A REVIEW OF AREAS AND VOLUMES IN EARTHWORKS"

BY

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DEDICATION

This thesis is fondly dedicated to:

My sister

Mrs Deborah 'Funke Babayemi

for her love, affection and concern

for my welfare always.

DECLARATION

I hereby declare that this thesis has been composed by myself and that it is a record of my own research work. It has not been accepted in any previous application for a higher degree. All quotations are distinguished by quotation marks and the sources of information are specifically acknowledged by means of references.

Ogundana, Joseph Akinola

CERTIFICATION

This thesis entitled "A Review of Areas and Volumes in Earthworks" by Ogundana, J. A. meets the regulations governing the award of the degree of Master of Science of Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

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ABSTRACT

Various approaches for computing cross sectional areas in earthworks are treated. Proofs of different formulae in the calculations are given, including some examples on the applications.

Each method is analysed with sufficient depth in order to easily infer its merit and limitations. The methods, considered comparatively, are synthesized and their relative stregth and advantages effectively enunciated with examples.

Estimation of earthworks volumes and rationalization of earthmoving patterns by the use of the Mass Haul Diagrams in construction economy are also treated.

Recommendations *on* methods to adopt on every situation and circumstance are contained in the conclusion. Finally, advice is given about the professional relationship that should exist on site between the Civil Engineer and the Engineering Surveyor when executing an earthworks project.

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NOMENCLATURE

b Finished read width

h Centre height i.e. depth of excavation of embankment of the section measured vertically along the centre line.

W1,W2 side widths, measured horizontally from either side of the centre line fixing the position of the slope stakes

1 in m Side slope of 1 vertical to m horizontal

1 in n side slope of 1 vertical to n horizontal

1 in k Existing ground slope (up or down gradient)

W_1 + W2 Plan width

N ordinate

E abscissa

CHAPTER ONE

INTRODUCTION

The aim of the dissertation is to make a comprehensive review of areas and volumes in earthworks. In mass construction all earth moving plans and analysis are based on sound knowledge of various methods of finding areas and volumes of earthworks. All such computations are based on the consultant engineer's design. The Engineering Surveyor is thus, occasionally, confronted with this responsibility of estimating the volume of the "materials" involved in a construction scheme and the planned pattern of movement on site.

The next chapter (chapter two) gives a review of the different methods of finding areas in earthworks right from the elementary aspects like areas of a square figure to the more complicated types like the cross sectional areas of excavations, embankments or the cut and fill sections on a hillside. Calculations are treated by use of different kinds of approach like direct application of formulae, the analytical or trigonometrical method of solution, the graphical solution and so forth. After being used to these methods the computor intuitively determines the most expedient method of solving any area problems in surveying.

Chapter three deals with volumes. Three different types of problems in volumes of earthworks are treated, viz: Volumes from cross sections, volumes from spot heights and volumes from contours. Simple illustrative examples are used to explain the application of each type.

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AHMADU

The principles of the Mass Haul Diagrams are given in chapter four. It is treated in detail becaurse of its importance in earthworks.

CHAPTER TWO

AREAS IN EARTHWORKS

2.1 Areas enclosed by straight lines i.e. cross sectional areas resulting from gridding of a terrain.

2.1.1 Areas of triangles

(i) Given the base a of triangle ABC and a height h,

Area = $\frac{1}{2}$. base x height sq. units

(ii) Given the three sides a,b,c of the triangle

Area = $\sqrt{s(s-a)(s-b)(s-c)}$ sq. units

where s = $\frac{1}{2}$ (a+b+c)

(i i) Given two sides a, b and the included angle C

Area = $\frac{1}{2}$. a. b. SinC sq. units

2.1.2 Areas of squares

Given the side of a square as x

Area = x.x = x^2 sq. units

2.1.3 Areas of rectangles

Given length a and breadth b of a triangle,

Area = a.b sq. units

2.1.4 Areas of parallelogram

Given the two adjacent sides a,b and the included angle B of a parallelogram ABCD

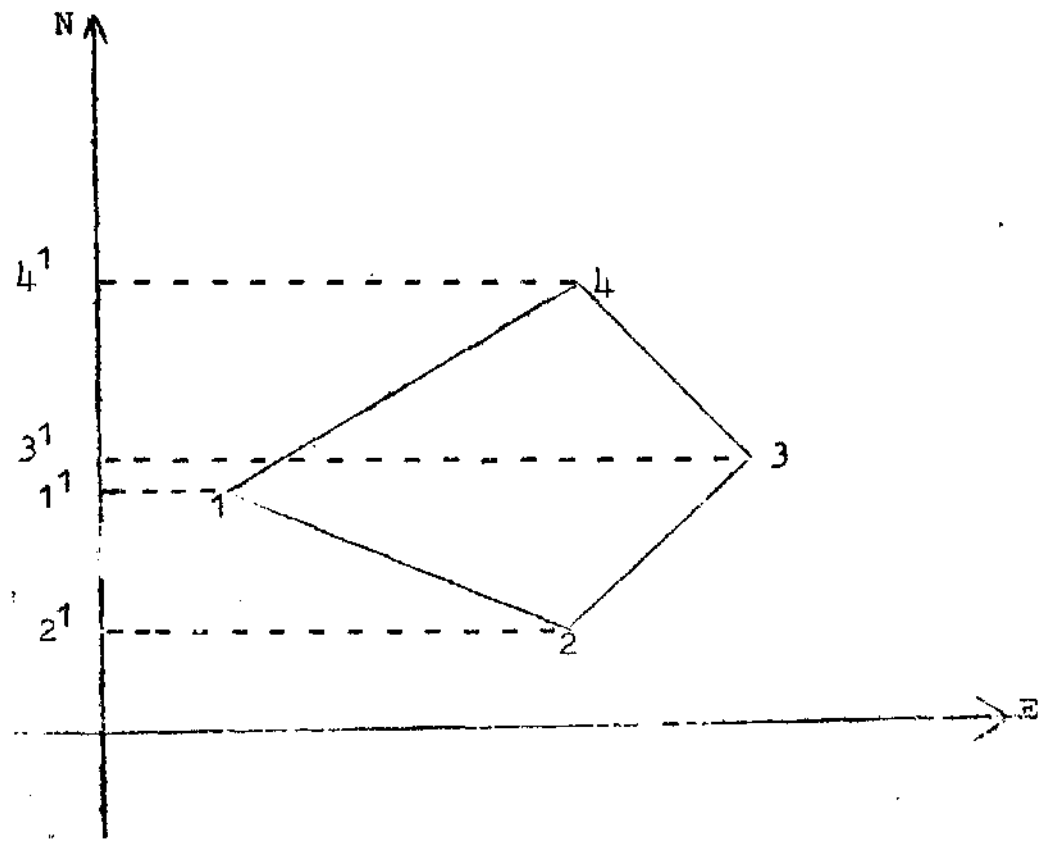
Area = a.b. Sin B

2.1.5 Areas from Coordinates

A traverse survey of an area can be made and the coordinates of the junction points calculated. The area of the figure can subsequently be computed using the method illustrated below:

2.1.5(i) Figure 2.1 shows a four-sided figure with vertices 1, 2, 3, 4.

