PRACTICAL ACCURACIES OF NUMERICAL RELATIVE ORIENTATION ON WILD - A7 AUTOGRAPH

A project thesis submitted in partial fulfillment of the requirements for the award of the Master of Science degree in land surveying of the Ahmadu Bello University, Zaria, Nigeria.

BY

FELICIA KASHIENNE ONUORAH

MAY, 186

D E C L A R A T I O N

I, Felicia Kashienne Onuorah, do hereby declare that this project thesis has been entirely composed by me and is a record of my research work. All observations were done in accordance with the regulations of Surveying. To the best my knowledge, this thesis has not been accepted before for the award of any higher degree. Sources of information are acknowledged by referencing.

C EcR T I F I C A T I O N

This project report entitled "PRACTICAL ACCURACIES OF NUMERIAL RELATIVE ORIENTATION ON THE WILD - A7 AUTOGRAPH" by MISS FELICIA KASHIENNE ONUORAH, meets the regulations governing the award of the degree of Master of Science (M.Sc.) in land surveying of Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

(Name and Signature) A.A. BAMMEKE Date:- 23/10/86
Chairman, Supervisory Committee.

(Name and Signature) I.I.K. ADAMINDA Date: 23/10/86
Member, Supervisory Committee.

(Name and Signature) _____ Date: _____
Member, Supervisory Committee.

(Name and Signature) _____ Date: _____
Head of Department.

(Name and Signature) _____ Date: _____
Dean, Postgraduate School.

ACKNOWLEDGEMENTS

I seize this opportunity to express my appreciation to my supervisor, Mr. A.A. Bammeke, for his never tiring support advice and encouragements. His suggestions have proved most valuable in seeing this project through to its final conclusion.

I also acknowledge, Mr. S.A.A. Okpaise, the Surveyor General of Bendel State who, not only encouraged me to take an interest in photogrammetry, but also threw open the doors of his library for my own extensive use.

My special acknowledgements go to the following people; Mr. S.J. Ojeikere, one time Surveyor-General of Bendel State and currently Managing Director of Esjay surveys, a consulting and licensed surveyors firm in Benin, Mr. Kayode Aregbeyen, technologist in charge of the photogrammetry laboratory in the land surveying department, Ahmadu Bello University, for getting me well acquainted with the Wild-A7; Messers Umar, Abdul Jule, Gunbola Oyawoye Akin Sawyerr for their support and Mrs. Nwude, for her motherly advice always.

I am also very grateful to all my lecturers without whose tutoring I will probably not have gone this far in surveying. To all those whose names do not appear here but who have been in one way or the other the support I needed to see me through this work, I say thank you all and God bless.

I acknowledge my beautiful parents Mr. Felix Iduka Onuorah and Mrs. Maria Ekwutozea Onuorah, my lovely and very loving sisters, Katherine, Maryann, Bridget, Winifred, and Maudlin. Mr. Francis Izuka Oduah (Ogbuefi), for spurring me on Dr. C.A. Okuofu. I also thank Mr. James O. Ameje of the Geography Department, Ahmadu Bello University, Zaria for getting this typed in record time. To my best friend Miss Anne Nordi (Nma) I send my greatest gratitude for taking care of the homefront for me while I am away.

ABSTRACT

Relative orientation is one of the three steps required to orient a stereoscope plotting instrument in order to obtain a true stereo model. The first of these steps is interior orientation followed by relative orientation and then absolute orientation. In aerotriangulation and plotting methods if the models are not parallax-free (possible only when relative orientation has been completed), the absolute orientation that follows will be greatly affected.

The main objective of this thesis is to investigate the accuracy of the numerical relative orientation on the Wild-A7 autograph in the photogrammetry laboratory of the Department of Land Surveying, Faculty of Engineering, Ahmadu Bello University, Zaria.

Five orientation points may be sufficient for the relative orientation of a model to obtain a parallax-free model. An increase in points is imperative for an adjustment requiring reduction observations.

Accuracy analysis is based on the distribution and the density of points in the model for relative orientation procedures. In the case of density, points are increased from six to nine to twelve then to fifteen. The effect on the accuracy of the determination of the correction to each element of relative orientation is examined. In the case of distribution, five describable positions or locations in the model were used and the effects on the accuracy of the determination of the correction to each element of relative

orientation are examined. The accuracy of the determination is based on the standard error of measurement.

Therefore the method of adjustment adapted is the least squares routine. A least square programme developed by Bammeke of the Department of Land Surveying, Ahmadu Bello University, Zaria, for the Hewlett Packard Calculator (HP 9830A) number 1303A02077 available in the photogrammetry laboratory was used.

In all the procedures and measurement, vertical or near vertical photographs with fairly flat terrain were assumed. The mathematical model used is from first principles as developed by Hallert (1960) and slightly modified to suit this project based on the peculiarities of the Wild-A7 autograph in the photogrammetry laboratory.

It is found that increase in number of points does not necessarily mean better accuracy and also the location of the points in the model has a great part to play in the accuracy.

TABLE OF CONTENTS

Chapter		Page
	Title Page	i
	Dedication	ii
	Certification	iii
	Acknowledgement	iv
	Abstract	vi
	Table of Contents	viii
	List of Tables	xi
	List of Figures	xii
	Symbols	xiii
1.	INTRODUCTION	1
1.1	Definition and Purpose of Relative Orientation	1
1.1.1	Definition	1
1.1.2	Purpose of Relative Orientation ...	1
1.2	Literature Review	2
1.2.1	Methods of Relative Orientation ...	2
	Theoretical Minimum Density and Distribution of Pass Points Required	3
	Elements and Sequences of Observation	7
1.2.2	Numerical Relative Orientation ...	14
1.2.3	Emprical Relative Orientation ...	17
1.2.4	Accuracy of Relative Orientation ...	19
1.2.5	Adjustment Procedure	21
2.	RESEARCH MATERIALS/EQUIPMENT AND PROCEDURES	22
2.1	Materials/Equipments	22
2.1.1	The Input	22
2.1.2	The Acquisition Unit	23

Chapter		Page
	Description of the Wild A7 ...	23
	Measuring System of Wild A7 ...	23
	Cordinate System of Wild A7 ...	24
2.1.3	Data Reduction and Data Processing Units	28
	Matrix Operations	30
2.2	Procedures on the Wild A7 ...	31
2.2.1	Preparatory Procedures	31
	Procurement of Preliminary Data ...	32
	Preparing the Instrument for Observation	33
	Selection of the most favourable scale	34
	Determination of gear Combinations ...	36
	Determination of Mean Projection Height(Zmm)	37
	Determination of Initial Model Base	38
	Settings of Instrument fittings ...	38
	Settings of Optical and Mechanical Fittings	38
	Settings of Orientation Elements ...	40
2.2.2	Relative Orientation Procedures on the Wild A7	40
	Inner Orientation Procedures ...	40
	Empirical Relative Orientation Procedures on Wild A7	42
	Relative Orientation of Independent Pairs of Photographs	43
	Numerical Relative Orientation on the Wild A7	46
3.	COMPUTATIONS AND ADJUSTMENT ...	49
3.1	Computations	49
3.2	Adjustments	51
3.2.1	Explanation and Excecution of the Least.Square Programme	54

Chapter		Page
4.	ANALYSIS:	59
4.1	Effects of the Distribution of points on Accuracy:	59
	Effects of the Densities of points on Accuracy:	61
5.	SUMMARY AND CONCLUSION:	64
5.1	Summary:	64
5.2	Conclusion:	65
	References	67
	Appendix A Derivation of the differential formulas:	68
	Appendix B Observation and Data Reduction: ...	78
	Appendix C Least Square Programme 1.4: ...	104
	Appendix D Analysis Tables (Corrections to Elements of Relative Orientation and their Standard Errors)	105

LIST OF FIGURES

Figure		Page
1.1	Standard Points Position	4
1.2	Effects Produced in the Model Space by Small Increments in the Elements of Relative Orientation	7
2.1	Coordinate System of the Wild A7 ...	24
2.2	Autograph and Drawing Table Assembled ...	25
2.3	The Model 9380A Calculator	27
2.4	Determination of the Angle of View ...	34
2.5	Determination of H'max and H'min ...	35
2.6	Disposition of Points in the Model ...	44
2.7a	Location of twenty-five points in a Model	46
2.7b	Direction of Observation in the Model ...	48
4.1	Distribututions and Densities of Points in the Model	50
A.1	Single photograph and projecting plane situation:	68
A.2	Two photographs and the projecting plane situation:	72

SYMBOLS

The following symbols, unless otherwise stated represent the meaning as indicated and as used in this thesis.

w' is the rotation of the left projector about the X-axis and also a rotational element of relative orientation for the left projector.

w" is the rotation of the right projector about the X-axis and also a rotational element of relative orientation for the right projector.

ϕ' is the rotation of the left projector about the Y-axis and also a rotational element of relative orientation for the left projector.

ϕ'' is the rotation of the right projector about the Y-axis and also a rotational element of relative orientation for the right projector.

k' is the rotation of the left projector about the Z-axis and also a rotational element of relative orientation for the left projector.

k" is the rotation of the right projector about the Z-axis and also a rotational element of relative orientation for the right projector.

bx' is the translation of left projector along the X-axis.

bx" is the translation of the right projector along the X-axis.

by' is the translation of the left projector along the Y-axis and also a translational element of relative orientation for the left projector.

by" is the translation of the right projector along the Y-axis and also a translational element of relative orientation for the right projector.

bz' is the translation of the left projector along the Z-axis and also a translational element of relative orientation for the left projector.

bz" is the translation of the left projector along the Z-axis and also a translational element of relative orientation for the right projector.

O_1 is the perspective centre of the left projector and assumed as the origin of the model.

O_2 is the perspective centre of the right projector.

f is the focal length or principal distance.

z is the projection distance.

n is the overcorrection factor

bx is the model base.

Xi x - coordinate of point i } measured at the instrument.

Yi x - coordinate of poing i }

Σ is summation.

σ_o^2 unit variance.

σ_{xx}^{-2} is variance covariance.

bxi is the reading of the bx knob at point i after clearing x-parallax at point i.

byi is the reading of the by" knob at point i at the elimination of y-parallax at i.

X_i, Y_i } X - and Y- coordinates of point i reduced to the origin O_1 of the model (

Xo, Yo } X and Y coordinates of the origin O_1 of the model.

d is a small change in any quantiy.

σ is standard error.

w is weight.

V is residual.

RMSE is root mean square error.

A^T means transpose of matrix A .

$(A^TA)^{-1}$ means inverse of normal matrix A^TA.

xs are the computed corrections to the elements of relative orientation.

Xi_m is the mean of the X-coordinates of point i measured at the instrument.

Yi_m is the mean of the X-coordinates of point i measured at the instrument.

byi_m is the mean of by" readings at point i, for i = 1, 2, 3.....n, where n varies from six to nine to twelve to fifteen.

KASHIM IBRAHIM LIBRARY
AHMADU BELLO UNIVERSITY
ZARIA, NIGERIA.

Chapter 1

1. INTRODUCTION

1.1 Definition and Purpose of Relative Orientation.

1.1.1 Definition: Relative orientation is "the determination (analytically or in a photogrammetric instrument) of the position and the attitude of one of a pair of overlapping photographs with respect to the other photograph." Slama 1966.

1.1.2 Purpose of Relative Orientation

The purpose of relative orientation is to reconstruct as nearly as possible the perspective conditions which existed between a pair of overlapping photographs at the instance of photography. These conditions are generally not known. However relative orientation procedure is based on the fact that each point on the ground is the intersection as well as the origin of a pair of corresponding rays each to one exposure station, and must therefore intersect in the stereo-plotting instrument.

For this to happen, the projectors are manipulated by applying rotational and translational movements to them while observing the images on a suitable model plane until all subsequent pairs of rays intersect simultaneously in the model space. Hence all corresponding images are made to coincide over the entire model space forming a continuous model space. This condition will only be fully satisfied when relative orientation has been completed if (a) both the optical lens system of the taking camera and projectors are free from

UNIVERSITY

distortions, (b) there is no geometric distortion introduced in the photographic image during photographic processes, (c) light rays are taken to have travelled in straight lines through the atmosphere.

These conditions (a), (b) and (c) are never in practice truly satisfied and therefore the purpose of relative orientation, for all practical purposes too, is to orient the stereopairs so that the condition of intersection may be achieved as nearly as possible.

1.2 Literature Review

1.2.1 Methods of Relative Orientation

The Relative orientation between two overlapping stereo-pairs of photographs is achieved only when all the corresponding rays from the two projectors bearing the photographs intersect simultaneously in the model space forming a continuous three-dimensional model similar geometrically to the photographed object. This condition is achieved by the manipulation of the projectors by changing the necessary elements of orientation to remove y-parallax. Ghosh (1968).

There are two types of relative orientation:

(a) The Independent Orientation (otherwise known as Swing-Swing) - where elements of both cameras are used to remove y-parallax,

(b) The Dependent Orientation- where the elements of one camera are used to remove y-parallax while keeping the other camera fixed.

The three main groups of relative orientation are:

(a) Numerical Relation Orientation: This is the most sophisticated of the three groups. The parallax equations are used here. Parallax measurements at the instrument are used in the parallax equations to compute, preferably by a least-squares adjustment, the differential changes to be applied to the initial (approximate) orientation elements to obtain a parallax-free model of a stereopair of photographs.

(b) Empirical Relative Orientation: In this group the differential changes, say dw' dφ', dκ', dby', dbz', to be applied to w', φ', κ', bz' respectively are obtained by the optical-mechanical procedures at the instrument. This theoretically makes the model parallax - free immediately. Ghosh (1968)

(c) Graphical Relative Orientation: The differential changes obtained are used for the derived parallax equations for a particular instrument's coordinate system, from simple diagrams based on parallax observations. Van der Weele (1960).

Theoretical Minimum Density and Distribution of Pass Points Required

It is sufficient to observe y-parallaxes in five points in a model so as to determine the necessary corrections to the five chosen elements of orientation.

The position of these five points may in principle be chosen arbitrarily provided that not more than three of these points are mutually collinear. However six points are usually chosen in the positions illustrated below for the following reasons:

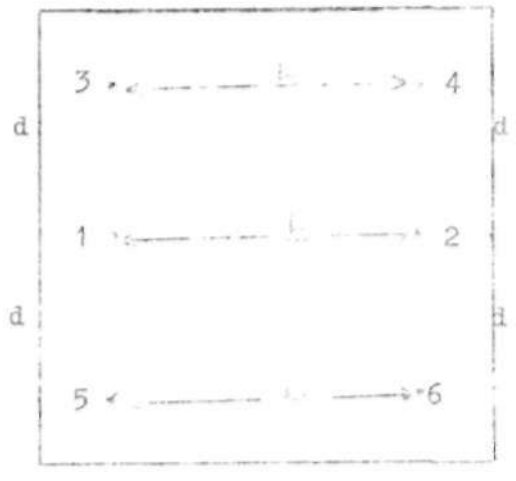


Fig. 1.1 Standard Points Position

(a) Theoretically, if parallax is removed from five points, there will be no parallax anywhere else in the model, barring all residual parallaxes resulting from observational errors, imperfect photographs, instruments' optical and mechanical imperfections. The presence of residual parallaxes can only be ascertained and checked for by an additional observation at a sixth point. This then provides for the possibility of increase in accuracy of orientation by the application of an adjustment.

(b) The parallax equation which expresses the mathematical relationship between observed y-parallaxes and movements of the various elements

of orientation provides for a system of five equations which are necessary to compute the corrections to these elements. It is a basic mathematical concept that when solving a set of simultaneous equations, the precision of determination of unknowns increases with increasing differences between the proportions of the coefficients of these unknowns in the different equations. Van der Weele (1960)

(c) For a system of equations to have a solution, the determinant of the coefficents must not equal to zero, which is the value of this determinant if the proportions in the two of the necessary number of equations are identical.

Therefore at least one of the five necessary equations should contain a maximum possible coefficient for each element and another should contain the minimum possible coefficient for the same element.

The coefficients of the unknowns are determined by the model coordinates of points at which parallax observations are taken. And in the case of a flat terrain, the maximum and minimum values of these coefficients may be obtained by an appropriate choice of the X- and Y- coordinates in the model coordinate system. Van der Weele (1960.

It will be apparent that this condition will be fulfilled if points are chosen in such a way that for each coordinate X and Y, there are combinations containing both a

zero and a maximum (and for y also a minimum) value corresponding with each such value of the other coordinate.

This leads automatically to the point positions of the six standard points. Therefore points 1 and 2 are chosen as the nadir points of the photograph approximately, while 3, 4, 5, and 6 are distributed symetrically on opposite sides of 1, 2 towards the boundaries of the model inorder to obtain the maximum and minimum X and Y coordinates.

Choice of Relative Orientation Elements And Sequences of Observation

Before dealing with the procedures of relative orientation, it is necessary to examine the effects produced in the model space by small changes in the orientation elements.

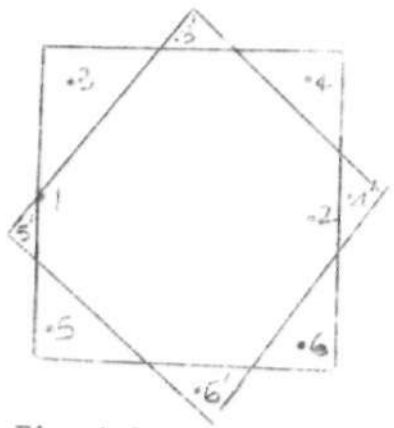


Fig. 1.2a Effect of dk'

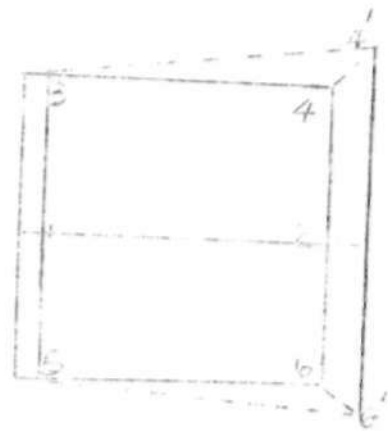
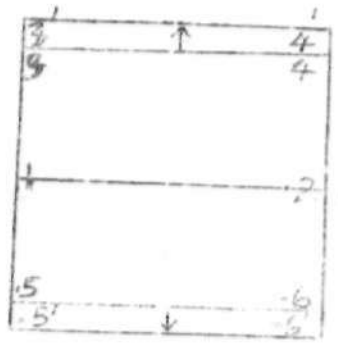


Fig. 1.2b Effect of dφ'



' is the new position after a small increment



Fig. 1.2c Effect of dw'

Fig. 1.2d Effect of dbx'



Fig. 1.2f Effect of dbz'

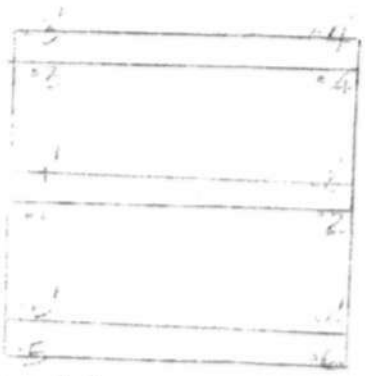


Fig. 1.2e Effect of dby'

Figure 1.2: Effects Produced in the Model Space by Small Increments in Orientation Elements Using the Six Standard Points.

The rotations are dk', dØ', dw' and the translations are dby', dbx' dbz' for the left hand projector. Double subscripts will indicate the right hand projector. Each projector has six degrees of freedom. See table below.

Table 1.1 Elements of Orientation

Projector	Rotational Elements	Translational Elements
Left	dw', dØ', dk'	dby', dbx', dbz'
Right	dw' dØ dk"	dby", dbx", dbz"

The effects illustrated in figure 1.2 (a) to (f) are explained as follows:

(a) Figure 1.2a shows that rotation dk' produces no effect, that is no y-parallax at point 1 but has maximum effects at points 2, 4 and 6. Similarly dk" will have no effect at point 2 but will produce maximum y-parallax at points 1, 3 and 5.

(b) Figure 1.2b shows that dØ'produces no y-parallax along the base line but produces maximum effect with opposite signs at points 4 and 6. In like manner dØ'will produce maximum y-parallax at points 3 and 5 but with opposite signs and no effect along the base line.

(c) Figure 1.2c shows that the rotation dw' and of-course dw" will have same effects. All the model

points will show introduction of some y-parallax, but the magnitude of the effects increases with the distance from the base line.

(d) Figure 1.2d shows that dbx' and consequently dbx" have same effects.

They introduces y-parallax into the model.

(e) Figure 1.2e shows that the translations dby' and dby" have same effects. They introduce the same amount of y-parallax on all the model points.

(f) Figure 1.2f illustrates the effect produced by the translation dbz' which produces an overall change in the scale of the projector 1 and hence no parallax is introduced at point 1 but all other points experience the introduction of y-parallax in linear proportion to their y-coordinate value. A similar effect centred on point 2 is produced by dbz".

From the foregoing, it can be seen that in the case where the elements of the left hand projector are used to relative the relative orientation of a model with the right hand projector fixed, there are two translations in X and Y directions and three rotations. The elements to be used are w', ϕ', k' bz' and by'. If the right hand projector is used with the left fixed, then w", ϕ" k" bz" and by" are used.

Considering the observations stated in (a) through (f) above, it is now obvious how fortunate the choice of the six standard points has been. This actually reduces the problem of determining a sequence of movements by which the influences of the different elements of orientation on observed y-parallax may be distinguished and separated from each other.

From these observations (a) to (f)many systematic ways can be developed for removing y-parallax in any model depending on elements used, an example of which is shown below using k', bz', Ø', w', by':

Step 1: Since k' is the only element which can affect the Δdifference between y-parallaxes at points 1 and 2, it is possible to correct k' by using it to make y-parallaxes at 1 and 2 equal. And therefore solve k'.

Step 2: Similarly move the element bz' until the y-parallax at 3 equals y-parallax at 5 bz' is solved.

Step 3: Next move Ø' until y-parallax at 4 equals y-parallax at 6 to solve Ø'.

Step 4: Move w' until y-parallax at 1 equals that at 3. Hence w' is solved (OR. y-parallax at 2 equals that at 4; y-parallax at 2 equals that at 5; or y-parallax at 2 equals that at 6).

Step 5: Move by' until y-parallax at all points equal zero by' is solved.

It is on the sequence above that the onpirical relative orientation procedure for wild A7 is based.

Depending on the instrument in use, the type of work, the capabilities of the operator as well as the time factor, a number of $\angle^{choices}$ are at the disposal of the operator as earlier stated.

For example the wild A7 has w', ∅', k', by', bz' for the left projector and e", ∅", k", by", bz" for the right projector, where w', ∅', k', w", ∅", k" are rotational elements and $\angle^{by', bz', by", bz"}$ are translational elements.

Those ten elements are all elements of orientation and are all incorporated in the differential formular for parallax. Although the choice of elements is guided by the method in use as well as the instrument used, the most important factor is that at least one element must be a member of one group such that all the groups are represented as in equations 23 and 24.

The relationship between y-parallax (want of correspondence) observed at any arbitrarily chosen point in the model and the corrections to be applied to the orientation elements is given by the parallax formular developed for vertical (or near vertical photographs) in a right handed coordinate system for the wild system as follows: Hallert (1960).

KASHIM IBRAHIM LIBRARY
AHMADU BELLO UNIVERSITY

$$Py = -dby' + dby'' - \frac{Y}{Z}dbz' + \frac{Y}{Z}dbz'' + Xdk' - (X - b)dk''$$
$$-\frac{XY}{Z}d\phi' + (\frac{X-b}{Z})Yd\phi'' + (1 + {}^{Y^2}/_{Z^2})Zdw'$$
$$-(1 + {}^{Y^2}/_{Z^2})Zdw'' - - - \quad 22a$$

There are ten elements of orientation in the above formular, only five of which is required to produce a parallax-free model. The parallax equation above may be written in a simpler from as follows

$$Py = -(dby' + dby'' + bdk'') + X(dk' - dk'') - \frac{Y}{Z}(dbz' - dbz'' - bd\phi'') - \frac{XY}{Z}(d\phi - d\phi'') + (1 + {}^{Y^2}/_{Z^2})Z(dw' - dw'') - - - - - 23$$

Let the following psuedoelements replace the items in brackets as follows:

$$\Delta A = (-dby' + dby'' + bdk'')$$
$$\Delta B = (dk' - dk'')$$
$$\Delta C = (dbz' - dbz'' - bd\phi'')$$
$$\Delta D = (d\phi' - d\phi'')$$
$$\Delta E = (dw' - dw'')$$

Such that equation 23 becomes

$$Py = \Delta A + X\Delta B - {}^{Y}/_{Z}\Delta C - \frac{XY}{Z}\Delta D + (1 + {}^{Y^2}/_{Z^2})\Delta E \text{ --- } 24$$

Therefore the model may contain five parallaxes that are proportional to ΔA, X, $-{}^{Y}/_{Z}$, $-\frac{XY}{Z}$ and $(1 + \frac{Y^2}{Z^2})Z$ respectively. Van der Weele (1968).

