ERROR ANALYSIS IN THE SOLUTION PROCESSES OF LITERAL EQUATIONS

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SEPTEMBER, 1984

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IN PARTIAL FULFILLMENT OF THE MASTERS IN EDUCATION (M.ED.)

DEPARTMENT OF EDUCATION
AHMADU BELLO UNIVEKSITY
ZARIA

SEPTEMBER 1984

APPROVAL SHEET

This research project has been read and approved by the undersigned as meeting the requirement for the award of the Masters degree in Education (M.Ed.) of the Department of Education, Ahmadu Bello University, Zaria,

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ACKNOWLEDGEMENT

I thank God my heavenly father for the abundant grace He supplied to me, without which, this work wouldn't have come to a fruition.

My sincere appreciation goes to my supervisor Dr. L.H. Shirley for his meticulous attention, a wealth of pertinent comment, readiness and good counsel from the start to the end of this piece of work.

I ewe thanks also to the Acting Dean of Faculty of Education Dr. A.A. Sambo, for his indept suggestions in the early stage of this inquiry; to the postprimary teachers of Mathematics in the Zaria educational zone for their co-operation; to the students who, by making errors in the scripts of this inquiry, provided a material means for analysis and the subsequent remediation; to the typist Mr. C.S. Bakoshi for a job carefully done.

Finally, I warmly acknowledge with the same tenderness, the practical care, love and encouragement which steadily flowed from my wife to me all through the research. To have such an understanding wife is a blessing beyond measure.

1984 INEKWE O.I.

DEDICATION

This project is dedicated

unto the Lord Jesus Christ.

Next to Bet, Blessing and Victory.

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CHAPTER 1

INTRODUCTION

1.1 Introduction to the Problem

Now that Nigeria is directing her policy on secondary education towards equiping students to live effectively in a modern age of science and technology, many researchers have been looking into the causes of poor performance in mathematics among our students. Reasons ranging from lack of adequate teaching force through lack of instrumental aids to lack of funds have been advanced. These could be generalized under the term: the "nation's inabilities". On the part of the students themselves, reasons range from lack of interest through poor attitude to group or peers' influence. These could also be termed "student's inabilities". But, not many investigations have been carried out on the major causes of these "student's inabilities" to supplement the national causes for their poor performance.

Moderators of the West African Examination Council (WAEC) have commented generally on the steady descent of students performance ever the recent past. They have also come out with what they feel are the causes of student's inabilities. The striking thing is that since

1977 till 1984 the yearly comments almost seem to be duplications of each other. For example, the report on May/June WASC examination (1977) read thus:

> "The standard of the question paper compared well with that of the previous year. The level of candidate's performance continues to fall. Fewer candidates scored good marks. The number of candidates scoring no marks in the whole paper was a definite indication that the subject (mathematics) is not being taught properly in many schools... Many candidates found it difficult to simplify easy arithmetical and algebraic expressions correctly..."

For the May/June 1980 reports see (Section 2.4).

In 1981 May/June examination the reports read thus:

> The standard of the paper compared well with that of the previous year. The syllabus was generally well covered but the performance of the candidates were ill-prepared and showed lack of understanding of the basic principles required by the syllabus. They exhibited total ignorance of some of the topics set e.g. log, surds, loci and also showed a lot of carelessness....
>
> This poorer performance is not due to the variation in the pattern of question setting, but, as could be seen from candidate's answers, it is due to:
>
> (i) Lack of understanding of basic principles.
>
> (ii) Poor manipulation of arithmetic and algebraic calculations.
>
> (iii) Unpreparedness for the paper.

(iv) Lack of self confidence and the preparedness of many candidates to cheat.

Some candidates merely copied the questions and did nothing further. This is a clear indication of complete unpreparedness for the paper. Some candidates cancelled their own correct workings to some questions to copy other wrong workings to the same questions from other candidates.

Even though this study is mainly concerned with "students inabilities", some problems of teacher deployment relate rather directly to the cause of the difficulties, A pilot study carried out by the researcher revealed some of these other important but often unmentioned causes of students' inabilities which are also problems to mathematics learning in Nigeria. One of these is the constant turnover of teachers due to social and economic reasons. As a result of rapid changes of personnel, and students sometime being taught by inexperienced teachers, (some filling in gaps each year e.g. NYSC(s)), the students do not receive continuity of teaching. Each new teacher has to adjust to the teaching pattern of the school and then assess each child's competence in mathematics. No sooner are these tasks accomplished than the teacher is off to another school. This seems to be the situation because teaching posts in Nigeria (unlike some other countries, e.g. U.K.) are not difficult to obtain. If the were, that would have fostered stability in schools and with it, a more

positive approach to the teaching of mathematics to allow teachers to determine the requirements of their students in their classes.

Another underated problem is that many principals of secondary schools think that a more qualified teacher must be attached to the senior classes and the less qualified teachers to the junior classes. This natural phenomenon of matching senior with senior and junior with junior is seriously and adversely affecting such continuous and accumulate subjects like mathematics. For example a teacher who passed out from any of the Teacher Training Colleges and finally finds himself to teach in a secondary school will naturally be told to take up the lower classes, the implication of which is that basics or fundamentals are sacrificed. This shaky foundation or lack of proper grounding makes itself manifest in the senior classes thus disarming their senior teachers who are left in the dilemma of facing the syllabus before them or going back to fill up the missing blocks. The researcher is of the opinion that teachers with more educational methodology should handle especially the first and second years and the next priority be given to the final years. Many mathematics enthusiasts testify that their interest to the subject started either in their elementary school days or their junior secondary school days due to how they were handled.

So if professionally qualified teachers handle the first and second years, then like a well warmed-up motor engine which can continue to run well with gradual releasing of the throttle, these children can build up to a considerable degree of independence even if little expertise is made available to them in their senior forms. In support of the above fact, the West African Examination Council reported in 1980 that "good teaching and good preparation of candidates for the paper (mathematics) are the only solutions to the poor performance of candidates in the paper, as candidates from a few good and well staffed schools continue to do very well in the paper.

In most other schools, only a few number of candidates, generally the specially talented ones, work up on their own to reach the credit pass level. The attention of all educational authorities concerned should now be directed to the ways and means of improving the standard of mathematics teaching and learning in schools."

Adetula (1979) in his study on modes of presentation and sequencing of mathematical ideas to secondary students, came out that one of the factors responsible for the student's fear is the dull, vague, barren and uninspiring presentation. His study was based on

Bruner's (1964) theory that at any level, a student can learn any concept in some intellectually honest manner if he works with concrete or enactive objects, iconic or pictorial mode and finally with abstract or symbolic modes. Adetula therefore observed that presentation and sequencing of ideas can make students find them accessible.

Adetula's work is relevant to this study because it borders on the causes of "students' inabilities" which is the concern of this research. Yet the researcher is of the opinion that not every mathematical idea can be presented according to Bruner's modes of presentation. For Piaget, Copeland (1974) the symbolic mode of presentation is only understandable by a child at the formal operations stage. According to Piaget, the formal operations stage i.e. 12 years and above is referred to as the hypothetic - deductive operational stage when the child can reason or hypothesize with symbols or ideas without representations of the physical environment as his basis; and has attained new mental structures and can construct new operations.

In line with the above belief, this study on error analysis in Literal Equations is concerned with the students in the formal operation's stage since the topic involves mostly symbolic ideas.

Also, besides, enumerating factors that generally affect mathematics students in Nigeria, it is high time researchers in this field started looking into the existing secondary mathematics content and isolating topics which are basically and fundamentally of great importance. Hence this study focuses on "Literal Equations" as one of those indispensible topics, having the widest application in the world of science and technology.

1.2 Statement of the Problem

This research is designed to investigate and analyse the errors students often make in the solution processes of literal equations.

A pilot study carried out among the teachers of mathematics in eight representative institutions in Zaria Educational zone clearly reveals that secondary students make a lot of errors in literal equations. In addition, a pilot test carried out at a special school (Demonstration Secondary School, Ahmadu Bello University) confirms the same fact.

The researcher has therefore come out with the following conjectural or hypothetical questions: Are the causes of students' errors in literal equation solving due to:

(a) lack of proper understanding or comprehension

of the actual meaning of literal equations;

(b) lack of understanding of basic mathematical operations involving square roots, fractions and transposition of letters;

(c) inability to collect like terms and to factorize;

(d) the confusing abstraction of using letters and symbols more than number?

1.3 Justification of the Study

Numerous justifications exist for an analysis of the problems of learning how to handle literal equations due to the need for literal equations in physics, engineering, advanced mathematics, business, nursing, medicine, even agriculture and many areas.

1.3.1 W.A.E.C. Reports on Literal Equation

Besides this, past examinations and observations from the researcher's own teaching experience reveal that students fumble a lot in the manipulation of literal equations. This is especially true when it involves "change of subject" or expressing a letter or symbol in terms of others in a given equation. Many seem to be more confused when such equations involve more than one, or two operations, for example, in May/June WASC examination of 1983, question 6(b) read thus "Given that $T = 2\pi\sqrt{\frac{l^2+k^2}{2gl}}$, (i) make k the subject of the formula. (ii) find the value of k when $l = 1$ and $T^2 = \frac{4\pi^2}{g}$.

The report analysis of this question stressed that even though the question attracted many students, yet numerical errors prevented many from arriving at the correct answer. Also many students did not know that it was necessary to square both sides of the equation to remove the square root sign. But in the November/ December WASC examination of 1983, question 2(a) read thus: "If T-mg = mf and pg-T = pf, find f in terms of m, g and p only." The report analysis of this question went thus:

> Many candidates did badly in this part because they could not discern that T was the variable that should be eliminated if f has to be expressed in terms of m, g and p.

Furthermore, in a conference coordination and marking exercise held in July this year at Ahmadu Bello University Zaria, May/June WASC examination, in which the researcher was a participant, the chief coordinator made a challenging statement to the participating teachers over question 3(a) of May/June West African School Certificate (WASC) examination 1984, which reads "make t the subject of the relation $x = \frac{y(1-t^2)}{1+t^2}$. He exclaimed "What are you people doing in this topic? West African Examination Council (WAEC) is always giving report over this topic and because of no improvement in the students performance over literal equations, it is being set every year. How am I sure that if this question is set for you some of you will not get it wrong!"

It is therefore clear from these reports that this mathematical fibre for the science subjects, literal equations, has been a problem to students over the years. The apparent inadequacy among students in manipulating arithmetical and algebraic expressions correctly, borders much on literal equations. Lack of this mathematical ability or skill is one of the inadequacies of the science students.

The National Policy on Education stipulates one of the nations goals for the secondary level as "to equip students to live effectively in a modern age of science and technology". How can this goal be achieved if science students are lacking the skill of solving literal equations? It is unquestionable that we are living in a century when computers and calculators are already doing much of the mechamical mathematical work. While we cannot abandon the mechanical processes, it is still the man that does the thinking of the processes involved while the calculator carries out the operations to find the numerical answer. Hence the need to dwell more, now, on the basic mathematical processes, the how and why of student's thinking processes than on the answers. Lack of knowledge of proper process in the solution of a problem leads to inability to interprete and blind acceptance of the numerical answer got by the calculator. Even when the calculator is faulty or giving a wrong

answer, the student is ignorant too, not bulging and not showing any eye brow.

1.3.2 Indispensible Nature of Literal Equations

Grove (1960) said that equation is the basic concept of mathematics and particularly algebra. There are several practical values of algebra in everyday life. In a similar manner, literal equations are applied in numerous occupations and in daily living so the student can see the purpose of his study of algebra now and its increasing value to him in the future. Specific examples are shown below to buttress this point.