Figure 2.1 : A four-sided Coordinated figure



You draw perpendiculars from the traverse junction points 1, 1, 3, 4 to meet the north axis.

a + 1^1 , 2^1, 3^1, 4^1.

Area 1234 = A

A = (Trap. 2^1233^1+3^1344^1)-(2^1211^1+1^1144^1)

$2A = (E_2+E_3)(N_3-N_2)+(E_3+E_4)(N_4-N_3)$

$- (E_1+E_2)(N_1-N_2)-(E_4+E_1)(N_4-N_1)$

By collecting terms

$2A = E_2(N_3-N_2+N_1+N_2)+E_3(N_3- N_2+ N_4- N_3)$

$- E_4(-N_4+N_1+N_4-N_3)+E_1(N_2-N_1+N_4+N_1)$

$= E_1(N_2-N_4)+E_2(N_3-N_1)+E_3(N_4-N_2)+E_4(N_1-N_3)$

Similarly if we draw perpendiculars to the east axis. We shall have

$$2A = N_1(E_2-E_4)+N_2(E_3-E_1)+N_3(E_4-E_2)+N_4(E_1-E_3).$$

Ex. use this formula to compute the area enclosed by the vertices A (0,0), B(63,63), C (136, 150), D (-20, 271).

$$2A = E_1(N_2-N_4)+E_2(N_3-N_1)+E_3(N_4-N_2)+E_4(N_1-N_3).$$

$$= 9450 + 28288 + 3000 = 40738$$

$$A = 20369 \text{ sq. units}$$

2.1.5 (ii) An alternative method of computing areas from coordinates is the double area method.

You multiply the algebraic sum of the northings of each station and the one following by the algebraic difference of the eastings of each station and the one following. The area is thus half of the algebraic sum of the products. The area is best computed from a tabulation as in the following example.

Ex. Find the area of the figure ABCD, A (0,0), B (63, 63), C (136, 150), D (-20, 271).

Table 2.1 : Double area method of finding areas from Coordinates.

Stations	N	E	Sum of N	Difference of E	Double Area +	Double Area -
A	0	0	63	-63		3969
B	63	63	213	-73		15549
C	150	136	421	156	65676	
D	271	-20	271	-20		5420
A	0	0				
					65676	24938
					24938	

Algebraic Sum = $40738m^2$

2. Area ABCD

= $40738m^2$

Area ABCD = $20369m^2$

as before

Relative merits of the methods:

The method in 2.1.5(i) can be checked by reversing the Coordinates and repeating the calculation. The areas obtained on both occasions should be the same. The method in 2.1.4(ii) enables computation to be made from the simple table.

2.1.5 (iii) Method of Cross multiplication.

This is another method of calculating areas from coordinates. From 2.1.5(i) we obtained the following expression:

$$2A = N_1(E_2-E_4)+N_2(E_3-E_1)+N_3(E_4-E_2)+N_4(E_1-E_3).$$

$$= N_1E_2-N_1E_4+N_2E_3-N_2E_1+N_3E_4-N_3E_2+N_4E_1-N_4E_3$$

By rearranging, we have

$$2A = N_1E_2 + N_2E_3 + N_3E_4 + N_4E_1$$

$$- N_2E_1 - N_3E_2 - N_4E_3 - N_1E_4$$

or, since we also had that

$$2A = E_1(N_2-N_4) + E_2(N_3-N_1) + E_3(N_4-N_2) + E_4(N_1-N_3)$$

$$= E_1N_2 - E_1N_4 + E_2N_3 - E_2N_1 + E_3N_4 - E_3N_2 + E_4N_1 - E_4N_3$$

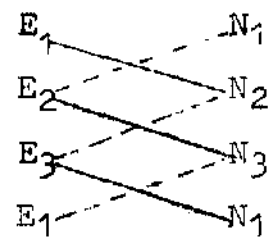
By rearranging we also have

$$2A = E_1N_2 + E_2N_3 + E_3N_4 + E_4N_1$$

$$- E_2N_1 - E_3N_2 - E_4N_3 - E_1N_4$$

The relative merit of this method is that it can easily be remembered from a tabulation. This is done for

a triangle below, the initial coordinates being repeated at the end:



$= 2\Delta$

The oblique full lines are multiplied and the products are added. The other oblique pecked lines are multiplied and the products added. The difference between these two sums is twice the area.

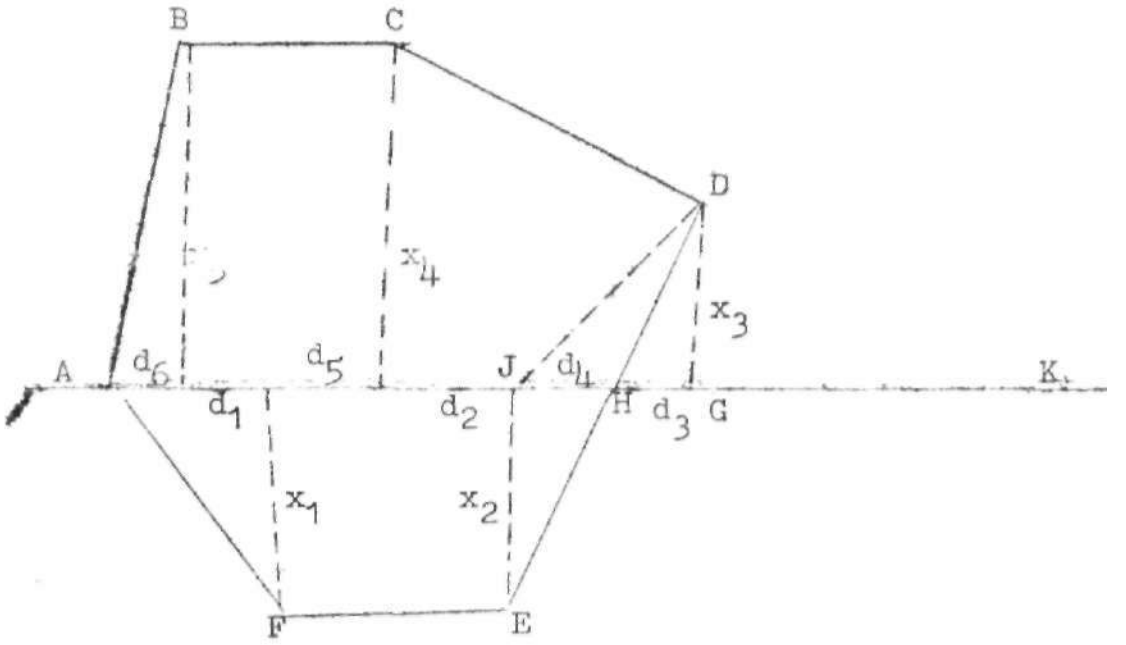
This method of cross multiplication is applicable to any coordinated figure of any number of sides. It can be chekced by reversal of coordinates. The numerical values of the two answers will be the same but they will differ in sign.