In order to obtain a number of equations of the form of 23, the location of the points where parallax measurements are observed can be changed. Then using five of such equations, the psuedoelements can be solved for.

Since there are five terms only in the quation of 24, it is therefore not necessary to increase the numb r of equations except where an adjustment is desired. It is therefore safe to state that not less than five elements are needed to obtain a parallax free model. However the elements are to be selected such that one out of each group of ΔA, ΔB, ΔC, ΔD and ΔE is used. For example, the following elements may be used for relative orientation dk" dk', dφ", dϕ' dw" from ΔA, ΔB, ΔC, ΔD and ΔE respectively.

It can be seen from the foregoing that a number of combinations can be obtained for the removal of y-parallax in a model, since at least one element fr m each group is to be used. Here parallax will then be eliminated. However residual parallaxes which are as a result of optical and mechanical faults of the instrument as well as defects in the photography due to lens distortions, film shrinkage etc. may still exist. An increase in the number of points used in clearing y-parallax will provide for the use of the least square principle which will take care of the residual y-parallax by distributing it all over the model space in a theoretically acceptable manner.

In the groups of pseudoelements, some elements appear in more than one group, making it obvious that these elements can only be used once for the elimination of a certain type of parallax. Van der Weele (1968)

From groups in ΔC and ΔD the following five combinations are possible:

$d\phi'$ and $d\phi''$
dlz' and dy''
dlz'' and $d\phi''$
dlx'' and $d\phi'$
dbz' and $d\phi''$

} First set.

and the following five combinations from ΔA and ΔB

dby'' and dk'
dby' and dk'
dk'' and dk'
dk' and dk''
dby'' and dk''

} Second set.

Each combination from the first set may be combined with any other combination from the second set making a total of twenty five combinations. These twenty-five possible combinations may be associated with either dw' or dw'' from ΔE group to produce a total of fifty possible combinations. Therefore there is a wide choice of elements depending on the instrument and the capabilities of the operator.

1.2.2 Numerical Relative Orientation

Because of the wide choice of combinations of elements that may be used to carry out relative orientation, there will be a number of variations of the numerical relative

orientation. These variations differ from one another so long as the handling of different elements and different cameras, the selection of points, their number and their location give differences in the final results. Ghosh (1968).

The numerical methods of relative orientation solve the relative orientation problem in a systematic way and any residual parallaxes are distributed over the model space in accordance with the principle of least squares. Although there are many variations in the methods, it is advisable to use only one set of operations.

It has already been stated that the minimum number of points necessary for a complete relative orientation of a model is five well located points. However in order to improve the orientation it has been suggested that either the number of points be increased, Ghosh (1968), or the number of observations at a point be increased. Hallert (1960). In the first suggestion, the principle of least squares is recommended for use for the adjustment.

Theoretically speaking, the increased number of points will automatically lead to better precision since the computation of five unknowns from say six, nine, twelve or fifteen observation is an adjustment problem for which the least square principle is employed. Ghosh (1968).

From the parallax equations for the dependent numerical relative orientation, (using by", bz", k", ϕ and w")and measuring parallaxes at the six standard points with

by" we shall obtain

$$- Py = -dby" + {}^{Y}/_{Z}dbZ" + Z(1 + {}^{Y^2}/_{Z^2})dw"$$

$$- (\frac{X - b}{Z}) Yd\phi" - (X - b)dk" \text{ ---------} 1.1$$

if the parallax observations are made at six points in the model, then observed parallax $\underline{Py}_i$ and existing parallax Pyi are related by

$$\underline{Pyi} + vi = Pyi \quad - \quad - \quad - \quad - 1.2$$

where vi are the residuals or errors of observation. For the least square principle, the equation will then become an observation equation as

$$- v = \underline{Py} - dby" + {}^{Y}/_{Z}dbZ" - (X-b)dk" + Z(1 + {}^{Y^2}/_{Z^2})dw"$$

$$- (\frac{X-b}{Z})Yd\phi" \text{ ---- } 1.3$$

This will be written for each of the six standard points and from the consequent five normal equations obtained, the solution of which gives dk", dϕ", dw" dbz" and dby". These values are obtained in radians and must be multiplied by the appropriate conversion factor to get the results relating to the rotations in either degrees or grads.

Procedures according to Ghosh (1968) are:

Step 1: Note the values of z, b and d in the model.

Step 2: Measure the parallaxes Py_1 to Py_6 with by' (or by"). This may be done at the instrument by the method of elimination of parallax with by' (or by") at each point in the model and noting the reading each time.

The average value of the by" readings at all points used in the elimination of parallax is then substracted from the individual by reading at the individual points. This gives the y-parallax at that point.

Step 3: Compute the changes to the elements (i.e. corrections), dk", dw", dØ", dbz" and dby". Set the final values (i.e. initial plus correction) in the instrument.

Step 4: Measure the residual parallaxes at the six points again with by' (or by") set by' (or by") to the average value of these six readings.

In the independent method of numerical relative orientation say on the wild A7, the following elements may be used k" k', Ø', Ø" and w" and parallax measurements are done with by". The procedures are same as dependent but the parallax equation differs. The elements to be used determine the nature of the applicable equations. Ghosh (1968).

1.2.3 Emprical Relative Orientation

In the dependent method, the elements of the right camera (or left camera) may be used with the left (or right) camera fixed. If only rotations are used, the number of possible combinations will reduce from fifty

to two. There are however many possibilities of varying the sequence and detail of empirical procedures, for any element may be solved by a procedure in any of the categories, there are many variations possible within each category, and there is a freedom of choice of sequence. Van der Weele (1960).

Below is an example of the dependent relative orientation (Empirical) using the elements of the right hand projector only while the left projector is left fixed Elements used are by'', bz'' k'' ϕ'' and w''. A flat terrain is assumed:

(1) Move by'' or w'' to make parallax at 2 = 0

(2) Move k'' to make parallax at 1 = 0

(3) Move bz'' to make parallax at 4 = 0

(4) Compute the over correction factor n for point 6 from $n = \frac{z^2}{c^2} = \frac{f^2}{a'^2}$

where f = focal length

a' = distance of point from base line on the photograph

(5) Read Initial w'' call it w''

Move w'' to make parallax at 6 = 0 read w''

Set $w'' = w''_m = \frac{w''_i + w''_f}{2}$ This brings back some parallax at this point. Move bz'' to make parallax at 6 = 0 Set $w''_m = n/2(w''_f - w''_i)$ to introduce some parallax at this point. Move bz'' to make parallax at 6 = 0

(6) Make parallax at 3 = 0 by moving ϕ'' Read ϕ''_i

(7) Move ϕ'' to make parallax at 5 = 0 Read ϕ''_f

$$\text{Set } \phi'' \text{ to } \phi''_m = \frac{\phi''_1 + \phi''_2}{2}$$

This should give a parallax-free model theoretically. However, iterative processes will give a parallax-free model in practice. This above procedure can be done for the left camera with right camera fixed by changing double subscripts to single and replacing the points 2, 2, 4, 6, 3, 5 with 1, 2, 3, 5, 4, 6, respectively. This procedure is fashioned after the explanations given in chapter 1.2.1.2. The parallax equation will contain the elements dk", dø", dw" dbz", dby" and their coefficients only.

A similar sequence can be development for the independent omirical relative orientation using say, dk', dk", dw" dø" and dø' noting all the observations in 1.2.1.2. The parallax equation will contain only the listed elements and their coeficients.

In the case of the empirical, the elements are solved for at the instrument and solution may be by iterative processes if residual parallaxes exist. The numerical method makes use of adjustments, and iterative processes may be employed where residual parallaxes still exist. This makes the numerical methods rather cumbersome. Van der Weele (1960).

1.2.4 Accuracy of Relative Orientation

Accuracy estimation may be determined from the following measures of precision:

(a) Maximum error

(b) Probable error

(c) Average error

(d) Root mean square error which gives rise to the standard deviation or standard error.

However, it has been found and accepted that only the theoretical mean of the square errors gives the fundamental measure of the precision of measurement. Hirvonen (1971).

Relative orientation is a photgrammetric procedure involving some measurements whose accuracy may be very vital in plotting and aerotriangulation methods. It's accuracy depends to a large extent on the following factors:

(a) The ability of the operator

(b) The quality of the restitution instrument

(c) The quality of the photographs

(d) The theoretically considered observational errors. Hallert (1960).

These are also regarded as sources of errors in the fundamental operations of relative orientation. Errors due to the inexact adjustment of relative orientation are regarded as systematic errors. (a), (b) and (c) are beyond the scope of this thesis, only (d) will be used for the accuracy estimation.

The accuracy estimation is therefore based on the standard error of the measurement of each element of the relative orientation. Hallert (1960).

1.2.5 Adjustment Procedure

No matter how carefully the relative orientation is performed, there would still exist residual parallaxes arising from sources such as lens distortion, optical and mechanical imperfections of the restitution instruments, systematic deformations of the plotting instruments as well as imperfect adjustment of relative orientation. The distribution of these residual parallaxes is of fundamental importance for the error propagation in general. It follows that if the relative orientation is not adjusted exactly a systematic error will arise. The least squares adjustment makes it possible to correct the y-parallaxes in the orientation points as completely as possible.

It was stated and proved that only five points are not only suitable but also sufficient to carry out the relative orientation of a model to obtain a parallax-free model. However since the distribution of the residual y-parallaxes in the orientation points is by least squares methods, there will therefore be a number of redundant observations which will require an increase in the number of points used for y-parallax observations in the model (i.e. more than five points are used). The object of the least square method is to produce unique values for the unknowns, which will be of maximum probability and indicate theprecision with which the unknowns have been determined. This precision determines the accuracy of any measurement or procedure. Van der Weele (1960)

Chapter 2

RESEARCH MATERIALS/EQUIPMENT AND PROCEDURES

2.1 Materials/Equipments

The following materials/equipments were used for this project:

(a) The input - this consisted of aerial photographs (film diapositives and paper prints).

(b) The Data Acquisition Unit - the wild-A7 autograph number 3268 available in the photogrammetry laboratory of the land survey department of Ahmadu Bello University, Zaria.

(c) The Data Reduction and Data Processing Units which were:

(i) The Casio Scientific Calculator fix 3600P - for data reduction;

(ii) The Hewlett Packard 98300A desk calculator model 30 also available in the photogrammetry laboratory.

2.1.1. The Input:

These were supplied by the department. They are three black and white aerial photographs with exposure numbers 77906-60, 77906-61, 77906-62 respectively. These were supplied as paper prints and film diapositives. They make up the first and second models (photograph numbers 77906-60, 77906-61 and 77906-61 . 77906-62

respectively) of the first strip of Akwanga. They were obtained with a taking camera of focal lenghh 152.67mm and at an exposure time of 8.21 a.m. on 6th June, 1977 (see 2.2. 1.1 for further details).

2.1.2 The Acquisition Unit

The main acquisition unit is the wild-A7 autograph number A7-3268.

Description of the Wild A7

The Wild A7 is a universal, versatile, high precision plotter capable of being used for aerotriangulation as well as for plotting details using either aerial or terrestrial photographs. Its focal length ranges between 98mm and 215mm, although it has only two projectors, the possession of base-in and base-out facilities makes it capable of aerotriangulation. The maximum image format is 23mm x 23cm. Hallert (1960.

Each of the two projectors has six degrees of freedom; three rotations and three translations (w, ϕ, κ and bx, by, bz respectively) The space rods pass through the gimbals located at the perspective centre of the dispositives so that images are projected mechanically through the space rods.

The Measuring System of Wild A7

The wild A7 autograph uses a pair of half marks (viz measuring marks) like all other universal plotters. These measuring marks or plotting marks are placed on

each in the optical path of each projector between the diapositive and the eyepiece. Two handwheels impart the motion of x and y to these half marks. The z - motion is imparted to the reference marks with a foot disc.

The spatial coordinates of any point in the model can be read and recorded manually by means of the x -, y - and z - counters. Hallert (1960).

The Coordinate System of Wild A7

The Wild A.7 autograph employs the Wild autograph system as coordinate system (see fig. 2.1).

This system is utilised in the development of the differential parallax equations used for this thesis.

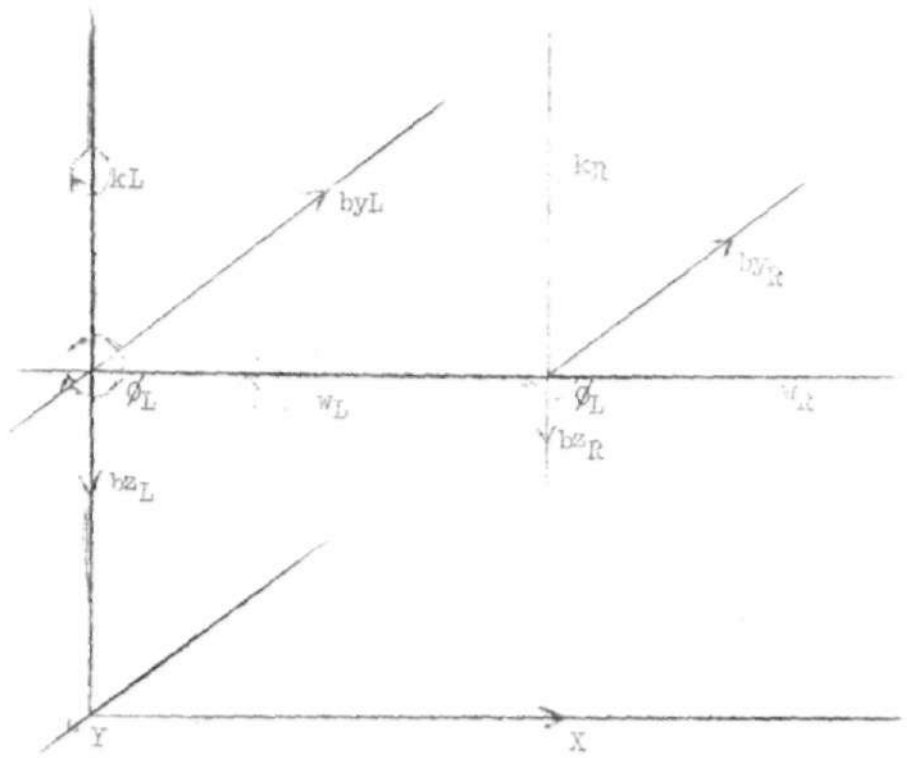


Fig. 2.1 Coordinate system of the Wild A.7 and x indicate left and rith respectively.

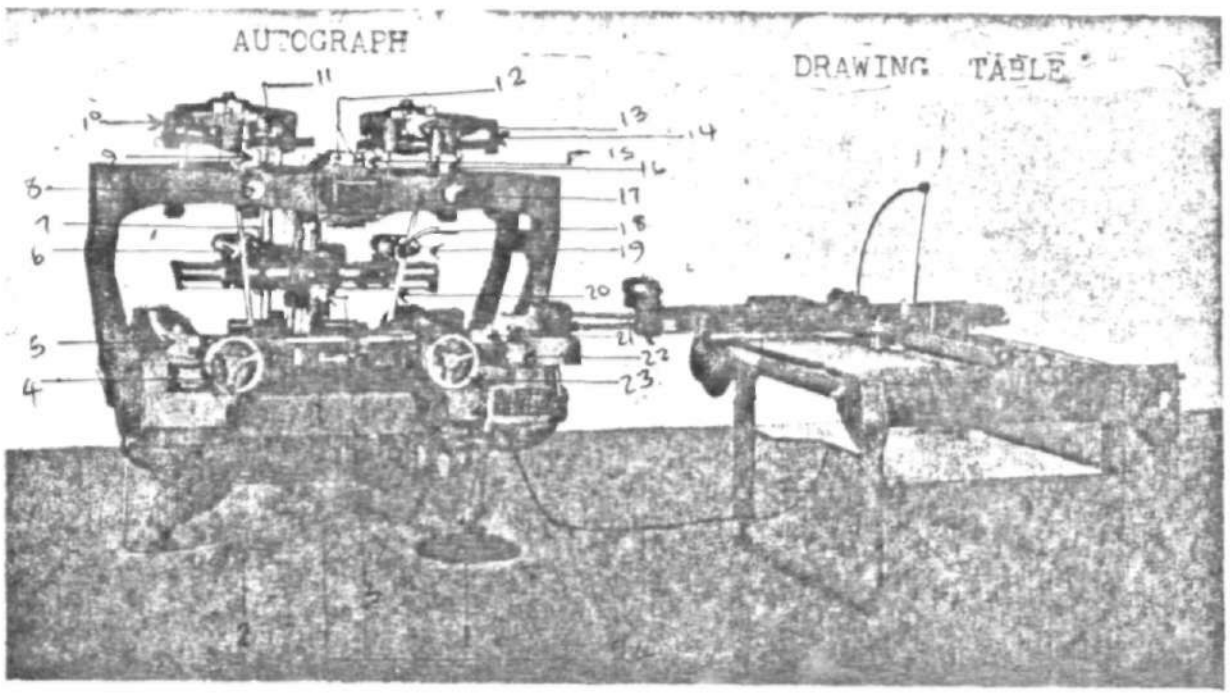


Fig. 2.2. Autograph and drawing table assembled
Après Manual of Wild A7 No. A7-3268.

1. Foot disc for z-motion
2. Pedal for pencil lift
3. bx - guide and slow motion
4. x - handwheel
5. x - counter
6. by' knob and counter
7. bz' knob and counter
8. w' knob
9. w'
10. left plotting camera
11. k'
12. dove prisms
13. k"
14. right plotting camera
15. Ø"
16. w"
17. w" knob
18. bz" knob and counter
19. by" knob and counter
20. space rod
21. Y-counter
22. Z-counter
23. Y-handwheel
24. Ø'

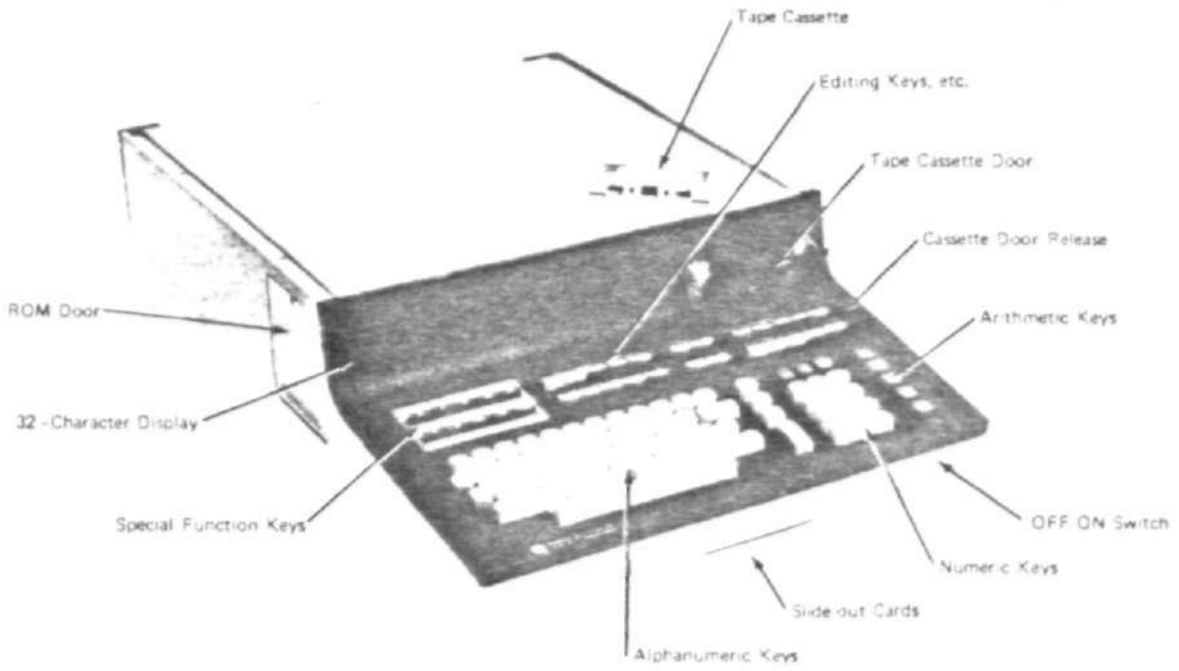


Fig. 2.3 The Model 30 Calculator

Apre s Manual of Model 30 Calculator No. 1303A02077.

The following observations are made for this coordinate system:

(i) It is a right handed coordinate system when the numerals on the x - and z - counters are black and the numerals on the y - counter are red.

(ii) The rotations are w, ϕ, k about their x, y^w, $z^{w\phi}$ axes respectively

(iii) All rotations that are clockwise when viewed from the positive end of their respective axes are regarded as positive.

(iv) The origin of the system is at the left camera's perspective centre.

For positive coordinate directions the following are also observed:

Scales: The fixed coordinate scales on the Wild A7 have divisions numbered as follows:

x - scale increases from left to right

y - scale increases towards the observer

z - scale increases from top to bottom.

The x - and y - counters correspond to the direction of the fixed scales. If the direction of one of the two counters is to be reversed, the normal (black numerals) counter disc and drum must be covered by others having inverted divisions (red).

height counter gives increasing readings when the base carriage moves upwards. Manual of A7.

2.1.3 Data Reduction and Data Processing Units

The data reduction and data processing units used are:

(i) The pocket calculator casio scientific calculator fx-3600p which is programmable. However it is not quite versatile and although it can be used for solving statistical problems, it cannot perform matrix inversion and matrix transposition. It is a ten-digit model that be used for extensive data reductions.

(ii) The Howlett Packard 9830A calculator (model 30) This is a programmable desk calculator that makes use of the BASIC programming language. With the exception of the following statements, other statements are used in the same way or with minor variations from other BASIC languages:

'Implied' LET	GRAD	STANDARD	Go To & GoSuB with oF
DISP	WAIT	FIXED N	Multiple-Line Functions
WRITE	RAD	FLOAT N	Programmable Tape Commands
FORMAT	DEG		

while the following have additional features not common in many BASIC languages:

STOP	DIM	READ. .	DATA with RESTORE
END	COM		

The manual of the Hewlett Packard 9830A calculator gives an extensive description of each statement.

The model 30 is modelled in the from of a typewriter and the keyboard is divided into five specified areas as follows:

(1) Alphanumeric keys - These keys appear like those of a standard typewriter.

(2) Numeric keys.

(3) Arithmetic keys.

(4) Special function keys.

(5) Editing keys.

The tape cassette compartment makes it possible for programmes to be recorded and stored for future use. The calculator is also designed for statistical problems and it is capable of calculating statistical measures for sets of observations on any number of variables. Statistics for each variable include mean, standard deviation (STD DEV) and standard error (STD ERR). For two variables, the covariance and correlation coefficient can also be calculated.

The matrix operations on the Hewlett Packard 9830A calculator (model 30) are fullytreated in the manual. The Least Square Programme 1.4 (see appendix C) developed by Mr. A.A. Bamboke, is based on the following matrix operations below with the appropriate explanations:

Matrix Operations

DIM	10 A(20,30), B(10,5)	Reserves space for a matrix of the specified physical dimensions. Initially, the working size is the same as the physical size for a matrix, as specified in the DIM statement.
MAT READ	30 MAT READ A	Reads an entire matrix from DATA statements. The matrix is filled in conventional row-column order.
MAT.	60 MAT C=A-B 61 MAT X=X-Y	Matrix substraction. The same matrix can appear on both sides of the equals sign.
SCALAR*	80 MAT A=(1/5)*B 81 MAT B=(2*PI)*B	Each element in matrix B is multiplied by the scalar value (expression in parenthesis). The same matrix can appear on both sides of the equals sign.
MAT*	90 MAT A=B*C	Matrix multiplication. If dimensions of A=(P,N) and B=(N,Q), the resulting matrix has dimensions (P, Q). The same matrix cannot appear on both sides of the equals sign.

