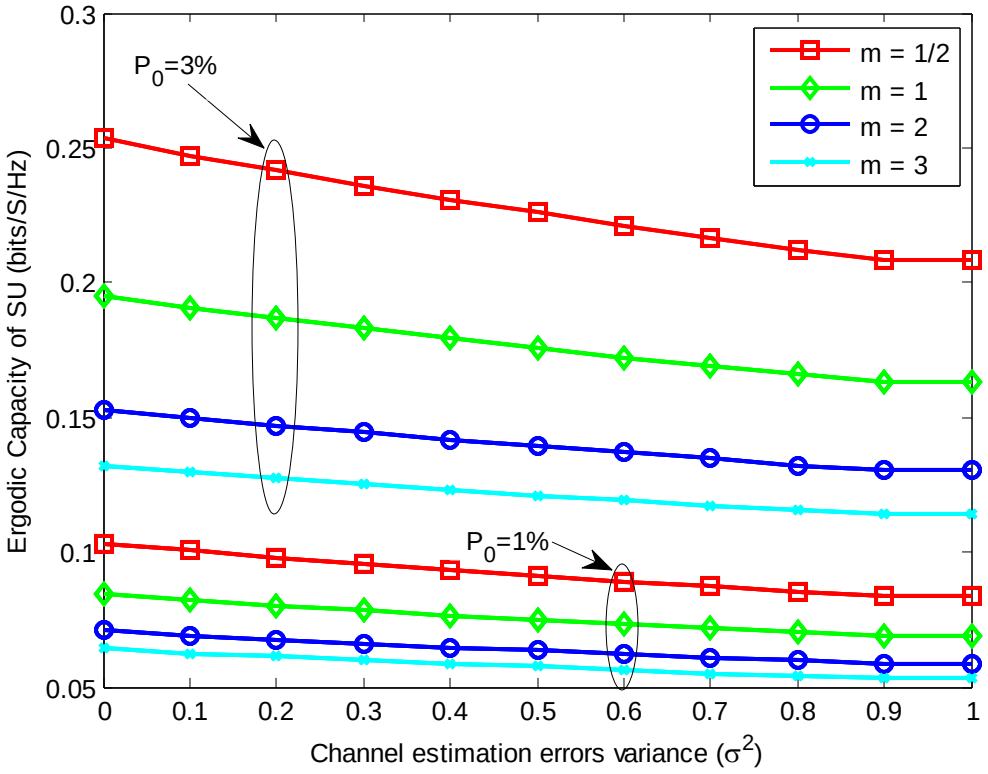


Figure 4.1: Ergodic Capacity of the SU against σ^2 for Different m Values

The percentage capacity gain from $\sigma^2 = 1$ to $\sigma^2 = 0$ under different m-parameters in Figure 4.1 is presented in Table 4.1. The percentage capacity gain at no estimation errors is expressed as:

$$C_{gain}\,(\%) = \frac{C_0 - C_1}{C_1} \times 100\% \quad (4.1)$$

where C_0 is the ergodiccapacity at $\sigma^2 = 0$, C_1 is the channel capacity at σ^2=1

For m=1 at 3% P_0

$$C_{gain\ at\ m=1} = \frac{0.1950 - 0.1629}{0.1629} \times 100\% = 19.71\%$$

Table 4.1: Capacity Gain for Different m Valuesfromto $\sigma^2 = 0$ to σ^2=1

Fading Parameter (m)	P_0 at 3%			P_0 at 1%		
	C_1 (bit/S/Hz)	C_0 (bit/S/Hz)	C_{gain}(%)	C_1 (bit/S/Hz)	C_0 (bit/S/Hz)	C_{gain}(%)
1/2	0.2083	0.2543	21.65	0.0835	0.1032	23.59
1	0.1629	0.1950	19.71	0.0690	0.0844	22.32
2	0.1302	0.1529	17.43	0.0585	0.0708	21.03
3	0.1138	0.1318	15.82	0.0533	0.0641	20.26

4.3 Impact of σ^2 on SU Capacity at m=2

The plot of theergodic capacity of the SU under the PIP constraint against σ^2 at different values of ρ , β , P_0 and m=2 is shown in Figure 4.2 using the Matlab code presented in appendix C2. The interference path experiences a Rayleigh channel, and the cognitive path experiences a Nakagami-m fading (at m=2). It was observed in Figure 4.2 that the SU capacity decreases as σ^2 increases. In addition, it can be seen that as β decreases, the SU capacity increases significantly, which means that increasing the distance between the STx and the PRx can be used to mitigate the impact of channel estimation errors and feedback delay on the ergodic capacity of the SU. The SU capacity in Rayleigh/Nakagami-m (at m=2) fading channel is significant lower than the results obtained in a Rayleigh/Rayleigh fading scenario in Xu*et al.* (2013b) as shown in the Figure 4.3. The Figure 4.3 was plottedusing equation (3.16) as in the Matlab code presented in appendix D2.