Grove went further to say "In our modern scientific world, formulas are used by many professions. The deep sea driver's helmet and clothing are specially designed to protect him under water and the formula for finding water pressure $P = 0.433h$ is therefore very important in the making of his clothing. $E = mc^2$ is the famous formula developed by Albert Einstein. Today scientists all over the world use this formula in their study of atomic energy". Literal equations cannot be avoided in physics. For example in the simple laws of motion, $S = ut + \frac{1}{2}ft^2$ where S stands for distance, it might be required that the acceleration 'f' should be determined while other values are given. A student who cannot express f in terms of u, t and s will either get stuck

or fail the problem. In the section of physics on optics such equations like $\frac{1}{u}+\frac{1}{v} = \frac{1}{f}$ where u and v are object and image distances from the lens and f the focal length respectively can be used to determine an unknown focal length of a given lens by expressing f in terms of u and v and substituting their values obtained by placing an object in front of the lens. Many more examples exist in physics. All these formulas are literal equations.

In Advanced Mathematics exist such equations like the cosine formula $a^2=b^2+c^2-2bcCosA$ where a, b, c are sides of triangle ABC and A the angle included between sides b and c. From this literal equation (formula) any side or angle can be obtained when other letters are known. This will still require some mathematical/ logical transpositions to express the required letter in terms of the given ones. A basic and thorough knowledge is still required for such advanced mathematics topics like functions and their inverses, formula for sequences and series; statistical formulas for arithmetic mean, median, standard deviation, correlation coefficient and regression lines. Even in ordinary level mathematics, literal equation is a basic topic for easier assimilation of such topics as mensuration, variation, and simple graph.

A further look into the fields of Business and Insurance reveal such literal equations, e.g. to determine discount (d = rl where r stands for the rate and L, the list price), simple interest (I = PRT), compound interest; Actual Interest rate on installment buying is denoted by $MP.R\frac{S}{12}$ where MP is the monthly payment, I is the carrying charge or the difference between cash price and installment price; R is the actual interest rate and S, total number of months for which each payment is owed. A businessman who knows his rate of interest as well as his simple interest for a certain time, may not even be able to say correctly the principal amount he put in, if he can't express P (principal) in terms of IRT i.e. $P = \frac{I}{RT}$.

Among the numerous mathematical but medical formulas is that for determining the amount of iron a body requires in terms of its weight and the haemoglobin concentration; Fe = 4.5WD + 500mg where Fe = Iron, W = weight in kg, D = haemoglobin concentration. Suppose D is required when Fe, and W are known, it is of fundamental importance to state $D = \frac{Fe - 500}{4.5W}$ before substitution is made to evaluate D. The simple conversion from centigrade to Fahrenheit scales of temperature used by nurses requires a literal equation: $F = \frac{9c}{5} + 32$ from $C = \frac{5(F-32)}{9}$. If instead of $C = \frac{5(F-32)}{9}$, a nurse works with $C = \frac{9(F-32)}{5}$

this will lead to a wrong estimation of the patients temperature which might lead to under dose or overdose prescription which might in turn lead to the deterioration of the patient's ill-health or even death.

In cartography, a mapmaker uses many complicated equations and formulas. An example is $HD = \frac{688.R.F.V.I}{A}$ where HD = Horizontal distance between the contours on any map. It is important to be able to express F in terms of R, V, I, HD and A as $F = \frac{HD.A}{688.R.V.I}$

Numerous engineering equations are literal. In finding the velocity of the satelite, an electrical engineer uses $V = R\sqrt{\frac{g}{R+r}}$ where R = radius of the earth

g = gravitational constant

r = distance from the earth's surface.

Therefore he can determine the value of r when V, R and g are known. But to find the distance (r) from the transmitter to the reciever in relation to the period t i.e. the time it takes to go round or come back to its starting point, he uses

$$r = \left[\sqrt[3]{\frac{R^2 tg}{4\pi^2}} - R\right]$$ To make R the subject of this literal equation is a problem to many secondary students.

Furthermore, to find the classical buckling load (P_c) of a steel column with one end fixed, a civil

engineer uses $P_c = \frac{\pi^2 EI}{4L^2}$ where L = the length of the steel element.

I = second moment of area

E = youngs modulus.

If on the other hand he wants to determine the length of the steel column probably after it had been covered by a river, he will simply express L in terms of P_c, E and I and by correct substitution L can be determined. A more complicated literal equation (formula)

$$Q = \frac{Qa}{\sqrt{1 + \frac{A(h/t)}{1.3QaS}}}$$ can be used to determine the steady state slope (S) for the actual discharge of water Qa from the general discharge of water Q drawn from a given stream of cross sectional area A with height h and t time of discharge.

It is of basic importance that students should be able to simplify or evaluate such equations with ease by following logically mathematical processes instead of using poor algorithm. Especially for science students, it is clear and obviours that this topic is indispensible.

1.3.3 Attention Paid to Literal Equations

The numerous mistakes and errors students make in changing the subject of a literal equation or in its evaluation show that this topic is important, not only as a problematic area of difficulty, but as an area that

needs investigation as to the nature of such errors, their causes and how to correct the situation. A study of this nature however cannot come out without recommending ways of remedying these difficulties.

One of such areas of difficulty in literal equation solving process has been stated by Lewis (1978, p.300). "... our greatest difficulty when solving Literal Equations is the problem of determining the coefficient of the variable for which we are solving". It's startling that some secondary students (seniors) can't defferentiate between an expression and an equation or between a variable and coefficient.

Of concern is the fact that despite the wide applicability of this mathematics topic, especially now that the country is scientifically hungry and technologically thirsty, the curriculum planners of the present new mathematics syllabus pampered it. It is a great negligence (or weakness) as far as this research is concerned and as far as our so called prospective scientists (who yet hate mathematics) are concerned. The present syllabus treats it on the assumption that 'equation' as a general topic is enough, as if an equation of one variable and or two variables (for graphs) are all that should be encountered in either advanced mathematics or science.

Meanwhile, this weakness is opposed by the National Council of Teachers of Mathematics (1980) who in their executive summary of the Prism Project gave the strongest support of 93% for including such algebraic topics as evaluation of formulas, 93% for signed numbers, 90% for linear equations and 83% for writing equations to solve word problems. It is clear therefore from these that literal equations is of greatest priority.

Treatment of the topic by authors of mathematics texts used in Nigeria is scanty. Specific example Ordinary Level Mathematic by Clark - 4 pages, Macmillan Secondary Mathematics Book 3, a bit elaborate yet assumed certain factors; - 10 pages, New General Mathematics Book 3 - 5 pages.

So far, no earlier treatment of this content has been found in the history of Mathematics Education researches in Nigeria. This scarcity of studies of student difficulty in this area also justifies the need for this study.

In order to improve student performance in our examinations we need to study specific areas of difficulty and find out ways of overcoming them.

1.3.4 Further Benefits

This study also should help teachers, students and curriculum planners or curriculists as well as textbook

writers:

(i) The study should make recommendations on how best to teach the topic in particular.

(ii) Textbook writers can get ideas from the emphasis and treatment of the topic.

(iii) It can lead to student's improved performance on tests, involving arithmetic and algebraic expression and hence reduce the anxiety level which bemoans a lot of students in this area.

The use of interviews of both teachers and students is of uttermost importance and can reveal much to the teachers concerning the variable nature of students individual thought processes, learning styles and individual differences. The result of the interviews will be a great asset to the classroom teachers to improve their own instructions if several of the students in class are making the same type of errors. It can also serve as a guide for effective individual remediation. More on the need for interviewing will be considered on the next chapter of literature review.

1.4 Scope and Delimitation of the Study

This study is limited to the post primary schools in Zaria educational zone. Concentration was on year 4 science students randomly sampled in each school chosen.

Year 4 students are chosen because the pilot study carried out showed that the topic under investigation is treated by many schools in different forms but in most schools it is in the form 4 syllabus. Also attention was paid only to those schools that have treated literal equations. About 120 students were sampled and tested with the test items constructed by the researcher, i.e. about 40 students were sampled in each of the 3 schools. Further details on the design and administration of the test are given in chapter 3.

Since this is a fact finding research on errors made by students on a mathematical topic and not a study on attitude, the scope is restricted to the skills involved and the errors in applying these skills and not opinions or other affective measures. The study is not measuring achievement or comparing boys' with girls' performances or even doing item analysis rather it is only looking for errors and their causes.

In mathematical study of this type, typing errors cannot be completely avoided. But effort is being made to reduce them to the barest minimum.

1.5 Definition of Terms

To avoid ambiguity, the following terms which are used in the study are here defined:

A Literal Equation is one in which some or all of the known quantities as well as the unknown are represented by letters.

A Formula - is a literal equation that expresses a definite principle or relationship.

Subscripts - are small numerals written following and slightly below a letter to indicate corresponding values in two or more different situations.

Solving a Literal Equation - is the process of expressing each letter in terms of the other quantities in the equation.

Evaluating a Literal Equation is the process of substituting known values and solving for the unknown.

Error - is not simply a mistake, but a wrong process carried out by one in problem solving which leads to a wrong solution, after one has been taught the right processes.

Mistake - an oversight that may lead to an error in problem solving, which is not due to one's lack of knowledge of the correct algorithm. It may be due to one's physical condition.

W.A.E.C. - West African Examination Council.

WASC - West African School Certificate

From these definitions it can be deduced that while all formulas are literal equations, all literal equations are not formulas. This research is not concentrating on

types of formulas, rather on literal equations which broadly incorporates formulas.

Many students are poor both in the solving and evaluation processes, hence many fail problems dealing with 'change of subject'. For example problems on variation, substitution with directed numbers are often a problem to many. Hence seeing the need for this crucial topic Grove (1960, p.94) emphasized that "since the prices we pay, the money we earn, the areas of things around us, the machines and electricity we use, and even the air we breath, all change or operate in accordance with formulas, it is important that we learn to handle them properly".

From the ongoing 'justifications, one can thought-fully say that if a business man makes a mistake in manipulating his business formulas, he can lose a lot of money to his customers, if an engineer makes a mistake in his bridge construction formulas, few lives can be lost, and if a medical nurse or doctor makes a mistake in his drug prescribing formulas, more lives can be lost but if a teacher makes mistake in his teaching of literal equations most lives will be lost. Hence the need to have a closer investigation of student's problems in this topic and the causes of such difficulties.

CHAPTER 2

SURVEY AND DISCUSSION OF LITERATURE

2.1 General Causes and Effects of Error in Mathematical Computation

The natural tendency is to label errors straight away as mistakes. Every mistake can be regarded as an error but every error is not due to mistake. If a student in carrying out the operation 458-253 got 200, but in another situation in time or place, he arrived at the correct solution 205, then it should be regarded that initially he made a mistake in subtracting 3 from 8 which led to the erroneous result 200. Causes of this mistake may be traced to some physical factors. For example, perhaps the board was very far from the student, or he has bad sight of which his eye glasses made him feel that the 8 is 3; or perhaps he was tired after the boring physical excercise outside, just before the being given problem; or he was feeling sleepy; or he was in a hurry; or many other possibilities. Each of these is equally likely to be a cause for the mistake that led to the error. But when he was well relaxed, he succeeded with the problem. He got the problem because he knew the correct operation. If this child makes this error in many other situations, the error is no

more due to mistake, but rather to other causes. Then one might think the cause then should be due to ignorance. But if he had been thoroughly taught how to carry out such operations, ignorance is no more a genuine cause of the error. Yet other causes need be sought or investigated.

Gershard Ashlock (1972) identified four error categories or "failure strategies" as:

(i) Wrong operation: whereby a student attempts to respond by performing an operation other than the one that is required to solve the problem.

(ii) Obvious computational Error: whereby he applies the correct operation but bases it on recalling wrong basic number facts.

(iii) Defective Algorithm: whereby he applies the correct operation but makes errors other than number fact errors in carrying through the necessary steps.

(iv) Random Response: wherein his response shows no discernable relationship to the given problem.