MAT	TRN	100	MAT A= TRN (B)	Transposes a matrix. If the dimensions of B= (P,N), the resulting matrix has dimensions (N,P). The same matrix cannot appear on both sides of the equals sign.
MAT	INV	140	MAT A= INV (B)	Inverts a square matrix.
DET	150	IF DET(A) = 0	THEN 180	Returns the value of the determinant of a square matrix to an expression.

2.2 Procedures on the Wild A7

The procedures on the Wild A7 autograph involved the preparatory steps and the actual observations for relative orientation.

2.2.1 Preparatory Procedures

This was executed in two stages:

(a) The procurement of the preliminary data.

(b) The preparation of the instrument for observation.

Procurement of Preliminary Data

All the necessary input (the photographs) as well as the index were examined for any information that could be useful to the project. Table 2.1 is a list of relevant information and their respective sources.

The photograph base was determined on the photographs (paper prints). This was done by joining the opposite fiducial marks at the sides of the photographic prints. The intersection of the lines joining these fiducial marks defined the position of the principal point of each photographic print. The principal point of each image was then transfered to the adjacent image as the conjugate principal point. A line was drawn on each photographic print to join the principal point and its conjugate. This line defined the base of each photograph.

Table 2.1: Table of Preliminary Data

	ITEM	INFORMATION	SOURCE OF INFORMATION
1	Camera	Ag 277	Contact (paper) print
2	Photograph format (1 x 1 cm^2	23 x 23cm^2	Contact (paper) print
3	Focal length of taking Camera (fmm)	152.676mm	Contact (paper) print
4	Flight Direction	East-West	Contact (paper) print
5	General fore and aft overlap	between 57% & 62%	determined from the overlap of paper print
6	General lateral overlap	between 28% and 30%	
7	Total area covered by photography	not specified	
8	Photoscale (SP)	1:8000	Index
9	Average photo base bmm	86.5mm	determined from paper print
10	Date of photography	6/6/77	Index
11	Approximate time of photography	8.21 a.m.	Index
12	Height of:		
	(a) Heighest control in the block hmax AMSL	498.72m	List of coordinate of photo control points.
	(b) Lowest control in the block hmin AMSl (AMSL = above mean sea level)	429.07m	
13	Optimum projection distance of plotter to be used (the Wild A7)	490mm	Wild A7 Instrument Manual
14	Desired Mapping Scale	1:2500m	as per client's requirements.

ABUBAKAR TAFAWA BALEWA LIBRARY
AHMADU BELLO UNIVERSITY,
ZARIA, NIGERIA.

... Preparing the Instrument for Observation

The preparation of the instrument for observation involved the determination and selection of the required

instrument settings as obtaible from the manual of wild A7, as well as the actual settings of the obtained data at the instrument. These were determined as follows:

. Selection of the most favourable scale.

This is dependent on the following:

(i) The optimum model scale (S^{o} m)

(ii) Angular field of view.

(iii) Maximum and minimum flight heights (H'max and H'min respectively).

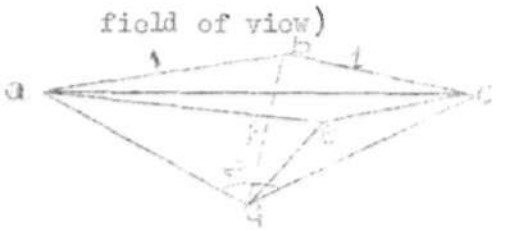
(iv) Availability of horizontal gear combinations for the desired mapping scale (i.e. if mapping is required).

(i) Optimum model scale (S^{o} m)

$S^{o}m = Zo. Sp = \frac{490mm}{152.676mm} \times 1/8000$

$S^{o}m = 1/2,493.$

(ii) Angular coverage of taking camera (angular field of view)



$l = 23cm = 230mm$

$f = 152.676mm$

Fig. 2.4. Determination of Angle of view.

From the above,

$\tan \theta/2 = \frac{\frac{1}{2}(2l^{2})^{\frac{1}{2}}}{f}$

$\theta = 2\tan^{-1}\left(\frac{l}{f(2)^{\frac{1}{2}}}\right) = 2\tan^{-1}\left(\frac{230}{152.676(2)^{\frac{1}{2}}}\right) = 93^{o}.62.$

The photography was wide angle since ϕ is more than 75° and less than 100°.

75

(iii) Maximum and minimum flight altitide above ground (H'max and H'minimum respectively)

H'max = average flight height (H') - Relief of the lowest point above mean ground level (Δ hmin)

H'min = average flight height (H') - Relief of the highest point above mean ground level (Δhmax)

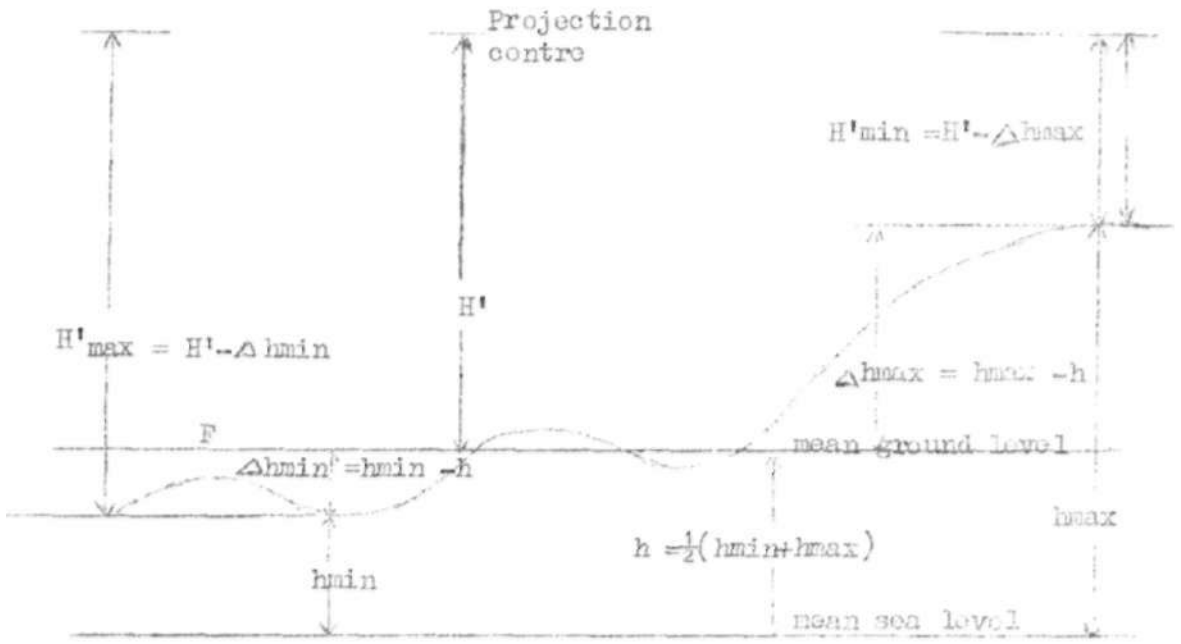


Fig. 2.5. Determination of H'max and H'min (maximum and minimum flight altitude above ground respectively).

Average flight height (H') is given by H' = f. $^{1}/_{Sp}$

= 152.676mm x 8000

= 1221.408m.

$h = {}^{1}/_{2}(h_{min} + h_{max})_{m} = \frac{1}{2}(429.07 + 498.72)_{m} = 463.90m$

$\Delta h_{min} = h_{min} - h = -34.83$

$\Delta h_{max} = h_{max} - h = 34.83$

$H'_{max} = H' - \Delta h_{min} = 1221.408 - (-34.83)_{m} = 1256.23m$

$H'_{min} = H' - \Delta h_{max} = 1221.408 - 34.83 = 1186.58m.$

(iv) Horizontal gear ratio combinations.

Table 2.2 Table for the selection of autography scales and gear ratios.

Normal Angle	Wide Angle				Gear Combination	Smallest division(value of the least division(m.)	Auto graph Scale 1:		Desired Mapping Scale
	Metres		Feet						
	H'max	H'min	H'max	H'min					1:2500
	1200	600			Bd	.01	3000		N/A
	1330	670			C	.1	3333		3-1-1
	1500	750			Cd	.01	3750		1-2-1
	1600	800			Db	.1	4000		N/A

The autograph (model) scale 1/3750 was selected as the most suitable model scale for the project because under wide angle photography; it is the closest even scale to the optimum model scale 1/2493 which also meets the pertinent requirement viz $1186 < H' < 1256m$ and has horizontal gear combinations.

. Determination of Gear Combination.

(i) Horizontal gear combination and gear ratio:

From the table above (table 2.2), the horizontal gear combination applicable to this project was obtained as 1-2-1 by looking up row of autograph scale 1/3750 and colum of mapping scale 1/2500.

The gear ratio is obtained from the manual as

Gear Ratio	Position of Pull out control knob		
:1	I	II	II
	1	2	1

(ii) Vertical gear combination and value of smallest division of height counter dials:

The vertical gear combination Cd and the value of the smallest division of the height contour dails 0.01m were found from table 2.2. to apply to this project. These were obtained by by

(i) looking up the row of the autograph scale 1/3750 against the column of combination to obtain the vertical gear combination, and

(ii) looking up the row of the autograph scale 1/3750 against the column of smallest interval to obtain the value of the smallest division of the height contour diels.

2. Determination of the Mean Projection height (Z)mn

$$Z = Sn \times H' = f \times Sn \times H'/f = f \times Sn/Sp$$

$Z = f \times {}^{Sn}/_{Sp}$ where ${}^{Sn}/_{Sp}$ is enlargement ratio.

$Z = 152.676 \times \frac{1}{3750} \div {}^{1}/_{8000}$

$Z = 152.676 \times \frac{8000}{3750} = 325.7088mm$

$Z = 325.709mm$

4. Determination of initial Model Base $(bx)mm$

bx was computed from

$bx = Sn/Sp \times b$

where S_n =model scale = 1/3750

Sp = photograph scale = 1/8000

b = 86.5mm as determined from photo prints (see table of preliminary data in 2.2.1.1).

$bx = 86.5 \times \frac{1}{3750} \times \frac{8000}{1} = 184.533mm.$

(b) Settings of Instrument Fittings:

The instrument fittings as determined in section 2.2.1.1 were set strictly in the following order:

(i) Height gear : Cb

(ii) The average projection height (325.709mm) on the Z - column

(iii) The height gear combination: Cd

(iv) The horizontal gear combination: 1-3-1

(v) Model base of 184.533mm

(c) Settings of optical and mechanical fittings:

Table 2.5 Table of the optical and mechanical fittings of Wild A7

	Negatives		Positives	
Base			Inside	Outside
Dove prism: Indices:			Outside	Inside
Triangulation lever:			back	front
Diapositive lever:			left	left
Sense of rotation of hand wheels				
X - knob			in	in
Y - knob			out	out
Y - Z - knob			back	back
Transmission to drawing Table:				
X - collar :			in	in
Y - collar :			out	out
X - Counters: numerals/ figures			black	black
Y - Counters: numerals/ figures			red	red

Dove prisms, triangulation lever, diapositive lever, sense of rotation of the handwheels and sense of transmission to the drawing table were read from table above.

(c ...ll the orientation elements were next set in their z ro positions as follows:

(i) $by' = by'' = bx'' = bx'' = 100.00mm$

(ii) $k' = k'' = 400.00^{g}$

(iii) $w' = w'' = \phi' = \phi' = 100.00^{g}$

With these final settings, the instrument was then ready for observation procedures to be performed.

2.2.2 Relative Orintation Procedures on Wild A7

There are three steps required to orient a stereoscopic plotting instrument in order to obtain a true stereomodel. The first of these is inner or orientation followed by relative orientation and then absolute orientation. Therefore to carry out relative orientation on the Wild A7, the interior (inner) orientation was performed first.

Inner Orientation Procedures

This involved two stages:

(i) centering the diapositives

(ii) setting off the principal distance in each restitution camera.

(i) Centering the diapositives: The paper prints were examined for numbering and establishment of direction of flight (left to right or East to West) and sequence of exposure (increase from east to west). The diapositives were examined to find out their emulsion face (by the unorthodox method of using a razor blade to make a small scratch on

the margin of both faces). The photo carriers were then brought down from the instruments and placed over the light box (provided with the Wild A7). The diapositives were then placed over the photograph carriers bearing the following in mind:

(a) Emulsion side of positives must always face downwards.

(b) The diapositives were so oriented that the direction of flight lies parallel to the X - direction.

(c) When the instrument was to be used in the base - in mode (i.e. for model 1) the overlap of photographs was outside while in the base-out mode (i.e. for model 2) the base was inside.

(d) The fiducial marks of the photograph are brought into perfect coincidence with those on the photo carriers using a magnifyer so as to ensure that the principal points of the diapositives coincide exactly with their corresponding centres on the photo carriers. Manual of A7.

The photo carriers were then returned to the instrument.

(ii.) Setting off the Principal distance; This part of inner orientation was completed only when the principal distance of each plotting camera was precisely set to the same value as the calibrated constant (focal length of 152.676mm) of the taking camera, (see preliminary data table in 2.2.1.).

This was done by adjusting, raising or lowering the diapositive image plane by means of the graduated column ring. This was done with an accuracy of 0.01mm on the Wild A7. This completed the inner orientation on Wild A7.

... Empirical Relative Orientation Procedures on Wild A7

Having set up the model at the instrument, the model was viewed through the eyepiece where a three dimensional image was seen through the optical system. The points in the model were located with the movements of the X and Y handwheel which impart the motion of X and Y to the half marks which are the measuring marks. Since the model was not parallax free at this stage, the half marks did not fuse together rather they seemed to float in space.

Out of the ten available independent elements of exterior orientation, only five are required to carry out relative orientation as earlier stated.

Usually the base components by', by", bz', bz" were kept at a constant zero. Hence the base in each of the two models lay in the direction of the X – axis.

There are two types of Empirical relative orientation that may be transformed on the *Wild A7*.

(i) Relative Orientation of independent pairs of photographs.

(ii) Relative Orientation of subsequent pairs of photographs.

The relative orientation of independent pairs of photograph was used in each of the two models.

(i) Relative Orientation of Independent Pairs of Photographs

The parallax Equation was given as

$$Py = -\frac{XY}{Z}d\phi' + \frac{X-b}{Z}Yd\phi'' + Xdk' - (X-b)dk'' - (1+\frac{Y^2}{Z^2})Zdw''$$

(see Appendix A for the derivation)

Therefore the elements used were ϕ', ϕ'', k', k" and w" to clear parallax in the model. The two cameras were oriented to the fixed base and this consequently makes the absolute orientation that might follow relative orientation simpler since the base components in the X– and Y-directions are zero.

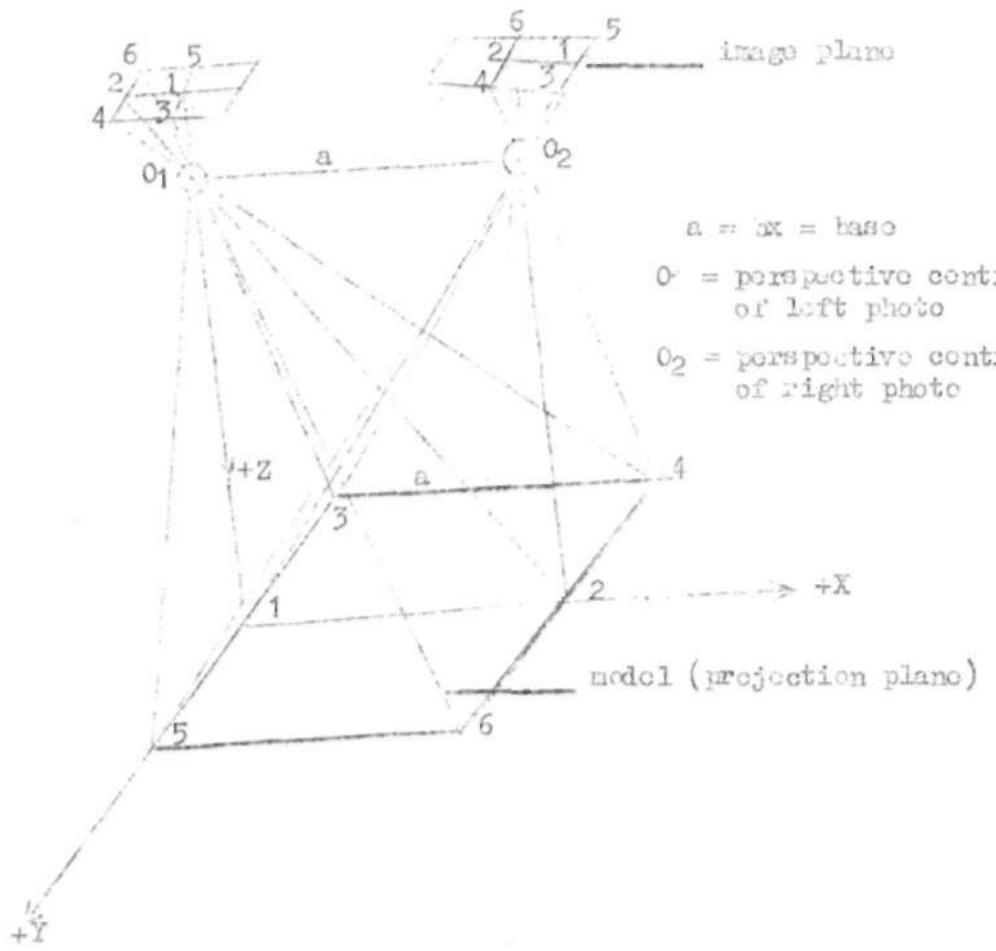


Fig. 2.6. Disposition of points in the model.

The figure above shows the correct disposition of points in each of the two models. The procedrue below make use of these points as they are located.

Procedure:

Step 1: At point 1 parallax was eliminated with k'

Step 2: At point 2 parallax was eliminated with k"

Step 3: At point 4 parallax was eliminated with ϕ'

Step 4: At 3 parallax was eliminated with ϕ''. w" read as w_3 (i.e. $w'' = w_3$)

Step 5: At point 5 parallax was eliminated with w". w" read as w_5 (i.e. $w'' = w_5$)

Step 6: At point 6 parallax was eliminated with w". w" read as w_6 (ie. $w'' = w_6$)

Step 7: w" set at $w_o = w_3 + \left\{\frac{w_5 + w_6}{2} - w_3\right\}n$

where n = over correction factor computed

from: $n = \frac{1}{2}\left(1 + f\,{}^{d}/_{2}\right)$. Manual of A7.
$\quad\quad\quad

The empirical relative orientation carried out on the Wild A7 by this author is used here as a check on the results of the small incrementsoon the elements of the relative orientation obtained by numerical method.

Numerical Relative Orientation on the Wild A7

The derieved parallax equation on the Wild A7 system is:

$$Py = \frac{-XY}{Z}d\phi' + \frac{X-b}{Z}Yd\phi'' + Xdk' - (X-b)dk'' - (1+\frac{Y^2}{Z^2})Zdw''$$

(see Appendix A for derivation)

using the elements ϕ', ϕ'', k', k", and w" for independent method of relative orientation. Here none of the elements of orientation of either restitution camera is known. The two cameras are oriented to the base.

Procedure:

Step 1: all elements were set to their zero positions as in 2.2.1.2.

Z was fixed at 325.709mm as computed in section 2.2.1.1. This was set by moving the z - column to read 325mm with the foot disc while making the z counter to read 4995. Then the z - counter was gently reduced to 4994.292mm approximately. This made the reading on the z column 325.709 since the height counter gives decreasing readings when the base carriage moves downwards (that is increase in reading on z - column), manual of A7.

Step 2: The extent of the model in X and Y directions was determined by moving the X- and Y- carriages with the aid of the X- and Y- handwheels to reach the four extremes of the model.

Step 3: The extent was demarcated mentally such that not less than twenty-five (25) points would be conveniently accommodated within the chosen model.space and evenly or symmetrically distributed all over the model space in a 5 x 5 dimensional spread. *see* fig 2.7a.

Step 4: Parallax measurements were performed using the by"-knob at each of the twenty five points.

The model was scanned in X and Y directions with the X and Y handwheels and these twenty-five points which had been chosen as natural points (points at road junctions, farm edges, shrub edges, edges of houses etc.) were then used to clear Y-parallax. X-parallax was removed with the bx-knob.

Step 5: at each point the following were recorded for each a point (a) X- coordinate (b) Y- coordinate (c) bx (d) by". Observations at the principal point of the left camera were done as at other points and the X- and Y- coordinates (X_o, Y_o) recorded as those of the origin of the model.

The following precautions were taken:

(a) operational steps were carried out very carefully to avoid blunders.

(b) all observations were done in one direction only as shown in fig 2.7b.

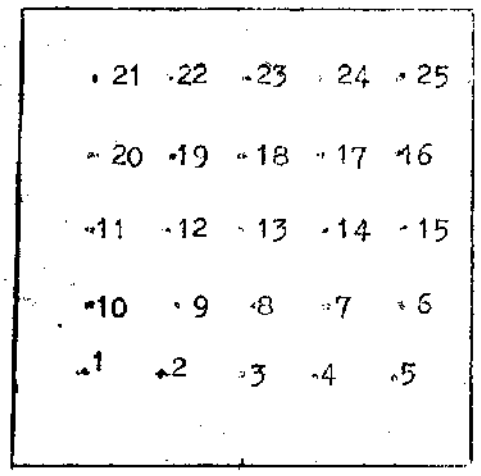


Fig. 2.7a Location of Twenty-five points in a Model

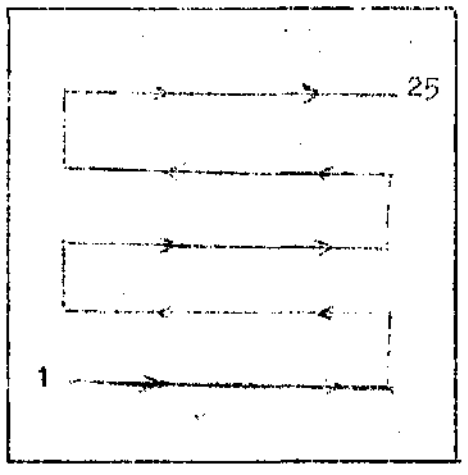


Fig. 2.7b Direction of Observation in the Model

(c) In the elimination of the X and Y- parallaxes at a point, elimination was done such that the larger of the two parallaxes was eliminated first

(d) care was taken not to upset the setting of Z = 325•709mm in the Z - column by not touching the foot disc all through the observations so as to maintain a constant Z.

Chapter 3

COMPUTATIONS AND ADJUSTMENTS

3.1 Computations

This included

(i) The reduction of the raw data,

(ii) The computations of the coefficients of the elements of relative orientation to be used in the parallax equations.

(i) Reduction of raw data:

The pocket calculator was used for all reductions of the raw data.

Two sets of readings were taken for each of the parameters measured during the numerical relative orientation. Each set contains Xi, Yi, bxi, byi, The average of each of the two readings was obtained by rounding up and recorded as shown in form 1.

The parallax equation developed for the Wild A7 system (as applicable to this project) is given by

$$Py = \frac{-XY}{Z}d\phi' + \left(\frac{X-b}{Z}\right)Yd\phi'' + Xdk' - (X-b)dk'' - \left(1 + {}^{Y^2}/_{Z^2}\right)Zdw''$$

(see appendix A for derivation)

where

$Py = byi - by_m$ Ghosh (1960)

$by_m = \frac{1}{n}\sum_{i=1}^{n} by_i$ with n = 6, 9, 12, 15 as the case may be

$X = Xi - Xo$

$Y = Yi - Yo$

$b = bxi$

Xi, Yi are the instrument (raw) coordinates for point i for i = 1, 2, - - - - - - - - -, 25.