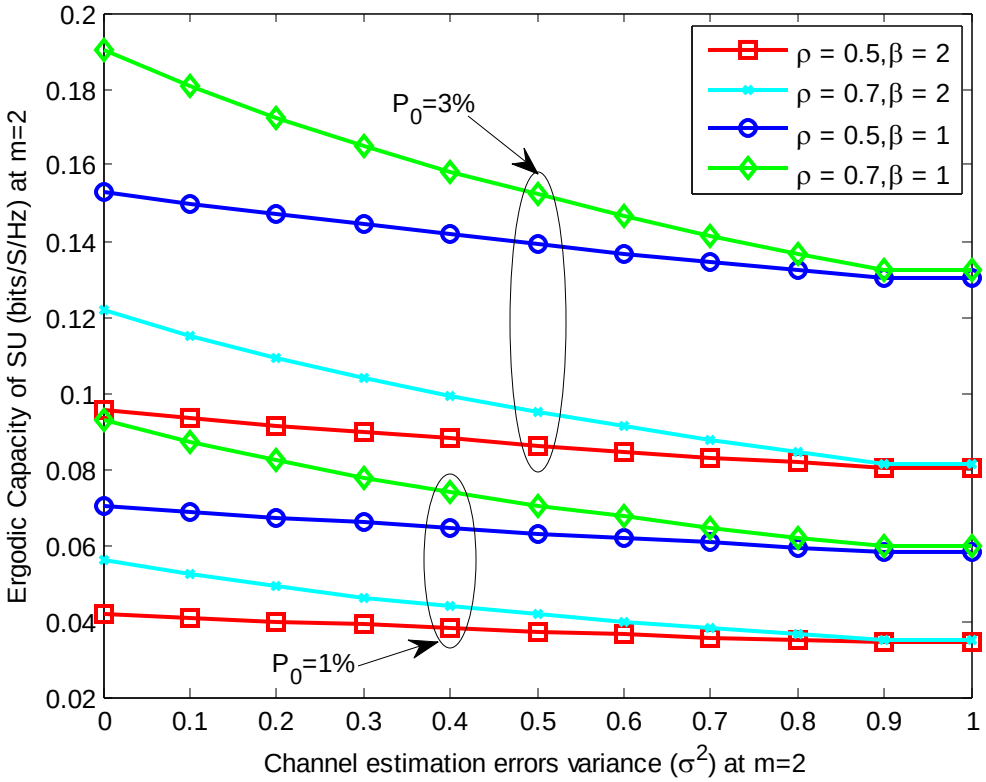


Figure4.2**:** Ergodic Capacity of the SU against σ^2 at m=2

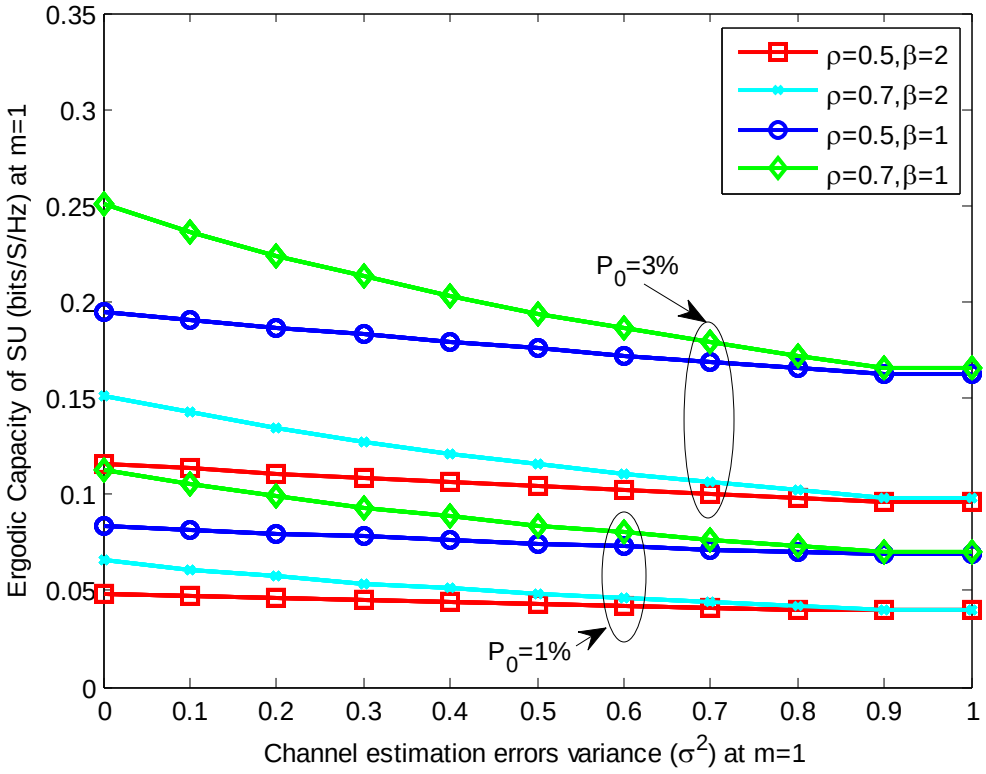


Figure4.3**:** Ergodic Capacity of the SU against σ^2 at m=1

4.4 Impact of ρ on the SU capacity under different m-values

In Figure 4.4, the plot of the SU capacity under the PIP constraint against the correlation coefficient ρ, at differentvalues of m was presented using the Matlab code in appendixE2. The interference path experience a Rayleigh channel, and the Cognitive Radio path experiences aNakagami-m fading at m=1/2, 1, 2 and 3. The value of $\beta = 1$, $\sigma^2 = 0.1$ and $Q_0 = 0$ dB. It was observed that the SU capacity increases with an increasing ρ .This indicates that significant gains can be achieved at low feedback delay. Also, a small interference outage results in a slight increase in the ergodic capacity. In addition, as the value of fading parameter of the cognitive path m increases, there is a

decrease in the SU capacity. This also indicates that the cognitive path is harmful to the capacity gain irrespective of the ρ values.

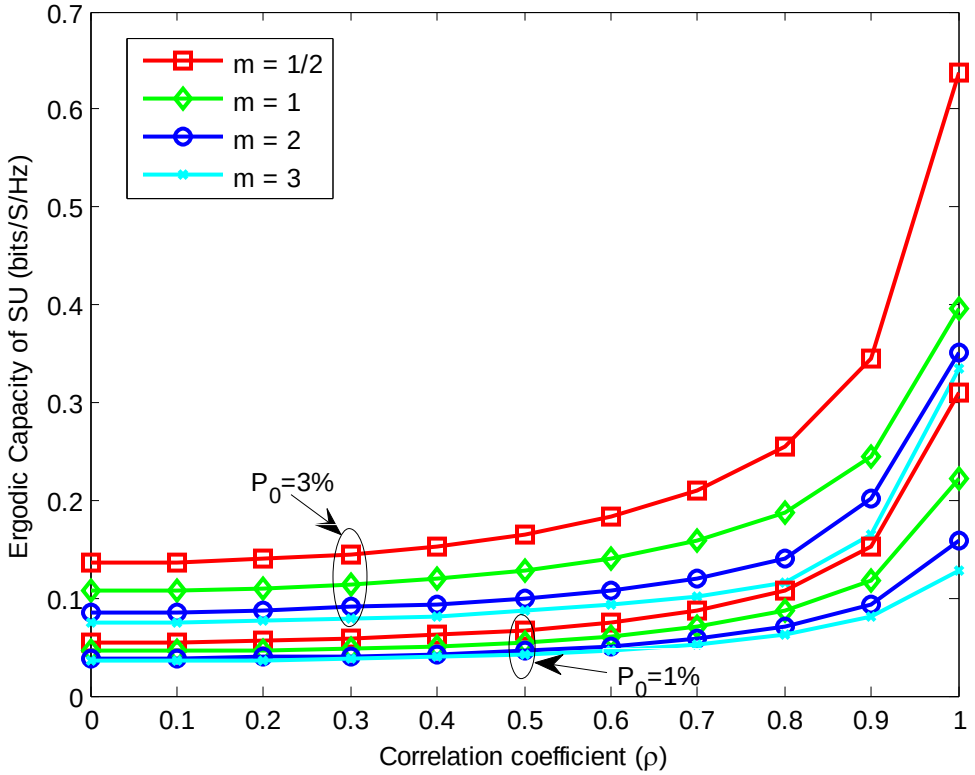


Figure 4.4: Ergodic Capacity of the SU against ρ for Different m Values

At $\rho = 0$ implies the CSI at the STx is completely random, while at $\rho = 1$ implies the SU has a perfect CSI of the link between the STx and SRx. Table 4.2 shows the capacity gain from $\rho = 1$ to $\rho = 0$ from Figure 4.4. The capacity gain in bit/S/Hz from $\rho = 1$ to $\rho = 0$ is expressed as:

$$C_{gain} = C_{\rho_1} - C_{\rho_0} \quad (4.2)$$

where C_{gain} is the capacity gain, C_{ρ_1} is the capacity at $\rho = 1$ and C_{ρ_0} is the capacity at $\rho = 0$