Therefore errors in computation are not necessarily due to careless mistakes or not knowing how to proceed (i.e. by ignorance). However some students hold tenaciously to faulty procedures even during remedial

instruction and when such defective algorithms become part and parcel of theirs, it becomes hard to correct like habit formation. So while mistakes are situational and shortlived, errors are much deeper and more long-lived. If errors are due to mistakes and ignorance alone, then the majority of West African Examination Council (WAEC) repeat candidates would have been crossing the line, but the reverse seems to be the case, and even where they cross the line, it is only marginal.

Drawing from the above considerations it is then of great relevance to review what are likely to be the general causes of errors in literal equations.

2.1.1 Symbolic Inhibition

Skemp (1971) has wisely and logically given a lot of reasons and benefits for using symbols. For example they save time and space when compared to worded problems. Symbols are mathematical shorthands and without them mathematics learning would be very cumbersome to grapple with. In the same line of thought, Polya (1973) suggested that students however, should acquire fairly early experience of problems 'in letters', because the latter have a great advantage over purely numerical problems. Secondly if the problem is given in letters' its result is accessible to several tests to which a problem purely 'in numbers' is not succeptible to show,

Without controverting Skemp's views Buxton (1980) feels that the symbols used in mathematics can themselves be great inhibitors to mathematics learning, due to their nature, together with their notations. Buxton's opinion arose from what he describes as a "marked distaste for mathematics among the adult population" that ranges from 80-90%. He views this situation and ironically feels that over the years, we have developed special skills in preventing people from learning mathematics.

Since literal equations comprises a lot of alphabetic and other symbols, it is necessary to ask at this juncture; what is the function of alphabetic, Greek and other symbols in forming barriers to understanding this topic? There seem to be two possible levels, one cognitive and one affective. In the cognitive level, for instance, if a student is not familiar with such symbols as θ, α, β, γ and thoroughly accustomed to their use in mathematics, there seems to be no schema to which to attach the problem in order to make a start. Without such a connection, no progress is made. At the affective level, it appears that unfamiliar symbols or letters are very likely to induce anxiety because both mystery and fear seem to be attached to them. Once this anxiety is induced it also impedes progress seriously. This is evidenced even in our classroom teaching where for example the application of

θ to represent any angle in the teaching of say geometry hinders the understanding of some, who might be wondering what the symbol θ really means. Some are so preoccupied with this thought that they fail to grasp what else is happening in the lesson. So children can record fear, not simply lack of understanding, when they first encounter the symbol 'x' in algebra. It is easier to state that x equals that and that, and produce evidence of it, than to say why it should be so.

Buxton (1980) also asked a group of 15 students, which is the strangest or most unusual letter of the alphabet. Seven of them said "q". It is strange but not unfamiliar. Against this α and β are unfamiliar but not strange whereas ξ is both. There seems therefore to exist a distinction between symbols which are merely unfamiliar and those that are strange, and the latter seem to be the strongest inhibitors to understanding. This same idea according to him is true of operation signs. Of the four main processes, $\div$ (division sign) looks strange but not unfamiliar and $\surd$ is both strange and unfamiliar and almost forbidden in every respect to some students. It is pertinent to consider Buxton's exclamation on page 19 that "we are in the realm of feeling, not of rationality and logic, but it is there that we need to search for why people cannot learn mathematics".

This study is investigating in general what the causes of error might be in the solution processes of literal equations. But from the above discussion, an investigation into the affective effects of letters and symbols in the solution processes of literal equation is still a recommendable field of study.

In addition, an interview focused on how children view the equal sign, carried out by Merlyn et al (1980) revealed that of the mathematical symbols which children encounter early in their learning of mathematics, e.g. +, = and such symbols, = has several meaning to adults. One is an intuitive notion of 'sameness' which arises from experience with equivalent sets of objects. The second notion of equality though more sophisticated is "an equivalence relation" which arises as a result of teaching. But the first notion is what we would like children also to exhibit. Merlyn and his group discovered that some children viewed $2 + 4 = \square$ as not being the same meaning as $\square = 2 + 4$; as if addition sentences must be of a particular order to be considered true by children. They also observed in the children's behaviour an extreme rigidity about written sentences, an insistence that statements be written in a particular form, and a tendency to perform actions (e.g. add) rather than reflect, make judgements and infer meaning. Moreover they concluded by saying on page 15,"we have

some evidence to suggest that children do not change in their thinking about equality as they get older".

The above mentioned lack of reflection, making judgments and infering meaning to a given problem is a great cause of error in mathematics and literal equations in particular. For instance, after an exciting lesson "multiplication of algebraic symbols" by a student teacher, in a form one class of a certain secondary school, the researcher asked the students a question based on a solution of their evaluative tests whose solution is 12abc. Are the following expressions the same viz. 12abc, 12bca, 12cba, 12bac, a(12)bc, abc(12)..? Many of them looked thoughtfully at the board but shook their heads in disagreement. About 2 of them raised their hands inagreement but when they were asked to say why the expressions are the same they declined and joined the majority because they could not say why. So even the position of letters or symbols affect the students thought processes in algebraic problems. Hence the need to equip them earlier with the knowledge of commutative, associative and distributive principles of algebra.

In another encounter, the researcher asked a remedial class of repeating WAEC candidates what is the coefficient of b in the following expressions 2ab, 2ba. In each case their response was 2. None had the boldness

to say 2a because they felt the coefficient of any variable must be a constant. All these practical experiences with student also reveal some causes of the problems they encounter in solution processes of literal equation.

The idea attributed to Merlyn et al, that children do not change in their thinking about equality as they get older, is true not only for equality signs but also in the previous modes of instruction they acquired earlier in their life and accepted as unchangeable. Unbalanced or immature modes of instruction can cause habitual errors in the learner's approach to problem solving. Such errors cause a great obstacle to remediation.

2.1.2 Teachers Attitude to Errors

It is not a balanced discussion to heap all the causative factors of errors, on the learners alone. The teachers and textbook writers have a great share of the causes. Most of what is taught in the schools comes directly from the classroom teachers or the textbooks and not as a reaction to pupils perceptions. Meyerson (1976) complained that as teachers we often glide through a lesson with satisfaction as long as our pupils are feeding back what we want them to learn. When they make an error, we simply correct it and forge ahead. Very rarely do we capitalise on an error as an incident of

high potential. Of course we don't often deem it important to permit students to make errors as they learn. Consciously or unconsciously we set up a situation in which students are so fearful of making errors that they never get started at all. Due to this fear of failure many students fear to try examples. The idea expressed here is not that this is an exhaustive reason why many students don't try examples. However, instead of encouraging trial and error and experimentation many teachers often frown too much over mistakes as if the students are robots or computers.

There are yet some reasons for this teacher attitude according to Henry Wallman (seminar participant as reported by Cardwell (1961). He said that many secondary school teachers are themselves quite weak in the use of mathematical imagination. They and their teachers before them, and their teachers' teachers have reached their positions, as a result of a competitive system of examinations in which the whole emphasies was placed on correctness, reliability and orderliness. These qualities are so important to them that they will find it difficult to accept the assertion that mathematical imagination is just as important as accuracy for engineering and science students.

Learning however is restructuring, and its aim is to have a free creative response to situations which

bring new experience. But when this very freedom, which is essential for the exploration of new ideas, is made frightening to some people, it negates the teacher's role of being the student's security, and that of giving him confidence to learn freely.

2.1.3 The Teacher's Instructional Errors

In addition to the teacher's negative attitude to students' errors are his own teaching errors. Meyerson (1976) discussed extensively on this, but only one aspect will be discussed here. Many a time we as teachers of mathematics have told our students that "whatever we do to the left hand side (LHS) of an equation, we must also do to the right hand side (RHS).

The following justifications can be possible students' responses, as to why $(a+b)^2 = a^2+b^2$:

(i) By applying the above principle, changing the negative sign on the L.H.S. of the equation $(a-b)(a+b) = a^2-b^2$ to a positive sign, and changing the negative sign on the RHS to a positive sign gives $(a+b)(a+b) = a^2+b^2$

(ii) By induction: $(a+b)^1 = (a^1+b^1) \Rightarrow (a+b)^2 = a^2+b^2$.

(iii) By considering pythagoras theorem $a^2+b^2 = c^2$, their argument can be, if $c = a+b$ then $a^2+b^2 = (a+b)^2$.

(iv) By saying the following very fast. The sum of the squares equals the square of the sum. The attraction here anyway is that it sounds correct.

(v) Since $(ab)^2 = a^2b^2$ and since multiplication is just a quick form of addition then $(a+b)^2 = a^2+b^2$. This also agrees with the principle quoted above.

These faulty explanations have two things in common. One which is very apparent is that in each case the student has used some previously learned principle; while the other is that the student has the compelling notion that in mathematics, the future must look like the past. These in themselves are often neglected, causes of error in solving literal equation. Hence the need that both positive and negative examples should be well taught for better and more logical application of any principle introduced, instead of singing the principle and forging ahead with only positive examples. Hence there is also the need for occasional interviews by classroom teacher among his pupils to detect their modes of thinking and which principle they are applying, for better remediation.

In a study of another aspect of the problem Choat (1978) noted that understanding through conceptualization should proceed computation but that rigid approaches of

instruction could discourage opportunities for children to conceptualise. Such terms as "borrow", "turn upside down and multiply", "add a nought on the end", "cross multiply", "minus 4" (instead of "negative ", etc., used by some unskilled teachers are all sources of error. Many go through these motions without any idea of what they are doing. Hence the idea that, mechanical learning cannot be referred to as education. This is what Skemp (1976) referred to as Instrumental Understanding i.e. 'rules without reasons'. In the same view Fletcher (1976) seemed to reiterate that a number of items in modern syllabuses make sense if theyare used as a means to cultivate conceptual understanding, while they make nonsense if they were merely practised as skills. In the short term view however, it seems to matter little whether a child learnt something by rote or not, but in the long term view, it matters considerably because in the long term view, we are concerned with the learner's ability to adapt knowledge and to extend it, which is what Skemp referred to as "relational understanding" i.e. knowing both what to do and why.

Skemp's concepts of instrumental and relational understanding are relevant for emphasis here, but not the question of which should be aimed at during instruction. Rather, it is the opinion of the researcher here that both of them play important roles in the

learner's learning. According to Skemp, therefore instrumental understanding usually involves a multiplicity of rules rather than fewer principles of more general application. Instrumental understanding according to him is what many pupils and their teachers take as understanding when they are in possession of such a rule and ability to use it. Students whose objective is to understand instrumentally when taught by a teacher who wants them to understand relationally face fewer problems (short term), but to the teacher, it is frustrating. In such a class, pupils are not interested in the teacher's careful groundwork of explanation given in preparation for whatever is to be learnt next. All they want is some kind of rule for getting the answer. As soon as they get it, they latch on to it and ignore the rest. If peradventure the teacher asks them a question that does not quite fit the rule, of course, they get it wrong.

From practical experience, the researcher has seen students who detest long explanations and note-making during mathematics lessons, but sooner or later they will ask sir is that the answer or sir please give us the formula so that it saves our time. It is funny that this same set of students are those who get scared when they neet mathematics problem that uses some physics terminologies. They will complain that this is

not a physics class as if it is forbidden to talk physics in a mathematics class, and vice versa. For instance a class which has been taught "linear equations" instrumentally might not be able to relate the knowledge to solve literal equations. On the other hand, a more damaging effect is experienced when students whose goal is to understand relationally are taught by a teacher who is instrumentally oriented. Hardly can he satisfy the longing of the students and both parties can be frustrated, while many may be absenting themselves from the class to tie themselves to textbooks or the library instead.

One can therefore deduce that instrumental understanding cannot last while relational understanding lasts. It is in line to suggest therefore that instrumental teaching will lead to more error than the latter in solving mathematical problems. However, each type has its merits and demerits. According to Skemp advantages of instrumental teaching include:

(a) It is easier and quicker to understand in its own context and more reliable since less knowledge is involved. Some topics too are very difficult to be taught relationally, e.g. multiplying two negative numbers or dividing a number by a fraction.