Xo, Yo are the instrument (raw) coordinates of the left perspective centre which is the origin of the model.

Z = constant projection distance already set at the instrument at 325.709mm.

The above parameters were computed and recorded as shown in

(ii) The computations of the coefficients of the elements of the relative orientation:

These were computated as follows using the pocket calculator.

Element	Coefficient
$d\phi'$	$-\frac{XY}{Z} = \frac{-(X_i-X_o)(Y_i-Y_o)}{Z}$
$d\phi''$	$(\frac{X-b}{Z})Y = \frac{(X_i - X_o - bx_i)(Y_i-Y_o)}{Z}$
$d\kappa'$	$X = (X_i - X_o)$
$d\kappa''$	$-(X-b) = X_i - X_o - bx_i$
dw''	$-(1+Y^2/Z^2)Z = \left\{ 1+ \frac{(Y_i-Y_o)^2}{Z^2} \right\} Z$

See table of coefficients in Appendix B for the computations above.

3.2 Adjustments

The adjustment was done using the least square routine.

From 3.1, the parallax equation may now be written in the form

$$by_i - by_m = -\frac{(X_i - X_o)(Y_i-Y_o)}{Z} d\phi' + \frac{(X_i-X_o-bx_i)(Y_i-Y_o)}{Z} d\phi''$$

$$+(X_i-X_o)dk'-(X_i-X_o-bx_i)dk''-\left(1+\frac{(Y_i-Y_o)^2}{Z^2}\right) Z dw'' \quad \text{.....(a)}$$

This is an observation equation which is in the form

$$Ax = B \quad \text{.....(b)}$$

where A is the matrix of the coefficients and of dimention M x N where

M is the number of equations

N is the number of unknowns

Note that $by_m = \frac{1}{n}\sum_{i=1}^{n} by_i$

where N = 6, 9, 12, 15 in the respective cases

$$A = \begin{bmatrix} -\frac{(X_1-X_o)(Y_1-Y_o)}{Z} & \cdots & -\left(1+\frac{(Y_1-Y_o)^2}{Z^2}\right) Z \\ \vdots & & \vdots \\ -\frac{(X_M-X_o)(Y_M-Y_o)}{Z} & \cdots & -\left(1+\frac{(Y_1-Y_o)^2}{Z^2}\right) Z \end{bmatrix} \quad \text{.....(c)}$$

M varied between 6, 9, 12 and 15 in the respective 6, 9, 12 and 15 points cases.

X isthe column vector of unknowns as follows:

$$x = \begin{bmatrix} d\phi' \\ d\phi'' \\ dk' \\ dk'' \\ dw'' \end{bmatrix} \qquad \text{.....(d)}$$

B is the matrix of contants byi - by_m

so that

$$B = \begin{bmatrix} by_1 - by_m \\ by_2 - by_m \\ \vdots \\ by_n - by_m \end{bmatrix} \qquad \text{.....(e)}$$

From equation (b)

$Ax = B$ multiplying both sides by A^T

then $A^TAx = A^TB$ if weighted becomes

$A^TWAx = A^TWB$ .....(f)

$x = (A^TWA)^{-1} A^TWB$ .....(g)

Since the observations in this project were done under the same conditions of (i) within a few hours for each model, (ii) on the same day, (iii) under the same weather conditions (instruments were in an airconditioned room) and (iv) by the same operator the observations are taken to be of unit weight

$x = (A^TA)^{-1} A^TB$ .....(h)

A^TA is the N x N positive, definite and symmetrical normal matrix which features in the normal equations.

X is now substituted in the original equation to solve for V (residuals); $Ax - B = v$ .....(i)

Each observation equation has a residual such that

$$by_i - by_m + v_i = - \frac{(X_i - X_o)(Y_i - Y_o)}{Z} d\phi' + \frac{(X_i - X_o - bx_i)(Y_i - Y_o)}{Z} d\phi''$$

$$+ (X_i - X_o) dk' - (X_i - X_o - bx_i) dk'' - \left\{ 1 + \frac{(Y_i - Y_o)^2}{Z^2} \right\} Z dw'' \quad \ldots\ldots (j)$$

from $v = Ax - B$

$$v_i = - \frac{(X_i - X_o) d\phi' (Y_i - Y_o)}{Z} + \frac{(X_i - X_o - bx_i)(Y_i - Y_o) + (X_i - X_o) dk'}{Z}$$

$$- (X_i - X_o - bx_i) dk'' - \left\{ 1 + \frac{(Y_i - Y_o)^2}{Z^2} \right\} Z\, dw'' - (by_i - by_m) \quad \ldots\ldots\ldots (k)$$

Estimation of accuracy of relative orientation is based on the standard error of measurement or determination of each element of relative orientation.

Standard error is computed from the variance. After solving for the residuals (vs) variance σ_o^2 is computed from

$$\sigma_o^2 = \frac{V^T W V}{M - N} \quad \ldots\ldots\ldots (l)$$

and for an observation of unit weight,

$$\sigma_o^2 = \frac{V^T V}{M - N} \quad \ldots\ldots\ldots (m)$$

and standard error is given by

$$\sigma_o = \left(\frac{V^T V}{M - N} \right)^{\frac{1}{2}} \quad \ldots\ldots\ldots (n)$$

The least square programme 1.4 (courtesy of Mr. A.A. Rambeko) was used to solve for

(i) The unknown (Xs)

(ii) The residuals (Vs)

(iii) the root mean square error (RMSE)

(iv) The standard errors (σs) or I1,. . . ., I5)

See Appendix C for the least square programme.
Appendix B gives the final results.

The least square programme is explained as follows:

3.2.1 Explanation and Execution of the Least Square Programme 1.4

(1) DISP means display

(2) DEG means degree, that is progrmme will give answers in degrees, therefore all unknowns (X(1);.....; X(N) as well as the standard deviations (I1; .....; IN) are converted to their respective values in grads by multiplying by K, where K = 200:pie (PI).

(3) FIXED 5 means working to five places of decimal.

(4) DIM means dimension where

A is the matrix of coefficient

B is the column matrix of constants

C is the normal matrix

D is the transpose of A multiplied by the column vector of constants B

E is the inverse of C

F is the transpose of A

X is the column vector of the unknowns (dø, dø", dk', dk", dw",)

V is the matrix of residuals

M is the number of observations.

N is the number of unknowns.

(5) Steps 70, 80, 90 allow for adequate space for data input.

(6) TRN means transpose

(7) MAT means matrix

(8) DET means determinant.

(9) INV means inverse of

(10) In step 230 MAT E is the variance covariance matrix.

(I) is the unit weight variance

E is the weight coefficient matrix

(11) In step 260 I1, ... IN means the standard deviation of each determination or measurement of the number of unknowns.

(12) / means division except in the line 290 where it means or.

(13) * means multiplication

(14) ↑2 is square of

(15) ↑0.5 means square root of

(16) R.M.S.E means Root mean square error SOLUTION.

KASHIM IBRAHIM LIBRARY
AHMADU BELLO UNIVERSITY
ZARIA, NIGERIA.

310299

(i) The solution of the unknowns (X S):

$(A^TA)^{-1}A^TB$; MATF in line 100 gives A^T

MAT in line 120 gives A^TB

MATC in line 110 gives A^TA

MATE in line 140 gives $(A^TA)^{-1}$

line 150 solves X as MAT X = E

(ii) Solution of the residuals:

$V = Ax - B$

lines 180 and 190 help to solve x i.e.

line 180 gives Ax i.e. MAT V = A X

line 190 gives AX - B MAT V = V - B

(iii) The unit variance σ_o^2 and variance-covariance σ_{xx}^2 are solved in lines 220 and 230 respectively.

In each of the cases of six points, nine points, twelve points and fifteen points, there were six, nine, twelve and fifteen observation equations respectively in five unknowns ($d\phi'$, $d\phi''$, dk', dk'', dw'') or (X S). These five unknowns gave a 5 x 5 normal matrix (A^TA) in each case. Therefore the number of redundant observations (M - N) in each case was (6 - 5)=1 for six point case, (9 - 5)=4 for nine point case, (12 - 5)=7 for twelve point case and (15 - 5)=10 for fifteen point case.

The residuals were six, nine, twelve and fifteen in number for the respective cases of six nine, twelve and fifteen points.

The variance σ_o^2 is obtained from line 220 while the standard errors are obtained from line 260.

Execution of the programme is performed by first switching on the instrument and then the programme is typed in.

There after the data were fed in in the following order:

1. All the coefficients in the M number of equations from 1 to 2 to ..... M

2. The constants (Bs) from equation 1 to 2 ..... to M and required results were obtained finally.

See Appendix B for final results.

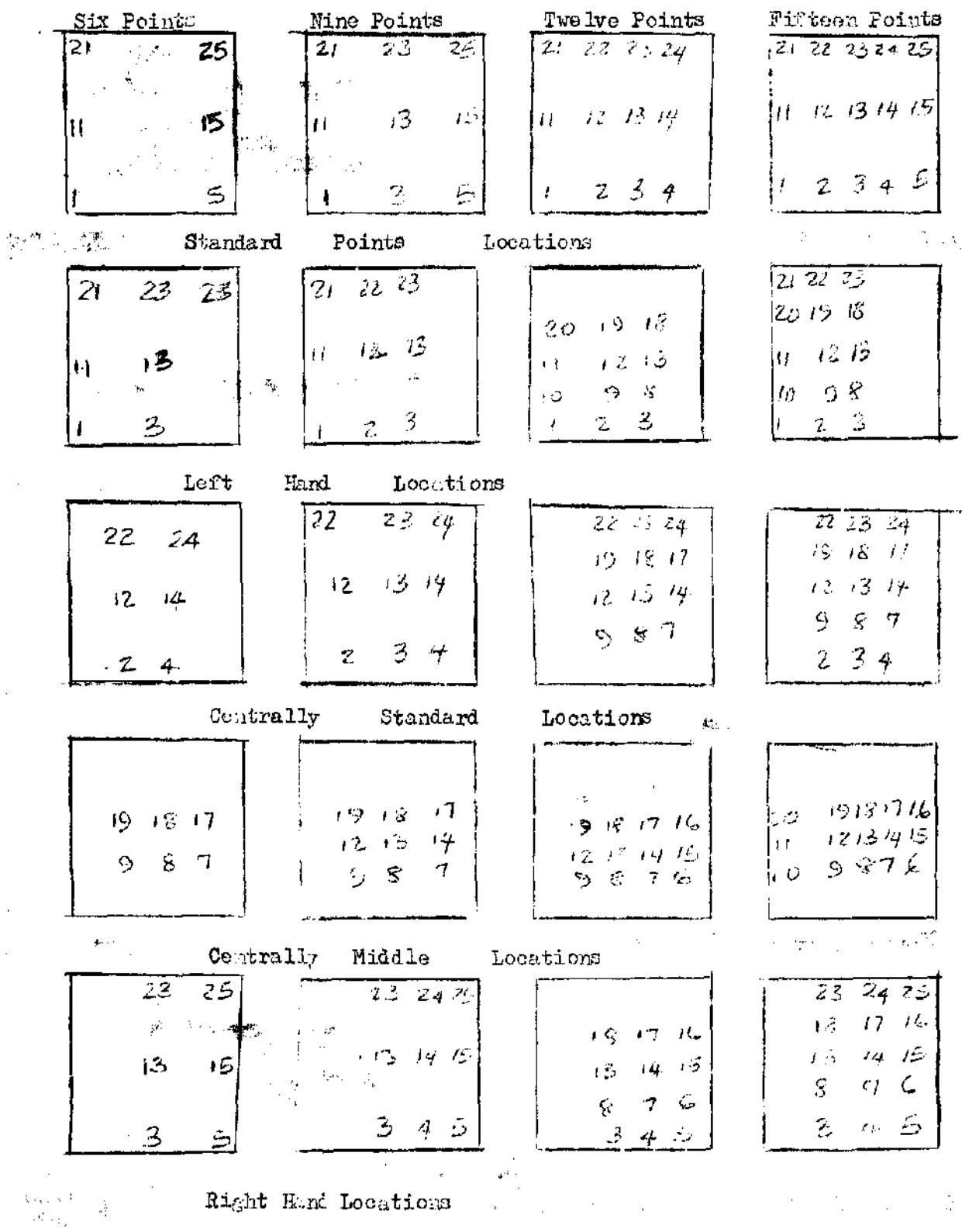


Fig. 4.1 Distribution and Densities of Points in the Model.

Chapter 4

ANALYSES OF RESULTS

The analyses are based on the following:

(a) The effects of the distributions of the points on accuracy.

(b) The effects of the densities of the points on accuracy.

In chapter three, it was stated that accuracy estimation is determined from root mean square error(which gives rise to standard error). Therefore the accuracy estimation is based on the standard error of measurement or determination of each element of relative orientation.

4.1 Effects of the Distribution of Pass Points on Accuracy

In this case, five describable locations (shown in figure 4.1) were chosen in the model as follows:

(a) Standard Points Locations: These points were located as closely as possible to the points commonly referred to as standard points in a model.

(b) Left hand side location.

(c) Centrally standard location

(d) Centrally middle location

(e) Right hand side location.

The main criterion for choosing these locations was that it was absolutely necessary to maintain a uniformity in the location of points, irrespective of the number of points involved, for convenience and ease of analyses. Only these five locations can conveniently accommodate the six points case, the nine points case, the twelve points case as well as the fifteen points case.

Both models were analysed together. The effects of the distribution of points on accuracy of determination of each element of relative orientation are investigated. The analyses are as follows: (See table D.1 and D.2 in Appendix D) $d\emptyset'$: The standard point position yielded the least standard error in about seventy-five percent (75%) of the cases while in all the cases, the centrally middle locations yielded the highest standard error hence the poorest accuracy.

$d\emptyset''$: Here also very good accuracy can be observed in the standard points location for seventy-five percent (75%) of the cases. The centrally middle locations recorded the poorest accuracy with very high standard error. As can seen from the Table D.1, the left hand side, right hand side and sometimes the centrally standard locations gave fairly good results which are comparable to those of the Standard point location. For example, in the nine point density in the second model, the left and right hand side locations gave better results than those of the standard points locations.

dk': Except for the left hand side position in the second model, in both models standard locations yielded good results generally. Also the right hand side in densities fifteen in model one, nine and fifteen in model two gave good results, as well as the centrally standard in model one.

dc": Out of the eight available densities (four densities for each model), the standard point location yielded excellent results in three as follows: nine and twelve in model one and twelve in model two. In all others, the different locations yielded very good results except the centrally middle which gave very poor results and the right hand side in six, nine and twelve densities in model two.

dw": In about fifty percent (50%) of the cases, the standard points location yielded the best results, followed closely by the left and right hand locations and the centrally standard locations. The centrally middle location yielded the worst results in all the cases, and produces the same poor accuracy for all the other elements as can be seen from Tables D .1 and D .2 (Appendix D).

4.2 The Effects of Densities of the Pass Points on Accuracy

The consideration of densities is based on the number of points used. Four cases are treated as follows:

(a) Six points case.

(b) Nine points case.

(c) Twelve points case.

(d) Fifteen points case.

Here the case that yielded the least standard error of the determination of correction to each element of relative orientation gave the best results. The analyses are as follows for each correction to element of relative orientation.

$d\phi'$: The least standard error of the determination of this correction to the element of relative orientation (ϕ') was obtained in the six points case with the next best result occurring in the fifteen point case in both models. The worst results were obtained in the twelve point case in about eighty percent of the locations.

$d\phi''$: Here again the best result was obtained in the six points case for both models. The fifteen points case and nine points case gave fairly good results. Twelve points case did not give good results.

dk': The fifteen points case and the six points case produced good accuracies in that order since both case gave small standard errors. It can be seen that the twelve point case in model two gave better results than that of the nine point case although in model one it gave the worst results.

dk": The six point case produced very good accuracy and also the fifteen point case. The twelve point case did not yield good accuracy.

dw": Generally the fifteen points case and the six points case produced good results for the determination of this correction to the element w". The nine points case produced

some fair results. Here again the twelve points case gave very poor results.

It should be noted that in the six points case in both models the centrally middle location yielded extremely poor results which are not comparable to any other results in the analyses tables. The explanation for this is given in chapter five (see 5.1).

Chapter 5

SUMMARY AND CONCLUSION

5.1 Summary

From the table of analyses, it can be seen that the centrally middle locations in all the cases yielded very poor results especially in the six point case. In the centrally middle locations, points were located very close to the base line (or were clustered around the line joining the nadir points of each model). The very poor results may be explained right away as having been caused by the fact that the points were so closely located to each other that the X - and Y - coordinates of such points were too close to give a good solution. According to Van der Weele (1960), the precision of the determination of the unknowns increases with increasing differences between the proportions of the coefficients of the unknowns in the different equations. This condition will not hold when these points are located close to each other since the coefficients of the unknowns are obtained from the X- and Y- coordinates of points used in the clearing of parallax. Therefore the precision of the determination of such unknowns will be very poor. From the table of analyses, it can be seen that that is what obtained in all the cases treated.

For all the cases, the standard error of the determination of $d\phi'$ and $d\phi''$ almost tallied while that of dk' and dk'' almost tallied. This is expected since $d\phi'$ and $d\phi''$ produced similar effects in a model that is no effect along the base line but

maximum y-parallax at points four and six with opposite signs for dØ' and at points three and five with opposite signs for dØ" (Sec 1.2.1) Same explanation holds for dk' and dk" where each has effects that compliment the other (Fig. 1.2).

In general the standard points location using six points and fifteen points gave fairly good results. The results in the nine points were fair enough. The twelve points gave very poor results generally. From figure 4.1, it can be seen that the spread of the twelve points case is slightly different from all others and may therefore not yield results that may be compared with the others.

Looking at the analyses tables, (Appendix D), it can be observed that for all locations and for all the elements increase in points produced better accuracy of determination of dØ', dØ", dk', dk" and dw" in most cases (say in about eighty five percent (85%) of the cases).

5.2 Conclusion

From the foregoing it then can be said that:

(i) Increase in number of points does not necessarily mean an increase in accuracy.

(ii) The location in of the points in the model has a great part to play in the accuracy expected.

The small discrepancies may be due to difficulties encountered in the accurate location of points in both models since it was difficult to choose some points due to (i) poor quality of photograph (ii) the position of some natural points.

In other words, both models were very difficult ones.

In conclusion therefore, the accuracy of the numerical relative orientation on the Wild A7 autograph depends on the density and the distribution of points in the model barring all systematic errors due to camera and lens distortions, poor quality photographs, capability of the observers and an assumption that all light rays travelled in straight lines.

In performing relative orientation on the instrument by numerical methods, six points may be sufficient to clear the model of parallaxes, except in extremely very precise work where more than six may be used. However for very difficult models, it is best to examine the model properly and choose the best natural points for removing parallax. These points must however be as close as possible to the six standard lpoints location (figure 1.1) universally accepted for clearing y-parallax in a model. The reason for this is that the nadir points (points one and two in figure 1.1) have the simplest relations between the y-parallax and the orientation elements and secondly the four corner points (3, 4, 5, 6, of figure 1.1) being farthest away from the nadir points and from each other will help in the solution of which will give best results.

REFERENCES

1. Allan, A.L., Hollway, J.R., Hayness, J.H.B. (1958) Practicial Field Surveying and Computations. William Heinemann Ltd., London, England. pp. 215-219.

2. Bomford, G. (1977), Geodesy Oxford University Press. Oxford England. pp. 604-633.

3. Born, J.C., Makarovic, B. and Speakman, E. (1980) Plotting Instruments with mechanical and optical trains. In: Born, J.C. (Ed.) Manual of photogrammetry. Fourth Edition - American Society of photogrammetry 105 W. Virginia Ave. Falls Church Va 22046, U.S.A.

4. Cooper, M.A.R., (1974) Fundamentals of survey measurements and analysis. Crosby Cockwood Staples, London. England. pp. 1-43.

5. Ghosh, S.K. (1968). The Theory of strereophoto grammetry. Edward Brothers Inc. Michigan, U.S.A.

6. Hallert, B. (1960). Photogrammetry. Basic principles and General Survey. McGraw-Hill Book Company, New York. U.S.A. pp. 257-294.

7. Hirvonen, R.A. (1971). Adjustment by least squares in Geodesy and photogrammetry. Federick Ungar Publishing Company, New York. U.S.A. pp. 1-19.

8. Rainsford, H.R. (1957). Survey Adjustment and Least Squares. Constable and Company Ltd., London. England. pp. 12-74.

9. Slama, C.S., Ebner, H., and Fritz, L. (1980). Aerotriangulation. In: Slama, C.S. (Ed.) Manual of photogrammetry. Fourth Edition - American Society of photogrammetry. 105 W. Virginia Ave. Falls Church, Va 22046 U.S.A. pp. 459-460.

10. Van Der Weele, Ir. A.J. (1960). The Principles of relative orientation and orientation procedures. Unpublished lecture notes. International Institute for Aerial Surveys and Earth Science (ITC) Delft Holland.

11. Wolf, P.R. (1974) Elements of photogrammetry McGraw-Hill International Book Company. Tokyo. Japan. pp. 291-300.

12. The manual of casio scientific calculator fx 3600p

13. The manual of Hewlett Packard desk calculator 9830A model 30.

14. The manual of Wild A7 autograph.

Appendix A.

Derivation of the Differential Formulas for Single Photographs From First Principles, Hallert (1960)

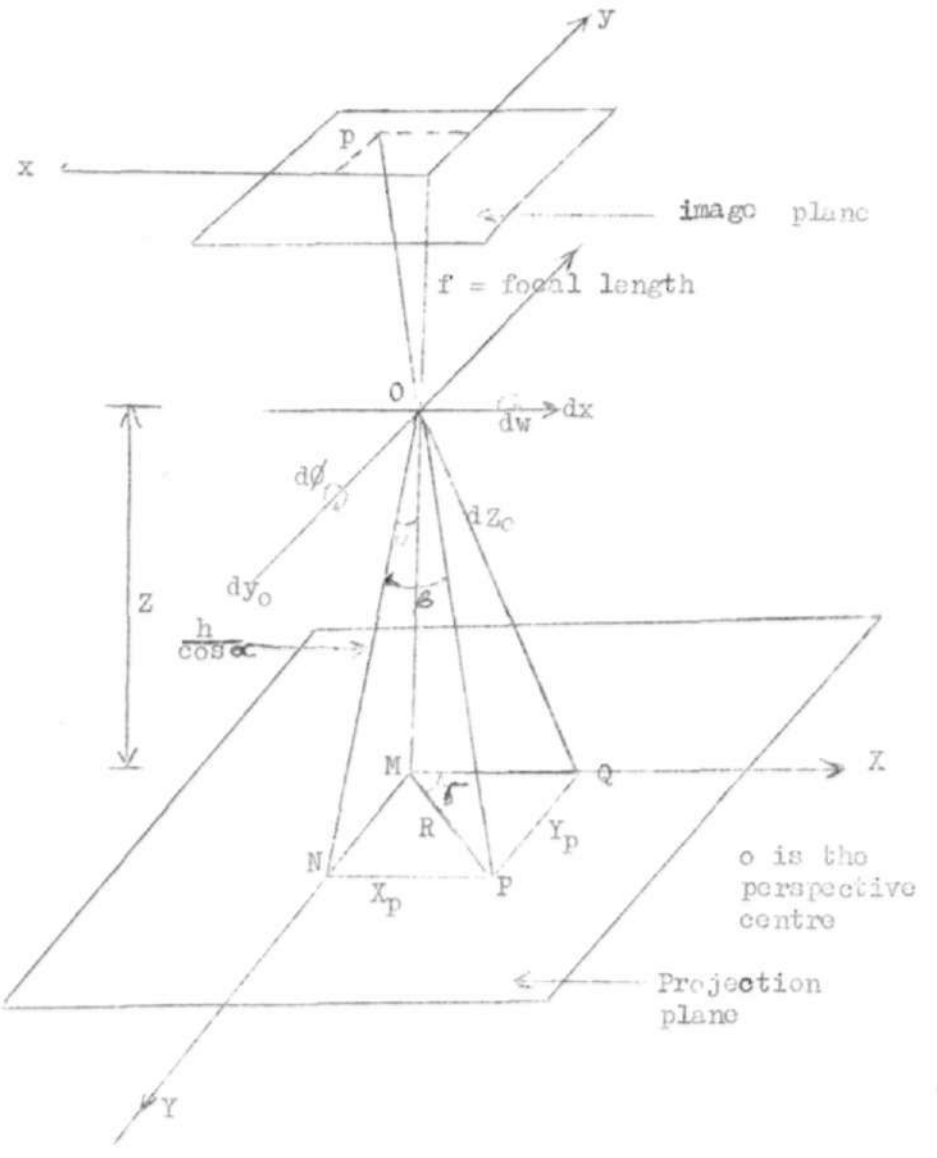


Fig. A.1 Single photograph and projection plane situation.