For m=1/2 at 3% P_0

$$C_{gain\ at\ m=1/2} = 0.6369 - 0.1362 = 0.5007$$

Table 4.2: Capacity Gain for Different m Values from $\rho = 0$ to $\rho = 1$

Fading Parameter (m)	P_0 at 3%			P_0 at 1%		
	$C_{\rho 1}$ (bit/S/Hz)	$C_{\rho 0}$ (bit/S/Hz)	C_{gain} (bit/S/Hz)	$C_{\rho 1}$ (bit/S/Hz)	$C_{\rho 0}$ (bit/S/Hz)	C_{gain} (bit/S/Hz)
m=1/2	0.6369	0.1362	0.5007	0.3093	0.0545	0.2584
m=1	0.3955	0.1067	0.2888	0.2222	0.0451	0.1771
m=2	0.3514	0.0854	0.2660	0.1594	0.0383	0.1211
m=3	0.3344	0.0748	0.2596	0.1280	0.0349	0.0931

4.5 Impact of ρ on the SU Capacity at m=2

In Figure 4.5, the plot of the ergodic capacity of the SU against ρ, at different values of σ^2, β, P_0 and m=2 was presented using the Matlab code in appendix F2. The interference path experiences a Rayleigh channel, and the cognitive path experiences a Nakagami-m fading (at m=2). It was observed that, the SU capacity increases with increasing ρ, however, at lower values of ρ $(\rho \leq 0.4)$, the curves for the ergodic capacity almost overlap. This implies that the ergodic capacity of the SU is insensitive to the feedback delay and channel estimation errors when the feedback delay is high which is also the case in existing studies (Xu*et al.*, 2013b) under a Rayleigh/Rayleigh fading scenario. The SU capacity is also lower in the Rayleigh/Nakagami-m (at m=2) fading

compared to the Rayleigh/Rayleigh fading scenario of Xu*et al.* (2013b) as shown in the Figure 4.6. TheFigure 4.6 was plotted using the Matlab code appendix G2.

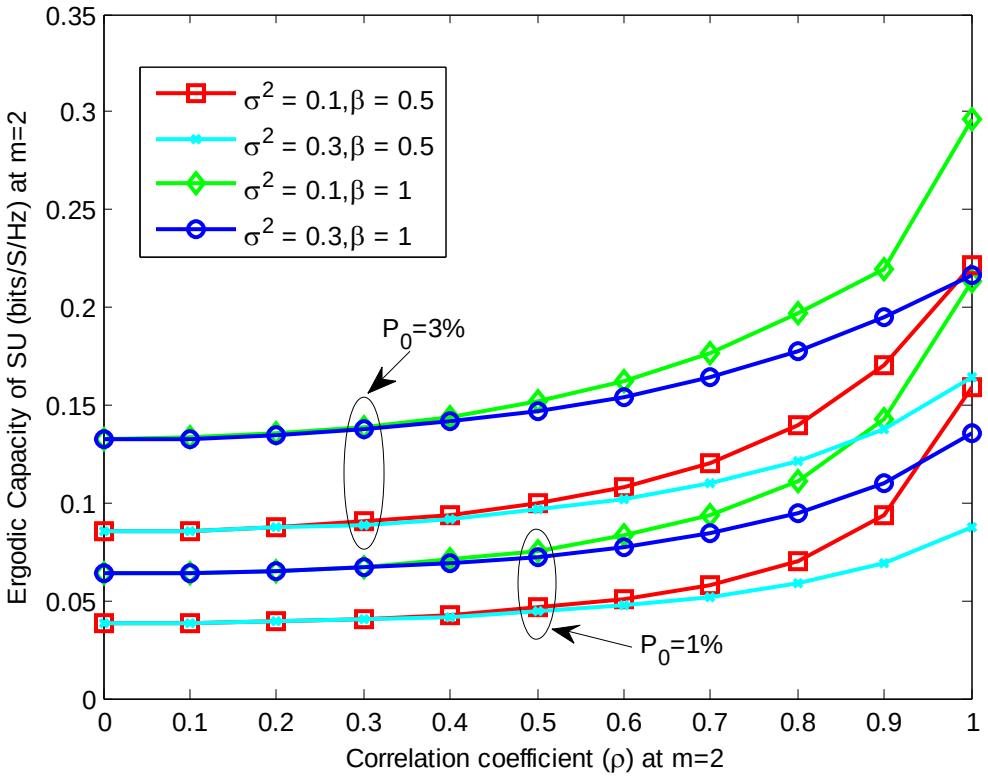


Figure4.5: Ergodic Capacity of the SU against ρ at m=2

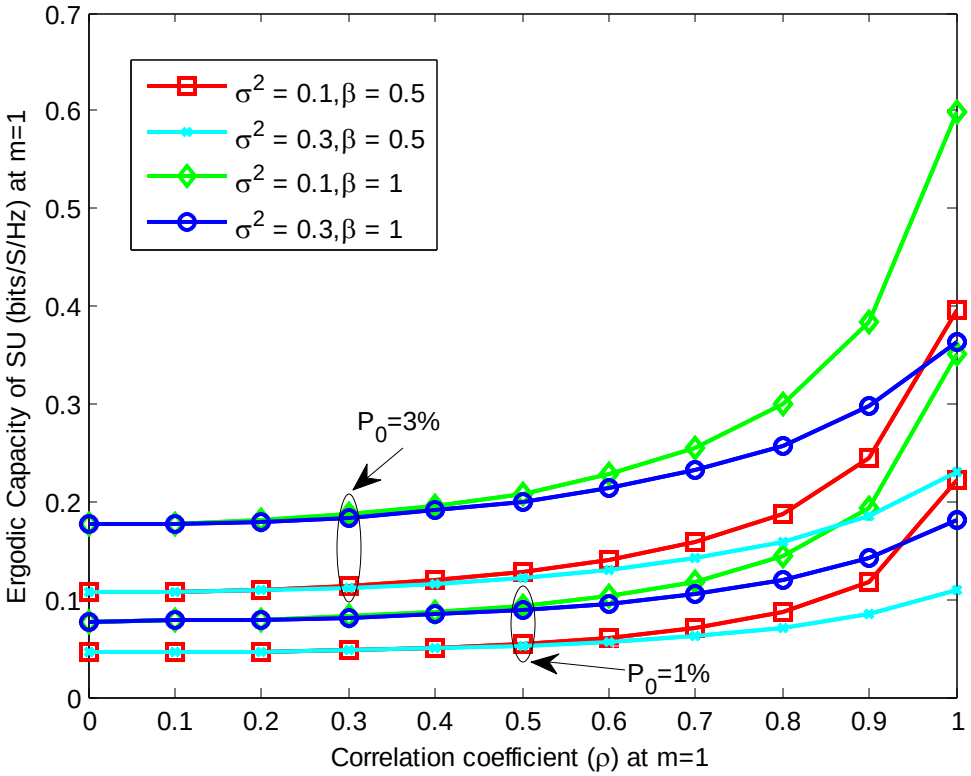


Figure4.6: Ergodic Capacity of the SU against ρ at m=1

4.6 Validation

Validation was carried out to compare the results obtained in this work with that of Xu*et al.*, (2013b). It can be seen in the illustrations in Figure 4.1, Figure 4.3, Figure 4.4 and Figure 4.6 at m=1, the results of the ergodic capacity obtained under the PIP constraintagree with that inXu*et al.*, (2013b).

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

This research has proposed todevelop an improved Ergodic capacity of underlay CR with imperfect CSI. This chapter summarises the whole dissertation. Conclusion and significant contributions during the course of theresearch. Recommendations for further research are also suggested.