(b) The rewards are much more apparent and immediate. For instance, in form 3 of our secondary school career, students are introduced to pure sciences like physics, chemistry etc. where they encounter some formulas that require pre-knowledge of literal equation solving. In order to satisfy the need in physics lessons, there is an immediate need to teach literal equations instrumentally especially in those schools that fix literal equations as a class 4 or 5 topic in their syllabus. Hence the recommendation that this fundamental topic should be taught as early as possible. Such earlier treatment will both enhance its relational applicability and save the half-baked understanding of the topic manifested in other science lessons.

Advantages of relational teaching according to Skemp include:

(a) It is more adaptable to new tasks.

(b) It is easier to remember though harder to learn.

(c) It can be effective as a goal in itself.

(d) Relational schemas are organic in quality.

Yet situational factors contribute to the difficulty of teaching mathematics relationally and cause many to teach it instrumentally. Factors outlined by Skemp include:

(a) The backwash effect of examinations. This is true in the Nigerian situation. In view of the importance attached to examinations for future employment, one can hardly blame pupils if success in these is one of their major aims. So the way students study or work is being influenced by the goal for which they are working. This is a natural phenomenon.

(b) Over-burdened syllabuses: Part of the trouble here is the high concentration of the information content of mathematics. A mathematical statement may condense into a single line what another subject syllabus may state in one or two paragraphs. This may be why most mathematics teachers are very fast especially when the West African School Certificate (WASC) is at the door, in order to beat the time. In the Nigerian context it is true but with the emergence of the new secondary school syllabuses specially prepared for the 6-3-3-4 tier system, the situation is likely to improve if they are followed strictly. Secondly, the six years instead of the former 5 is also an advantage.

(c) Difficulty of assessment of whether a person has understood relationally or instrumentally.

It is always difficult to make valid inference from the marks a student makes on paper, concerning his mental process by which he arrived at them. However, talking with students seems almost certainly the best way to find out but, with the great pupil-teacher ratio plagueing our schools it is very difficult to find the time. This is a national problem staring at the face of the recently propagated "continuous assessment programme" in our secondary schools.

(d) The great psychological difficulty for teachers to accommodating and restructuring their existing and long-standing mental schemas. This was the problem that, among others, adversely affected the introduction of "modern mathematics" in the country.

To conclude this discussion one comment made by Skemp is pertinent here:

> If people are taught instrumentally, then a traditional syllabus will probably benefit them more. They will at least acquire proficiency in a number of mathematical techniques which will be of use to them in other subjects (especially science subjects) and whose lack has recently been the subject of complaints by teachers of science, employers and others...

> In contrast learning relational mathematics consists of building up a conceptual structure (Schema) from which it's possessors can (in principle) produce an unlimited number of plans for getting from any starting point within his Schema to any finishing points" Skemp (1976 p. 25).

Having so far considered error sources on the part of the teachers and the taught, it is still in line to consider errors due to difficulty of the problem.

2.2 The Notion of Problem Difficulty

Certain mathematical operations and processes are regarded as more difficult than others. Some are said to be too difficult for a certain age group. Some students experience difficulties in certain problems while others do not. Many a time difficulty is cited as an explanation for high frequency of errors and at other times difficulty is regarded as itself calling for explanation. Thyne (1954) sees the concept of difficulty as especially liable to emerge in a discussion of error. When errors are made, there is a tendency to associate some difficulty giving rise to them or to suppose that such difficulty exists. To many students these days any mathematical problem they see is difficult even when they have not read it; while the slogan in vogue is 'maths is difficult'.

But according to Thyne, the word difficulty is an ambiguous concept which does not provide a single due

to the nature of an error. He therefore considered the concept in 3 relevant meanings.

1. A quality of the experience of the learner or performer, a psychical entity, something which pupils experience, find or meet with.

2. A task is said to be difficult when the performer, continues at it for some time without success. If after a student gives his whole attention to it and yet does not solve it, the task is said to be too difficult for him. So difficulty here is referred to, not as a quality of the performer's experience but the length of time taken to complete the task compared with that taken by other pupils.

But the argument here then is that some people may take quite a long time to complete a task and yet experience no strain or frustration or difficulty in the sense of the word while others may experience considerable difficulty and perspiration and yet finish the work or task in a very short term. Embedded in this argument are "difficulty of tasks" and "difficulty felt by pupil" which now emerge as distinct aspects of the meaning.

3. By using the phrase "Order of difficulty", which is established by regarding frequency of error as a measure of difficulty. We would be implying therefore, for instance, that equations with high error frequency are relatively difficult, which is not completely true since other factors can still account for such errors. So two separate entities exist here i.e. error-frequency on one hand and difficulty on the other.

   If, for instance, 96 pupils out of 100 erred in the solution of $x+y = a$, find y and we say therefore the question is relatively difficult, it is not quite appropriate because difficulty cannot be cited as a reason or cause of error, for we do not know anything about difficulty apart from the fact that many pupils made errors.

From the earlier definition of "error" in this study, difficulty cannot be taken as a cause of error because it is going to use a sample of students who have already treated the topic and who don't have any physical/mental (apparent) inhibition or inability. So difficulty has been taken account of; and yet there is need for other forms of investigating errors.

It is necessary therefore to conclude here that since the concept of difficulty can be seen as a quality of the learner's experience or a quality of the length of time taken to complete the task or a measure of error frequency, one should be careful and specific when using it.

2.3 The Need for Interviews

The knowledge that an answer is wrong is not the same as the knowledge of how the pupil arrived at it. Also the relations between the things one thinks about are not the same as the relations between his different aspects or modes of thinking. And again every problem does not provoke the same type and degree of thinking. Thought processes are unique with individuals. If two students fail a problem it is improper to attribute the failure to the same error, and even if they make the same error on the paper, it is improper to account for the error in the same way for both. (Hence Knight and Ford [Thyne (1954, p.21)] said that "treating all errors alike is analogous to treating all fevers alike".

Yet treating same errors alike will be the only gateway when we restrict our investigation to the raw, marked test papers of the students. On the other hand, it should be born in mind that in a given test some pupils can guess some answers correctly. How do we account for that? In a research of this nature therefore

it will be insufficient if attention is concentrated only on the written test. Hence it is of vital importance to employ interview techniques to investigate further causes of error, in order to be able to provide a description of the difference between modes of obtaining the answers. Without structured interviews any inferences about the child's thinking drawn from the written responses alone represent littlemore than guesswork on the part of the researcher. Hence Clement (1980) pointed out in same line that written responses can suggest to a teacher reasons for errors but structured interviews must be conducted with the child before consistent patterns of errors can be determined with any degree of certainty. Lankford (1974) noted that individual oral interviews with several people can make a teacher aware of the wide variations in computational strategies employed by children. This can help him then plan his teaching either to encourage and reinforce such variations.

Yet, it is rare in our secondary schools to see a classroom teacher interviewing his students face to face as to how they think along in problem solving. Much dependence is only on the paper. All the same the reasons outlined earlier as to why relational teaching of mathematics is rare are still valid for the above.

But the question is: do we indeed have the intention or desire for such?

2.4 Related Works on Error Investigation

A review of literature in this study revealed that a lot of work has been carried out in this area of study. Most of such works were done in America and Australia and focused mainly on the elementary schools and junior secondary schools.

Clement (1980) reported reviews by Hollander (1978) who summarised the methods and results of American investigations of the thought processes employed by children attempting verbal arithmethic problems. In one of the summaries 100 elementary school children were given one step or two step problems on verbal arithmetic. Before this, error categories were defined and pupils' errors analyzed in terms of these definitions. Later, structured interviews were conducted in order that pupils thought processes could be clarified.

Clement gave some of Hollander's summaries on the research of Rice (1920) who pointed out that the main difficulty of children in grades 3 - 8 in attempting verbal arithmetic problems was the choice of appropriate mathematical operations; John (1930) [Hollander (1978)] defined error categories as (Reasoning, Fundamentals, Reading and Miscellaneous), and came out that Reasoning

errors were most common among grade 4 to 6 followed by errors due to lack of fundamentals. Doty (1940) [Hollander (1978)] found that four main factors which affected the success of grade 4 to 6 pupils on verbal arithmetic problems were: (i) ability in computation; (ii) ability to gain mathematical implications from language forms; (iii) understanding of the various mathematical processes and (iv) effective procedures with problems. Among the summaries also was Corle's (1950) interview with 6th grade pupils after giving them the one and two step problems which went thus

1. "What is your answer?
2. How did you work the problem?
3. Why did you work it that way?
4. Do you think your answer is reasonable?
5. Why do you think so?
6. Do you think your answer is right?
7. Why do you think so?" Clement (1980 p. 6).

After this Corle classified his data into 3 major areas - insight, thought processes and number relationships, and computational skills. He concluded that good problem solvers tended to have superior insights than the rest, while both had similar employment of desirable computational procedures. This conclusion was challenged by Clement on the ground that his three major categories were vague.

Some American studies on pupils' written scripts were, according to Clement, carried out by Arthur (1950) Cox (1975) and Knifong and Hollan (1976). These, as earlier discussed, are still insufficient without structured interviews.

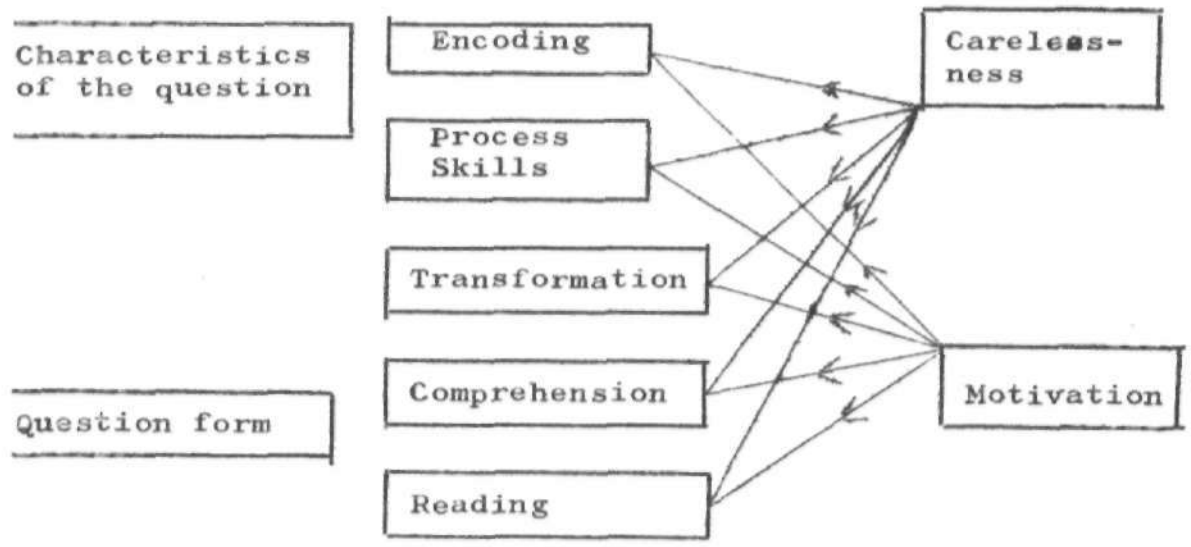
A large scale work of a similar kind was carried out in Melbourne Australia. Here Newman (1976) gave a 40 item mathematics test to 917 grade 6 pupils in 31 classes, in 19 schools. The scripts were quickly marked and the five lowest performing children in each of the classes were interviewed by Newman himself, or some mathematics Diploma students who were given a 4-hours training in interview techniques by Newman. The interviews were structured according to an "error analysis guideline" drawn by Newman. According to this guideline, the interviewer would ask a pupil to attempt the question once again, and would wait until the pupil unassisted, had attempted the question once again, and would then ask the pupil to respond to other questions that follow. This was an extensive study and his heirarchy for analysing errors, has been widely used by other researchers like Clement (1980) and Casey (1978), though in a modified form.