The advantage of this method over the method of 2.1.5(ii) is that once the coordinates are arranged in order the answer can be obtained without writing any other figures down.

2.1.5(iv) Areas by method of departures and total latitudes.

This method is applicable also to any polygon of any number of sides. We shall first derive this method from first principles before giving the tabular form of the method.

Figure 2.2 : Departures and total latitudes



The departures and total latitudes are show in Fig. 2.2 with respect to station A of polygon ABCDEFA. The departures of the lines are shown as d_1, d_2, d_3 .... d_6 and the total latitudes with respect to A are shown as x_1, x_2, x_3 .....

AK is parallel to the east - west axis.

By using the formulae for the area of a triangle and quadrilateral respectively;

Area of polygon ABCDEFA i.e. A

$$= \frac{d_1 \cdot x_1}{2} + d_2\left(\frac{x_1+x_2}{2}\right) + d_4\left(\frac{x_3+x_4}{2}\right) + d_5\left(\frac{x_4+x_5}{2}\right)$$

$$+ \frac{d_6 \cdot x_5}{2} + \text{triargle EJH - triangle GDH}$$

But triangle EJH - triangle GDH

= triangle EJD - triangle JDG

$$= \frac{d_3 \cdot x_2}{2} - \frac{d_3 \cdot x_3}{2}$$

$$= d_3\left(\frac{x_2 - x_3}{2}\right)$$

$$\text{i.e. } A = \frac{d_1 \cdot x_1}{2} + d_2\left(\frac{x_1+x_2}{2}\right) + d_3\left(\frac{x_2-x_3}{2}\right) + d_4(x_3+x_4)$$

$$+ d_5\left(\frac{x_4+x_5}{2}\right) + \frac{d_6 \cdot x_5}{2}$$

By rearranging this expression

$$2A = x_1(d_1+d_2) + x_2(d_2+d_3) + x_3(-d_3+d_4)$$

$$+ x_4(d_4+d_5) + x_5(d_5+d_6).$$

This formula may be stated in words thus:
Twice the area of a polygon is the algebraic sum of the products of the total latitudes of each station and the algebraic sum of the departures of the two lines adjoining the station.

The above formula is most conveniently applied in the tabulated form as shown in the following illustration.

Take figure ABCDEFA in fig. 2.2 above in which the latitudes ($\triangle N^S$) and departures ($\triangle E^S$) are as in the table 2.2.

We want to find the area of the figure.

Table 2.2 : Tabular form of calculating areas by departures and total latitudes

Line	Latitude $\triangle N$	Departure $\triangle E$ (d)	Station	Total Latitude(x)	Sum of adjacent departures (d+d)	PRODUCTS	
						+	-
AB	-48.7	+56.9	A	0	-		-
BC	+31.6	+41.3	B	-48.7	+98.2		4782.34
CD	+44.5	+26.4	C	-17.1	+67.7		1157.67
DE	+39.3	-16.3	D	+27.4	+10.1	276.74	-
EF	+19.2	-54.8	E	+66.7	-71.1		4742.37
FA	+85.9	-61.9	F	+85.9	-116.7		10024.53

Sum = 2A = 20430.17

A = 10215.085 sq. m

or A = 1.0215085 ha

The method of working is explained thus:-

The total latitude x_1 of station B with respect to A (assumed origin) is the latitude of the line itself; i.e. - 48.7 in table 2.2.

The total latitude x_2 of station C is the total latitude of B plus the latitude of the line BC, i.e. - 48.7 + 31.6 = - 17.1 and the process continues in that manner.

The total latitude of the final point must always be the same as the latitude of the last line with the sign changed. This is a very reliable check for if this condition is not obtained an error of calculation has been incurred.

The sum of adjacent departures is obtained by adding together the departures of the two lines on either side of the station concerned:

+ 56.9 + 41.3 = + 98.2

+ 41.3 + 26.4 = + 67.7

+ 26.4 - 16.3 = + 10.1

- 16.3 - 54.8 = - 71.1

- 54.8 - 61.9 = -116.7

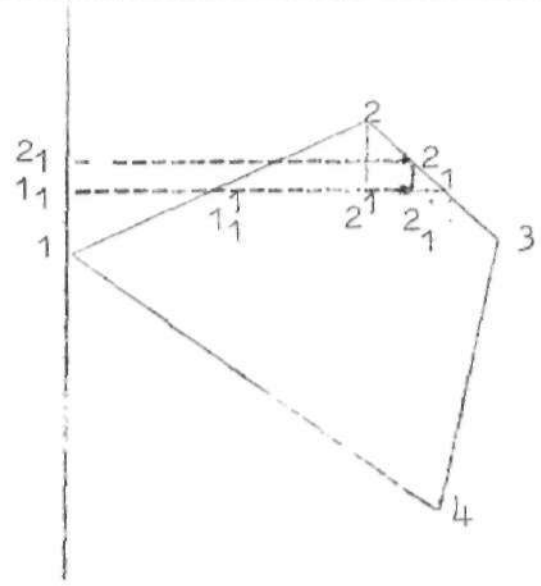
The figures in the x column are multiplied by the figures in the (d+d) column as shown in table 2.2. Their algebraic Sum forms twice the area.

Another good check is to repeat the whole calculation after interchanging the columns of $\triangleright E^C$ and $\triangleright N^S$. This rotates the figure through 90° without any effect on the area.

2.1.5 (v). Area by double longitude Method:

Definition of longitude. In the co-ordinate system every line has a longitude with respect to a meridian. The longitude of a line is the porpendicular distance from a convenient meridian to the centre point of the line. In practice the most westerly point in the traverse is assumed as the reference meridian, e.g. point 1 in figure 2.3.

Figure 2.3 : Illustration of longitude of a line.



Consider the traverse 12341 in figure 2.3.

The midpoint of line 1,2 is $1_1^{\,1}$ and the mid-point of 2,3 is $2_1^{\,1}$ and so forth.