5.2 Conclusion

The impact of estimation errors and correlation coefficient on the secondary link in a spectrum sharing system with different values ofm-parameter of Nakagami-m fading channel was studied. A setting where the CR user was constrained bypeak interference powerconstraint set by the PU was considered. A power allocation scheme to overcome the constraint violation by the imperfect channel condition was provided. The imperfect channel condition was due to channel estimation errors and feedback delay.A closed form expression of the ergodic capacitywas developed. The results obtained shows that cognitive radio path affects the SU ergodic capacity and that significant capacity gains are achieved for Rayleigh fading channel as compared toNakagami-m fading channel.

5.3 Significant Contributions

The significant contributions of this research work can be summarized as follows:

1. Deriving aclosed form expression of the Ergodic capacity of an underlay cognitive radio with an imperfect channel state information under the peak interference power constraint.
2. Examining the impact of channel estimation errors under different m values, the ergodic capacity decreased with increasing channel estimation error. At no channel estimation error with an interference outage probability of 3% for the different m valuesat m=1/2,1,2, and 3,capacity gains of 21.65%, 19.71%, 17.43% and 15.82% were achieved, while at 1% outage probability, capacity gains of 23.59%, 22.31%, 21.02% and 20.26% were also achieved for therespective m values.
3. Examining the impact of channel correlation coefficient under different m values, the ergodic capacity increased with increasing channel correlation coefficient. At unity channel correlation coefficient with an interference outage probability of 3%for different m values at m=1/2,1,2 and 3, spectral efficiencies in (bit/S/Hz) of 0.5007,0.2888, 0.2660 and 0.2596 were achieved, while at 1% outage probability, spectral efficiencies in (bit/S/Hz) of 0.2548,0.1771,0.1211,0.0931was also achieved for the respective m values.

5.4 Recommendations for Further Work

1. A cognitive relay can be introduced on the system model to enhance secondary user ergodic capacity.
2. The work can be extended to show the effect of increasing the no of SU's on the ergodic capacity.

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APPENDIX A

CDF OF $\frac{\breve{Q}_0}{Q_0}$

$$F_{\frac{\breve{g}_0}{g_0}}(y)=\int_0^y f_{\frac{\breve{g}_0}{g_0}}(z)\,dz$$

$$=\int_0^y \frac{\left(2\alpha\left(1-\sigma^2\right)z+1\right)\Gamma\left(\frac{3}{2}\right)}{\pi\left(1-\sigma^2\right)\left[z^2+\left(\frac{2}{\left(1-\sigma^2\right)}+4\rho\right)z+\frac{1}{\left(1-\sigma^2\right)^2}\right]^{\frac{3}{2}}}dz$$

$$=\alpha\left(\frac{\frac{1-\sigma^2}{2\left(1-\left(1-\sigma^2\right)\rho^2\right)\sqrt{\frac{1}{\left(1-\sigma^2\right)^2}}}+\frac{-\left(1-\sigma^2\right)\left(1-\left(1-\sigma^2\right)y\right)}{2\left(1-\left(1-\sigma^2\right)\rho^2\right)\sqrt{\frac{1-\left(1-\sigma^2\right)y\left(-2-\left(1-\sigma^2\right)y\right)-4\left(1-\sigma^2\right)^2\rho^2y}{\left(1-\sigma^2\right)^2}}}}{\left(1-\sigma^2\right)^2}\right)$$

$$=\alpha\left(\frac{\frac{\left(1-\sigma^2\right)^2}{2\alpha}-\frac{\left(1-\sigma^2\right)^2\left(1-\left(1-\sigma^2\right)y\right)}{2\alpha\sqrt{1+2\left(1-\sigma^2\right)y+\left(1-\sigma^2\right)^2y^2-4\left(1-\sigma^2\right)\rho^2y}}}{\left(1-\sigma^2\right)^2}\right)$$

$$=\frac{1}{2}-\frac{1-\left(1-\sigma^2\right)y}{2\left(1-\sigma^2\right)\sqrt{\left(y+\frac{1}{\left(1-\sigma^2\right)}\right)^2-4\rho^2y}}$$

APPENDIX B1

MATHEMATICA CODE FOR SU CAPACITY AGAINST σ^2

```
a=1-Range[0,1,0.1];
ρ1=0.5;β=1
α1=1-a*Subscript[ρ, 1]^2;
P01=0.03; P02=0.01;
b01=1-2P01;
c01=1-P01;b02=1-2P02;c02=1-P02;
Qpk=1;
Ω̂pk01=((α1-b01*Sqrt[α1*(1-a*Subscript[b, 01]^2*Subscript[ρ,
1]^2)])/(2P01*c01)+1-2α1)*a*Qpk;
Ω̂pk02=((α1-b02*Sqrt[α1*(1-a*Subscript[b, 02]^2*Subscript[ρ,
1]^2)])/(2P02*c02)+1-2α1)*a*Qpk;
m1=1/2;
n01=Ω̂pk01/(a*β);
cer01=n01/(n01-1)*(Log2[n01]+EulerGamma+PolyGamma[0,m1])
n11=Ω̂pk02/(a*β);
cer11=n11/(n11-1)*(Log2[n11]+EulerGamma+PolyGamma[0,m1])
m2=1;
cer02=n01/(n01-1)*(Log2[n01]+EulerGamma+PolyGamma[0,m2])

cer12=n11/(n11-1)*(Log2[n11]+EulerGamma+PolyGamma[0,m2])
m3=2;
cer03=n01/(n01-1)*(Log2[n01]+EulerGamma+PolyGamma[0,m3])
cer13=n11/(n11-1)*(Log2[n11]+EulerGamma+PolyGamma[0,m3])
m4=3;
cer04=n01/(n01-1)*(Log2[n01]+EulerGamma+PolyGamma[0,m4])
cer14=n11/(n11-1)*(Log2[n11]+EulerGamma+PolyGamma[0,m4])
```