The following assumptions are made:

(a) Approximate vertical photographs are assumed

(b) The wild system of coordinates is assumed (see Fig. 2.1)

(c) w, ∅, k -- rotations are about their respective x y^{w} and $z^{w\emptyset}$ axes

(d) x, y, z are for the image plane while X, Y, Z are for the projection plane

(e) Derivation is on the projection plane.

In the above figure, let P' a point in the image plane (i.e the photograph) be projected in P in the projection plane (i.e. the model plane).

It is required to determine the changes dx and dy of the coordinates of point P as functions of the small stranslations dx_o, dyȯ, dz_o of the perspective centre O and the photograph as well as of the small rotations d∅, dw, d' around their respective dy_o, dx_o dz_o axes.

From first principles from the diagram for point P on the projection plane

$$X = R \cos r \qquad \text{---------- (1)}$$

$$Y = R \sin r \qquad \text{---------- (2)}$$

$$\cos \alpha = \frac{Z}{ON} \qquad \therefore ON = \frac{Z}{\cos \alpha}$$

$$\tan B = \frac{X}{ON} = \frac{X \cos \alpha}{Z}$$

$$X = \frac{Z}{\cos \alpha} \tan B \qquad \text{---------- (3)}$$

$$\tan \alpha = \frac{Y}{Z}$$

$$Y = z \tan \alpha \qquad \text{——————————} (4)$$

from similar triangles

$$X = \frac{xZ}{f} \qquad \text{——————————} (5)$$

$$Y = \frac{yZ}{f} \qquad \text{——————————} (6)$$

from equations (1) and (2) differentiating from first principles.

$$dX = - R \operatorname{Sin} r \, dr$$

but $Y = R \operatorname{Sin} r$

$$dX = - Y dr \qquad \text{——————————} (7)$$

Also $dY = R \cos r \, dr$

but $X = R \cos r \, dr$

$$dY = X \, dr \qquad \text{——————————} (8)$$

differentiating equations (3) and (4) will result in

$$dX = \frac{Z \tan B \operatorname{Sin} \alpha \, d\alpha}{\cos^2 \alpha} \qquad \text{——————————} (9)$$

$$dY = \frac{Z}{\cos^2 \alpha} \, d\alpha \qquad \text{——————————} (10)$$

$$\cos \alpha = \frac{Z}{NO}$$

$$\operatorname{Sin} \alpha = \frac{Y}{NO}$$

from pythogoras theorem

$$NO^2 = Z^2 + Y^2 = Y^2 + Z^2$$

$$NO = (Y^2 + Z^2)^{\frac{1}{2}}$$

$$\cos \alpha = \frac{Z}{(Y^2 + Z^2)^{\frac{1}{2}}} \qquad \text{——————————} (11)$$

$$\operatorname{Sin} \alpha = \frac{Y}{(Z^2 + Y^2)^{\frac{1}{2}}} \qquad \text{——————————} (12)$$

Similarly $\tan \beta = \frac{X}{NO} = \frac{X}{(Y^2 + Z^2)^{\frac{1}{2}}}$ ..... (13)

Substituting (11) and 12) and (13) ijto equations (9) and (10) will result in

$$dX = \frac{XY}{Z} \quad d\alpha \quad \text{..... (13a)}$$

$$dY = (1+ {}^{Y^2}/_{Z^2})\, d\alpha \quad \text{..... (13b)}$$

Differentiated equations (5) and (6) become

$$dX = \frac{x}{f} \quad dZ \quad \text{..... (14)}$$

$$dY = {}^{y}/_{f} \quad dZ \quad \text{..... (15)}$$

according to equations (5) and (6) for

$$dz = - dzo$$

and $\frac{x}{f} = \frac{X}{Z}$; ${}^{y}/_{f} = \frac{Y}{Z}$

$$dx = - \frac{X}{Z} \quad dZo \quad \text{..... (16)}$$

$$dY = - \frac{Y}{Z} \quad dZo \quad \text{..... (17)}$$

Similarly it is found that

$$dX = (1+ {}^{X^2}/_{Z^2})\, d\beta \quad \text{..... (18)}$$

$$dY = \frac{XY}{Z} \quad d\beta \quad \text{..... (19)}$$

from equations 7, 8, 13a, 13b through to 19 are expressed the complete diffential formulas for dx and dY

The following substitutions are made

dw for $d\alpha$, $d\phi$ for $d\beta$

dk for dr, dbx for dxo

dby for dyo, dbz for dzo

and collecting all the like terms in equations (9), (8), (13a), (13b). (14) to (19) the following are obtained

$$dX = dbx - \frac{X}{Z}dbz - \frac{XY}{Z}d\phi - (1+\frac{X^2}{Z^2})Zd\phi + \frac{XY}{Z}dw \quad \text{----------(20)}$$

$$dY = dbY - \frac{Y}{Z}dbz + Xdk - \frac{XY}{Z}d\phi + (1+\frac{Y^2}{Z^2})Zdw \quad \text{------(21)}$$

The Differential Formula for Y - Parallaxes as Applied to Two Photographs (a model):

Fig. A.2 Two photographs and the projection plane situation.

In the above diagram, a point shown on the 2 images planes p' and p" is projected onto the projection plane at p

b = base

subscript one and two denote the left and right photographs respectively.

Here Y - Parallaxes is defined as

$$Y_1 - Y_2 = dY_1 - dY_2$$

or $Y_1 - dY_1 = Y_2 - dY_2$. Therefore Y - Parallax is the difference between the Y - coordinates of points as measured on two photographs

dY_1 and dY_2 are the small changes in the Y - coordinates of the point. This is so because if no parallax exists in the model both Y - coordinates of the point measured on the left and right photographs should be equal so that

$$\triangle Py = \triangle Y_1 - \triangle Y_2 = dY_1 - dY_2$$

or in words

The change in the Y - parallax equals the change to the Y - coordinate of left hand projection minus change to the Y - coordinate of the right hand projection.

From equation (20) it will be found that

$$Py = -dbY' + dbY'' - {}^{Y}/_{Z}dbz' + {}^{Y}/_{Z}dbZ'' + Ydk'$$
$$- (X - b)\,dk'' - \frac{XY}{Z}d\phi' + \left(\frac{X-b}{Z}\right)Yd\phi''$$
$$+ (1 + {}^{Y^2}/_{Z^2})\,Zdw' - (1 + {}^{Y^2}/_{Z^2})\,Zdw'' \quad \text{---------- (22)}$$

Here $Py = dY' - dY''$

$Y = Y_1 \simeq Y_2$ and $X_2 = X_1 - b = X - b$

X is the coordinate of P in the model plane with origin at the perspective centre of the left projector.

Y is the coordinate of P in the model plane with origin at the perspective centre of the left projector.

Z the projection distance, is the same for all points in the model for a flat terrain.

It therefore follows that if the coordinates of the perspective centre of the left projector are known then the coordinates of a point (say P) in the model plane to be used in equation (22) will be given by

$$Xp - Xo$$
$$Yp - Yo$$

where Xo, Yo, and Xp, Yp are the instrument coordinates of the perspective centre O_1 and point P respectively.

Development of Parallax Equation Used in the Thesis

Taking a parallax observation, generally implies that some observation perhaps of the orientation element should be recorded following the elimination of Y - parallax in a certain point with that element.

Each projector of the Wild A7 stereoplotter has six degrees; three translations by, bx, bz and three rotations w, ∅, k, to make a total of 12 degrees of freedom.

Since it has been shown that dbx' and dbx" introduce no Y- parallax into the model, (see Figure 1.2). It therefore follows that for the elimination of parallax in a model there are two translations and three rotations for each projector. Numerical Relative Orientation on Wild A7 ((Equation used).

In using the parallax equation developed for the right-handed system of the coordinate system of the Wild A7, the elements borne in mind were w', ∅', k', w", ∅", k", by', by", bz' and bz".

The parallax equation is

$$Py_i = -dby' + dbY'' - {}^{Y}/_{Z}dbz' + {}^{Y}/_{Z}dbZ'' + Xdk'$$

$$- (X-b)dk'' - \frac{XY}{Z}d\emptyset' + (\frac{X-b}{Z})Yd\emptyset'' + (1+{}^{Y^2}/_{Z^2})Zdw'$$

$$- (1+{}^{Y^2}/_{Z^2})Zdw'' \qquad \text{---------- (23)}$$

where $Py_i = by_i - by_m$. Ghosh (1960)

by_m is the mean of the parallax measurement read on by (reading on the element used for elimination of parallax)

at all points used for parallax measurement, and read on by". by_i is the parallax measurement at each point (i). Ghosh (1968).

The X-parallax is removed either with the foot disc or the bx. In this thesis X-parallax is removed with the bx.

Z is the projection distance which is assumed constant for flat termin and this holds for this thesis.

In the above equation:

$X = X_i - X_o$

$Y = Y_i - Y_o$

bx_i is read and recorded at all points for i = 1, 2 - - - - 25 after removing X-parallax with the bx at point i

$X\text{-}b = X_i - X_o - bx_i$

where X_i Y_i are the instrument coordinates of point i

Recalling equation (23) and rewriting as follows with

$Py_i = dY_i - dY_i"$

$PY_i = (-dby'+dbyY" +bdk") + X(dk'-dk")$

$- \frac{Y}{Z} (dbz'-dbZ"-db\phi") - \frac{XY}{Z} (d\phi'-d\phi")$

$+ (1+ {}^{Y^2}/_{Z^2})Z (dw'-dw")$

with the psuedoelements as follows:

$\triangle A = (-dby' +dbY"+bdk")$

$\triangle B = (dk' - dk")$

$$\Delta C = (dbZ' - dbz'' - bd\phi'')$$

$$\Delta D = (d\phi' - d\phi'')$$

$$\Delta D = (dw' - dw'')$$

In order to have five parallaxes proportional to ΔA, X, $-Y/Z$, $-\frac{XY}{Z}$ and $(1+Y^2/Z^2)Z$ in the model, it is necessary to choose at least one element from each of the groups of psuedoelements to be used to clear parallax from the model, Von der Weele (1960.

from ΔA _____ dk''

from ΔB _____ dk'

from ΔC _____ $d\phi''$

from ΔD _____ $d\phi'$

from ΔE _____ dw''

Then the parallax equation will reduce to

$$PY = -\frac{XY}{Z} d\phi' + Xdk' + (\frac{X-b}{Z})Yd\phi'' - (X-b)dk''$$

$$-(1+Y^2/Z^2)Z\, dw'' \qquad \text{_____} \quad (24)$$

Obtained by collecting all the coefficients of each element chosen for relative orientation (23) as follows:

$d\phi'$	_____	$\frac{-XY}{Z}$
$d\phi''$	_____	$\frac{XY}{X}$ $\frac{-bY}{Z} = (\frac{X-b}{Z})Y$
dk'	_____	X
dk''	_____	$-X + b = -(X-b)$
dw''	_____	$(1+Y^2/Z^2)Z$

Equation (24) is the equation used for this project.

In section 1.2.1.2, it was stated that dby' and dby" have the same effects on the model. They introduce the same amount of Y - parallax on all model points as illustrated in figure 1.2.

Therefore either by' or by" can be used to remove y - parallax from all the points in the model space effectively. w' or w" may be used to remove y - parallax in the model since both have same effects. The only constraint is that an additional determination has to be done. The by' and by" are well located on the Wild A7. They can easily be reached from any convenient sitting position infront of the plotter. The by" knob is used in this thesis to clear parallax at all model points.

APPENDIX B

OBSERVATIONS AND DATA REDUCTIONS

Table B.1: Empirical Relative Orientation and final Results for Model 1

Strip Number : 1 Model Number : 1
Instrument : Wild A-7 Autograph ; Instrument Number : 3268
Area : Akwanga
Operator : Onuorah, F. K. (Miss)
Date : 28-3-86 ; Time : 10 am

Approximate Scale of Photographs : 1/8000
Focal length (f) mm : 152.676mm ; d = 84mm
Overcorrection factor (n) = $(1+\frac{f^2}{d^2})^{\frac{1}{2}} = 2.152$

Photographs :
Base : in ; bx = 184.533mm
Left photograph number : 77906-60
Right photograph number : 77906-61

Elements / Values	RELATIVE ORIENTATION ELEMENTS					
	K'	K''	ϕ'	ϕ''	ω''	over-correction
Initial	400.000	400.000	100.000	100.000	100.000	0.985
First iteration	400.234	399.096	99.237	99.392	100.985	0.072
second iteration	402.013	400.860	100.168	100.265	101.057	0.000
third Iteration	402.140	400.977	100.169	100.382	101.057	

Final values of Orientation Elements

$\omega' = 100.000^g$; $\omega'' = 101.057^g$
$\phi' = 100.169^g$; $\phi'' = 100.382^g$
$K' = 402.140^g$; $K'' = 400.977^g$
$by' = 100.000_{mm}$; $by'' = 100.000mm$
$bz' = 100.000_{mm}$; $bz'' = 100.000mm$

Table B·2: Empirical Relative Orientation readings and final Results for Model 2

Strip Number: 1 Model Number : 2
Instrument: Wild A·7 Autograph ; Instrument Number : 3268
Area : Akwanga
Operator : Onuorah, F.K. (Miss)
Date : 30-3-86 ; Time : 11am

Approximate Scale of Photographs : 1/8000
Focal Length(f) mm : 152.676mm ; d = 84mm
Overcorrection factor (n) = $(1+f^2/d^2) \div 2 = 2.152$

Photographs:
Base : out ; bx = 184.533mm
Left photograph number : 77906-62
Right photograph number : 77906-61

Elements / values	Relative Orientation Elements					
	κ'	κ''	φ'	φ''	ω''	overcorrection
initial	400·00	400·000	100·000	100·000	100·000	0·466
First iteration	401·591	401·097	101·478	100·989	100·466	0·056
second iteration	401·475	400·988	101·114	100·602	100·552	-0·002
third iteration	401·326	400·827	101·037	100·517	100·550	-0·008
fourth iteration	401·326	400·845	101·037	100·517	100·542	0·000
fifth iteration	401·326	400·843	101·052	101·497	100·542	

Final Values of Relative Orientation Elements

ω' = 100.00g ; ω'' = 100·542g
φ' = 101.052g ; φ'' = 101.497g
κ' = 401.346g ; κ'' = 400.543g
by' = 100.000mm ; by'' = 100.000mm
bz' = 100.000mm ; bz'' = 100.000mm

Table B.3a: Numerical Relative Orientation Readings

Model: 1 ; Instrument: Wild A7; autograph number: 3268

Operator: Onuorah, F.K. (Miss); Date: 28-3-86; Time - 10.45am

Focal length (f): 152.676mm; bx = 184.533mm; Base : in

$w' = w'' = \phi' = \phi'' = 100.00^g$; $k' = k'' = 400.00^g$; $by' = by'' = bz' = bz'' = 100.00mm$

Left Photo number: 77906-60; Right photo number: 77906-61

$X_o = 5006.920mm$; $Y_o = 5143.118mm$; $Z = H = 325.709mm$

Point	x-coordinates (mm)		Y-coordinates (mm)		bzi (mm)		byi (mm)	
	First reading	second reading	First	Second	First	second	First	second
1	5006.106	5006.106	4970.242	4970.241	186.126	186.126	95.395	95.395
2	5051.106	5061.106	4970.242	4970.242	186.910	186.910	96.168	96.168
3	5096.106	5096.106	4970.242	4970.242	188.240	188.240	97.145	97.145
4	5141.106	5141.106	4970.242	4970.242	189.340	189.340	97.990	97.990
5	5186.106	5186.106	4970.242	4970.242	188.920	188.920	98.821	98.821
6	5186.106	5186.106	5055.242	5055.242	187.042	187.042	100.088	100.088
7	5141.106	5141.106	5055.242	5055.242	187.275	187.275	99.334	99.334
8	5096.106	5096.106	5055.242	5055.242	186.721	186.720	98330	98.330
9	5051.106	5051.106	5055.242	5055.242	185.968	185.968	97.618	97.619
10	5006.106	5006.106	5055.242	5055.242	185.408	185.408	96.762	96.762
11	5006.106	5006.106	5140.242	5140.242	183.049	183.049	97.391	97.391
12	5051.106	5051.106	5140.242	5140.242	184.120	184.120	98.070	98.070
13	5096.106	5096.106	5140.242	5140.242	183.582	183.582	99.000	99.000
14	5141.106	5141.106	5140.242	5140.242	184.160	184.160	99.790	99.790
15	5186.106	5186.106	5140.242	5140.242	184.420	184.420	100.775	100.775
16	5186.106	5186.106	5225.242	5225.242	181.865	181.865	100.450	100.450
17	5141.106	5141.106	5225.242	5225.242	183.103	183.103	99.712	99.711
18	5096.106	5096.106	5225.242	5225.242	183.850	183.850	98.916	98.916
19	5051.106	5051.106	5225.242	5225.242	183.580	183.580	98.116	98.116
20	5006.106	5006.106	5225.242	5225.242	183.409	183.409	97.325	97.325
21	5006.106	5006.106	5310.242	5310.242	180765	180.765	96.390	96.390
22	5051.106	5051.106	5310.242	5310.242	181.845	181.845	97.170	97.170
23	5096.106	5096.106	5310.242	5310.242	182.665	182.665	98.160	98.160
24	5141.106	5141.106	5310.242	5310.242	182.690	182.690	98.810	98.810
25	5186.106	5186.106	5310.242	5310.242	182.340	182.340	99.590	99.590

Table B3b: Numerical Relative Orientation Readings

Model : 2 ; Instrument : Wild A7 autograph number 3268

Operator : Onuorah, F. K. (miss) ; Date: 30-3-86 ; Time : 11.30am

Focal length(f) = 152.676mm ; bx = 184.533mm ; Base : aut

$\omega' = \omega'' = \phi' = \phi'' = 100.000^g$; $\kappa' = \kappa'' = 400.000^g$; $by' = by'' = bz' = bz'' = 100.000mm$

Left photo. number : 77906-62 ; Right photo number : 77906-61

$X_o = 5007.953mm$; $Y_o = 5001.101mm$; $Z = H = 325.709mm$

POINTS	X-coordinates (mm)		Y-coordinates (mm)		bx (mm)		by (mm)	
	First reading	Second reading	First reading	Second reading	First reading	Second reading	First reading	Second reading
1	5004.286	5004.285	4829.577	4829.577	181.494	181.494	94.410	94.410
2	5049.286	6049.286	4829.577	4829.577	181.272	181.272	94.400	94.400
3	5094.286	5094.286	4829.577	4829.577	181.309	181.309	94.560	94.560
4	5139.286	5139.286	4829.577	4829.577	180.425	180.425	94.675	94.625
5	5184.286	5184.286	4829.577	4829.577	179.423	179.423	94.827	94.827
6	5184.286	5184.286	4914.577	4914.577	180.601	180.601	94.978	94.978
7	5139.286	5139.286	4914.577	4914.577	181.560	181.560	94.798	94.798
8	5094.286	5094.286	4914.577	4914.577	182.417	182.417	94.490	94.490
9	5049.286	5049.286	4914.577	4914.577	182.540	182.540	94.255	94.255
10	5004.286	5004.286	4914.577	4914.579	182.100	182.100	94008	94.008
11	5004.286	5004.286	4999.577	4999.577	181.120	181.120	93.421	93.421
12	5049.286	5049.286	4999.577	4999.577	181.105	181.105	93.820	93.820
13	5094.286	5094.286	4999.577	4999.577	181.643	181.643	94.060	94.060
14	5139.286	5139.286	4999.577	4999.577	181.427	181.427	94.440	94.440
15	5184.286	5184.286	4999.577	4999.577	180.875	180.875	94.866	94.866
16	5184.286	5184.286	5084.577	5084.577	182.600	182.600	94.260	94.260
17	5139.286	5139.286	5084.577	5084.577	182.937	182.937	93.768	93.768
18	5094.286	5094.286	5084.577	5084.577	182.937	182.937	93.330	93.330
19	5049.286	5049.286	5084.577	5084.577	182.127	182.127	92.848	92.848
20	5004.286	5004.286	5084.577	5084.577	182.186	182.186	92.440	92.440
21	5004.286	5004.286	5169.577	5169.577	184.876	184.876	90.988	90.988
22	5049.286	5049.286	5169.577	5169.577	188.788	188.788	91.490	91.490
23	5094.286	5094.286	5169.577	5169.577	187.319	187.319	92.071	92.071
24	5139.286	5139.286	5169.577	5169.577	185.637	185.637	92.665	92.665
25	5184.286	5184.286	5169.577	5169.577	185.856	185.856	93.186	93.186

Table B·4a: Reduction of Readings For Model 1

Equation: $Py_i = -\frac{XY}{Z}d\phi' + (x-\frac{b}{Z})Yd\phi'' + Xd\kappa' - (x-b)d\kappa'' - (1+\frac{Y_2^2}{Z^2})Zdw''$

$X_o = 5006.920mm$; $Y_o = 5143.118mm$; $Z = H = 325.709$

$X = X_{im} - X_o$; $Y = Y_{im} - Y_o$; $b = X - bx_{im}$

Point	X_{im}	Y_{im}	bx_{im}	by_{im}	$x = X_{im} - X_o$	$Y = Y_{im} - Y_o$	$b = X - bx_{im}$
1	5006·106	4970·242	186·126	95·395	-0·814	-172·876	-186·940
2	5051·106	4970·242	186·910	96·168	44·186	-172·876	-142·724
3	5096·106	4970·242	188·240	97·145	89·186	-172·876	-99·054
4	5141·106	4970·242	189·340	97·990	134·186	-172·876	-55·154
5	5186·106	4970·242	188·920	98·821	179·186	-172·876	-9·734
6	5186·106	5055·242	187·042	100·088	179·186	-87·876	-7·856
7	5141·106	5055 242	187·275	99·334	134·186	-87·876	-53·089
8	5096·106	5055·242	186·721	98·330	89·186	-87·876	-97·535
9	5051·106	5055·242	185·968	97·619	44·186	-87·876	-141·782
10	5006·106	5055·242	185·408	96·762	-0·814	-87·876	-186·222
11	5006·106	5140·242	183·049	97·391	-0·814	-2·876	-183·863
12	5051·106	5140·242	184·120	98·070	44·186	-2·876	-139·934
13	5096·106	5140·242	183·582	99·000	89·186	-2·876	-94·396
14	5141·106	5140·242	184·160	99·790	134·186	-2·876	-49·974
15	5186·106	5140·242	184·420	100·725	179·186	-2·876	-5·234
16	5186·106	5225·242	181·865	100·450	179·186	82·124	-2·679
17	5141·106	5225·242	183·103	99·712	134·186	82·124	-48·917
18	5096·106	5225·242	183·850	98·916	89·186	82·124	-94·664
19	5051·106	5225 242	183·580	98·116	44·186	82·124	-139·394
20	5006·106	5225 242	183·409	97·325	-0·814	82·124	-184·223
21	5006·106	5310·242	180·765	96·396	-0·814	167·124	-181·579
22	5051·106	5310·242	181·845	97·170	44·186	167·124	-137·659
23	5096·106	5310·242	182·662	98·160	89·186	167·124	-93·476
24	5141·106	5310·242	182·690	98·810	134·186	167·124	-48·504
25	5186·106	5310·242	182·340	99·590	179·186	167·124	-3·154

Table B-4b: Reduction of Readings for Model 2

Equation: $P_{y_i} = -\frac{X}{Z}Y d\phi' + (X-b)\frac{Y}{Z}d\phi'' + X d\kappa' - (X-b)d\kappa' - \left(1+\frac{Y^2}{Z^2}\right)Z d\omega''$