Then the longitude of 1,2 = $1_1\ 1_1^{\,1}$ (= ½ departure 1,2)

Longitude of 2,3 = $2_1 2_1^{\,1}$

$= 1_1 1_1^{\,1} + 1_1^{\,1} 2^1 + 2^1 2_1^{\,11}$

= Long. 1,2 + ½ departure 1,2

+ ½ departure 2,3.

∴ In general the longitude of line N = long. of line (N-1)

+ ½ departure of line (N-1)

+ ½ depature of line N.

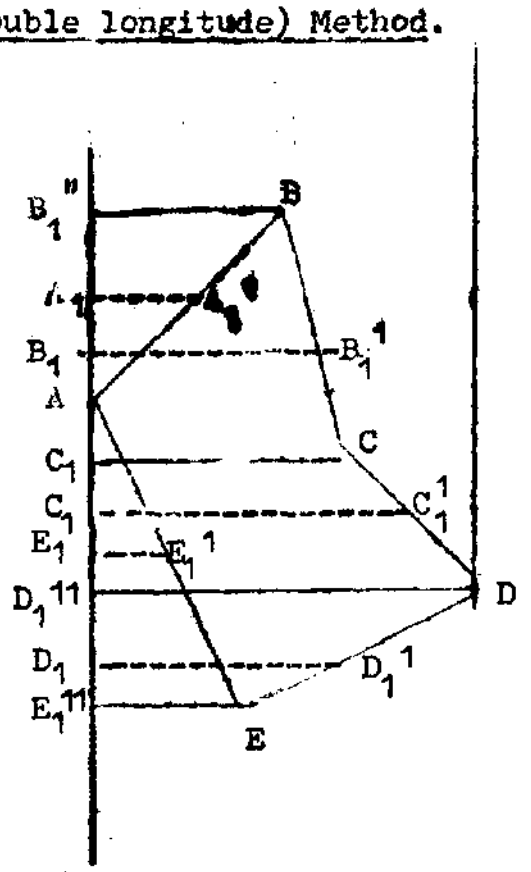
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The longitude of any line is equal to the longitude of the previous line plus half the depature of the previous line plus half its own depature. The summing must be strictly with due regard to the sign, i.e. algebraically. However, by taking the most westerly station as the reference meridian no longitude could be negative.

Proof of Double longitude Method

In figure 2.4 point A is the most westerly station through which the reference meridian passes. We are to obtain area for ABCDEA.

Figure 2.4 Illustration of latitude and longitude (double longitude) Method.



Area ABCDEA = trapezoid B_1^{11} BCC_1^{11} + trapezoid C_1^{11} CDD_1^{11}

+ trapezoide D_1^{11} DEE_1^{11} - triangle B_1^{11} BA

- - triangle AEE_1^{11}.

$$= B_1^{11} C_1^{11} \times B_1 B_1^1 + C_1^{11} D_1^{11} \times C_1 C_1^1 + D_1^{11} E_1^{11} \times D_1 D_1^1 - AB_1^{11} \times AA_1^1 - E_1^{11} A \times E_1 E_1^1$$

or = Latitude BC x longitude BC + latitude CD x longitude CD + latitude DE x longitude DE - latitude AB x longitude AB - latitude EA x longitude EA.

i.e. the algebraic sum of the products of latitude and longitude gives the required area. For convenience of working from tabulation it is convenient to compute in double longitude and then to divide the answer by two. Double Longitude of a line = double logitude of previous line + departure of previous line + departure of line under consideration.

This is best illustrated by an example. The calculation is made easy as the area can be obtained from a table as shown in table 2.3.

We want to calculate the area of a gigure ABCDEFA whose coordinates are listed in the table below:

Table 2.3: Double Longitude Method in tabular form.

Station	Eastings	Northings	Double Longitude (E+E)	▷N	PRODUCTS +	PRODUCTS -
A	160.0	110.4				
B	86.4	167.3	246.4	56.9	14020.16	
C	134.3	189.9	220.7	22.6	4987.82	
D	205.2	213.6	339.5	23.7	8046.15	
E	168.8	241.2	374.0	27.6	10322.40	
F	223.5	177.4	392.3	-63.8		25028.74
A	160.0	110.4	383.5	-67.0		25694.50

Algebraic Sum = 2A

= 13346.71

$\therefore$ A = 6673.355 m^2

The repetition of the initial coordinate (station A) at the bottom of the list is to simplify calculation.

For large coordinates a whole number constant could be subtracted from the eastings before summation in the double longitude column.

The (E + E) column is tabulated from the eastings column, e.g.

160.0 + 86.4 = 246.4

86.4 + 134.3 = 220.7

134.3 + 205.2 = 339.5 and so on.

The $\triangleright$N column contains normal departures or $\triangleright N^S$ thus:

167.3 - 110.4 = 56.9

189.9 - 167.3 = 22.6

213.6 - 189.9 = 23.7 etc.

The double longitude method commends itself because the calculation can be checked by repeating the calculation with the coordinates interchanged. The cross multiplication method (see 2.1.5(iii))can also be used to calculate the area as a cheque.

2.1.5(vi): Sanyaolu's analytical methods and automatic checks for area computation. This method is very much akin to the cross-multiplication method in 2.1.5(iii). It is based on the following concepts:

(a) Two co-planar vectors 1-2 and 3-4 moving at random in their plane generate space or successive geometrical figures.

(b) The area of each of the successive figures spanned (or space) is given by the slopes of the two vectors and this is half the difference of the cross products of the gradient elements of the two vectors

- Sanyaolu, A; 1975.

From (b); if such space is a quadrilateral area A is given by

$$A = \tfrac{1}{2}\left((x_1-x_2)(y_3-y_4)-(y_1-y_2)(x_3-x_4)\right)$$

$$\text{or} = \tfrac{1}{2}\left(\Delta x_{12}\,\Delta y_{34} - \Delta y_{12}\,\Delta x_{34}\right).$$

(c) Two elements that are critically essential in calculating coordinates are the lengths of the lines and the directions. If the directions of two vectors B_1 and B_2 and their lengths 1_1 and 1_2 are known the arising area spanned = $\tfrac{1}{2}$

$$(1_1 \text{ S in } B_1 \cdot\ 1_2 \text{ Cos } B_2 - 1_1 \text{ Cos } B_1 \cdot 1_2 \text{ S in } B_2)$$
$$= \tfrac{1}{2}\, 1_1\, 1_2 \text{ Sin } Q$$

Where Q is the angle between the vectors.