APPENDIX B2

M-FILE FOR THE PLOT INFIGURE 4.1

```
%% m-file for plotting the impact of channel correlation coefficient
and channel estimation errors under PIP constraint

%plot for ergodic capacity at different values of m
clc;
%v=variance
v=0:0.1:1;
%PIP Capacity at P_0=0.03, beta=1, rho=0.5, m=1/2
C_er01=[0.2534,0.2473,0.2416,0.2361,0.2309,0.2259,0.2212,0.2167,0.2124,
0.2083,0.2083];
%PIP Capacity at P_0=0.01, beta=1, rho=0.5, m=1/2
C_er11=[0.1032,0.1005,0.0980,0.0956,0.0933,0.0911,0.0891,0.0871,0.0853,
0.0835,0.0835];
%PIP Capacity at P_0=0.03, beta=1, rho=0.5, m=1
C_er02=[0.1950,0.1907,0.1867,0.1828,0.1791,0.1756,0.1722,0.1690,0.1659,
0.1629,0.1629];
%PIP Capacity at P_0=0.01, beta=1, rho=0.5, m=1
C_er22=[0.0844,0.0823,0.0803,0.0785,0.0767,0.0750,0.0734,0.0719,0.0704,
0.0690,0.0690];
%PIP Capacity at P_0=0.03, beta=1, rho=0.5, m=2
C_er03=[0.1529,0.1499,0.1471,0.1444,0.1418,0.1393,0.1369,0.1346,0.1323,
0.1302,0.1302];
%PIP Capacity at P_0=0.01, beta=1, rho=0.5, m=2
C_er13=[0.0708,0.0692,0.0676,0.0661,0.0647,0.0634,0.0621,0.0608,0.0597,
0.0585,0.0585];
%PIP Capacity at P_0=0.03, beta=1, rho=0.5, m=3
C_er04=[0.1318,0.1295,0.1273,0.1251,0.1231,0.1211,0.1192,0.1174,0.1156,
0.1138,0.1138];
%PIP Capacity at P_0=0.01, beta=1, rho=0.5, m=3
C_er14=[0.0641,0.0626,0.0613,0.0600,0.0587,0.0575,0.0564,0.0550,0.0543,
0.0533,0.0533];

plot(v,C_er01,'r-s','LineWidth',2)
holdon
plot(v,C_er02,'g-d','LineWidth',2)
holdon
plot(v,C_er03,'b-o','LineWidth',2)
holdon
plot(v,C_er04,'c-x','LineWidth',2)
holdon
plot(v,C_er11,'r-s','LineWidth',2)
holdon
plot(v,C_er22,'g-d','LineWidth',2)
holdon
plot(v,C_er13,'b-o','LineWidth',2)
holdon
plot(v,C_er14,'c-x','LineWidth',2)
xlabel('Channel estimation errors variance (\sigma^2)')
ylabel('Ergodic Capacity of SU (bits/S/Hz)')
legend('m = 1/2','m = 1','m = 2','m = 3')
```

APPENDIX C1

MATHEMATICA CODE FOR SU CAPACITY AGAINST σ^2 AT m=2

```
a=1-Range[0,1,0.1];
ρ1=0.5;ρ2=0.7;
α1=1-a*Subscript[ρ, 1]^2;
α2=1-a*Subscript[ρ, 2]^2;
P01=0.03; P02=0.01;
b01=1-2P01;
c01=1-P01;b02=1-2P02;c02=1-P02;
Qpk=1;
Q̂pk01=((α1-b01*Sqrt[α1*(1-a*Subscript[b, 01]^2*Subscript[ρ,
1]^2)])/(2P01*c01)+1-2α1)*a*Qpk;
Q̂pk02=((α2-b01*Sqrt[α2*(1-a*Subscript[b, 01]^2*Subscript[ρ,
2]^2)])/(2P01*c01)+1-2α2)*a*Qpk;
Q̂pk03=((α1-b02*Sqrt[α1*(1-a*Subscript[b, 02]^2*Subscript[ρ,
1]^2)])/(2P02*c02)+1-2α1)*a*Qpk;
Q̂pk04=((α2-b02*Sqrt[α2*(1-a*Subscript[b, 02]^2*Subscript[ρ,
2]^2)])/(2P02*c02)+1-2α2)*a*Qpk;
β1=2;m1=2;
n01=Q̂pk01/(a*β1);
cer01=n01/(n01-1)*(Log2[n01]+EulerGamma+PolyGamma[0,m1])
n02 =Q̂pk02/(a*β1);
cer02=n02/(n02-1)*(Log2[n02]+EulerGamma+PolyGamma[0,m1])
n03=Q̂pk03/(a*β1);
cer03=n03/(n03-1)*(Log2[n03]+EulerGamma+PolyGamma[0,m1])
n04=Q̂pk04/(a*β1);
cer04=n04/(n04-1)*(Log2[n04]+EulerGamma+PolyGamma[0,m1])
β2=1;
n05=Q̂pk01/(a*β2);
cer05=n05/(n05-1)*(Log2[n05]+EulerGamma+PolyGamma[0,m1])
n06 =Q̂pk02/(a*β2);
cer06=n06/(n06-1)*(Log2[n06]+EulerGamma+PolyGamma[0,m1])
n07=Q̂pk03/(a*β2);
cer07=n03/(n03-1)*(Log2[n07]+EulerGamma+PolyGamma[0,m1])
n08=Q̂pk04/(a*β2);
cer08=n08/(n08-1)*(Log2[n08]+EulerGamma+PolyGamma[0,m1])
```

APPENDIX C2

M-FILE FOR THE PLOT INFIGURE 4.2

```
%% m-file for plotting the impact of feedback delay and channel
estimation errors under PIP constraint
clc;
% v=variance
v=0:0.1:1;
% PIP Capacity At P_0=0.03, beta=2,rh0=0.5, m=2
C_er01=[0.0955,0.0935,0.0915,0.0897,0.0879,0.0862,0.0846,0.0831,0.0816,
0.0801,0.0801];
%PIP Capacity At P_0=0.03, beta=2, rho=0.7, m=2
C_er02=[0.1218,0.1151,0.1093,0.1040,0.0993,0.0951,0.0912,0.0877,0.0845,
0.0815,0.0815];
%PIP Capacity At P_0=0.03, beta=1, rho=0.5, m=2
C_er03=[0.1529,0.1499,0.1471,0.1444,0.1418,0.1393,0.1368,0.1346,0.1323,
0.1302,0.1302];
%PIP Capacity At P_0=0.03, beta=1, rho=0.7, m=2
C_er04=[0.1901,0.1809,0.1726,0.1652,0.1584,0.1523,0.1466,0.1415,0.1367,
0.1323,0.1323];
%PIP Capacity At P_0=0.01, beta=2, rho=0.5, m=2
C_er05=[0.0419,0.0409,0.0399,0.0390,0.0381,0.0373,0.0365,0.0357,0.0350,
0.0343,0.0343];
%PIP Capacity At P_0=0.01, beta=2, rho=0.7, m=2
C_er06=[0.0558,0.0522,0.0490,0.0463,0.0439,0.0417,0.0398,0.0380,0.0364,
0.0350,0.0350];
%PIP Capacity At P_0=0.01, beta=2, rho=0.5, m=2
C_er07=[0.0704,0.0688,0.0672,0.0658,0.0644,0.0630,0.0618,0.0606,0.0594,
0.0582,0.0582];
%PIP Capacity At P_0=0.01, beta=2, rho=0.7, m=2
C_er08=[0.0929,0.0872,0.0822,0.0778,0.0740,0.0705,0.0674,0.0645,0.0620,
0.0596,0.0596];

plot(v,C_er01,'r-s','LineWidth',2)
holdon
plot(v,C_er02,'b-o','LineWidth',2)
holdon
plot(v,C_er03,'g-d','LineWidth',2)
holdon
plot(v,C_er04,'c-x','LineWidth',2)
holdon
plot(v,C_er05,'r-s','LineWidth',2)
holdon
plot(v,C_er06,'b-o','LineWidth',2)
holdon
plot(v,C_er07,'g-d','LineWidth',2)
holdon
plot(v,C_er08,'c-x','LineWidth',2)

xlabel('Channel estimation errors variance (\sigma^2) at m=2')
ylabel('Ergodic Capacity of SU (bits/S/Hz) at m=2')
legend('\rho = 0.5,\beta = 2','\rho = 0.7,\beta = 2','\rho = 0.5,\beta
= 1','\rho = 0.7,\beta = 1')
```