The design of this study is a modification of Newman's heirarchy of error causes. Newman's technique has been described as an invaluable one for gross

diagnostic purposes. Hence it has been used by many researchers as an effective tool.

According to Clement, the error classification devised by Newman (1977) has an advantage for the teacher in that it provides a clear framework for questioning the pupil and for analysing any errors. It can also be used with a wide range of mathematical problems and is based on a plausible model of how a child goes about solving a problem. The model postulates a sequence of steps, and failures at different stages are shown as different errors.

Newman's Heirarchy



The sequence of steps are Reading - Comprehension - Transformation - Process skills - Encoding.

Watson (1980) summarized the associated criterion for the above error causes with sub-categories under each main

category, which explains them further.

1. Reading ability - Can the pupil read the question?
   (i) Word recognition; (ii) Symbol recognition.
2. Comprehension - Can the pupil understand the question?
   (i) General understanding; (ii) Understanding of specific terms or symbols.
3. Transformation - Can the pupil select the mathematical processes required to obtain the solution?
4. Process skills - Can the pupil perform the mathematical operation necessary for the task?
   (i) Random response, (ii) Wrong operation, (iii) Faulty algorithm, (iv) Faulty computation, (v) No responses.
5. Encoding - Can the pupil write the answer in acceptable form? The three other blocks in the model do not reflect the sequence of steps, even though they are possible causes of error as well. They are.
6. Motivation - The pupil could have correctly solved the problem had he or she tried.
7. Carelessness - The pupil could do all the steps but made a careless error which is unlikely to be repeated i.e. mistake.

These two 6 and 7 according to the model can produce errors at any of the five stages outlined above.

8. Question form - The way the problem has been presented and its characteristics. (May be the question is vague or ambiguous).

Newman's Interview Procedure

1. Please read the question to me. If you don't know a word leave it out.
2. Tell me what the question is asking you to do.
3. Tell me how you are going to find the answer.
4. Show me what to do to get the answer. Tell me what you are doing as you work.
5. Now write down the answer to the question.

Each of these requests corresponds to the level of the heirarchy discussed.

In 1977 Clement trained some teachers on in-service education courses to use Newman's Error Analysis Guideline on a 36-item test known as Monash Assessment of Mathematical Performance (MAMP) containing 20 numeration items, 8 spatial items and 8 logical items for children aged between 9-14 years. A total of 542 pupils were later interviewed by the teachers. Unlike Newman's work every child was interviewed. His report showed that reading and reading comprehension difficulties cause fewer errors in higher grades.

In another study with the same purpose Casey's data are different from Newman's and Clement's. He constructed his own instrument for classifying errors, which was a modification of Newmans heirarchy. He used a many step problem tests, unlike Newman's one step type. He analyzed errors of 120 grade 7 pupils in one school. He trained 30 Diploma in Maths Education students to interview 4 children each and classify errors according to his guideline. His test was a 15-item type. Of importance is that Casey was the normal mathematics teacher of the pupils tested and had not specifically taught the children the topics covered by the questions on the test. The result of his findings differed from Corle's, that good and poor problem solvers' characteristics, bear close resemblance.

Watson (1980) worked with thirty children in grade 2 (their 3rd year at primary school) in Melbourne with a test of sixteen arithmetic problems, and over the next three weeks, fifteen of the children were interviewed. The Newman error analysis guide was used to detect their errors. The children's ages ranged between 6½ to 7½ years. He interviewed 10 of the less able group and 5 of the more able group. These were children whose mathematics abilities were known before because he had taught 25 of the sample and the other five came from other classes. His result shows that younger children

show a high percentage of errors at the stages of reading and comprehension than did the elder children studied by Newman and Clements.

In another study, Newman found with 124 sixth grade low-achievers, aged between 11 and 13 years, that 47% of errors occured in the first three stages of solving the problems (stages - Reading, Comprehension and transformation). Clements found similar proportions for 5th grade children and for low achievers in grade 7.

A more careful and systematic mode of interview for error investigation was carried out by Lankford (1972) on some computational strategies of 7th grade children, 176 in number, enrolled in schools in America. The interviews were conducted in a room where a single pupil and the interviewer could work undisturbed. The pupil was given a set of computational exercises and asked to do them as he usually did, but to say out loud his thinking as he computed. Lankford's exercises comprised 4 operations with whole numbers and fractions. The interview lasted for about 40 or 50 minutes each. The pupil was not hurried, he simply completed as many exercises as he could in the alloted class period or time. A verbatim record of each interview was made on cassette tape for future analysis.

From the above study Lankford discovered that computational strategies vary greatly among the 7th grade children. Also that good computers tended to follow conventional algorithms rather consistently; remembered what they had been taught to do and followed the orthodoxy of classroom teacher and textbook quite closely; and appeared to be less dependent on the vertical or horizontal arrangement of an exercise to provide a clue to the appropriate algorithm than did the poorer computers. The latter tended to rely more on mechanical aids to memory and often switched to something else that would produce an answer when they encountered difficulty with an improvised algorithm regardless of how remote it might be from the actual correct procedure. Worst of all was that they often supported themselves with faulty reasons, a pointer that they were often likely to be confused.

This confusion among poor problem solvers has been strongly emphasised by moderators of (W.A.E.C.) examination as indicated below:

> The standard of the paper compares well with that of the last year. Though the questions are simple, they require the understanding of basic principles which is conspicuously lacking in the answers of many candidates. Candidates performance is generally poor as the modal marks for most centres lie between 0 and 10 marks. Evidence of poor preparation for the paper is aboundantly manifested in the answers of many candidates, as some of the candidates did not go beyond writing down the questions. It is unnecessary and a waste of precious time(an indication for total confusion) for a candidate to write down a question before answering it in the examination hall. WAEC report (1980).

These works reviewed, bring to mind whether similar results on similar studies could be obtained among the secondary school children in the Nigerian context and setting. Though the works are foreign, yet for the benefit of comparative education it is necessary to compare them with Nigerian student's result. Such comparisons can reveal some variations and similarities which themselves can lead to another research that delves into the causes and nature of those variations and similarities.

2.5 Brief Summary of Survey

The general causes and effects of error in mathematical computation have been discussed followed by

specific causes like symbol inhibition. In the use of symbols, it has been found that some appear strange or unusual to students while some appear familiar. Such symbols as the equality θ, α, β, y and their effects were looked into. The failure of many teachers to capitalise on an error as an incident of high potential in their classroom instruction, was discussed as another specific cause of students' errors. Possible justifications that could be rendered by students due to teachers' instructional errors were outlined according to Meyerson (1976). Skemp's (1976) relational and instrumental modes of teaching and learning were discussed in detail, and brought to the Nigerian context. The implication being that many of our students seem to prefer instrumental understanding which means "rules without reasons", to relational understanding which is learning that is more adaptable to new task though harder to acquire. This preference of the former to the later also seemed to be another cause of erring among students, especially in literal equation solving. The concept of difficulty was next considered as having many meanings, and as such could not be regarded as a cause of error in this study, to avoid ambiguity in its use.

The need for interviews after written tests as a vital part of the study was also discussed. The modified form of structured interviews by Newman (1977), and Lankford (1972) were discussed and applied in the study. It is therefore of educational importance to ask: how do Nigerian students react to symbols? Which symbols are strange and which are familiar? What are teacher's attitudes with regard to error investigation and interview of their students from time to time? How do Skemp's instrumental and relational categories fit Nigerian student's learning? Finally what are the effects of these on literal equation solving? The need to find these led to this study.

CHAPTER 3

DESIGN AND METHODOLOGY

Subsequent to the reviews of literature on other studies carried out by other researchers in error investigation, this study follows a 3-phase design viz a pilot survey, a written test and an interview. It is strongly based on Newman's model for its data collection and analysis with necessary modifications. These modifications in the design will be clearly pointed out in this chapter with their subsequent reasons. Before this, the design and method of the pilot study will be discussed.

3.1 The Pilot Survey

The design of the pilot survey followed a three-phase process viz a pre-interview, questionnaire and test.

. . In the first phase the researcher carried out some oral interviews with teachers of mathematics in two secondary schools. This interview as an initial step sought their perception on literal equations with respect to its importance, difficulty levels, reasons for students error in the topic and their views of how such errors could be corrected or minimised. In order to have a more general

perception as viewed by teachers it became necessary to construct some questionnaires.

A 5-point scale questionnaire was administered to more than 60 teachers of mathematics in 7 representative secondary schools and one school of Basic Studies. This questionnaire (see design in appendix) sought the opinion of both degree, diploma and non-degree holders in mathematics including long experienced teachers as to whether students make a lot of errors in literal equations; the probable causes and forms of such errors, as well as the probable forms of the equation that are more likely for students to make errors.

To confirm some of the teachers' responses it was necessary to construct and administer a pilot test on the topic (literal equations) to a sample of students.

The pilot test was administered to a form three class of 40 students (boys and girls) in a special secondary school (Demonstration Secondary School, Ahmadu Bello University, Zaria). Before they were tested, it was confirmed through their teacher that they have just finished the topic and were about entering into a new topic of the syllabus. The test lasted for a lesson-period of 35 to 40 minutes and

comprised 11 items. Their scripts were marked and their performances analysed. Through the analysis of the entire results of the piloting, the researcher arrived at the stating of the hypothetical questions under (section 1.2). It also contributed much to a clearer construction of the actual test.

3.2 The Nature of the Study Sample

In the actual study, a total of 115 science students were randomly sampled out of 3 representative schools comprising 1 Boys, 1 Girls and 1 Mixed School. The level tested was form 4 while attention was focused mainly on the science students. The representative schools chosen are only those where form 4 science students have already covered the topic (literal equations). Form 4 science students were the focus-sample of the study because from the piloting, it was found that this topic is mostly treated in class 4 by many schools except the exceptional ones who are able to treat it in form 3. Science students were randomly sampled because the topic (literal equations) has most applications in science than arts subjects as could be seen under (section 1.3).

35 girls were randomly sampled in the Girls School, 40 boys were randomly sampled in the Boys school and 40 in the Mixed school (comprising 34 boys and 6 girls). In the girls school the science students were not up

to 40 in number, hence some arts students were also sampled making a total of 35 girls.

3.3 The Written Test

The actual test consisting of 12 items was constructed and administered to form 4 secondary school students majoring in sciences, to point out their ability and difficulties in solving literal equations. Each item, required the solution of a typical problem from the area of literal equations - changing the subject or substituting or evaluating. Each also had a number of steps of operation leading to the solution.

An ample time of not more than 1 hour was allowed for the test in each school. One hour was much because form 3 students in the pilot test of 11 items finished the solutions in less than 40 minutes. The implication here is that time factor was eliminated as an extraneous variable that could affect the students' performance.

Each candidate's script was quickly and carefully marked, while various errors were pinpointed and analysed. In the marking process, a big stroke was legibly ticked for the right methods and solutions at any juncture in each item. There was no stroke at all for wrong or partial solutions. All marking was visibly done in red ink.

This marking process was adopted in order not to discourage those who will be interviewed later when they see their scripts. Secondly it could master some more positive self-concept and boldness in the students during the interviews. The results of the test were collected and used for analysing student performance, detecting types of errors and reasons for such errors. Then to account for the errors beyond the guess-work of the researcher, it was necessary to carry out an oral interview based on their performances.

3.4 The Interview

The interview was structured and carried out with individual students to a maximum of 10 in each school, making a total of 30 students. Each encounter with a student during the interview lasted between 10 - 20 minutes as the case may be. Those who were interviewed were selected judging from their performance in the written exercise. More attention was paid to the items which a student either failed completely or got partially right. The purpose of the interview was to find out how they actually arrived at their erroneous solutions, individual's line of thought leading to such solutions, and the algorithms used. Yet mostly those who showed signs of knowing what they were doing were interviewed, as there was no need interviewing many who demonstrated complete ignorance

of what was set.