For purposes of brevity Sanyaolu's method is not in any way significantly different from the traditional methods earlier given in this section. It however commends itself while the computor is faced with a polygon of large number of vertices because:

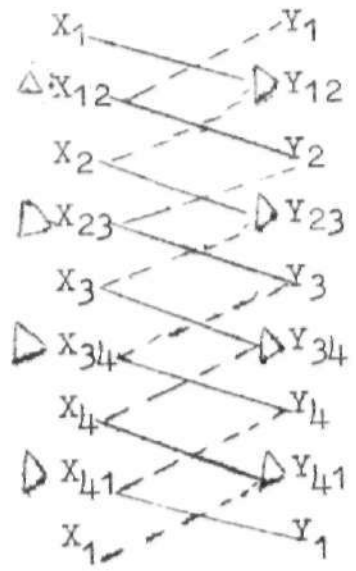
(a) The polygon can be sub-divided among any number of computors.

(b) Checking can be effected at any point during the calculations.

An example will explain this better.

Take a qudrilateral 1, 2, 3, 4.

Sanyaolu gives the area and countercheck calculations in the form of cross multiplication method as done below:



Where (x_1,y_1) , (x_2,y_2) , (x_3,y_3), and (x_4,y_4) are the coordinates of 1, 2, 3, 4.

From the table,

$$\triangle x_{12} = x_1 - x_2 \; ; \quad \triangle y_{12} = y_1 - y_2 \quad \text{etc.}$$

If we pair the elements of the Coordinates as in 2.1.5(iii) we shall calculate the area of the quadrilateral, A as

$$2A = x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1 - x_2y_1 - x_3y_2 - x_4y_3 - x_1y_4$$

From the table all the oblique full lines are positive elements in the cross multiplication. The oblique pecked lines are the negative elements in the cross multiplication.

In the countercheck the following expression must be true for the number of vertices.

$$x_1 (y_1-y_2) + y_2 (x_1-x_2) + x_2 (y_2-y_3) + y_3 (x_2-x_3)$$

$+ x_3 (y_3-y_4) + y_4 (x_3-x_4) + x_4 (y_4-y_1) + y_1 (x_4-x_1) =$

$y_1 (x_1-x_2) + x_2 (y_1-y_2) + y_2 (x_2-x_3) + x_3 (y_2-y_3) + y_3 (x_3-x_4)$

$+ x_4 (y_3-y_4) + y_4 (x_4-x_1) + x_1 (y_4-y_1) = 0$ and so on.

Depending on the number of sides of the polygon.

As an example we want to compute the area 'A' of the quadrilateral made up of the first four vertices of the polygon shown in table 2.3 thus:

Easting	Northing
160.0	110.4
86.0	167.3
134.3	189.9
205.2	213.6

$$2A = x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1$$
$$- x_2y_1 - x_3y_2 - x_4y_3 - x_1y_4$$

Substituting accordingly.

$$2A = (160.0 \times 167.3) + (86.4 \times 189.9) +$$
$$(134.3 \times 213.6) + (205.2 \times 110.4)$$
$$- (86.4 \times 110.4) - (134.3 \times 167.3)$$
$$- (205.2 \times 189.9) - (160.0 \times 213.6)$$
$$= 26768.0 + 16407.36 + 28686.48 + 22654.08$$
$$- 9538.56 - 22468.39 - 38967.48 - 34176.0$$
$$= 10634.51$$
$$A = 5317.255 \text{ m}^2$$

A check is now made at the 4^{th} vertex, say we resort to the 'check' cross multiplication formula earlier given. Considering the L. H. S. of the expression, i.e.

$x_1 (y_1-y_2) + y_2 (x_1-x_2) + x_2 (y_2-y_3) + y_3 (x_2-x_3)$

$+ x_3 (y_3-y_4) + y_4 (x_3-x_4) + x_4 (y_4-y_1) + y_1 (x_4-x_1)$

Substituting accordingly

$(110.4 \times 73.6) + (86.4 \times -56.9) + (167.3 \times -47.9)$

$+ (134.3 \times -22.6) + (189.9 \times -70.9) + (205.2 \times -23.7)$

$+ (213.6 \times 45.2) + (160.0 \times 103.2)$

$= 8125.44 - 4916.16 - 8013.67 - 3035.18 - 13463.91$

$- 8463.24 + 9654.72 + 16512.0 = 0$

Considering the R.H.S. of the expression for counter-check.

$y_1 (x_2-x_2) + x_2 (y_1-y_2) + y_2 (x_2-x_3) + x_3 (y_2-y_3)$

$+ y_3 (x_3-x_4) + x_4 (y_3-y_4) + y_4 (x_4-x_1) + x_1 (y_4-y_1)$

$= (160.0 \times -56.9) + (167.3 \times 73.6) + (86.4 \times -22.6)$

$+ (189.9 \times -47.9) + (134.3 \times -23.7) + (213.6 \times -70.9)$

$+ (205.2 \times 103.2) + (110.4 \times 45.2)$

$= -9104.0 + 12313.23 - 1952.64 - 9096.21 - 3182.91$

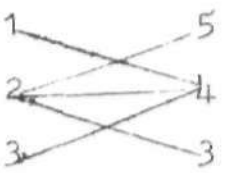
$-15144.24 + 21176.64 + 4990.08 = 0$ as expected.

The above confirms the notion that the checking process is rather long and time consuming. It becomes even more tedious if we are dealing with large coordinates. One could have preferred obtaining the area using the double longitude or departures and total latitude method and their respective conventional checking procedure earlier indicated in this section.

However, for simplicity purposes, the counter-check given by Sanyaolu can also be conveniently reduced to a table. For this reason his method of computing areas is acceptable.

Generalisation of Sanyaol:'s Method:

The idea is to divide the polygon into a chain of quadrilaterals. The cross multiplication involved is as arranged below, depending on the number of vertices of the polygon. If n (number of vertices) is odd, then, the last number in the chain of quadrilaterals will be $\frac{n+1}{2}$ on either side of the arrangement. For example for a polygon of five vertices the combination of quadrilaterals will be as follows:



This cross multiplication will give the following result (in determinant form). A is the area of the figure.

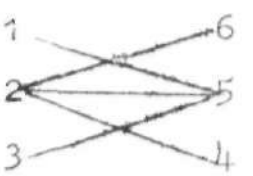
$$A = \frac{1}{2}\begin{vmatrix} \triangle x_{44} & \triangle y_{14} \\ \triangle x_{25} & \triangle y_{25} \end{vmatrix} + \frac{1}{2}\begin{vmatrix} \triangle x_{23} & \triangle y_{23} \\ \triangle x_{34} & \triangle y_{34} \end{vmatrix}$$

$$A = \tfrac{1}{2}\left((x_{14}\ y_{25} - x_{25}\ y_{14}) + (x_{23}\ y_{34} - x_{34}\ y_{23})\right)$$

$$A = \tfrac{1}{2}\left((x_1 - x_4)(y_2-y_5) - (x_2-x_5)(y_1-y_4) + (x_2-x_3)(y_3-y_4) - (x_3-x_4)(y_2-y_3)\right).$$

The substitutions are then made from the Coordinates (x_1,y_1), (x_2,y_2), (x_3,y_3), (x_4,y_4), and (x_5,y_5) of the five vertices of the polygon.