APPENDIX D1

MATHEMATICA CODE FOR SU CAPACITY AGAINST σ^2 AT m=1

```
a=1-Range[0,1,0.1];
ρ1=0.5;ρ2=0.7;
α1=1-a*Subscript[ρ, 1]^2;
α2=1-a*Subscript[ρ, 2]^2;
P01=0.03; P02=0.01;
b01=1-2P01;
c01=1-P01;b02=1-2P02;c02=1-P02;
Qpk=1;
Q̂pk01=((α1-b01*Sqrt[α1*(1-a*Subscript[b, 01]^2*Subscript[ρ,
1]^2)])/(2P01*c01)+1-2α1)*a*Qpk;
Q̂pk02=((α2-b01*Sqrt[α2*(1-a*Subscript[b, 01]^2*Subscript[ρ,
2]^2)])/(2P01*c01)+1-2α2)*a*Qpk;
Q̂pk03=((α1-b02*Sqrt[α1*(1-a*Subscript[b, 02]^2*Subscript[ρ,
1]^2)])/(2P02*c02)+1-2α1)*a*Qpk;
Q̂pk04=((α2-b02*Sqrt[α2*(1-a*Subscript[b, 02]^2*Subscript[ρ,
2]^2)])/(2P02*c02)+1-2α2)*a*Qpk;
β1=2;m1=1;
n01=Q̂pk01/(a*β1);
cer01=n01/(n01-1)*(Log2[n01]+EulerGamma+PolyGamma[0,m1])
n02 =Q̂pk02/(a*β1);
cer02=n02/(n02-1)*(Log2[n02]+EulerGamma+PolyGamma[0,m1])
n03=Q̂pk03/(a*β1);
cer03=n03/(n03-1)*(Log2[n03]+EulerGamma+PolyGamma[0,m1])
n04=Q̂pk04/(a*β1);
cer04=n04/(n04-1)*(Log2[n04]+EulerGamma+PolyGamma[0,m1])
β2=1;
n05=Q̂pk01/(a*β2);
cer05=n05/(n05-1)*(Log2[n05]+EulerGamma+PolyGamma[0,m1])
n06 =Q̂pk02/(a*β2);
cer06=n06/(n06-1)*(Log2[n06]+EulerGamma+PolyGamma[0,m1])
n07=Q̂pk03/(a*β2);
cer07=n03/(n03-1)*(Log2[n07]+EulerGamma+PolyGamma[0,m1])
n08=Q̂pk04/(a*β2);
cer08=n08/(n08-1)*(Log2[n08]+EulerGamma+PolyGamma[0,m1])
```

APPENDIX D2

M-FILE FOR THE PLOT INFIGURE 4.3

```
%% m-file for plotting the impact of feedback delay and channel
estimation errors under PIP constraint
clc;
% v=variance
v=0:0.1:1;
% PIP Capacity At P_0=0.03, beta=2,rh0=0.5, m=2
C_er01=[0.0955,0.0935,0.0915,0.0897,0.0879,0.0862,0.0846,0.0831,0.0816,
0.0801,0.0801];
%PIP Capacity At P_0=0.03, beta=2, rho=0.7, m=2
C_er02=[0.1218,0.1151,0.1093,0.1040,0.0993,0.0951,0.0912,0.0877,0.0845,
0.0815,0.0815];
%PIP Capacity At P_0=0.03, beta=1, rho=0.5, m=2
C_er03=[0.1529,0.1499,0.1471,0.1444,0.1418,0.1393,0.1368,0.1346,0.1323,
0.1302,0.1302];
%PIP Capacity At P_0=0.03, beta=1, rho=0.7, m=2
C_er04=[0.1901,0.1809,0.1726,0.1652,0.1584,0.1523,0.1466,0.1415,0.1367,
0.1323,0.1323];
%PIP Capacity At P_0=0.01, beta=2, rho=0.5, m=2
C_er05=[0.0419,0.0409,0.0399,0.0390,0.0381,0.0373,0.0365,0.0357,0.0350,
0.0343,0.0343];
%PIP Capacity At P_0=0.01, beta=2, rho=0.7, m=2
C_er06=[0.0558,0.0522,0.0490,0.0463,0.0439,0.0417,0.0398,0.0380,0.0364,
0.0350,0.0350];
%PIP Capacity At P_0=0.01, beta=2, rho=0.5, m=2
C_er07=[0.0704,0.0688,0.0672,0.0658,0.0644,0.0630,0.0618,0.0606,0.0594,
0.0582,0.0582];
%PIP Capacity At P_0=0.01, beta=2, rho=0.7, m=2
C_er08=[0.0929,0.0872,0.0822,0.0778,0.0740,0.0705,0.0674,0.0645,0.0620,
0.0596,0.0596];

plot(v,C_er01,'r-s','LineWidth',2)
holdon
plot(v,C_er02,'c-x','LineWidth',2)
holdon
plot(v,C_er03,'b-o','LineWidth',2)
holdon
plot(v,C_er04,'g-d','LineWidth',2)
holdon
plot(v,C_er05,'r-s','LineWidth',2)
holdon
plot(v,C_er06,'c-x','LineWidth',2)
holdon
plot(v,C_er07,'b-o','LineWidth',2)
holdon
plot(v,C_er08,'g-d','LineWidth',2)

xlabel('Channel estimation errors variance (\sigma^2) at m=2')
ylabel('Ergodic Capacity of SU (bits/S/Hz) at m=2')
legend('\rho = 0.5,\beta = 2','\rho = 0.7,\beta = 2','\rho = 0.5,\beta
= 1','\rho = 0.7,\beta = 1')
```

APPENDIX E1

MATHEMATICA CODE FOR SU CAPACITY AGAINST ρ

```
P01=0.03; P02=0.01;
v1=0.1;
ρ=Range[0,1,0.1];
a1=(1-v1)*ρ^2;
a2=(1-v2)*ρ^2;
α1=1-a1;
α2=1-a2;
b01=1-2P01;b02=1-2P02;c01=1-P01;c02=1-P02;Qpk=1;
Q̂pk01=((α1-b01*Sqrt[α1*(1-(1-v1)*Subscript[b,
01]^2*ρ^2)])/(2P01*c01)+1-2α1)*(1-v1)*Qpk;
Q̂pk02=((α1-b02*Sqrt[α1*(1-(1-v1)*Subscript[b,
02]^2*ρ^2)])/(2P02*c02)+1-2α1)*(1-v1)*Qpk;
β=1;
m1=1/2;
n01=Q̂pk01/((1-v1)*β);
cer01=n01/(n01-1)*(Log2[n01]+EulerGamma+PolyGamma[0,m1])
n11=Q̂pk02/((1-v1)*β);
cer11=n11/(n11-1)*(Log2[n11]+EulerGamma+PolyGamma[0,m1])
m2=1;
cer02=n01/(n01-1)*(Log2[n01]+EulerGamma+PolyGamma[0,m2])
cer12=n11/(n11-1)*(Log2[n11]+EulerGamma+PolyGamma[0,m2])
m3=2;
cer03=n01/(n01-1)*(Log2[n01]+EulerGamma+PolyGamma[0,m3])
cer13=n11/(n11-1)*(Log2[n11]+EulerGamma+PolyGamma[0,m3])
m4=3;
cer04=n01/(n01-1)*(Log2[n01]+EulerGamma+PolyGamma[0,m4])
cer14=n11/(n11-1)*(Log2[n11]+EulerGamma+PolyGamma[0,m4])
```