The interview was carried out in some secluded corners of the school where the interviee's attention could not be distracted by either noise or physical passing-by of fellow students. With the co-operation of their mathematics head teacher, each was called according to his or her turn, in any particular school.

3.4.1 The Model of the Interview

The interview model of this study is a modified form of Newman's heirarchy of error causes discussed under (section 2.3). Before arriving at the required model, the following modifications were very necessary. The need for modifications arose because unlike Newman and others whose tests dealt with many mathematics topics, this study dealt with only one - literal equations. Newman used a one-step verbal problem while this dealt with a many-step non verbal but written type.

He used a less mature sample from the primary school, while the researcher here dealt with a mere mature sample from the secondary school. For these reasons, certain categories in his model were not very necessary for this study. For example the fifth category in his hierarchy was Encoding which implies whether the children can write the answer in acceptable form. This category was not necessary in this study judging from its sample. Encoding can be a problem

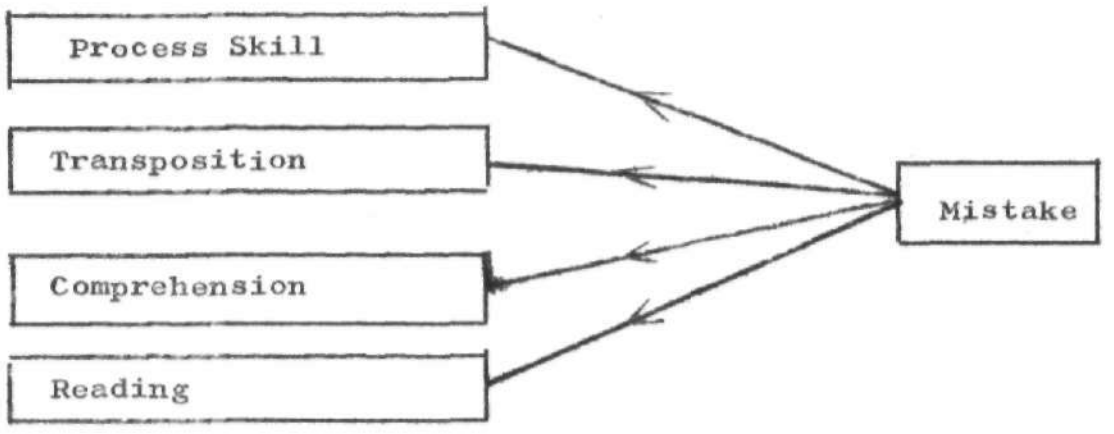
to a kid in the primary school but not a senior science adult in a secondary school.

This study did not also include Motivation which according to Newman's heirarchy implies whether the pupil could have correctly solved the problem if he or she had tried. It is left out here because lack of motivation has been shown by many studies as one of the causes of poor performance in mathematics in this country. It is not therefore regarded as testable source of error in this study.

Carelessness which was his 7th category was replaced with the word mistake (see definition). The term Transformation which was his 3rd category, was transformed to transposition, since the solution processes of literal equations require this ability greatly.

According to Newman's heirarchy, question form, which implied that the way a problem was set or presented could cause a student to err, was one of the less emphasized blocks. It is left out completely in this study because not only that the actual test was piloted upon, it was also reviewed by expert teachers of mathematics after construction before being administered.

The resulting model for this study after the subsequent eliminations, transformations and redefinitions, looks thus:



3.4.2 The Interview Techniques and Procedure

In order to make each level of the heirarchy correspond with the interviewers' requests, the following questions,were asked in the procedure: This still involved some few modifications again in Newman's interview procedure. For example, question 3 of his interview procedure was deleted since question 4 of his, covers it. Also question 5 of his procedure was left out since encoding was earlier proved as a non significant cause of error to be tested in this study. The questions in the procedure were:

1. Please read the question to my hearing
2. Can you really tell me what the question is asking you to do? Explain.
3. Show me what you did here to arrive at this answer. Say out what you are doing as you work.

In recording the interview, where a student could not read well the researcher wrote down the letter 'R'

against the question under consideration. Similarly if he shows lack of comprehension of what he was asked, C was written. Where a student cannot transpose letters or symbols correctly T was written against the question while P implied poor process skills. Where a student fails in any combination of the error categories, such combination was also stated against the question, for example, RC, RTP or CTP or TP as the case may be. But where a student was able to correct himself from his written solution, it was regarded as a mistake 'M' otherwise it is categorised as error. Later the responses were sorted out with respect to their frequency of occurence, and analysed.

The researcher tried to eliminate any psychological barrier as much as possible. For example each student recieved a bright and cheeful smile and was greeted first by the researcher before such a one was asked to sit.

The student's script was placed in such a manner that the big strokes for right solutions could be conspicuous to him or her before the interview commenced. Students were given enough time to think and answer without being hurried. It was only at the end of the student's responses that the researcher explained some of his or her errors and taught him or her briefly how such errors could be avoided, or remedied. Lastly the overall data gathered from the three-phase design

of the study was analysed with respect to the interview model and the hypothetical questions associated to each category. The results of the entire analysis of data were fully recorded and discussed in the proceeding chapter 4.

CHAPTER 4

PRESENTATION AND ANALYSIS OF DATA

This chapter presents a detailed account of the analysis of data collected during the study. It contains the result of the pilot study, the written test and structured interview. The presentation here includes both the major and minor causes of error in literal equation solving confirmed by the interview. There are some other findings of great educational importance included in the chapter which, themselves, are not the purpose of the study. Enclosed at the appendix is a copy of the test-item constructed with brief solutions of the problems, and the questionnaire used in the study. Included in the questionnaire is the average response for each item in terms of agreement. In other words, the average of the positive response of teachers are recorded against each item, instead of the original rating: SA for strongly agree, A for Agree, U for Undecided, D for Disagree, and SD for Strongly Disagree. At the end of this chapter there is a brief summary of the entire analysis.

4.1 Results of the Pilot Survey

This preliminary survey among the teachers of mathematics showed some striking results. It brought

to surface an existing inconsistency in the levels at which literal equation is being taught in various secondary schools. It was found that such a fundamental mathematical topic especially for the science students is being taught in form three in some schools, or in form four or five as the case may be. This is a great anomally to be corrected because it is a big destruction to basic. No builder would like to place the foundation stones at the middle or on top of the building, or else the building is baseless and cannot stand the test of time.

The pilot also revealed that there is a high degree of agreement among teachers that a lot of errors are being made by students in solving literal equations, even after they have been taught it. This finding was confirmed by the pilot test because not only was the test carried out in a specially designed school among students who have just finished treating the topic, yet there were numerous partial attempts, an indication that this is a problematic topic the cause of which is yet to be found out.

Through the same analysis, the common opinion of the teachers was that students are likely to make errors in literal equations especially those which involve square roots, fractions, negative number substitution or decimals. They also felt that the causes of these errors are due to the students (1) lack of proper understanding or comprehension of

the actual meaning of literal equations; (2) poor knowledge or ability in solving ordinary linear equations; (3) inability to collect like terms together; and (4) difficulty with factorisation. From the pre-interview, it was found that many expatriate teachers have the strong opinion that automatic promotion and poor admission processes are some of the causes of error in literal equation solving. It is true that a study of this nature dealing with causes of error in a specific area of mathematics cannot be divulged completely from causes of error in mathematics in general, yet automatic promotion and poor admission processes are more of a general cause of poor performance in mathematics than in literal equations specifically.

4.2 Result of the Written Test And The Structured Interviews

Before a detailed descussion of the major and minor error causes, it is of interest to note that from the written tests, the girls from the girls schools were fastest in their work, very detailed and yet performed worst, in comparison with the other schools. The science students from the mixed school were faster with many partially correct solutions, while the science students from the boys school were a bit slower than those mentioned above. They were more

methodical and systematic in their approaches, but still many had the relatively few of the items they attempted partially correct.

From the interview it was discovered that in a global sense, most of the students were bold in their responses. This can be attributed to the fact that many were eager and willing either to see their marked scripts and know their performance or to interact with the interviewer whom they might have esteemed as an expert from the University. It is interesting as well to note that the students from the girls school and the mixed school were bolder, more fluent and most willing to be interviewed than the more methodical boys from the boys school. The researcher attributed this to the fact that the boys from the boys school were looking much younger in age than those of the other schools.

Generally speaking however, a greater percentage of those interviewed showed they know what to do but lacked in the operative how. Knowing something mentally without an observable action by the learner does not fully express that learning has really taken place. Hence from the taxonomy of educational objectives by Bloom et al (1969), the phrase "to know" falls short for describing a specific lesson objective. For Gagné (1977), learning is seen as a change in human disposition

or capability which can be retained and which is not simply describable to maturation. Since learning exhibits itself as a change in observable behaviour that happens under certain observable conditions, the major inference that real learning has occured in the student's treatment of literal equations, can be drawn from what they can exhibit after they have been taught the topic.

From the analysis of both the written test and the interview, the most consistent errors were due to lack of:

(i) Process skills } Major
(ii) Transposition ability }
(iii) Comprehension } Minor
(iv) Reading }

4.3 Errors Due to Poor Process Skills

Analysis of the test papers shows that poor strategies employed by the students were the major causes of error. This employment of poor process skills is heaviest in two sub-areas viz dealings with square root and fractions. Before a letter or symbol is to be made the subject of an equation there are substages that cannot be circumvented or skipped. They are like overlying rocks that must be removed before some precious mineral can be dug up. Some of these underlying stages are pervious or permeable for

some students while some are completely impermeable for them. It is this poor encoding of lower order concepts (according to Bruner) or previous schema (according to Skemp) that is the major source of error in the solution processes of literal equations as exhibited by majority of the students.

4.3.1 The Square Root Dilemma

There are obvious and apparent, interesting but painful (to the teacher) manifestations that square roots continue to pose a strong barrier in student's problem solving. Radicals still evoke poor perception among many, not only among the pupils in the concrete operational stage but also among the formal stage students.

Question 7 has the most remarkable worst attempt in terms of clearing the square root. To make x the subject of the equation $W = 1 - V\bar{x}$ was not correctly solved by any member of the entire sample. The consistent error was that they thought that to get rid of the square root sign should be by squaring each term singly. Hence they proceeded thus: $W^2 = 1^2-(V\bar{x})^2$ or $W^2 = 1 - x$. None could transform the equation in this form $W - 1 = -V\bar{x}$ or $V\bar{x} = 1-W$ or $(V\bar{x})^2 = (1-W)^2$ Had many students been able to say or write that $W^2 = (1-V\bar{x})^2$, eventhough it is a long process, yet one would have been convinced that at least they have a right

perception of an equation as a balance with equal weights. Squaring each item individually in an equation involving radicals, was also rampant in many items viz questions 6, 9 and 10, In question 6, many in an effort to make t the subject from $U = \sqrt{1-t}$, wrote $U^2 = 1^2-t^2$ or $U^2 = 1-t^2$. Yet through the interview many showed the knowledge that when you square a square root you obtain only the number or expression inside the square root sign. Acquisition of knowledge without ability to apply can be attributed to lack of practice. In questions 9 and 10, many students employed their strategy of squaring singly to dodge making a letter or symbol in a square root, the subject of the equation before substitution for other letters is made. Usually in trying to shortcut or circumvent the normal processes leading to the right solution, they get stuck on the way.

Question 5 revealed another student inadequacy in dealing with surds. Some in trying to find Q from $\frac{P}{\sqrt{Q}} = \frac{\sqrt{R}}{S}$ proceeded thus $PS = \sqrt{Q}\sqrt{R}$ then $\sqrt{Q} = \frac{PS}{\sqrt{R}}$ $\therefore Q = \sqrt{\frac{PS}{\sqrt{R}}}$. So some think that to eliminate or get rid of the root sign for Q you have to transfer it to the other side of the equation. From these indications therefore, squaring a square root number and finding the square root of a squared number are students' sources of error. Teachers should therefore spend time on this sub area in dealing with literal equations.