For an even number of vertices (e.g. six) the chain of quadrilaterals is obtained in a more straight forward manner as follows:



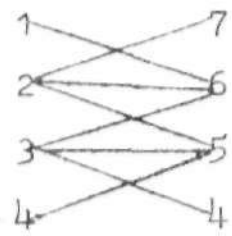
The above cross multiplication gives the following result (in determinant form). A is the area.

$$A = \frac{1}{2}\begin{vmatrix} \Delta x_{15} & \Delta y_{15} \\ \Delta x_{26} & \Delta y_{26} \end{vmatrix} + \frac{1}{2}\begin{vmatrix} \Delta x_{24} & \Delta y_{24} \\ \Delta x_{35} & \Delta y_{35} \end{vmatrix}$$

$$A = \frac{1}{2}\left((\Delta x_{15}\Delta y_{26} - \Delta x_{26}\Delta y_{15}) + (\Delta x_{24}\Delta y_{35} - \Delta x_{35}\Delta y_{24})\right)$$

$$A = \frac{1}{2}\left((x_1 - x_5)(y_2 - y_6) - (x_2 - x_6)(y_1 - y_5) + (x_2 - x_4)(y_3 - y_5) - (x_3 - x_5)(y_2 - y_4)\right).$$

For seven vertices the series of quadrilaterals will be in the following arrangement:



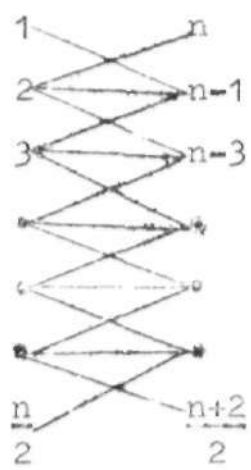
If A is the area

$$A = \frac{1}{2}\begin{vmatrix} \Delta x_{16} & \Delta y_{16} \\ \Delta x_{27} & \Delta y_{27} \end{vmatrix} + \frac{1}{2}\begin{vmatrix} \Delta x_{25} & \Delta y_{25} \\ \Delta x_{36} & \Delta y_{36} \end{vmatrix} + \frac{1}{2}\begin{vmatrix} \Delta x_{34} & \Delta y_{34} \\ \Delta x_{45} & \Delta y_{45} \end{vmatrix}$$

$$A = \frac{1}{2}\left((\Delta x_{16}\Delta y_{27} - \Delta x_{27}\Delta y_{16}) + (\Delta x_{25}\Delta y_{36} - \Delta x_{36}\Delta y_{25}) + (\Delta x_{34}\Delta y_{45} - \Delta x_{45}\Delta y_{34})\right).$$

$$A = \frac{1}{2}\left((x_1 - x_6)(y_2 - y_7) - (x_2 - x_7)(y_1 - y_6) + (x_2 - x_5)(y_3 - y_6) - (x_3 - x_6)(y_2 - y_5) + (x_3 - x_4)(y_4 - y_5) - (x_4 - x_5)(y_3 - y_4)\right).$$

We can therefore generalise for the area of any polygon of 'n' vertices using a similar chain of quadrilaterals as follows:

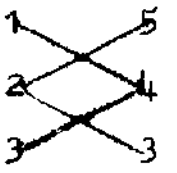


Where the last number $\frac{n+2}{2}$ is obtained from $\frac{n+1}{2}$ as earlier espoused in the section.

The area is obtained by arranging the determinants in the manner of the previous examples. We now use the novelty of Sanyaolu's determinant method to find the area of a polygon of five vertices with the following coordinates:

Vertex	Northing X	Easting Y
1	+72.923	+406.988
2	+81.871	+361.079
3	+88.913	+348.494
4	+109.987	+341.540
5	+111.022	+413.970

As usual, the chain of quads. can be obtained thus:



, A being the area.

$$A = \frac{1}{2}\begin{vmatrix} \Delta x_{14} & \Delta y_{14} \\ \Delta x_{25} & \Delta y_{25} \end{vmatrix} + \frac{1}{2}\begin{vmatrix} \Delta x_{23} & \Delta y_{23} \\ \Delta x_{34} & \Delta y_{34} \end{vmatrix}$$

$$A = \frac{1}{2}\left((\Delta x_{14}\,\Delta y_{25}) - (\Delta x_{25}\,\Delta y_{14}) + (\Delta x_{23}\,\Delta y_{34} - \Delta x_{34}\,\Delta y_{23})\right)$$

$$= \frac{1}{2}\left((x_1-x_4)(y_2-y_5) - (x_2-x_5)(y_1-y_4) + (x_2-x_3)(y_3-y_4) - (x_3-x_4)(y_2-y_3)\right).$$

$$= \frac{1}{2}(-37.064 \times -52.891) - (-29.151 \times +65.448) + (-7.042 \times +6.954) - (-21.074 \times +12.585)$$

$$= \frac{1}{2}(+1960.352 + 1907.875 - 48.970 + 265.216)$$

$$= \frac{1}{2} \times 4084.473$$

$$= 2042.2365\ m^2$$

As a check the 'conventional' double longitude method (see 2.1.4(v)) was adopted and it gives the same answer as above. Sanyolu's countercheck method used in the former example in this section can also be applied to check the answer.

2.2 Areas of Irregular Figures

2.2.1 Graphical Method (Counting of squares)

This method involves covering the whole drawing or plan with small squares or the use of transparent overlay of squared paper. The transparent squared paper is laid over the plan. The number of squares and parts of squares which fall completely inside the drawing (or plan) of the area is then counted. Since the plan scale is known, the area represented by each square is easily determined. Hence the total area can be computed.

The accuracy of this method depends on the size of the small squares. The method can give a high degree of accuracy if squares of very small grid is used.

Ex. If the plan scale is 1:200 and each small square is 1mm x 1mm, say.

Then the true area of small square is $(200 \times 200)\ mm^2$.