APPENDIX E2

M-FILE FOR THE PLOT INFIGURE 4.4

```
%%third part
%Ergodic capacity at different values of m
%rho is the feedback delay
clc;
rho=0:0.1:1;
%PIP capacity at P_0=0.03, beta=1, variance=0.1, m=1/2
C_er01=[0.1362,0.1371,0.1400,0.1452,0.1531,0.1649,0.1824,0.2094,0.2547,
0.3452,0.6369];
%PIP Capacity at P_0=0.01, beta=1, variance=0.1, m=1/2
C_er11=[0.0545,0.0549,0.0562,0.0584,0.0619,0.0670,0.0748,0.0869,0.1079,
0.1518,0.3093];
%PIP Capacity at P_0=0.03, beta=1, variance=0.1, m=1
C_er02=[0.1067,0.1074,0.1095,0.1132,0.1188,0.1272,0.1394,0.1578,0.1877,
0.2439,0.3955];
%PIP Capacity at P_0=0.01, beta=1, variance=0.1, m=1
C_er12=[0.0451,0.0454,0.0464,0.0481,0.0509,0.0549,0.0609,0.0702,0.0860,
0.1179,0.2222];
%PIP Capacity at P_0=0.03, beta=1, variance=0.1, m=2
C_er03=[0.0854,0.0859,0.0874,0.0901,0.0941,0.0999,0.1083,0.1206,0.1395,
0.2009,0.3514];
%PIP Capacity at P_0=0.01, beta=1, variance=0.1, m=2
C_er13=[0.0383,0.0386,0.0393,0.0408,0.0429,0.0461,0.0509,0.0581,0.0701,
0.0935,0.1594];
%PIP Capacity at P_0=0.03, beta=1, variance=0.1, m=3
C_er04=[0.0748,0.0752,0.0764,0.0785,0.0817,0.0863,0.0928,0.1020,0.1153,
0.1644,0.3344];
%PIP Capacity at P_0=0.01, beta=1, variance=0.1, m=3
C_er14=[0.0349,0.0351,0.0358,0.0371,0.0390,0.0417,0.0459,0.0521,0.0622,
0.0812,0.1280];

plot(rho,C_er01,'r-s','LineWidth',2)
holdon
plot(rho,C_er02,'g-d','LineWidth',2)
holdon
plot(rho,C_er03,'b-o','LineWidth',2)
holdon
plot(rho,C_er04,'c-x','LineWidth',2)
holdon
plot(rho,C_er11,'r-s','LineWidth',2)
holdon
plot(rho,C_er12,'g-d','LineWidth',2)
holdon
plot(rho,C_er13,'b-o','LineWidth',2)
holdon
plot(rho,C_er14,'c-x','LineWidth',2)
xlabel('Correlation coefficient (\rho)')
ylabel('Ergodic Capacity of SU (bits/S/Hz)')
legend('m = 1/2','m = 1','m = 2','m = 3')
```

APPENDIX F1

MATHEMATICA CODE FOR SU CAPACITY AGAINST ρ AT m=2

```
P01=0.03; P02=0.01;
v1=0.1;v2=0.3;
ρ=Range[0,1,0.1];
a1=(1-v1)*ρ^2;
a2=(1-v2)*ρ^2;
α1=1-a1;
α2=1-a2;
b01=1-2P01;b02=1-2P02;c01=1-P01;c02=1-P02;Qpk=1;
Q̂pk01=((α1-b01*Sqrt[α1*(1-(1-v1)*Subscript[b,
01]^2*ρ^2)])/(2P01*c01)+1-2α1)*(1-v1)*Qpk;
Q̂pk02=((α1-b02*Sqrt[α1*(1-(1-v1)*Subscript[b,
02]^2*ρ^2)])/(2P02*c02)+1-2α1)*(1-v1)*Qpk;
Q̂pk03=((α2-b01*Sqrt[α2*(1-(1-v2)*Subscript[b,
01]^2*ρ^2)])/(2P01*c01)+1-2α2)*(1-v2)*Qpk;
Q̂pk04=((α2-b02*Sqrt[α2*(1-(1-v2)*Subscript[b,
02]^2*ρ^2)])/(2P02*c02)+1-2α2)*(1-v2)*Qpk;
β1=0.5;m1=2;
n01=Q̂pk01/((1-v1)*β1);
cer01=n01/(n01-1)*(Log2[n01]+EulerGamma+PolyGamma[0,m1])
n02=Q̂pk02/((1-v1)*β1);
cer02=n02/(n02-1)*(Log2[n02]+EulerGamma+PolyGamma[0,m1])
n03=Q̂pk03/((1-v2)*β1);
cer03=n03/(n03-1)*(Log2[n03]+EulerGamma+PolyGamma[0,m1])
n04=Q̂pk04/((1-v2)*β1);
cer04=n04/(n04-1)*(Log2[n04]+EulerGamma+PolyGamma[0,m1])
β2=1;
n05=Q̂pk01/((1-v1)*β2);
cer05=n05/(n05-1)*(Log2[n05]+EulerGamma+PolyGamma[0,m1])
n06=Q̂pk02/((1-v1)*β2);
cer06=n06/(n06-1)*(Log2[n06]+EulerGamma+PolyGamma[0,m1])
n07=Q̂pk03/((1-v2)*β2);
cer07=n07/(n07-1)*(Log2[n07]+EulerGamma+PolyGamma[0,m1])
n08=Q̂pk04/((1-v2)*β2);
cer08=n08/(n08-1)*(Log2[n08]+EulerGamma+PolyGamma[0,m1])
```