4.3.2 Inconsistency in Clearing Fractions

It is surprising that majority of the students easily use the phrase "cross multiplication" and are very fast to apply it, yet in a sheepish and careless manner showing lack of the common sense behind it's usage or a proper understanding of the concept. Again it is interesting to discover that question 1 was mostly attempted by almost all, yet only two students (one from the boys school and the other from the mixed school) got the right process and right solution out of the entire sample. In attempting question 1 which says "make K the subject of the equation $T = \frac{mu^2}{K} - 5$", many wrote $TK = mu^2 - 5 \therefore K = \frac{mu^2-5}{T}$. During interview, they were asked why 5 was not affected by K in the process. In response some reasoned that "since 5 is an ordinary number and $\frac{mu^2}{K}$ comprises letters, it is impossible to subtract 5 from it; hence, 5 should be left as it is".

Although not many students responded specifically in this manner, many who were interviewed were not able to say why 5 should not be affected. Hence the above response in quotes, can be regarded as one of the thought processes of students which lead to error in the solution, processes of literal equations. That is perception of the incompatibility between ordinary numbers and letters in the alphabet might be a cause of

difficulties. Another noteworthy candidate is one who got question one correct by simply writing $T+5 = \frac{mu^2}{K}$ $\therefore K = \frac{mu^2}{T+5}$, yet when interviewed he couldn't explain what he did. This is related to the apparent fact that many have not grasped the real understanding of the concept "cross multiplication". They use it as a direct means of dealing with fractions instead of seeing it as a result of another fundamental operation which is multiplying every item in the equation by the least common multiple (LCM) of the denominators on either side of the equation. For example in item 1, multiplying both sides by K, the LCM, we have $TK = mu^2-5K$ before TK and 5K are collected together as like terms. This superficial knowledge of cross multiplication can only be applied without risk in such items as items 3, 5 and 9 unlike item 1. So students' overuse of cross multiplication while lacking the underlying operation is another source of error.

The inability to multiply through by the LCM of all denominators also lead to the many errors encountered in question 8. Some were even adding the numerators and denominators on the left hand side of the equation and since this is impossible for them, they simply consoled themselves by taking $\frac{1}{V} + \frac{1}{Y} = \frac{2}{Z}$ to lead to $\frac{2}{VY} = \frac{2}{Z}$. It appears also that inability to

find the least common multiple of letters rather than numbers, for example the least common multiple of V, Y and Z, is another cause of errors and fumbling in this topic among students. Teachers ought to expand this area much since it is one of the underlying schema in literal equation solving.

4.4 Error Due to Lack of Transposition Ability

Transposing a letter, symbol or number from one side of an equation to the other is indispensable in solving literal equations. But the poor knowledge of when and how to transpose is another cause of errors in solutions involved in this topic. There is a common sense that is required to know when to transpose. For example in question 7 which demands that x should be made the subject of the formula $W = 1 - \sqrt{x}$, isolating $\sqrt{x}$ to one side and W and 1 on the other side will shorten the length of the solution greatly. The same is true of question 10. In question 6, after squaring both sides as $U^2 = 1-t$, many were writing $\therefore t = U^2-1$, which is an incorrect transposition of letters/numbers to either side of the equation. This is where the song of "whatever you do to the L.H.S., do the same to the R.H.S.", criticised by Meyerson (1976) comes into play. It seems teachers often gloss over proper explanation of this rule to the detriment of the student. Poor transposition ability was manifested in

questions 1, 3, 4 and 11. For example in question 3 which stated thus "Given that $y = \frac{a-rx}{a+rx}$, express x in terms of y, a and r", after getting $y(a+rx) = a-rx$ and then getting $ya+yrx = a-rx$, the next step should be to collect the x terms together correctly. At this juncture, many will either abandon the problem or work any thing they like. Lack of this knowledge made many to fail question 4. Many couldn't isolate α from $\theta-\alpha = \psi$ before substituting for θ and ψ; rather they employed their shortcut of substituting directly in the given equation before finding α. However if they were able to substitute correctly, and then transpose correctly they will still arrive at the correct solution.

Many students cannot substitute a negative number for a negative letter or symbol e.g. substituting -3 for ψ after isolating α as $\alpha = \theta-\psi$ to obtain -(-3) which is +3. Improper substitution and fear to substitute especially negative numbers is a strong cause of error in the evaluation of literal equations.

So it can be discovered that students due to their poor abilities in the process skills and transposition skills of solving literal equation try to substitute first, before evaluation (see definitions).

4.5 Comprehension Errors

Lack of comprehension of the real meaning of literal equation has been labelled as a minor error for the fact that not many students manifested it when compared with the previous error sources mentioned. The interviews received bold responses from students showing that they are really free with the topic, and have a good mental knowledge of literal equations as different from ordinary equations. Yet in practice they demonstrate poor manipulative ability.

Perhaps one should expect a fairly high degree of comprehension of the topic since the sample consists of science students. But, on the contrary, this poor manipulative ability is a great snag for their future science learning, especially as they encounter similar equations in terms of formulas and rules in their sciences.

However there are a few cases of lack of comprehension that are noteworthy. The first is mentioned in the discussion under section 4.3.2. In question 4, a student on seeing the symbols θ, α and y started solving a trigonometrical problem not given because he apparently felt α and θ can only be used in trigonometry problems. Another student on seeing α and y started solving the problem as though it were a quadratic equation because he felt that α,

γ and β which are often used in finding the sums and products of roots of a quadratic equation are restricted to it. These are cases of symbolic inhibition and are important to be noted by the teachers. From the interviews as well it was noticed that subscripts are not inhibitions to students eventhough they could not read or pronounce say H_b as "Hsubb" but rather as "Hb". Also many were familiar with the symbols α and θ but found the symbol γ very strange. This didn't hinder their work however because they took it as ordinary y.

Question 11, being the only fully worded item, revealed that many students are poor in forming equations of their own before proceeding to get their solutions. The statement "D is subtracted from C" was interpreted and written as 'D-C' by some while the phrase "and the difference subtracted from B" was confusing to many. The causes of these errors spring from the fact that many students think the word difference means 'minus' rather than the result obtained after subtraction. Hence they didn't understand that C-D is the difference that should be subtracted from B to obtain A i.e. B-(C-D) = A. One can ponder to ask at this juncture: 'what is the grammartical or English language ability of the science students? What is their attitude toward the learning of English language?

4.6 Reading Errors

Reading errors have been regarded as the least source of error because, from the interview, the students could read well and interprete what they were told to do in each item.

However, it will not be too far removed from this study to say that about 90% or more of the students have some poor language mannerisms in reading out some mathematical phrases. For example, the sign of equality = in an equation say A = B is read "A is equals to B", instead of "A is equal to B" or "A equals B". A negative number is often read as 'minus' number when it is not being subtracted from any number. For example, -4 is read by many as "minus 4" instead of "negative 4". The dividing fraction E/F in item question 2 was read and applied by many as merely EF while disregarding the stroke / because they never knew it stood for 'division', and this led to many failing to invert it. But it was found that many could interprete E/F if written as $\frac{E}{F}$.

4.7 Summary of Results

Therefore poor process skills in clearing square root signs, fractions, incorrect transposition of letters/symbols or numbers from one side of an equation to the other have been found as the great causes of error in the solution processes of literal equations.

The analysis also showed that many students have a mental knowledge of the algorithms and yet in practice a lot of errors are still made. Comprehending the meaning of a literal equation and the symbols used as well as reading ability have not been discovered as causing any significant errors.

CHAPTER 5

SUMMARY, CONCLUSION AND RECOMMENDATIONS

This investigation of students' difficulties with literal equations based on the students' oral and written performances cannot be brought to an end without a focus at the salient findings. This chapter therefore is intended to summarise and draw the necessary conclusions. It will finally make recommendations based on the findings from the investigation, coupled with the inbedded remediation.

5.1 Summary

From the start, this study has been out to establish the fact that students make a lot of errors in solving literal equations. It then proceeded with some hypothetical questions as to why such errors are made, and which errors are the most common among the students. Justification of the importance of the area was drawn from several scientific fields ranging from engineering, medicine, advanced mathematics, agriculture to even the business field. More to this justification are the reports which the West African Examination Council (WAEC) issues every year concerning students' problem with literal equations.

Literal equations have been considered as occupying an indispensible position in the life career of every science-inclined student. This shows that the little attention paid to the treatment of this topic by some Nigerian authors and teachers has been a big draw-back to mathematical and scientific advancement in this country.

The literature survey attempted to relate other studies and research findings to the Nigerian context. This same review of literature pointed out generally some error categories or "failure strategies" in mathematics learning (section 2.1), though not attempting to specify what "failure strategies" that have been affecting literal equations in particular. Wrong operation, obvious computational error, defective algorithm, and random response have been considered by Gershard [Ashlock(1972)] and have been discussed in this study. Error sources were seen not only as coming from the students, but also from the symbols used, and from the teachers as well. In addition, reports from the West African Examination Council (WAEC) have stipulated that lack of understanding of basic mathematical principles and inability to manipulate arithmetical and algebraic expressions are the general causes of poor performance in mathematics.

Related works on error investigation were reviewed especially those of renowned American researchers like Clement (1980) and Hollander (1978); as well as those from Australia like Newman (1976) and Watson (1980). However, none of these works was on literal equations, but in general mathematics topics.

The design of this study is based on a modification of Newman's model of "error analysis guideline". The methodology followed in this study started by a "pre-interview" with teachers of mathematics, followed by an opinion seeking questionnaire; and an initial test to a sample of students. All these made up the pilot survey. From the results of the survey and the literature reviewed, the actual test was constructed and administered to test the student's ability in handling literal equations. This was followed with a structured interview to learn the students' thought processes, which lead to their working of the problems. Full details of this part of the study are in chapter 3.

The results of the analysis have been extensively discussed in the previous chapter from which some conclusions can be drawn.

5.2 Conclusions

From the finding reported in chapter 4, it is possible to make the following conclusions of the study.

It has been found that students fall into a lot of errors in their solution processes of literal equations due to lack of understanding of basic mathematical operations involving square roots and fractions. This poor process skill has been found to be consistent among students.

Another great source of error is inability to transpose letters or numbers from one side of an equation to another. This error was also consistent as could be seen from the mindless application of cross multiplication by many of them. This inability leads to the inability to collect like terms (especially in fractional equations containing compound fractions) and related difficulties in factorisation.

Lack of proper understanding or comprehension of the actual meaning of literal equations has not been found as a significant source of error. Also the seemingly confusing abstraction of using letters and symbols more than numbers has not been found to affect student's performance significantly.

Literal equations, as this study has revealed has a lot of loop-holes as to when and how it is taught. It has been found also that such a fundamental topic is taught as late as in class 5 in some schools and many present it in year 4. It is discovered that many teachers can teach many other mathematics topic

and may proudly quote the formulae involved without questioning why the students still fail the problem. This study has revealed literal equation as a little canker worm that can dilapidate a mansion. Students' hatred to mathematics in general and their failure in many mathematics topics especially in the higher secondary school can now be traced to their inability in this topic. Another point worth considering is that this study also revealed that many teachers have earlier on looked at literal equations as an insignificant area of mathematics and this is why many do not treat it seriously and some text writers just give it a little space. One might feel that this is why so far many have not come out with studies in this area in this country despite the yearly West African Examination Council (WAEC) reports sent to schools hammering on the poor performance of students in this topic.

5.3 Recommendations

By considering the key position that the topic literal equations occupies in scientific advancement, it is therefore worthy at this juncture to make the following recommendations to remedy most of these errors.

Firstly literal equations should be taught as early as year 2 in the secondary school to prepare students before they start choosing their West African School Certificate (WASC) subjects. A good treatment and

presentation of this topic will kindle a strong desire in them to do the sciences.