$X_o = 5007.953\,mm$; $Y_o = 5001.101\,mm$; $Z = H = 325.709\,mm$

$X = X_{im} - X_o$; $Y = Y_{im} - Y_o$; $b = X - bx_{im}$

Point	x_{im}	y_{im}	bx_{im}	by_{im}	$X = x_{im} - x_o$	$Y = y_{im} - y_o$	$b = X - bx_{im}$
1	5004.286	4829.577	181.494	94.410	-3.667	-171.524	-185.161
2	5049.286	4829.577	181.272	94.400	41.333	-171.524	-139.939
3	5094.286	4829.577	181.309	94.560	86.333	-171.524	-94.976
4	5139.286	4829.577	180.425	94.675	131.333	-171.524	-49.092
5	5184.286	4829.577	179.423	94.827	176.333	-171.524	-3.090
6	5184.286	4914.577	180.601	94.978	176.333	-86.524	-4.268
7	5139.286	4914.577	181.560	94.798	131.333	-86.524	-50.227
8	5094.286	4914.577	182.417	94.490	86.333	-86.524	-96.084
9	5049.286	4914.577	182.540	94.255	41.333	-86.524	-141.207
10	5004.286	4914.577	182.100	94.008	-3.667	-86.524	-185.767
11	5004.286	4999.577	181.120	93.421	-3.667	-1.524	-184.787
12	5049.286	4999.577	181.105	93.820	41.333	-1.524	-139.772
13	5094.286	4999.577	181.643	94.060	86.333	-1.524	-95.310
14	5139.286	4999.577	181.427	94.440	131.333	-1.524	-50.094
15	5184.286	4999.577	180.875	94.866	176.333	-1.524	-4.542
16	5184.286	5084.577	182.600	94.260	176.333	83.476	-6.267
17	5139.286	5084.577	182.937	93.768	131.333	83.476	-51.604
18	5094.286	5084.577	182.937	93.330	86.333	83.476	-96.604
19	5049.286	5084.577	182.127	92.848	41.333	83.476	-140.794
20	5004.286	5084.577	182.186	92.440	-3.667	83.476	-185.853
21	5004.286	5169.577	184.876	90.988	-3.667	168.476	-188.543
22	5049.286	5169.577	188.788	91.490	41.333	168.476	-147.455
23	5094.286	5169.577	187.319	92.071	86.333	168.476	-100.984
24	5139.286	5169.577	185.637	92.665	131.333	168.476	-54.304
25	5184.286	5169.577	185.856	93.186	176.333	168.476	-9.523

Table B.5a: Table of Coefficients of Small increments of Elements of Relative Orientation

Equation: $P_{y_1} = -\frac{XY}{Z}d\phi' + (X - \frac{X-b}{Z})Y d\phi'' + X d\kappa' - (X-b)d\kappa'' - (1+\frac{Y^2}{Z^2})Z d\omega''$

Model: 1

Point	$-\frac{XY}{Z}$	$(X-\frac{X-b}{Z})Y$	X	$-(X-b)$	$-(1+\frac{Y^2}{Z^2})Z$
1	-0.432045315	99.22181898	-0.814	186.940	-417.4660941
2	23.45252649	75.75336949	44.186	142.724	-417.4660941
3	47.3370982	52.57471947	89.186	99.054	-417.4660941
4	71.221670	29.27399275	134.186	55.154	-417.4660941
5	95.1062418	5.166498267	17.186	9.734	-417.4660941
6	48.3442742	2.119541849	179.186	7.856	-349.417867
7	36.203264	14.32336522	134.186	53.089	-349.417867
8	24.0623038	26.3148567	89.186	97.535	-349.417867
9	11.9213437	38.25265815	44.186	141.782	-349.417867
10	-0.219616479	50.24253082	-0.814	186.222	-349.417867
11	-0.007187594	1.62350438	-0.814	183.863	-325.734395
12	-.39016095	1.235612722	44.186	139.934	-325.734395
13	0.78750951	0.833513645	89.186	94.396	-325.734395
14	1.18485806	0.441268813	134.181	49.974	-325.734395
15	1.58220662	0.046216051	179.186	5.234	-325.734395
16	-45.17981101	-0.67548086	179.181	2.679	-346.415678
17	-33.83354793	-12.3338922	134.181	48.917	-346.415678
18	-22.48728486	-23.8685032	89.186	94.664	-346.415678
19	-11.14102178	-35.1466887	44.186	139.394	-346.415678
20	0.20524129	-46.44983L	-.0.814	184.223	-346.415678
21	0.44767017	-9.31696968	-0.814	181.579	-411.461716
22	-22.67220453	-70.6339791	44.186	137.659	-411.461716
23	-45.76207923	-47.9633139	89.186	93.476	-411.461716
24	-68.85195393	-24.8875062	134.181	48.504	-411.461716
25	-91.94182814	-1.618333366	179.186	3.154	-411.461716

Table B.5b: Table of Coefficients of Small Increments of Elements of Relative Orientation

Equation: $Py_i = -\frac{XY}{Z}d\phi' + \left(X - \frac{b}{2}\right)Y d\phi'' + X d\kappa' - (X-b)d\kappa'' - \left(1+\frac{Y^2}{Z^2}\right)Z d\omega''$

Model : 2

Point	$-\frac{XY}{Z}$	$+\left(\frac{X-b}{Z}\right)Y$	X	$-(X-b)$	$-\left(1+\frac{Y^2}{Z^2}\right)Z$
1	-1.931106	97.50899	-3.667	185.161	-416.036509
2	21.766674	73.694301	41.333	139.939	-416.036509
3	45.464453	50.016006	86.333	94.976	-416.036509
4	69.162232	25.852697	131.333	49.092	-416.036509
5	92.860012	1.627248	176.333	3.090	-416.036509
6	46.842539	1.133786	176.333	4.268	-348.693942
7	34.888371	13.342711	131.333	50.227	-348.693942
8	22.934204	25.524539	86.333	96.084	-348.693942
9	10.980036	37.511381	41.333	141.207	-348.693942
10	-0.974132	49.348664	-3.667	185.767	-348.693942
11	-0.0171580	0.864623	-3.667	184.787	-325.716131
12	0.193398	0.653996	41.333	139.772	-325.716131
13	0.403954	0.445958	86.333	95.310	-325.716131
14	0.614510	0.234391	131.333	50.094	-325.716131
15	0.825066	0.021252	176.333	4.542	-325.716131
16	-45.192406	-1.604170	176.333	6.267	-347.103074
17	-33.659351	-13.222560	131.333	51.604	-347.103074
18	-22.126295	-24.758651	86.333	96.604	-347.103074
19	-10.593240	-36.084112	41.333	140.794	-347.103074
20	0.939816	-47.552838	-3.667	185.853	-347.103074
21	1.896790	-97.525665	-3.667	188.543	-412.854773
22	-21.379878	-76.272466	41.333	147.455	-412.854773
23	-44.656515	-52.235945	86.333	100.986	-412.854773
24	-67.933212	-28.089247	131.333	54.304	-412.854773
25	-91.209879	-4.925860	176.333	9.523	-412.854773

87

Table B: b₀ Least Squares adjustment for Model 1: [The six points Cases]

Point	$-\frac{XY}{Z_0}$	$(\frac{X_0}{Z_0})Y$	X	$-(X-b)$	$-(1+\frac{X^2}{Z_0^2})Z$	by_i	by_m	$by_i - by_m$	x	x	σ	RMSE	V
1	-0.432045365	99.22181898	-0.814	186.940	-417.4660941	95.395	98.060	-2.665	dφ	-0.2757	0.0255	±0.062597	-0.01524
6	95.1062418	5.166498267	179.186	9.734	-417.4660941	98.821		0.761	dφ''	-0.3451	0.0248		+0.01544
16	-0.007187594	1.623504338	-0.814	183.863	-325.734395	97.391		-0.669	dK'	2.9227	0.0707		+0.03024
15	1.582206621	0.046216051	179.186	5.234	-325.734395	100.775		2.715	dK''	1.7615	0.0713		-0.03089
21	0.41767017	-93.16969368	-0.814	181.579	-411.461716	96.396		-1.670	dw''	1.1103	0.0537		-0.0513
25	-91.94182864	-1.61834366	179.186	3.154	-411.461716	99.590		1.530					0.01544
1	-0.432045365	99.22181898	-0.814	186.940	-417.4660941	95.395	97.247	-1.852	dφ	-0.3616	0.0675	±0.062608	0.01771
3	47.5370982	52.57471947	89.186	99.054	-417.4660941	97.145		-0.102	dφ''	-0.3514	0.0287		-0.01824
11	-0.007187594	1.623504338	-0.814	183.863	-325.734395	97.391		0.144	dK'	3.0749	0.0866		-0.03510
13	0.78750951	0.833513645	89.186	94.396	-325.734395	99.000		1.753	dK''	1.8971	0.0840		0.03648
21	0.41767017	-93.16969368	-0.814	181.579	-411.461716	96.390		-0.857	dw''	1.0402	0.0395		0.01768
23	-45.76207923	-47.96331391	89.186	93.476	-411.461716	98.160		0.913					-0.01824
2	23.45252649	75.75336949	44.186	142.724	-417.4660941	96.168	97.999	-1.832	dφ	-0.2610	0.0115	±0.01436	-0.00413
4	71.221670	29.27399275	134.186	55.154	-417.4660941	97.890		-0.010	dφ''	-0.3634	0.0108		0.00425
12	0.39016095	1.235612722	44.186	139.934	-325.734395	98.070		0.070	dK'	2.7368	0.0197		0.00809
14	1.184858806	0.441268813	134.186	49.974	-325.734395	99.790		1.790	dK''	1.5330	0.0197		-0.00844
22	-22.67220453	-70.63397391	44.186	137.659	-411.461716	97.170		-0.830	dw''	1.0129	0.0089		-0.00407
24	-68.85195393	-24.88878062	134.186	48.504	-411.461716	98.810		0.810					0.00423
3	47.3370982	52.57471947	89.186	99.054	-417.4660941	97.145	98.915	-1.770	dφ	-0.2610	0.0592	±0.011592	-0.0346
5	95.1062418	5.166498267	179.186	9.734	-417.4660941	98.821		-0.094	dφ''	-0.4444	0.0229		0.03405
13	0.78750951	0.833513645	89.186	94.396	-325.734395	99.000		0.055	dK'	2.5513	0.1611		0.00538
15	1.582206621	0.046216051	179.186	5.234	-325.734395	100.775		1.860	dK''	1.3378	0.1693		-0.00619
25	-45.76207923	-47.96331391	89.186	93.476	-411.461716	98.160		-0.755	dw''	1.0640	0.0598		-0.03427
25	-91.94182864	-1.61834366	179.186	3.154	-411.461716	99.590		0.675					0.03388
19	-11.14102178	-35.14668587	44.186	139.394	-346.415678	98.116	98.671	-0.555	dφ	-0.3342	0.6513	±0.011798	0.00765
18	-22.45728486	-23.86850382	89.186	94.664	-346.415678	98.926		0.245	dφ''	-0.3813	1.7743		0.01540
17	-33.83354793	-12.33385922	44.186	45.917	-346.415678	99.712		1.041	dK'	4.2036	17.7617		0.00770
9	11.92134737	38.25265815	89.186	141.782	-349.417867	97.619		-1.052	dK''	3.0456	17.8690		-0.04147
8	24.06233038	26.31485671	44.186	97.535	-349.417867	98.350		-0.341	dw''	1.9143	9.4665		0.09493
7	36.203264	14.32356522	89.186	53.089	-349.417867	99.334		0.663					-0.04746

Table B.6b least squares adjustment for Model 2a ["The six points cases"]

Point	-X/yZ	X=bZ	x	-(x-b)	-(1+ν/2)Z	byi	bym	bx-bxm	x	x	σ	RMSE	v
1	-1.931106	97.508499	-3.667	185.161	-416.036509	94.410	93.616	0.794	dφ'	0.5704	0.0337	±0.06541	0.04630
5	92.860012	1.627248	176.333	3.090	-416.036509	94.827		1.211	dφ''	1.1548	0.0318		-0.04537
11	-0.017158	0.864623	-3.667	184.787	-325.716131	93.421		-0.195	dK'	1.4661	0.0948		-0.0315
15	0.825066	0.021252	176.333	4.542	-325.716131	94.866		1.250	dK''	0.8785	0.0948		0.0907
21	1.896790	-97.525615	-3.669	188.543	-412.854773	90.988		-2.628	dw'	0.5723	0.0446		0.0460
25	-91.209879	-4.925860	176.333	9.533	-412.854773	93.186		-0.430					-0.0454
1	-1.931106	97.508499	-3.667	185.161	-416.036509	94.410	93.252	1.158	dφ'	0.5367	0.0127	±0.01152	0.0759
3	45.464453	50.016006	86.333	94.976	-416.036509	94.560		1.308	dφ''	1.1542	0.0051		-0.0878
13	0.403954	0.445958	86.333	95.310	-325.716131	94.060		0.808	dK'	1.4719	0.0166		0.0967
11	-0.017580	0.864623	-3.667	184.787	-325.716131	93.421		0.169	dK''	1.0326	0.0159		-0.1112
21	1.896790	-97.525615	-3.667	188.543	-412.854773	90.988		-2.264	dw'	0.5379	0.0076		0.0352
23	-44.656545	-52.235945	86.333	100.986	-412.854773	92.071		-1.181					-0.0502
22	-21.379878	-76.272466	41.333	147.455	-412.854773	91.402	93.582	-2.094	dφ'	0.5354	0.0505	±0.07614	0.0221
24	-61.933212	-28.089247	131.333	54.304	-412.854773	92.665		-0.917	dφ''	1.1459	0.0541		-0.0218
12	0.193338	0.663996	41.333	139.772	-325.716131	93.820		0.238	dK'	1.5272	0.1095		-0.0455
14	0.614510	0.234391	131.333	50.094	-325.716131	94.440		0.858	dK''	1.0269	0.1082		0.0428
2	21.766674	73.694301	41.333	139.939	-416.036509	94.400		0.818	dw'	0.5997	0.0503		0.0225
4	69.162232	25.852697	131.333	49.092	-416.036509	94.675		1.093					-0.0213
19	-10.593240	-36.084112	41.333	140.784	-347.103074	92.848	93.915	-1.067	dφ'	0.5042	0.1833	±0.03606	0.0983
18	-22.126295	-24.758651	86.333	96.604	-347.103074	93.330		-0.585	dφ''	1.0455	5.1070		-0.0497
17	-33.659351	-13.225560	131.333	51.604	-347.103074	92.768		-0.147	dK'	-0.4068	5.1471		0.0985
7	34.885371	13.342711	131.333	50.229	-348.693942	94.798		0.883	dK''	-0.9288	5.1433		-0.0109
8	22.934204	25.524539	86.333	96.084	-348.693942	94.490		0.575	dw'	-0.3562	2.6380		0.0218
9	10.980068	37.511381	41.333	141.207	-348.693942	94.265		0.340					-0.0110
23	-44.656545	-52.235945	86.333	100.986	-412.854773	92.071	93.928	-1.557	dφ'	0.5660	0.0299	±0.05537	-0.0171
25	-91.209879	-4.925860	176.333	9.523	-412.854773	93.186		-0.742	dφ''	1.1179	0.0595		0.0169
13	0.403954	0.445958	86.333	95.310	-325.716131	94.060		0.132	dK'	1.3625	0.0828		0.0345
16	0.825066	0.021252	176.333	4.542	-325.716131	94.866		0.938	dK''	0.8359	0.0847		-0.0337
3	45.464453	50.016006	86.333	94.976	-416.036509	94.560		0.632	dw'	0.5748	0.0274		-0.0171
5	92.860012	1.627248	176.333	3.090	-416.036509	94.827		0.899					0.0168

Table B·7a: Least square adjustment for Model 1: (The nine points cases)

Point	$-\frac{xy}{z}$	$(\frac{x-b}{z})Y$	X	$-(x-b)$	$-(1+\frac{y^2}{z^2})z$	by_i	by_m	by_i-by_m	x	X	σ	RMSE	V
1	-0·432046365	99·22181898	-0·814	186·940	-417·4660941	95·395	98·074	-2·679	dφ'	-0·2865	0·0312	±0·07032	-0·0089
3	47·33709 82	52·57471947	89·186	99·054	-417·4660941	97·145		-0·929	dφ''	-0·3571	0·0299		-0·0139
5	95·1062418	5.166498267	179·186	9·734	-417·4660941	98·821		0·747	dκ'	2·8431	0·0770		0·0215
11	-0·007187594	1·62350438	-0·814	183·863	-325·734395	97·391		-0·653	dκ''	1·6813	0·0783		0·0482
13	0·78750951	0·833513645	89·186	94·396	-325·734395	99·000		0·926	dw''	1·0727	0·0369		0·05349
15	1·58220662	0·046 46051	179·186	5·234	-325·734395	100·775		2·701					0·05582
21	0·41767017	-93·1696968	-0·814	181·579	-411·461716	96·390		-1·684					0·0307
23	-45·76207923	-47·9633139	89·186	93·476	-411·461716	98·160		0·086					0·0919
25	-91·94182864	-1·61834366	179·186	3·154	-411·461716	99·590		1·516					0·0605
1	-0·432045365	99·22181898	-0·814	186·940	-417·4660941	95·395	97·210	-1·815	dφ	-0·2865	0·03	±0·09036	-0·0133
2	23·4525264	75·75556949	44·186	142·724	-417·4660941	96·168		-1·042	dφ	-0·3495	0·0382		0·0633
3	47·33709 82	52·57471947	89·186	99·054	-417·4660941	97·145		-0·065	dκ'	3·0488	0·1050		-0·0495
11	-0·007187594	1·62350435	-0·814	183·863	-325·734395	97·391		0·181	dκ''	1·8704	0·0993		-0·0359
12	0·39016095	1·235612722	44·186	139·934	-325·734395	98·070		0·860	dw''	1·0275	0·0465		0·0992
13	0·78750951	0·833513645	89·186	94·396	-325·734395	99·000		1·790					-0·0138
21	0·41767017	-93·1696968	-0·814	181·579	-411·461716	96·390		-0·820					-0·01841
22	-22·67220453	-70·63397391	44·186	137·659	-411·461716	97·170		-0·040					0·0729
23	-45·76207923	-47·9633139	89·186	93·476	-411·461716	98·160		0·950					-0·0543
2	23·4525264	75·75336949	44·186	142·724	-417·4660341	96·168	98·034	-1·866	dφ	-0·2706	0·0592	±0·07820	+0·0169
3	47·33709 82	52·57471947	89·186	99·054	-417·4660941	97·145		-0·589	dφ	-0·3756	0·0554		-0·0425
4	71·22167 0	29·27392275	134·186	55·154	-417·4660941	97·994		-0·044	dκ'	2·7228	0·0836		0·0254
12	0·39016095	1·235612722	44·186	139·934	-325·734395	98·070		0·036	dκ''	1·5190	0·0591		0·0359
13	0·78750951	0·833513645	89·186	94·396	-325·734395	99·000		0·966	dw''	1·0059	0·0407		-0·0555
14	1·18485806	0·441268813	134·186	49·974	-325·734395	99·790		1·756					0·0196
22	-22·67220453	-70·63397391	44·186	137·659	-411·461716	97·170		-0·564					0·0492
23	-45·76207923	-47·9633139	89·186	93·476	-411·461716	98·160		0·126					0·1001
24	-68·85195393	-24·8878062	134·186	48·504	-411·461716	98·810		0·776					0·0511

Table B.7a, Continued

Point	$-\frac{XY}{Z}$	$\frac{(X-b)}{Z}Y$	X	$-(X-b)$	$-(1+\frac{Y^2}{Z^2})Z$	by_i	by_m	$by_i - by_m$	x_i	x	v	RMSE	V
7	36.203264	14.323365 22	134.186	53.089	-349.417867	99.339	98.765	0.569	dφ'	-0.2617	0.1082	±0.0712	-0.0369
8	24.0623038	26.3148569	89.186	97.535	-349.417867	98.330		-0.435	dφ''	-0.3877	0.1012		+0.0993
9	11.9213437	38.25265815	44.186	141.782	-349.417867	97.619		-1.146	dk'	2.1753	0.2948		0.0603
12	0.39016095	1.235612722	44.186	139.934	-325.734395	98.070		0.695	dk''	0.9931	0.2954		0.0475
13	0.78750951	0.833513645	89.186	94.396	-325.734395	99.000		0.235	dw''	0.8467	0.1598		-0.0540
14	1.18485806	0.441268813	134.186	49.974	-325.734395	99.790		1.025					0.0018
19	-11.14102178	-35.14668887	44.186	139.394	-346.415678	98.116		0.649					0.0130
18	-22.48728486	-23.8685032	89.186	94.664	-346.415678	98.916		0.151					0.0052
17	-33.83354793	-12.33389222	134.186	48.917	-346.415678	99.712		0.947					0.0100
3	47.3370982	52.57471947	89.186	99.054	-417.4660941	97.145	98.898	-1.753	dφ'	-0.2540	0.0363	±0.0778	-0.0367
4	71.221670	29.27399275	134.186	55.154	-417.4660941	97.990		-0.908	dφ''	-0.4074	0.0789		0.0043
5	95.1062418	5.166498267	179.186	9.734	-417.4660941	98.821		-0.077	dk'	2.4981	0.0866		0.0521
13	0.78750951	0.833513645	89.186	94.396	-325.734395	99.000		0.102	dk''	1.3541	0.0930		0.0354
14	1.18485806	0.441268813	134.186	49.974	-325.734395	99.790		0.592	dw''	1.0479	0.0414		0.0675
15	1.58220662	0.046216051	179.186	5.234	-325.734395	100.715		1.877					-0.1025
23	-45.76207923	-47.9623139	89.186	93.476	-411.461716	98.160		-0.738					-0.0515
24	-68.85195393	-24.88578062	134.186	48.504	-411.461716	98.810		-0.088					0.0464
26	91.94182864	-1.6183466	179.186	3.154	-411.461716	99.590		0.692					0.0104

Table B.7b: Least squares adjustment for Model 2: [The nine points Cases]

Point	$\frac{XY}{Z}$	$(\frac{X-b}{Z})Y$	X	-(X-b)	$-(1+\frac{Y^2}{Z^2})Z$	by_i	by_m	$by_i - by_m$	x	X	∇	RMSE	V
1	-1.931106	97.50899	-3.667	185.161	-416.036509	94.410	93.599	0.811	dφ	0.5641	0.0280	±0.06163	-0.046
3	45.464453	50.016006	86.333	94.076	-416.036509	94.560		0.961	dφ"	1.1478	0.0261		0.0533
5	92.860012	1.627248	176.333	3.090	-416.036508	94.827		1.228	dκ'	1.4407	0.0707		-0.0061
11	-0.1711580	0.864623	-3.667	184.787	-325.716131	93.421		-0.178	dκ"	0.9537	0.0703		0.0760
13	0.403954	0.445958	86.333	95.310	-325.716131	94.060		0.461	dω'	0.5609	0.0331		0.0612
15	0.825066	0.021252	176.333	4.542	-325.716131	94.866		1.267					-0.0717
21	1.896790	-97.525615	-3.667	188.543	-412.854773	90.988		-2.611		0.5386	0.0592		-0.0285
23	-44.656540	-52.235945	86.333	100.986	-412.854773	92.071		-1.528		1.1542	0.0223		0.0179
25	-91.209879	-4.925860	176.333	9.523	-412.854773	93.186		-0.413		1.5514	0.0662		0.0116
1	-1.931106	97.50899	-3.667	185.161	-416.036509	94.410	93.247	1.163	dφ	0.5386	0.0592	±0.05437	-0.0300
2	21.766674	73.694301	41.333	139.939	-416.036509	94.400		1.153	dφ"	1.1542	0.0223		-0.0549
3	45.464453	50.016006	86.333	94.076	-416.036509	94.560		1.313	dκ'	1.5514	0.0662		-0.0732
13	0.403954	0.445958	86.333	95.310	-325.716131	94.060		0.813	dκ"	1.1128	0.0630		0.0122
12	0.193398	0.653996	41.333	139.772	-325.716131	93.820		0.573	dω'	0.5761	0.0293		-0.0573
11	-0.0017580	0.864623	-3.667	184.787	-325.716131	93.421		0.174					0.0338
21	1.896790	-97.525615	-3.667	188.543	-412.854773	90.988		-2.259					-0.0238
22	-21.379878	-76.272466	41.333	147.455	-412.854773	91.490		-1.757					0.0412
23	-44.656545	-52.235945	86.333	100.986	-412.854773	92.071		-1.176					-0.0114
22	-21.379878	-76.272466	41.333	147.455	-412.854773	91.490	93.576	-2.086	dφ'	0.5260	0.0376	±0.04983	0.0193
23	-44.656545	-52.235945	86.333	100.986	-412.854773	92.071		-1.505	dφ"	1.1453	0.0337		0.0051
24	-67.933212	-28.089247	131.333	54.304	-412.854773	92.665		-0.911	dκ'	1.4852	0.0579		-0.0234
12	0.193398	0.653996	41.333	139.772	-325.716131	93.520		0.244	dκ"	0.9855	0.0573		-0.0699
13	0.403954	0.445958	86.333	95.310	-325.716131	94.060		0.484	dω'	0.5800	0.0267		0.0495
14	0.614510	0.234391	131.333	50.094	-325.716131	94.440		0.864					0.0173
2	21.766674	73.694301	41.333	139.939	-416.036509	94.400		0.824					0.0258
3	45.464453	50.016006	86.333	94.076	-416.036509	94.560		0.984					-0.0069
4	69.162232	25.852697	131.333	49.092	-416.036509	94.675		1.099					-0.0175