APPENDIX F2

M-FILE FOR THE PLOT INFIGURE 4.5

```
clc;
% v=variance
v=0:0.1:1;
% PIP Capacity At P_0=0.03, beta=2,rh0=0.5, m=1
C_er01=[0.1161,0.1135,0.1109,0.1085,0.1063,0.1041,0.1020,0.1000,0.0981,
0.0962,0.0962];
%PIP Capacity At P_0=0.03, beta=2, rho=0.7, m=1
C_er02=[0.1513,0.1423,0.1344,0.1274,0.1212,0.1156,0.1106,0.1060,0.1018,
0.0980,0.0980];
%PIP Capacity At P_0=0.03, beta=1, rho=0.5, m=1
C_er03=[0.0486,0.0474,0.0462,0.0451,0.0441,0.0431,0.0421,0.0412,0.0403,
0.0395,0.0395];
%PIP Capacity At P_0=0.03, beta=1, rho=0.7, m=1
C_er04=[0.0656,0.0611,0.0573,0.0540,0.0510,0.0484,0.0461,0.0440,0.0420,
0.0403,0.0403];
%PIP Capacity At P_0=0.01, beta=2, rho=0.5, m=1
C_er05=[0.1950,0.1907,0.1867,0.1828,0.1791,0.1756,0.1722,0.1690,0.1659,
0.1629,0.1629];
%PIP Capacity At P_0=0.01, beta=2, rho=0.7, m=1
C_er06=[0.2510,0.2367,0.2242,0.2130,0.2031,0.1941,0.1861,0.1787,0.1719,
0.1658,0.1658];
%PIP Capacity At P_0=0.01, beta=2, rho=0.5, m=1
C_er07=[0.0838,0.0818,0.0798,0.0780,0.0762,0.0746,0.0730,0.0714,0.0700,
0.0686,0.0686];
%PIP Capacity At P_0=0.01, beta=2, rho=0.7, m=1
C_er08=[0.1127,0.1053,0.0989,0.0933,0.0884,0.0840,0.0800,0.0765,0.0733,
0.0703,0.0703];

plot(rho,C_er06,'r-s','LineWidth',2)
holdon
plot(rho,C_er08,'c-x','LineWidth',2)
holdon
plot(rho,C_er02,'g-d','LineWidth',2)
holdon
plot(rho,C_er04,'b-o','LineWidth',2)
holdon
plot(rho,C_er05,'r-s','LineWidth',2)
holdon
plot(rho,C_er07,'c-x','LineWidth',2)
holdon
plot(rho,C_er01,'g-d','LineWidth',2)
holdon
plot(rho,C_er03,'b-o','LineWidth',2)

xlabel('Correlation coefficient (\rho) at m=2')
ylabel('Ergodic Capacity of SU (bits/S/Hz) at m=2')
legend('\sigma^2 = 0.1,\beta = 0.5','\sigma^2 = 0.3,\beta =
0.5','\sigma^2 = 0.1,\beta = 1','\sigma^2 = 0.3,\beta = 1')
```

APPENDIX G1

MATHEMATICA CODE FOR SU CAPACITY AGAINST ρ AT m=1

```
P01=0.03; P02=0.01;
v1=0.1;v2=0.3;
ρ=Range[0,1,0.1];
a1=(1-v1)*ρ^2;
a2=(1-v2)*ρ^2;
α1=1-a1;
α2=1-a2;
b01=1-2P01;b02=1-2P02;c01=1-P01;c02=1-P02;Qpk=1;
Q̂pk01=((α1-b01*Sqrt[α1*(1-(1-v1)*Subscript[b,
01]^2*ρ^2)])/(2P01*c01)+1-2α1)*(1-v1)*Qpk;
Q̂pk02=((α1-b02*Sqrt[α1*(1-(1-v1)*Subscript[b,
02]^2*ρ^2)])/(2P02*c02)+1-2α1)*(1-v1)*Qpk;
Q̂pk03=((α2-b01*Sqrt[α2*(1-(1-v2)*Subscript[b,
01]^2*ρ^2)])/(2P01*c01)+1-2α2)*(1-v2)*Qpk;
Q̂pk04=((α2-b02*Sqrt[α2*(1-(1-v2)*Subscript[b,
02]^2*ρ^2)])/(2P02*c02)+1-2α2)*(1-v2)*Qpk;
β1=0.5;m1=1;
n01=Q̂pk01/((1-v1)*β1);
cer01=n01/(n01-1)*(Log2[n01]+EulerGamma+PolyGamma[0,m1])
n02=Q̂pk02/((1-v1)*β1);
cer02=n02/(n02-1)*(Log2[n02]+EulerGamma+PolyGamma[0,m1])
n03=Q̂pk03/((1-v2)*β1);
cer03=n03/(n03-1)*(Log2[n03]+EulerGamma+PolyGamma[0,m1])
n04=Q̂pk04/((1-v2)*β1);
cer04=n04/(n04-1)*(Log2[n04]+EulerGamma+PolyGamma[0,m1])
β2=1;
n05=Q̂pk01/((1-v1)*β2);
cer05=n05/(n05-1)*(Log2[n05]+EulerGamma+PolyGamma[0,m1])
n06=Q̂pk02/((1-v1)*β2);
cer06=n06/(n06-1)*(Log2[n06]+EulerGamma+PolyGamma[0,m1])
n07=Q̂pk03/((1-v2)*β2);
cer07=n07/(n07-1)*(Log2[n07]+EulerGamma+PolyGamma[0,m1])
n08=Q̂pk04/((1-v2)*β2);
cer08=n08/(n08-1)*(Log2[n08]+EulerGamma+PolyGamma[0,m1])
```

APPENDIX G2

M-FILE FOR THE PLOT INFIGURE 4.6

```
%rho is the feedback delay
clc;
rho=0:0.1:1;
%PIP Capacity At P_0=0.03, variance=0.1, beta=0.5
C_er01=[0.1765,0.1775,0.1808,0.1866,0.1955,0.2084,0.2272,0.2555,0.3007,
0.3839,0.5994];
%PIP Capacity at P_0=0.01,variance=0.1, beta=0.5
C_er02=[0.0774,0.0779,0.0795,0.0824,0.0869,0.0935,0.1033,0.1184,0.1437,
0.1940,0.3519];
%PIP Capacity at P_0=0.03, variance=0.3, beta=0.5
C_er03=[0.1765,0.1773,0.1798,0.1842,0.1908,0.2002,0.2132,0.2313,0.2574,
0.2971,0.3640];
%PIP Capacity at P_0=0.01, variance-0.3, beta=0.5
C_er04=[0.0774,0.0778,0.0790,0.0812,0.0845,0.0893,0.0959,0.1054,0.1194,
0.1416,0.1815];
%PIP Capacity at P_0=0.03, variance=0.1, beta=1
C_er05=[0.1067,0.1074,0.1095,0.1132,0.1188,0.1272,0.1394,0.1578,0.1877,
0.2439,0.3955];
%PIP Capacity at P_0=0.01, variance=0.1, beta=1
C_er06=[0.0451,0.0454,0.0464,0.0481,0.0509,0.0549,0.0609,0.0702,0.0860,
0.1179,0.2222];
%PIP Capacity at P_0=0.03, variance=0.3, beta=1
C_er07=[0.1067,0.1072,0.1088,0.1117,0.1159,0.1219,0.1302,0.1420,0.1591,
0.1853,0.2304];
%PIP Capacity at P_0=0.01, variance=0.1, beta=1
C_er08=[0.0451,0.0453,0.0461,0.0474,0.0494,0.0523,0.0564,0.0622,0.0708,
0.0847,0.1099];

plot(rho,C_er06,'r-s','LineWidth',2)
holdon
plot(rho,C_er08,'c-x','LineWidth',2)
holdon
plot(rho,C_er02,'g-d','LineWidth',2)
holdon
plot(rho,C_er04,'b-o','LineWidth',2)
holdon
plot(rho,C_er05,'r-s','LineWidth',2)
holdon
plot(rho,C_er07,'c-x','LineWidth',2)
holdon
plot(rho,C_er01,'g-d','LineWidth',2)
holdon
plot(rho,C_er03,'b-o','LineWidth',2)

xlabel('Correlation coefficient (\rho) at m=1')
ylabel('Ergodic Capacity of SU (bits/S/Hz) at m=1')
legend('\sigma^2 = 0.1,\beta = 0.5','\sigma^2 = 0.3,\beta =
0.5','\sigma^2 = 0.1,\beta = 1','\sigma^2 = 0.3,\beta = 1')
```