If there are 475 whole squares approximately, then the plan area $= (475 \times 200 \times 200)\ mm^2$

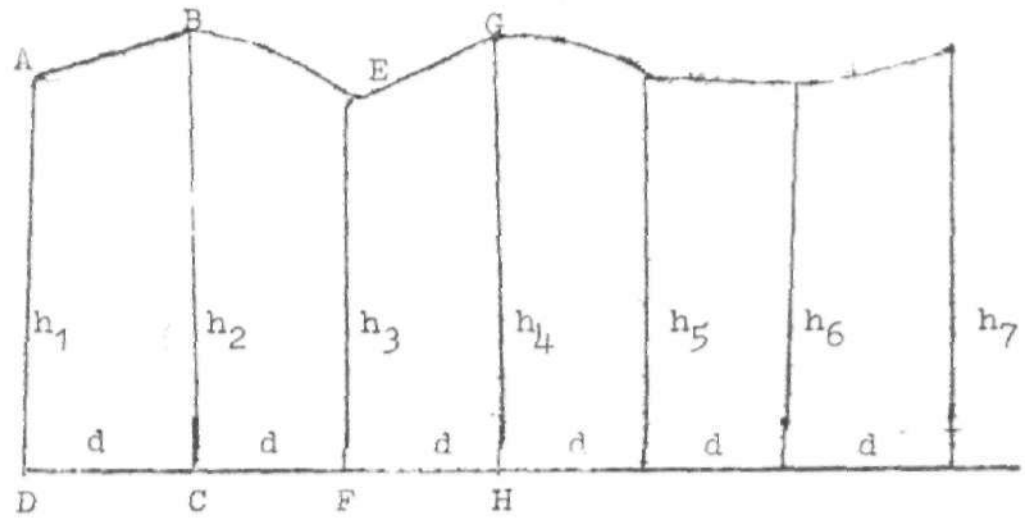
$$= \frac{475 \times 200 \times 200}{100 \times 1000}$$

$$= \frac{1900}{100}\ m^2$$

$$= 19.00\ m^2$$

2.2.2 Trapezoidal Rule

Figure 2.5 : Irregular figure using the trapezoidal rule



The irregular figure, (fig. 2.5) is reduced to series of four sided figures each of which is approximated to a trapezoid.

Area of trapezoid = $\frac{1}{2}$ (sum of parallel sides x width).

Area of trapezoid ABCD = $\frac{h_1+h_2}{2} \times d$

Area of second trapezoid BCEF = $\frac{h_2+h_3}{2} \times d$

Similarly Area of Trap. EGHF = $\frac{h_3+h_4}{2} \times d$ and so on.

Total area = Sum of the areas of the trapezoids.

$$A = \frac{h_1+h_2}{2} \times d + \frac{h_2+h_3}{2} \times d + \frac{h+h}{2} \times d + \ldots\ldots$$

$$= d \left(\frac{h_1+h_2}{2} + h_2 + h_3 + h_4 + h_5+h_6\right).$$

This formula is true even if the first or last ordinate is Zero. The zero quantity must still be included when substituting in the formula.

In general, for n number of offsets (or ordinates)

$$A = d \left(\frac{h_1+h_n}{2} + h_2 + h_3 + h_4 + \ldots + h_{n-1}\right)$$

Ex. The following offsets 7.5m apart, were measured at right angles from a traverse line to an irregular boundary 0, 3.6, 4.9, 9.2, 8.9, 7.8, 6.9, 4.6, 0 all in meters. To find the area between the traverse line and the irregular boundary.

From the trapezoidal formular

$$A = d\left(\frac{h_1+h_n}{2} + h_2 + h_3 + \ldots\ldots + h_{n-1}\right).$$

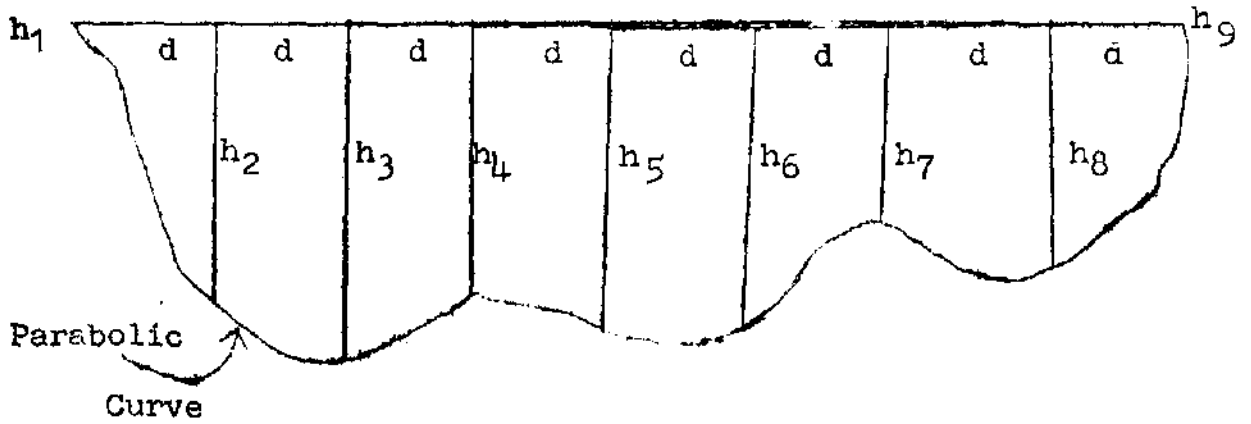
Substituting the above measurements

$$A = 7.5\left(\frac{0+0}{2} + 3.6 + 4.9 + 9.2 + 8.9 + 7.8 + 6.9 + 4.6\right)$$

$$= 344.25\ m^2$$

2.2.3 Simpson's Rule.

Figure 2.6: Irregular figure using Simpson's principle



"The Simpson's rule assumes a Curved boundary which is a better approximation to the shape of the irregular boundary and 's therefore more accurate than the trapezoidal rule. If the boundary was a parabola the formual would be exact".

Schofield W; 1979

The Simpson's rule considers offsets (or ordinates) in sets of three. It can thus be shown that the area between offset 1 and 3 is given by

$$\frac{d}{3}(h_1+4h_2+h_3).$$

That between offset 3 and 5 is,

$$\frac{d}{3}(h_3+4h_4+h_5)$$

and similarly $\frac{d}{3}(h_5+4h_6+h_7)$ for between 5 and 7.

Considering them together we obtain the area between offset 1 and 7 to be

$$= \frac{d}{3}(h_1+4h_2+h_3) + \frac{d}{3}(h_3+4h_4+h_5) + \frac{d}{3}(h_5+4h_6+h_7).$$

$$= \frac{d}{3}\left((h_1+h_7) + 4(h_2+h_4+h_6) + 2(h_3+h_5)\right).$$

Hence the formular can be generalized, thus:

Total Area = $\frac{d}{3}\big((h_1+h_n) + 4(\text{sum of even offsets}) + 2(\text{sum of odd offsets})\big)$.

i.e. one third of the distance between ordinates, multiplied by the sum of the first and last ordinates plus four times the sum of the even ordinates, plus twice the sum of the odd ordinates.