Table B.7b: Continued

Point	$-\frac{xy}{2}$	$(x-b)y$	X	$-(x-b)$	$-(1+\frac{y^2}{2})2$	by_i	by_m	$by_i - by_m$	x	x	σ	RMSE	V
23	-44.656545	-52.235945	86.333	100.986	-417.854773	92.071		-1.857	dd'	0.5600	0.0179		-0.0368
24	-67.933212	-28.089247	131.333	54.309	-417.854773	92.665		-1.263	dd"	1.1096	0.0166		-0.0268
25	-91.209879	-4.925860	176.333	9.523	-417.854773	93.186		-0.742	dK'	1.3471	0.0414		-0.0304
15	0.825066	0.021252	176.333	4.542	-325.716131	94.866		0.938	dK"	0.8155	0.0433		-0.0422
14	0.614510	0.234391	131.333	50.094	-325.716131	94.440		0.512	dw'	0.5672	0.0191		0.0168
13	0.403954	0.445958	86.333	95.310	-325.716131	94.060	93.925	0.132					0.0255
3	45.464453	50.016006	86.333	94.976	-416.036509	94.560		0.632					0.0234
4	69.162232	25.852697	131.333	49.092	-416.036509	94.675		0.747					0.0135
5	92.860012	1.627248	176.333	3.090	-416.036509	94.827		0.899				±0.036602	0.0102
19	+10.593240	-36.084112	41.333	140.794	-347.103074	92.848		-1.131	dd'	0.5647	0.0770		0.0269
18	-22.126295	-24.758651	86.333	96.604	-347.103074	93.330		-0.649	dφ"	1.0415	0.0700		-0.0116
17	-33.659351	-13.222560	131.333	51.604	-347.103074	93.768		-0.211	dK'	1.3445	0.2058		-0.01343
12	0.193398	0.653996	41.333	139.772	-325.716131	93.820		-0.159	dK"	0.8439	0.2083		-0.0626
13	0.403954	0.445958	86.333	95.310	-325.716131	94.060		0.081	dw'	0.5819	0.1114		-0.0483
14	0.614510	0.234391	131.333	50.094	-325.716131	94.440	93.979	0.461					-0.0140
9	10.980036	37.511381	41.333	141.207	-348.693942	94.255		0.276					0.00614
8	22.934204	25.524559	86.333	96.084	-348.693942	94.490		0.511				±0.050220	0.0232
7	34.888371	13.342711	131.333	50.227	-348.693942	94.798		0.819					-0.0357

B. 8a: Least squares adjustment for model 1: The twelve points cases)

Point	$-\frac{x^2}{2}$	$(\frac{x-b}{2})Y$	X	$-(x-b)$	$-(1+\frac{Y^2}{2})Z$	by_i'	by_m	$by_i - by_m$	x	X	σ	RMSE	V
1	-0.432045365	99.22181898	-0.814	186.940	-417.4660941	95.395		-2.728	dϕ	-0.2960	0.0446		+0.0208
2	23.45252764	75.75336949	44.186	142.724	-417.4660941	96.168		-1.455	dϕ'	0.3565	0.0293		0.0673
3	47.33709827	52.57471947	89.186	99.054	-417.4660941	97.145		-0.478	dκ'	2.8871	0.0719		-0.0363
4	71.221670	29.27399275	134.186	55.154	-417.4660941	97.990		0.367	dκ''	1.7278	0.0719		-0.0115
11	-0.007187594	1.62350438	-0.814	183.863	-325.734395	97.391		-0.232	dω'	1.0269	0.0357		-0.0765
12	0.39016095	1.23561272	44.186	139.934	-325.734395	98.070		0.447					0.0935
13	0.78750951	0.83351364	89.186	94.396	-325.734395	99.000		1.377					-0.0311
14	1.18485806	0.44126881	134.186	49.374	-325.734395	99.790		2.167					0.0146
21	0.41767017	-93.16965968	-0.814	181.579	-411.461716	96.390		-1.233					0.0090
22	-22.67220453	-70.63397911	44.186	137.659	-411.461716	97.170		-0.453					0.0592
23	-46.76207923	-47.96331391	89.186	93.476	-411.461716	98.160		0.537					-0.1085
24	-69.85135393	-24.88780622	134.186	48.504	-411.461716	98.810		1.187					0.0401
1	-0.432045365	99.22181898	-0.814	186.940	-417.4660941	95.395		-2.125	dϕ'	-0.1273	0.1121		-0.0253
2	23.45252764	75.75336949	44.186	142.724	-417.4660941	96.168		-1.352	dϕ''	-0.4456	0.0554		0.0662
3	47.33709827	52.57471947	89.186	99.054	-417.4660941	97.145		-0.375	dκ'	2.7935	0.1261		-0.0297
10	-0.219616477	50.24253082	-0.814	186.222	-349.417867	96.762		-0.758	dκ''	1.7609	0.1171		0.0016
9	11.92134371	38.25265815	44.186	141.782	-349.417867	97.619		0.099	dω''	1.0057	0.0624		-0.0503
8	24.06230388	26.31485647	89.186	97.535	-349.417867	98.330		0.810					0.0486
11	-0.007187594	1.62350438	-0.814	183.863	-325.734395	97.391		-0.129					+0.0214
12	0.39016095	1.23561272	44.186	139.934	-325.734395	98.070		0.550					0.1039
13	0.78750951	0.83351364	89.186	94.396	-325.734395	99.000		1.480					-0.1090
20	0.20524129	-23.86850328	-0.814	184.223	-346.415678	97.325		-0.195					0.0515
19	-11.14102178	-35.14668897	44.186	139.371	-346.415678	98.116		0.596					-0.0061
18	-22.48728486	-46.44983636	89.181	94.664	-346.415678	98.916		1.396					0.0331
7	36.20326464	14.32336522	134.186	53.089	-349.417867	99.334		0.748	dϕ	-0.2763	0.0834		-0.0545
8	24.06230388	26.31485647	89.186	97.535	-349.417867	98.330		-0.256	dϕ''	-0.3635	0.0753		0.0882
9	11.92134371	38.25265815	44.186	141.782	-349.417867	97.619		-0.967	dκ'	2.5579	0.1292		-0.0192
14	1.18485806	0.44126881	134.186	49.974	-325.734395	99.790		1.204	dκ''	1.3745	0.1305		0.0362
13	0.78750951	0.83351364	89.186	94.396	-325.734395	99.000		0.414	dω''	1.0265	0.0675		-0.0235
12	0.39016095	1.23561272	44.186	139.934	-325.734395	98.070		-0.516					0.0810

97.623
97.520
98.586

±0.07473
±0.07360
±0.07966

Table B.8a: Continued

Point	$-\frac{XY}{Z}$	$(\frac{X-b}{Z})Y$	X	$-(X-b)$	$-(1+\frac{Y^2}{Z^2})Z$	by_i	by_m	$by_i - by_m$	x	x	σ	Rinse	V
17	-33.83354793	-12.3338922	134.186	48.917	-346.415678	99.712	99.586	1.126					-0.0154
18	-22.48728486	-23.8685032	89.186	94.664	-346.415678	99.712		0.330					-0.0233
19	-11.14102178	-35.1466887	44.186	139.394	-346.415678	98.116		-0.470					-0.0506
24	-68.85195393	-24.8878062	134.186	48.504	-411.461716	98.810		0.224					+0.0584
23	-45.76207923	-47.9633139	89.186	93.476	-411.461716	98.160		-0.426					0.0974
22	-22.67220453	-70.6339791	44.186	137.659	-411.461716	97.170		-1.416					0.0676
6	48.34422942	2.119541849	179.186	7.856	-349.417867	100.080	99.413	0.675	dd'	-0.2305	0.0439	± 0.09005	-0.0684
7	36.20326464	14.32336522	134.186	53.089	-349.417867	99.334		-0.079	dd''	-0.5367	0.0802		-0.0742
8	24.06230388	26.31485674	89.186	97.535	-349.417867	98.330		-1.083	dk'	2.3536	0.0993		0.0206
15	1.582206622	0.046216051	179.186	5.234	-325.734395	100.775		1.362	dk''	1.1685	0.1015		-0.0846
14	1.184858062	0.441268813	134.186	49.974	-325.734395	99.790		0.377	dw'	1.0625	0.0522		0.0518
13	0.787509511	0.833513695	89.186	94.395	-325.734395	99.000		-0.413					-0.0185
16	-45.17981101	-0.675480862	179.186	2.679	-346.415678	100.450		1.037					-0.0237
17	-33.83354793	-12.3338922	134.186	48.917	-346.415678	99.712		0.299					-0.0045
18	-22.48728486	-23.8685032	89.186	94.664	346.415678	98.916		-0.497					0.0235
25	-91.94182864	-1.61834366	179.186	3.154	-411.461716	99.590		0.177					-0.0164
24	-68.85195393	-24.8878062	134.186	48.504	-411.461716	98.810		-0.603					0.0406
23	-45.76207923	-47.9633139	89.186	93.476	-411.461716	98.160		-1.253					+0.0401
16	-45.17981101	-0.675480862	179.186	2.679	-346.415678	100.450	99.183	1.267	dd'	-0.2687	0.0643	± 0.07718	0.0501
17	-33.83354793	-12.3338922	134.186	48.917	-346.415678	99.712		0.529	dd''	-0.3756	0.0879		-0.0706
18	-22.48728486	-23.8685032	89.186	94.664	-346.415678	98.916		-0.267	dk'	2.3587	0.2712		-0.0169
19	-11.14102178	-35.1466887	44.186	139.394	-346.415678	98.116		-1.067	dk''	0.8976	0.2712		-0.0512
15	1.582206622	0.046216051	179.186	5.234	-325.734395	100.775		1.592	dw''	1.0231	0.1464		-0.1032
14	1.184858062	0.441268813	134.186	49.974	-325.734395	99.790		0.607					0.0434
13	0.787509511	0.833513695	89.186	94.396	-325.734395	99.000		-0.183					-0.0167
12	0.390160855	1.235612722	44.186	139.984	-325.734395	98.070		-1.113					01836
6	48.34422942	2.119541849	179.186	7.856	-349.417867	100.088		0.905					0.0479
7	36.20326464	14.32336522	134.186	53.089	-349.417867	99.334		0.151					-0.0535
8	24.06230388	26.31485674	89.186	97.535	-349.417867	98.330		-0.852					0.0819
9	11.92134377	38.25265815	44.186	141.782	-349.417867	97.619		-1.564					-0.0790

95

B.8b: Least squares adjustment for Model 2: [The twelve points cases]

Point	$-\frac{XY}{Z}$	$(\frac{X-b}{Z})Y$	X	$-(X-b)$	$-(1+\frac{Y^2}{Z^2})Z$	by_i	by_m	$by_i - by_m'x$	x	σ	Rmse	V
11	-0.017580	0.864623	-3.667	184.787	-325.716131	93.421	93.417	0.004 dφ	0.5328	0.0299	±0.03782	0.0133
12	0.193398	0.653996	41.333	139.772	-325.716131	93.820		0.403 dφ'	1.1523	0.01846		-0.0555
13	0.403954	0.445958	86.333	95.310	-325.716131	94.060		0.643 dκ	1.5018	0.0497		0.044
14	0.614510	0.234391	131.333	50.094	-325.716131	94.440		1.023 dκ'	1.0313	0.0490		-0.0087
21	1.896790	-97.525615	-3.669	188.543	-412.854773	90.988		-2.429 dω	0.5678	0.0484		-0.0315
22	-21.379878	-76.272466	41.333	147.455	-412.854773	91.490		-1.927				0.0439
23	-44.656545	-52.235945	86.333	100.986	-412.854773	92.071		-1.346				0.0172
24	-67.933212	-28.089247	131.333	54.304	-412.854773	92.665		-0.752				-0.0290
1	-1.931106	97.50899	-3.669	185.161	-416.036509	94.410		0.993				-0.0415
2	21.766674	73.694301	41.333	139.939	-416.036509	94.402		0.983				-0.0498
3	45.464453	50.016006	86.333	94.976	-416.036509	94.560		1.143				0.0078
4	69.162232	25.852697	131.333	49.092	-416.036509	94.675		1.258				-0.0094
8	22.934204	25.524559	86.333	96.084	-348.693942	94.490		1.388 dφ	0.5220	0.0635	±0.03782	-0.0078
9	10.980036	37.511381	41.333	141.207	-348.693942	94.255		1.153 dφ'	1.0613	0.0242		0.0005
10	-0.974132	49.348664	-3.667	185.767	-348.693942	94.008		0.906 dκ'	1.8118	0.0700		0.0061
11	-0.017580	0.864623	-3.667	184.787	-325.716131	93.421		0.319 dω'	1.3426	0.0694		0.0171
12	0.193398	0.653996	41.333	139.772	-325.716131	93.820		0.718 dω''	0.6984	0.0563		-0.0527
13	0.403954	0.445958	86.333	95.310	-325.716131	94.060		0.958				0.0482
18	-22.126295	-24.758651	86.333	96.604	-347.103074	93.330		0.228				-0.0278
19	-10.593240	-36.084112	41.333	140.794	-347.103074	92.848		-0.254				0.0122
20	0.939816	-47.552838	-3.667	185.853	-347.103074	92.440		-0.662				-0.0064
21	1.896790	-97.525615	-3.669	188.543	-412.854773	90.988		-2.117				0.0236
22	-21.379878	-76.272466	41.333	147.455	-412.854773	91.490		-1.612				-0.0517
23	-44.656545	-52.235945	86.333	100.986	-412.854773	92.071		-1.031				-0.0192
3	45.464455	50.016006	86.333	94.976	-416.036509	94.580		0.139 dφ	0.5017	0.0299	±0.03744	-0.0129
4	69.162232	25.852697	131.333	49.092	-416.036509	94.675		0.254 dφ'	1.0965	0.0420		0.0115
5	92.860012	1.627748	176.333	3.090	-416.036509	94.827		0.406 dκ	1.0873	0.0503		-0.0035
6	46.842539	1.133786	176.333	4.268	-348.693942	94.978		0.557 dκ'	0.6551	0.0516		0.0347
7	34.888371	13.342711	131.333	50.227	-348.693942	94.798		0.377 dω''	0.5195	0.0261		-0.0374
8	22.934204	25.524539	86.333	96.084	-348.693942	94.490		0.069				0.0172

8b Continued

POINT	$-\frac{XY}{Z}$	$\left(\frac{X-b}{Z}\right)Y$	X	$-(X-b)$	$-(1+\frac{Y^2+Z^2}{Z})Z$	by_i	by_m	$by-by_m$	x	X	σ	RMSE	V
13	0.403954	0.445958	86.333	95.310	-325.716131	94.060	94.491	-0.367				1C 03144	-0.0187
14	0.614510	0.234391	131.333	50.094	-325.716131	94.440		0.019					0.0113
15	0.825066	0.023252	176.333	4.542	-325.716131	94.866		0.445					0.0451
16	-45.192906	-1.606170	176.333	6.267	-347.103074	94.260		-0.161					0.0105
17	-33.659351	-13.222560	131.333	51.604	-347.103074	93.768		-0.653					0.0264
18	-22.126295	-24.758651	86.333	96.604	-347.103074	93.330		-1.091					-0.0761
22	-21.379878	-76.272456	41.333	147.455	-412.854773	91.490	95.503	-2.013	dσ	0.5080	0.0471	1/- 0.04574	0.0247
23	-44.656545	-52.235945	86.333	100.986	-412.854773	92.071		-1.432	dσ''	1.1065	0.0427		0.0086
24	-67.933212	-28.089247	131.333	54.304	-412.854773	92.665		-0.838	dz''	1.6032	0.0840		-0.0223
19	-10.593240	-36.084112	41.333	140.794	-347.103074	92.848		-0.655	dσ'	1.0969	0.0824		-0.0149
18	-22.126295	-24.758651	86.333	96.604	-347.103074	93.330		-0.173	dω	0.6283	0.0433		-0.0199
17	-33.659351	-13.222560	131.333	51.604	-347.103074	93.768		0.265					0.0089
12	0.193398	0.653996	41.333	139.772	-325.716131	93.820		0.317					-0.0688
13	0.403954	0.445958	86.333	95.310	-325.716131	94.060		0.557					0.0566
14	0.614510	0.239391	131.333	50.094	-325.716131	94.440		0.937					0.0291
9	10.980036	37.511381	41.333	141.207	-348.693942	94.255		0.752					0.0206
8	22.934204	25.524539	86.333	96.084	-348.693942	94.490		0.987					0.0288
7	34.888371	13.342711	131.333	50.227	-348.693942	94.793		1.295					0.0521
2	21.766674	73.694301	41.333	139.939	-416.036509	94.410	94.873	0.313	dσ	0.4877	0.0379		-0.0567
3	45.464453	50.016006	86.333	94.976	-416.036509	94.400		0.302	dσ'	1.1071	0.0528		0.0651
4	69.162232	25.852697	131.333	49.092	-416.036509	94.560		0.462	dz'	1.3255	0.0879		-0.0038
7	34.888371	13.342711	131.333	50.227	-348.693942	94.788		0.700	dσ''	0.9461	0.0872		-0.0731
8	22.934204	25.524539	86.333	96.084	-348.693942	94.490		0.392	dω	0.5991	0.0452		0.0251
9	10.980036	37.511381	41.333	141.207	-348.693942	94.255		0.157					0.0371
12	0.193398	0.653996	41.333	139.772	-325.716131	93.820		-0.278					-0.0544
13	0.403954	0.445958	86.333	95.310	-325.716131	94.060		-0.039					0.0525
14	0.614510	0.234391	131.333	50.094	-325.716131	94.440		0.343					0.0090
17	-33.659351	-13.222560	131.333	51.604	-347.103074	93.768		-0.330					0.0032
18	-22.126295	-24.758651	86.333	96.604	-347.103074	93.330		-0.768					-0.0127
19	-10.593240	-36.084112	41.333	140.794	-347.103074	92.848		-1.250					0.0084

Table 9a: Least squares adjustment for Model 1: The fifteen points (case 5)

point	$-\frac{XY}{Z}$	$-(\frac{X}{Z}-b)Y$	X	$-(x-b$	$-(1+\frac{Y^2}{Z^2})Z$	by_i	by_m	by_i-by_m	x	α	σ	Rmse	V
1	-0.432045365	99.22181898	-0.814	186.940	-417.466094	95.395	98.044	-2.649	$d\phi$	-0.2801	0.0280	±0.07860	-0.0441
2	23.45252649	75.75336949	44.186	142.724	-417.466094	96.168		-1.876	$d\phi'$	-0.3571	0.0267		0.0570
3	47.33709827	52.57471947	89.186	99.054	-417.466094	97.145		-0.899	dk	2.7960	0.0637		-0.0336
4	71.22167078	29.27399275	134.186	55.154	-417.466094	97.990		-0.054	dk'	1.6253	0.0643		0.0026
5	95.10624184	5.166498267	179.186	9.734	-417.466094	98.821		0.777	$d\omega$	1.0485	0.0299		0.0185
11	-0.007187594	1.623504385	-0.814	183.863	-325.734395	97.391		-0.653					-0.0624
12	0.390160965	1.235612722	44.186	139.934	-325.734395	98.070		0.026					0.1133
13	0.787509551	0.833513645	89.186	94.396	-325.734395	99.000		0.956					-0.0017
14	1.184858064	0.441268813	134.186	49.974	-325.734395	99.790		1.746					0.0511
15	1.582206622	0.046216051	179.186	5.234	-325.734395	100.775		2.731					-0.1016
21	0.417670171	-93.16969685	-0.814	181.579	-411.461716	96.390		-1.654					-0.0022
22	-22.67222453	-70.63397917	44.186	137.659	-411.461716	97.170		-0.874					0.0482
23	-45.76207923	-47.96331397	89.186	93.476	-411.461716	98.160		0.116					-0.1189
24	-68.55195392	-29.88780625	134.186	48.504	-411.461716	98.810		0.766					0.0317
26	-91.34182864	-1.618343665	179.186	3.154	-411.461716	99.590		1.546					0.0414
1	-0.432045365	99.22181898	-0.814	186.940	-417.466094	95.395	97.464	-2.069	$d\phi$	-0.3699	0.0656	±0.06831	0.0066
2	23.45252649	75.75336949	44.186	142.724	-417.466094	96.168		-1.296	$d\phi'$	-0.3501	0.0261		0.0572
3	47.33709827	52.57471947	89.186	99.054	-417.466094	97.145		-0.319	dk	2.9526	0.0649		-0.0823
8	24.06230385	26.31485675	89.186	97.535	-349.417867	98.330		0.866	dk'	1.8048	0.0618		0.0747
9	11.92134375	38.25265815	44.186	141.782	-349.417867	97.619		0.155	$d\omega$	1.0339	0.0306		-0.0420
10	-0.219616479	50.29253082	-0.814	186.222	-349.417867	96.762		-0.702					-0.0074
13	0.787509551	0.833513645	89.186	94.396	-325.734395	99.000		1.536					-0.0242
12	0.390160965	1.235612722	44.186	139.934	-325.734395	98.070		0.606					0.1099
11	-0.007187594	1.623504385	-0.814	183.863	-325.734395	97.391		-0.073					-0.0526
18	-22.98728484	-23.86850332	89.186	94.664	-346.415678	98.916		1.452					0.0025
19	-11.14102178	-35.14668337	44.186	139.394	-346.415678	98.116		0.652					-0.0204
20	0.205241291	-46.44983361	-0.814	184.223	-346.415678	97.325		-0.139					-0.0493
23	-45.76207923	-47.96331397	89.186	93.476	-411.461716	98.160		0.696					-0.0640
23	-22.67220853	-70.63397917	44.186	137.659	-411.461716	97.170		-0.294					0.0819
21	0.417670171	-93.16969685	-0.814	181.579	-411.461716	96.396		-1.074					0.0097