Secondly teachers should stop treating this topic like mere linear equation. Eventhough the two topics may have the same processes, yet they don't have the same scope. This is because literal equations may be of any degree whereas linear equation as the name stipulates, is only of degree 1.

In teaching equations in general a teacher should give the young minds a concrete representation which will enable them have a right perception of equality. That is, that whatever is on the left handside of an equation is equal to what is on the right handside no matter what they look like. An improvised beam or weighing balance can be used to demonstrate this, using different, if possible queer objects on the two different arms demonstrating equality and also inequality. This will go a long way to erase such questions as "how can L be equal to all that on the right?" in such equation as $L = (\frac{B-C}{D})r$.

Next, teachers should spend time to make students assimilate the fact that any letter or symbol in literal equation can be used to represent any unknown. It must not only be x or y which are mostly used and hence more familiar. Students should be encouraged to use any symbol in problem solving instead of querrying

the meaning of that symbol or letter. It is hereby recommended that a study on symbolic inhibition be carried out to see if the use of other symbols other than the common x and y are of any significant effect in student's perception of and performance in mathematics learning.

In the treatment of literal equation it is very necessary to establish the real idea of coefficients of a variable. This will nullify the old idea held by many that coefficient of a variable must be a constant. For example saying that the coefficient of y in 3xy is 3 instead of 3x. Students should also be made aware that the coefficient of x in 3xy is not also 3 but 3y. So the position of the letters in the monomial does not necessarily matter. So commutative, associative and distributive properties must be established. Then they should be taught the use of coefficients. This is very important in change of subject. A student who does not know that (x+z) in (x+z)b or b(x+z) is the coefficient of b will not be bold or able to say that $b = \frac{m+n}{x+z}$ when he is told to make b the subject of the equation $bx + bz = m+n$.

It has been found that square roots pose a dilemma to many students. Squaring a square root number and finding the square root of a squared number is a major problem to many. Teachers therefore should

spend time in establishing the rubrics of radicals before going on in this topic.

Students can easily find the least common multiple (LCM) of ordinary numbers but do not find it so easy when using letters and symbols. This has been found as another cause of erring among students. Teachers ought to expound this area since it is one of the underlying schema in solving literal equations.

The idea of "whatever you do to one side do the same to the other" should be explained more. Some teachers feel that the above statement as it is, is enough for the students to assimilate the application. (Section 2.1.3) has discussed this assumption and it's dangers extensively. Rather, it is better to teach students that whichever number or letter you are adding to or subtracting from, or multiplying or dividing with, on one side of the equation, the same number or letter should be used to operate on the other side in the same manner. Lack of thorough explanation of this area both in written and concrete form is the major cause of the poor transposition ability which many students demonstrate.

Judging from the nature of this study, it is hereby strongly recommended that this study be carried out in a wider scale. A wider investigation of errors

in many mathematical topics of student's difficulty, carried out in more than one or two states of the federation will reveal more than this study has done. Results from such large-scale analysis can be of vital use to the curriculists, teachers of mathematics, textbook writers as well as the students.

If teachers will be meticulous in applying the above recommendations, then they will go a long way in reconstructing the broken walls of understanding basic mathematical principles lacking in the Nigerian Secondary School student. It will contribute in no small measure in enhancing student's ability in manipulating arithmetical and algebraic expressions correctly. It will reduce to a great extent the negative attitude many students have especially in those mathematical problems that involve algebraic expressions, equations which demand algebraic interpretations for example substitution and change of subject.

A positive attitude to the solution processes of such basic mathematical topics like literal equations can have great therapeutic effect on the anxiety level of students and on the desparate condition of the parents and the public over mathematics learning. It will put a stop to the repeated reports from West African Examination Council (WAEC) concerning

student's poor performance in literal equations. Moreover the school will then be fulfilling one of the national goals stipulated in the Nigerian Policy on Education for the secondary school level. Consequently, Nigeria will be heading towards a realistic scientific and technological advancement.

APPENDIX A

TEACHERS' QUESTIONNAIRE

A sincere response to each of these items will be highly appreciated for stronger reliability.

Thank you.

Instructions

Tick the box that best indicates your response.

1. Qualification: /Ph.D./ /M.Sc, M.E./ /B.Sc, B.Ed, HDME/ /NCE, DME, OND/ /Any other/

2. Years of experience in Mathematics teaching:

   /5yrs or less/ /between 6-10yrs/ /between 11-15/ /between 16-20yrs/ /More than 20yrs/

   Definition: A literal equation is one in which some or all of the known quantities, as well as the unknown, are represented by letters or non-numerical symbols. It broadly incoporates formulas.

3. At what level in the secondary school are literal equations first taught?

   Form /1/ /2/ /3/ /4/ /5/

4. At what level are they taught most?

   Form /1/ /2/ /3/ /4/ /5/

Instructions: For numbers 5-9, circle the response which indicates your opinion. Use the following ratings:

SA=Strongly Agree A=Agree U=Undecided D=Disagree
SD=Strongly Disagree.

5. Secondary school students make a lot of errors in solving literal equations. 3.84

6. One approach of solving literal equations is to express one letter in terms of others i.e. "Change of Subject".

In this approach, students are likely to make errors in such equations involving:

(a) fractions 3.81
(b) negative numbers 3.83
(c) negative fractions 4.20
(d) decimals 3.40
(e) negative decimals 3.83

7. Why do you think students make these errors in solving literal equations:

(a) Lack of proper understanding or comprehension of the actual meaning of literal equation 3.87
(b) Poor knowledge/ability in solving ordinary linear equations 3.91
(c) Lack of understanding of basic mathematical operations and transposition of letters. 4.11
(d) Inability to collect like terms and to factorise 3.70

(e) The use of letters and symbols more than numbers, makes the topic abstract and more confusing for the students 3.51

(f) Automatic promotion from one level to another 3.65

(g) Poor language ability 3.39

(h) Poor admission process into the secondary school 3.78

8. In which of these sample questions do you think students are likely to make errors as they attempt to solve it. Please Do Not solve the problems.

(a) Given that A = B-C-D make C the subject of the equation 2.88

(b) Given that $\frac{D}{E/F}$ = G express E in terms of D, F and G. 3.73

(c) The symbols θ, α and ψ are connected by the relation $\theta - \alpha = \psi$, find the value of α when $\psi = -3$ and $\theta = -\frac{1}{3}$ 3.30

(d) If $\frac{F}{\sqrt{Q}} = \frac{\sqrt{R}}{S}$ express Q in terms of F, R and S.

(e) Make t the subject of the equation $U = \sqrt{1 - t}$ 3.65

(f) From the formula $W = 1 - \sqrt{X}$ make X the subject 3.63

(g) From the relation $\frac{1}{V} + \frac{1}{Y} = \frac{1}{Z}$ find Z in terms of V and Y 3.65

(h) Given the expression $\frac{VX - VY}{f} = g$ find y when $x = \frac{1}{4}$, $f = -4$ and $g = 4$ 3.91

(i) If $X = \sqrt{y^2 + z^2} - 1$ express z in terms of X and Y. Hence find z when $x = 0.01$ and $y = 0.3$ 4.16

(j) Express T as the subject of the formula $P^2 T^3 = 8R$ 3.41

(k) $1 + \frac{1}{H_b} = 3 H_p H_q$ find H_b in terms of H_p and H_q. 4.05

9. Do you have any further responses for Nos 7 and 8? State below.

_____

_____

_____

APPENDIX B

STUDENTS' TEST

Name in full_____

Sex (Male or Female)_____ Test Number_____

Solve the following problems. Attempt all

1. Make K the subject of the formula

$$T = \frac{mu^2}{k} - 5$$

2. Given that $\frac{D}{E/F} = G$, express E in terms of D, F and G.

3. Given that $y = \frac{a - rx}{a + rx}$ express x in terms of y, a and r.

4. The symbols θ , α and γ are connected by the following relation $\theta - \alpha = \gamma$, find the value of α when $\gamma = -3$ and $\theta = -\frac{1}{3}$.

5. If $\frac{P}{Q} = \frac{R}{S}$ find Q in terms of P, R and S

6. Make t the subject of the equation $U = \sqrt{1 - t}$

7. From the formular $W = 1 - \sqrt{X}$, make X the subject.

8. From the relation $\frac{1}{V} + \frac{1}{Y} = \frac{2}{Z}$ find Z in terms of V and Y

9. Given the expression $\frac{\sqrt{X} - \sqrt{Y}}{f} = g$ find Y when $X = \frac{1}{4}$, $f = -4$ and $g = 4$

10. Y if $X = \sqrt{y^2 + Z^2} - 1$ express Z in terms of x and y. Hence find Z when $x = 0.01$ and $y = -0.3$

11. $1 + \frac{1}{H_b} = 3 H_p H_q$ find H_b in terms of H_p and H_q

12. The quantities A, B, C and D are connected by the fact that, when D is subtracted from C and the difference subtracted from B, the result equals A. Write down a literal equation connecting the quantities.

Solutions to the Written Test

1. $T = \frac{mu^2}{k} - 5$

$\Rightarrow TK = mu^2 - 5k \quad \Rightarrow TK + 5k = mu^2$

$\Rightarrow K(T+5) = mu^2$

$\Rightarrow k = \frac{mu^2}{T+5}$

2. $\frac{D}{E/F} = G \Rightarrow \frac{DF}{E} = G$

$\Rightarrow E = \frac{DF}{G}$

3. $y = \frac{a-rx}{a+rx} \Rightarrow y(a+rx) = a-rx$

$\Rightarrow ya+yrx = a-rx \Rightarrow yrx + rx = a-ya$

$\Rightarrow x(yr+r) = a-ya$

$\Rightarrow x = \frac{a-ya}{r(y+1)}$ or $\frac{a(1-y)}{r(1+y)}$

4. $\theta - \alpha = \psi$

$\Rightarrow \alpha = \theta - \psi$, by correct substitution,

$\therefore \alpha = -\frac{1}{3} - (-3) = 3 - \frac{1}{3} = 2\frac{2}{3}$

5. $\frac{P}{\sqrt{Q}} = \frac{\sqrt{R}}{S} \Rightarrow PS = \sqrt{Q}\sqrt{R}$ or $\sqrt{QR}$

$\therefore (PS)^2 = QR \Rightarrow Q = \frac{(PS)^2}{R}$

6. $U = \sqrt{1-t} \Rightarrow U^2 = 1-t \Rightarrow t = 1-U^2$

7. $W = 1-\sqrt{x} \Rightarrow \sqrt{x} = 1-w \Rightarrow x = (1-w)^2$ or $1-2w + w^2$

8. $\frac{1}{v} + \frac{1}{y} = \frac{2}{Z} \Rightarrow YZ + VZ = 2VY$

$\Rightarrow Z(Y+V) = 2vy \Rightarrow Z = \frac{2vy}{y+v}$

9. $\frac{\sqrt{x} - \sqrt{y}}{f} = g \Rightarrow \sqrt{x} - fg = \sqrt{y}$

$\Rightarrow y = (x-fg)^2 = x-2fg\ x + f^2g^2$ by correct substitution, $y = 272\frac{1}{4}$

10. $x = \sqrt{y^2+Z^2} - 1$

$\Rightarrow x+1 = \sqrt{y^2+Z^2} \Rightarrow y^2+Z^2 = (x+1)^2$

$\Rightarrow Z^2 = (x+1)^2 - y^2 = (x+1-y)(x+1+y)$

$\therefore Z = (x+1-y)(x+1+y)$ by correct substitution,

$Z = 0.9301$

11. $1+\frac{1}{H_b} = 3H_pH_q \quad H_b+1 = 3H_pH_qH_b$

$\Rightarrow H_b(3H_pH_q - 1) = 1$

$\therefore H_b = \frac{1}{3H_pH_q-1}$

12. $A = B-(C-D)$ or $B-C + D$.

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