B.9a Continued

Points	$-\frac{XY}{Z}$	$(\frac{X-b}{Z})Y$	X	$-(x-b)$	$-(1+\frac{Y^2}{Z^2})Z$	by_i	by_m	$by_i - by_m$	x	x	σ	RMSE	V
2	23.46252649	75.75336949	44.186	142.724	-417.466094	96.168		-2.121	dφ'	-0.2808	0.0605		0.0323
3	47.3370982	52.57471947	89.186	99.054	-417.466094	97.145		-1.144	dφ''	-0.3877	0.0567		-0.0388
4	71.221670	29.27399275	134.186	55.154	-417.466094	97.990		-0.299	dκ'	2.6401	0.0828		0.0174
9	11.9213437	38.25265815	44.186	141.782	-349.417867	97.619		-0.670	dκ''	1.4515	0.0840		-0.0335
8	24.0623038	26.3148567	89.186	97.535	-349.417867	98.330	97.289	0.041	dω''	1.0059	0.0401		0.0817
7	36.203264	14.32336522	134.186	53.089	-349.417867	99.334		1.045					-0.0504
12	0.39016025	1.235612722	44.186	139.934	-325.734395	98.070		-0.219					0.0742
13	0.78750951	0.833513645	89.186	94.396	-325.734395	99.000		0.711					-0.0271
14	1.18485806	0.441268813	134.186	49.974	-325.734395	99.790		1.501				±0.08903	0.0366
19	-11.14102178	-36.14668887	44.186	139.394	-346.415678	98.116		-0.173					-0.0389
18	-22.48728486	-23.86850332	89.186	94.664	-346.415678	98.916		0.627					-0.0115
17	-33.83354793	-12.33389222	134.186	48.917	-346.415678	99.712		1.423					0.0049
22	-22.67220953	-70.63397911	44.186	137.659	-411.461716	97.170		-1.119					0.1042
23	-45.76207923	-47.96331339	89.186	93.476	-411.461716	98.160		-0.129					-0.1796
24	-68.85195393	-24.88780682	134.186	48.504	-411.461716	98.810		0.521					0.0884
3	47.3370982	52.57471947	89.186	99.054	-417.466094	97.145		-1.982	dφ	-0.2489	0.0261		-0.0608
4	71.221670	29.27399275	134.186	55.154	-417.466094	97.990		-1.137	dφ''	-0.4208	0.0523		0.0609
5	95.1062418	5.166498267	179.186	9.734	-417.466094	98.821		-0.306	dκ'	2.3848	0.0586		0.0386
8	24.0623038	26.3148567	89.186	97.535	-349.417867	98.330		-0.797	dκ''	1.2274	0.0618		0.0915
7	36.203264	14.32336522	134.186	53.089	-349.417867	99.334		0.207	dω''	1.0313	0.0287		0.0521
6	48.3442242	2.119541849	179.186	7.856	-349.417867	100.088		0.961					0.0405
13	0.78750951	0.833513645	89.186	94.396	-325.734395	99.000		-0.127					0.0039
14	1.18485806	0.441268813	134.186	49.974	-325.734395	99.790		0.663					0.0440
15	1.58220662	0.046216051	179.186	5.234	-325.734395	100.775		1.648					-0.1170
18	-22.48728486	-23.86850332	89.186	94.664	-346.415678	98.916	99.127	-0.211				±0.06322	0.0122
17	-33.83354793	-12.33389222	134.186	48.917	-346.415678	99.712		0.585					0.0121
16	-45.17981101	-0.676480186	179.186	2.679	-346.415678	100.450		1.323					0.0113
23	-45.76207923	-47.96331339	89.186	93.476	-411.461716	98.160		-0.967					-0.0581
24	-68.85195398	-24.88780682	134.189	48.504	-411.461716	98.510		-0.317					-0.0482
25	-91.94182864	-1.618343666	179.186	3.154	-411.461716	99.590		0.463					0.0159

B.S.a continued

Point	$-\frac{XY}{Z}$	$(\frac{X-b}{Z})Y$	X	$-(X-b)$	$-(1+\frac{Y^2}{Z^2})Z$	by_i	by_m	$by_i - by_m$	x	x	σ	RMSE	V
6	48·3442242	2·119541849	179·186	7·856	-349·417867	100·088	98·779	1·309	dφ	-0·2795	0·0535	±0·07096	0·0272
7	36·203264	14·32336522	134·186	53·089	-349·417867	99·334		0·555	dφ"	-0·3752	0·0509		-0·0531
8	24·0623038	26·3148567	89·186	97·535	-349·417867	98·330		-0·449	dK'	2·4930	0·2215		-0·1014
9	11·92/3437	38·25265815	44·186	141·782	-349·417867	97·619		-1·160	dK"	1·3318	0·2222		-0·0411
10	-0·8196/6479	50·24253082	-0·814	186·222	-349·417867	96·762		-2·017	dω	1·0237	0·1203		-0·0331
11	-0·007187594	1·62350438	-0·814	183·863	-325·734395	97·391		-1·388					-0·0456
12	0·390/6095	1·235612722	44·186	139·934	-325·734395	98·070		-0·709					0·1189
13	0·78750951	0·833513645	89·186	94·391	-325·734395	99·000		0·221					-0·0017
14	1·18485806	0·441268813	134·186	49·974	-325·734395	99·790		1·011					0·0421
15	1·5822066 2	0·04621605/	179·186	5·234	-325·734345	100·725		1·996					-0·1164
16	-45·17981101	-0·67548086	179·186	2·679	-346·415678	100·450		1·671					0·0319
17	-33·83354793	-12·3338922	134·186	48·917	-346·415678	99·712		0·933					-0·0058
18	-22·48728486	23·8685032	89·186	94·664	-346·415678	98·916		0·137					0·0035
19	-11·14102178	-35·1466887	44·186	139·394	-346·415678	98·116		-0·663					-0·0060
20	0·2052 4129	-46·449836	-0·814	184·223	-346·415678	97·325		-1·454					-0·0223

B.9b : Least squares adjustment for Model 2 ; [The fifteen points cases]

Point	$-\frac{x^2}{2}$	$-\frac{(x-b)^2}{2}$	x	$-(x-b)$	$-(1+\frac{y^2}{2^2})z$	by_i	by_m	$by_i - by_m$	x	X	σ	RMSE	V
1	-1.931106	97.508899	-3.667	186.161	-416.036509	94410	93.592	0.818	dφ	0.5551	0.0197	±0.05183	0.0710
2	21.766674	73.694301	41.333	139.939	-416.036509	94.400		0.808	dφ'	1.1446	0.0185		0.0603
3	45.464453	50.016006	86.333	94.976	-416.036509	94.560		0.968	dκ	1.4731	0.0465		0.0282
4	69.162232	25.852697	131.333	49.092	-416.036509	94.675		1.083	dκ'	0.9829	0.0458		0.0180
5	92.860012	1.627248	176.333	3.090	-416.036509	94.827		1.235	dω	0.5761	0.0217		-0.0302
11	-0.017580	0.864623	-3.667	184.787	-325.716131	93.424		-0.171					0.0086
12	0.193398	0.663996	41.333	139.772	-325.716131	93.820		0.228					-0.0464
13	0.403954	0.445958	86.333	95.310	-325.716131	94.060		0.468					0.0668
14	0.614510	0.234391	131.333	50.094	-325.716131	94.440		0.848					0.0271
15	0.825066	0.021252	176.333	4.542	-325.716131	94.866		1.274					-0.0632
21	1.896790	-97.525665	-3.667	188.543	-412.854773	90.988		-2.604					0.0406
22	-21.379878	-76.272466	41.333	147.455	-412.854773	91.490		-2.102					0.0430
23	-44.656545	-52.235945	86.333	100.986	-412.854773	92.071		-1.521					0.0146
24	-67.933212	-28.089247	131.333	54.304	-412.854773	92.665		-0.927					-0.0287
25	-91.209879	-4.926860	176.333	9.523	-412.854773	93.186		-0.406					0.0138
2	21.766674	73.694301	41.333	139.939	-416.036509	94.400		0.689	dφ	0.5379	0.0287		-0.0132
3	45.464453	50.016006	86.333	94.976	-416.036509	94.560		0.849	dφ'	1.1205	0.0255		0.0024
4	69.162232	25.852697	131.333	49.092	-416.036509	94.675		0.964	dκ	1.4439	0.0414		-0.0220
7	34.888371	13.342711	131.333	50.227	-348.693942	94.798		1.087	dκ'	0.9346	0.0407		-0.0235
8	22.934204	35.524539	86.333	96.084	-348.693942	94.490		0.779	dω	0.6812	0.0197		0.0504
9	10.980036	37.511381	41.333	141.207	-348.693942	94.255		0.544					-0.0523
12	0.193398	0.663996	41.333	139.772	-325.716131	93.820	93.711	0.109					0.0376
13	0.403954	0.445958	86.333	95.310	-325.716131	94.060		0.349					-0.0789
14	0.614510	0.234391	131.333	50.094	-325.716131	94.440		0.729					0.0471
17	-33.659351	-13.222560	131.333	51.604	-347.103074	93.768		0.057					0.0219
18	-22.126295	-24.758651	86.333	96.604	-347.103074	93.330		-0.381					0.0051
19	-10.593240	-36.054112	41.333	140.794	-347.103074	92.848		-0.863					-0.0330
22	-21.379878	-76.272466	41.333	147.455	-412.854773	91.490		-2.221					-0.0297
23	-44.656545	-52.235945	86.333	100.986	-412.854773	92.071		-1.640					0.0312
24	-67.933212	-28.089247	131.333	54.304	-412.854773	92.665		-1.046					0.0144

B.9b continued

Point	$-\frac{\Delta Y}{Z}$	$\frac{(X-b)}{Z}Y$	x	$-(x-b)$	$-(1+\frac{Y^2}{Z^2})Z$	by_i	by_m	$by_i - by_m$	x''	$\bar{x}$	σ	RMSE	V
3	45·464453	50·016006	86·333	94·976	-416·036509	94·560	94·065	0·495	dφ	0·5488	0·0153	±0·036991	-0·0298
4	69·162232	26·852697	131·333	49·092	-416·036509	94·675		0·610	dφ	1·1026	0·0231		0·0016
5	92·860012	1·627248	176·333	3·090	-416·036509	94·827		0·762	dκ	1·2993	0·0350		-0·0066
6	46·842539	1·133786	176·333	4·268	-348·693942	94·978		0·913	dκ	0·7441	0·0363		0·0521
7	34·888371	13·342711	131·333	50·227	-348·693942	94·798		0·733	dω	0·5704	0·0172		-0·0192
8	22·934204	25·524539	86·333	96·084	-348·693942	94·490		0·425					0·0388
13	0·403954	0·445958	86·333	95·310	-325·716131	94·060		-0·005					0·0327
14	0·614510	0·234391	131·333	50·094	-325·716131	94·440		0·375					0·0196
15	0·825066	0·021252	176·333	4·542	-325·716131	94·866		0·801					-0·0436
16	-45·192406	-1·606170	176·333	6·267	-347·103074	94·260		0·195					-0·0327
17	-33·659351	-13·222560	131·333	51·604	-347·103074	93·768		-0·297					0·0090
18	-22·126295	-24·758651	86·333	96·604	-347·103074	93·330		-0·735					-0·0430
23	-44·656545	-52·235945	86·333	100·986	-412·854773	92·071		-1·394					-0·0128
24	-67·983212	-28·089247	131·333	54·304	-412·854773	92·665		-1·400					-0·0027
25	-91·209879	-4·925860	176·333	9·523	-412·854773	93·186		-0·879					-0·0009
1	-1·931106	97·508599	-3·667	185·161	-416·036509	94·410		1·0337	dφ	0·5584	0·0567		-0·0717
2	21·766674	73·694301	41·333	139·939	-416·036509	94·400		1·027	dφ	1·1338	0·0210		0·0305
3	45·464453	50·016006	86·333	94·976	-416·036509	94·560		1·187	dκ	1·4903	0·0586		-0·0328
8	22·934204	25·524539	86·333	96·084	-348·693942	94·490		1·117	dκ	1·0332	0·0560		0·0288
9	10·980036	37·511381	41·333	141·207	-348·693942	94·255		0·882	dω	0·5621	0·0274		0·0569
10	-0·974132	49·348664	-3·667	185·767	-348·693942	94·008		0·635					0·0852
11	-0·017580	0·869623	-3·667	184·787	-325·716131	93·421		0·048					0·0038
12	0·193398	0·653991	41·333	139·772	-325·716131	92·820		0·447					-0·0743
13	0·403954	0·445958	86·333	95·310	-325·716131	94·060		0·687					0·0157
18	-22·126295	-24·758651	86·333	96·604	-347·103074	93·330		-0·043					-0·0579
19	-10·593240	-36·084112	41·333	140·794	-347·103074	92·948		-0·525					-0·0182
20	0·939814	-47·552838	-3·667	185·853	-347·103074	92·440		-0·933					-0·0410
21	1·896790	-97·525615	-3·667	188·543	-412·854773	90·988		-2·385					-0·0078
22	-21·379878	-76·272466	41·333	147·455	-412·854775	91·490		-1·883					0·0622
23	-44·656545	-52·235945	86·333	100·986	-412·854773	92·071		-1·302					0·01543
							93·373					±0·05800	

KASHIM IBRAHIM LIBRARY
AHMADU BELLO UNIVERSITY
ZARIA, NIGERIA.

B.9b continued

Points	$-\frac{XY}{2}$	$\frac{(X-b)Y}{2}$	X	$-(X-b)$	$-(1+\frac{Y^2}{2^2})2$	bY_i	bY_m	$bY_i - bY_m$	x	x	σ	RMSE	V
6	46.842539	1.133786	-3.667	4.268	-348.693942	94.978	94.986	0.992	dφ	0.5214	0.0356	± 0.03938	0.0142
7	34.888371	13.342711	41.333	50.227	-348.693943	94.798		0.812	dφ'	1.0606	0.0280		-0.0424
8	22.934204	25.524539	86.333	96.084	-348.693942	94.430		0.504	dκ'	1.4114	0.1260		0.0272
9	10.980036	37.511381	41.333	141.207	-348.693942	94.255		0.269	dκ''	0.9056	0.1260		0.0101
10	-0.974132	49.348664	-3.667	185.767	-348.693942	94.008		0.022	dλ''	0.6499	0.0675		-0.0056
11	-0.017580	0.864623	-3.667	184.787	-325.716131	93.421		-0.565					-0.0105
12	0.193398	0.653996	41.333	139.772	-325.716131	93.820		-0.166					-0.0558
13	0.403954	0.445958	86.333	95.310	-325.716131	94.060		0.074					0.00
14	0.614510	0.237391	131.333	50.094	-325.716131	94.440		0.454					0.0366
15	0.825066	0.021252	176.333	4.543	-325.716131	94.866		0.880					-0.0434
16	-45.192406	-1.606170	176.333	6.267	-347.103074	94.260		0.274					-0.0233
17	-33.659351	-13.222560	131.333	51.604	-347.103074	93.768		-0.218					0.0188
18	-22.126295	-24.758654	86.333	96.604	-347.103074	93.330		-0.656					0.0034
19	-10.593240	-36.084112	41.333	140.794	-347.103074	92.848		-1.138					0.0240
20	0.939816	-47.552838	-3.667	185.853	-347.103074	92.440		-1.546					+0.0195

Appendix C

PROGRAMME 1.4

```
10    DISP "LEAST SQUARE PROGRAM"
20    STOP
30    DEG
40    FIXED 5
50    DIM A(M,N), B(M), C(N,N), D(N), E(N,N), F(N,M), X(N),
      V(M)
60    MAT READ A(MM,N), B(M)
70    DATA - - - - - - -
80    DATA - - - - - - -
90    DATA - - - - - - -
100   MAT F = TRN (A)
110   MAT C = F * A
120   MAT D = F * B
130.  IF DET (C) = 0 THEN 290
140   MAT E = INV (C)
150   MAT X = E * D
160   DISP x (1); X(2); - - - - - ; X(N)
170   STOP
180   MAT V = A * X
190   MAT V = V - B
200   DISP V(1); V(2); - - - - - -; V(M)
210   STOP
220   I = (V(1) 2 + V(2) 2+ - - - - - + V(M) 2)/(M-N)
230   MAT E = (I) * E
240   DISP "R.M.S.E. = +/- "I 0.5
```

```
250     STOP
260     DISP  "I1 = "E (1,1)   0.5; -- -  - -; "IN="E(N,N)  0.5
270     STOP
280     GO TO 300
290     DISP  "THE SYSTEM OF EQUATIONS HAS NO PRECISE/UNIQUE
        SOLUTION"
300     END IND
310     END
```

APPENDIX D

D.1. Corrections to Elements of Relative Orientation and their σs for Model 1

DENSITY	ELEMENT ± σ / DISTRIBUTION	dφ' ± σ	dφ'' ± σ	dk' ± σ	dk'' ± σ	dw'' ± σ
SIX	STANDARD POINT	-0.2757 ± 0.0255	-0.3451 ± 0248	2.9227 ± 0.0707	1.7615 ± 0.0713	1.1103 ± 0.0337
	LEFT HAND SIDE	-0.3616 ± 0.0650	-0.3514 ± 0.0287	3.0749 ± 0.0866	1.8971 ± 0.0840	1.0402 ± 0.0395
	CENTRALLY STANDARD	-0.2610 ± 0.0116	-0.3654 ± 0.0108	2.7368 ± 0.0137	1.5330 ± 0.0197	1.0129 ± 0.0089
	RIGHT HAND SIDE	-0.2610 ± 0.0592	-0.4144 ± 0.1229	2.5318 ± 0.1611	1.3878 ± 0.1698	1.0638 ± 0.0598
	CENTRALLY MIDDLE	-0.3342 ± 0.6523	-0.3813 ± 1.7473	4.2036 ± 17.7621	3.0456 ± 17.8700	1.9143 ± 9.4665
NINE	STANDARD POINT	-0.2865 ± 0.0312	-0.3571 ± 0.0299	2.8431 ± 0.0770	1.6813 ± 0.0783	1.0727 ± 0.0369
	LEFT HAND SIDE	-0.3595 ± 0.0968	-0.3495 ± 0.0382	3.0488 ± 0.1050	1.8704 ± 0.0983	1.0275 ± 0.0465
	CENTRALLY STANDARD	-0.2706 ± 0.0592	-0.3756 ± 0.0554	2.7228 ± 0.0885	1.5190 ± 0.0891	1.0059 ± 0.0407
	RIGHT HAND SIDE	-0.2590 ± 0.0363	-0.4074 ± 0.0789	2.4981 ± 0.0866	1.3541 ± 0.0930	1.0479 ± 0.0414
	CENTRALLY MIDDLE	-0.2617 ± 0.1082	-0.3877 ± 0.1012	2.1753 ± 0.2948	0.9931 ± 0.2954	0.8467 ± 0.1598
TWELVE	STANDARD POINT	-0.2960 ± 0.0446	-0.3565 ± 0.0293	2.8871 ± 0.0719	1.7278 ± 0.0719	1.0269 ± 0.0337
	LEFT HAND SIDE	-0.1273 ± 0.1121	-0.4456 ± 0.0554	2.7935 ± 0.1261	1.7609 ± 0.1171	1.0057 ± 0.0624
	CENTRALLY STANDARD	-0.2763 ± 0.0834	-0.3635 ± 0.0783	2.5579 ± 0.1292	1.3745 ± 0.1305	1.0205 ± 0.0675
	RIGHT HAND SIDE	-0.2305 ± 0.0439	-0.5367 ± 0.0862	2.3536 ± 0.0923	1.1625 ± 0.1019	1.0625 ± 0.0522
	CENTRALLY MIDDLE	-0.2687 ± 0.0643	-0.3756 ± 0.0879	2.3587 ± 0.2712	0.8976 ± 0.2712	1.0231 ± 0.1464
FIFTEEN	STANDARD POINT	-0.2801 ± 0.0281	-0.3571 ± 0.0267	2.7960 ± 0.0634	1.6253 ± 0.0643	1.0485 ± 0.0299
	LEFT HAND SIDE	-0.3699 ± 0.0656	-0.3501 ± 0.0261	2.9526 ± 0.0649	1.8048 ± 0.0618	1.0339 ± 0.0301
	CENTRALLY STANDARD	-0.2808 ± 0.0605	-0.3877 ± 0.0567	2.6401 ± 0.0828	1.4515 ± 0.0840	1.0059 ± 0.0401
	RIGHT HAND SIDE	-0.2489 ± 0.0261	-0.4208 ± 0.0573	2.3848 ± 0.0586	1.2274 ± 0618	1.0313 ± 0287
	CENTRALLY MIDDLE	-0.2795 ± 0.0535	-0.3750 ± 0.0509	2.4930 ± 0.2215	1.3318 ± 0.2222	1.0237 ± 0.1203

D.2 Corrections to Elements of Relative Orientation and their σs for Model 2

Density	Distribution \ Elements	$d\phi' \pm \sigma$	$d\phi'' \pm \sigma$	$dk' \pm \sigma$	$dk'' \pm \sigma$	$d\omega'' \pm \sigma$
SIX	STANDARD POINT	0·5704 ± 0·0337	1·1548 ± 0·0518	1·4661 ± 0·0707	0·9785 ± 0·0948	0·5723 ± 0·0446
	LEFT HAND SIDE	0·6367 ± 0·0127	1·1548 ± 0·0051	1·4719 ± 0·0176	1·0326 ± 0·0159	0·5329 ± 0·0072
	CENTRALLY STANDARD	0·5354 ± 0·0598	1·1459 ± 0·0541	1·5272 ± 0·1086	1·0269 ± 0·1082	0·5997 ± 0·0503
	RIGHT HAND SIDE	0·5660 ± 0·0299	1·1179 ± 0·0598	1·3625 ± 0·0828	0·8359 ± 0·0847	0·5768 ± 0·0274
	CENTRALLY MIDDLE	0·5042 ± 0·1833	1·0485 ± 5·147	-0·4068 ± 5·1471	-0·9288 ± 5·1433	-0·3562 ± 2·6980
	STANDARD POINT	0·5641 ± 0·0280	1·1478 ± 0·0261	1·4407 ± 0·0707	0·9537 ± 0·0700	0·5609 ± 0·0331
NINE	LEFT HAND SIDE	0·5386 ± 0·0592	1·1541 ± 0·0233	1·5514 ± 0·0662	1·1128 ± 0·0630	0·5860 ± 0·0293
	CENTRALLY STANDARD	0·5360 ± 0·0376	1·1453 ± 0·0337	1·4852 ± 0·0579	0·9855 ± 0·0573	0·5800 ± 0·0267
	RIGHT HAND SIDE	0·5600 ± 0·0179	1·1096 ± 0·0166	1·3471 ± 0·0414	0·8155 ± 0·0433	0·5672 ± 0·0191
	CENTRALLY MIDDLE	0·5647 ± 0·0770	1·0415 ± 0·0700	1·3445 ± 0·2088	0·8499 ± 0·2088	0·5819 ± 0·1114
	STANDARD POINT	0·5328 ± 0·0299	1·1523 ± 0·0185	1·5018 ± 0·0487	1·0313 ± 0·0490	0·5678 ± 0·0484
	LEFT HAND SIDE	0·5220 ± 0·0535	1·0613 ± 0·0242	1·8118 ± 0·0700	1·3426 ± 0·0694	0·6984 ± 0·0363
TWELVE	CENTRALLY STANDARD	0·5080 ± 0·0471	1·1065 ± 0·0427	1·6036 ± 0·0840	1·0969 ± 0·0834	0·6283 ± 0·0433
	RIGHT HAND SIDE	0·5017 ± 0·0229	1·0963 ± 0·0420	1·0873 ± 0·0503	0·5551 ± 0·0516	0·5195 ± 0·0261
	CENTRALLY MIDDLE	0·4877 ± 0·0579	1·1071 ± 0·0528	1·3293 ± 0·0879	0·8461 ± 0·0872	0·5991 ± 0·0452
FIFTEEN	STANDARD POINT	0·5551 ± 0·0197	1·1446 ± 0·0185	1·4731 ± 0·0465	0·9529 ± 0·0458	0·5761 ± 0·0217
	LEFT HAND SIDE	0·5284 ± 0·0567	1·1338 ± 0·0210	1·4903 ± 0·0586	1·0332 ± 0·0560	0·5621 ± 0·0274
	CENTRALLY STANDARD	0·5379 ± 0·0287	1·1205 ± 0·0255	1·4439 ± 0·044	0·9342 ± 0·0407	0·5812 ± 0·0197
	RIGHT HAND SIDE	0·5488 ± 0·0153	1·1026 ± 0·0331	1·2993 ± 0·0350	0·7441 ± 0·0363	0·5704 ± 0·0172
	CENTRALLY MIDDLE	0·5219 ± 0·0356	1·0606 ± 0·0280	1·4114 ± 0·1260	0·9085 ± 0·1260	0·6149 ± 0·0675