Before Simpson's Rule can apply the number of offsets must be odd.

Ex. The following are the offsets from an irregular boundary to a traverse line measured at right angles in meters at 6m interval 6.46, 13.80, 4.90, 8.60, 7.80, 10.60, 6.20, 7.80, 9.20 and 4.30.

There are ten offsets. We, therefore, use the Simpson's rule to find the area of the figure enclosed

between offset 1 and 9 and the trapezoidal rule for the area between offset 9 and 10.

$$\text{Area}_{1-9} = \frac{6}{3}\left((6.46+9.2) + 4(13.8+8.6+10.6+7.8) + 2(4.9+7.8+6.2)\right)$$

$$= 2(15.66 + 163.2 + 37.8)$$

$$= 433.32 \text{ m}^2$$

$$\text{Area}_{9-10} = \frac{(9.2 + 4.3)}{2}$$

$$= 40.5 \text{ m}^2$$

Therefore, Total Area = $\underline{473.82 \text{ m}^2}$

2.3 Calculation of Cross-sectional Areas by formulae.

Under this heading the various typical cross sections used on road schemes and other earthworks will be shown and the formulae for computing their areas derived from first principles.

In all the cross sections, the following standard symbols shall be adopted:

Nomenclature

b Finished road width

h Centre height i.e. the depth of excavation or embankment of the section measured vertically along the centre line.

W_1, W_2 side Widths, measured horizontally from either side of the centre line fixing the position of the slope stakes.

1 in m side slope of 1 vertical to m horizontal

1 in n side slope of 1 vertical to n horizontal

1 in k existing ground slope (up or down gradient)

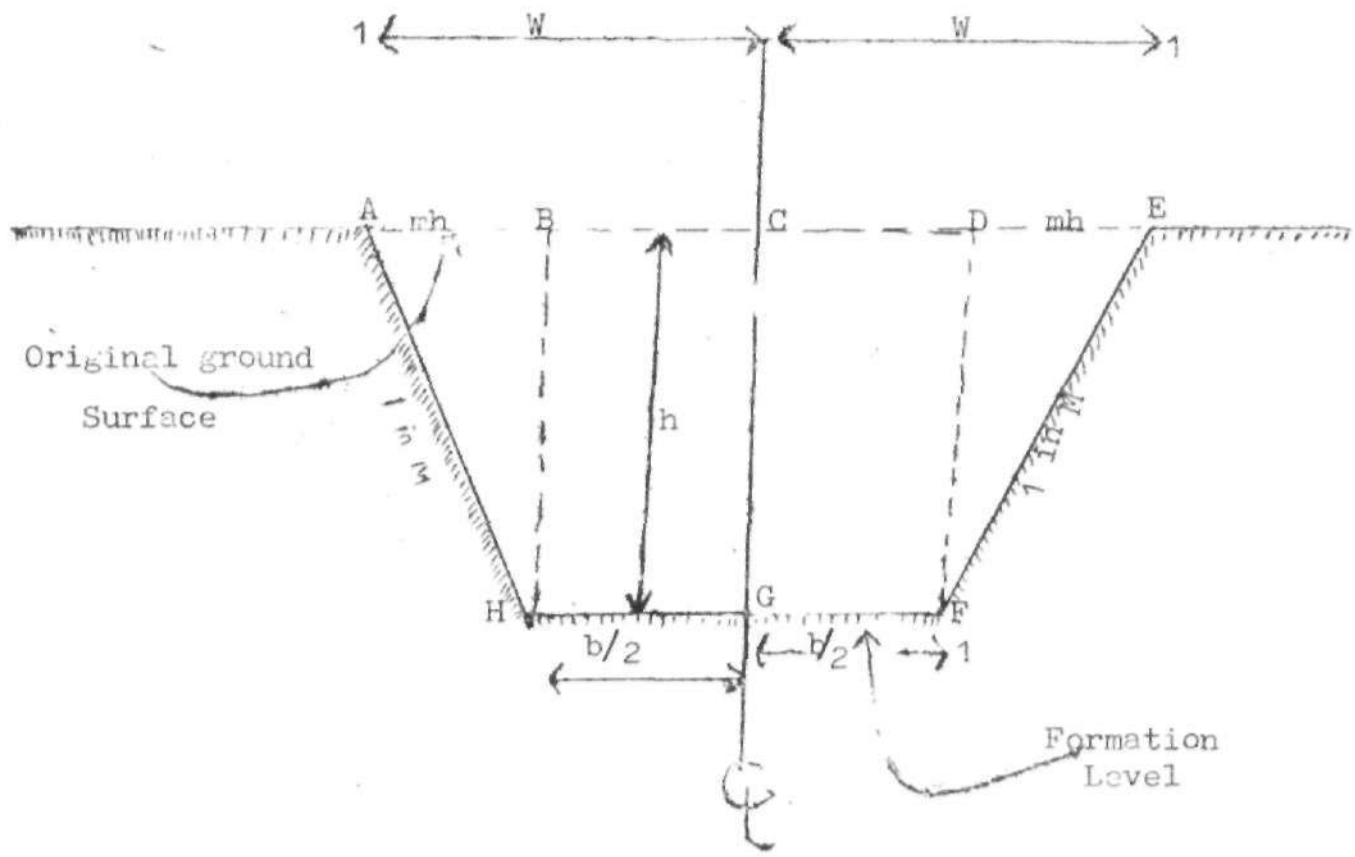
W_1+W_2 plan width.

2.31 Existing ground level horizontal

(See figure 2.7).

i.e. Side width = W; plan Width 2_W

Figure 2.7 : Cross Section of excavation with ground horizontal, side Width = W, Plan Width = 2_W



(i) Cross Section of excavation with ground horizontal

(See figure above).

Side Width = W = AC = CE

Since side slope AH = 1 in M

and the centre heigh h = CG = HB

∴ AB = mh . Similarly DE = mh

But BC = b/2

AC = W and W = b/2 + mh.

2_W = Plan Width = 2(b/2+mh)

Area ABH = Aread DEF

= ½ . mh . h

Rentangle BDFH = b . h

Area of Cross section AEFH

= Area ABH + Area DEF + Area BDFH.

= mh^2 + bh

= h(mh+b)

(ii) For the embankment, the cross section is simply inverted and the same formulae apply.

2.32 Existing transverse (ground) surface sloping

The Cross Section of excavation is represented by ACDE (see figure 2.8).

JF is the centreline,

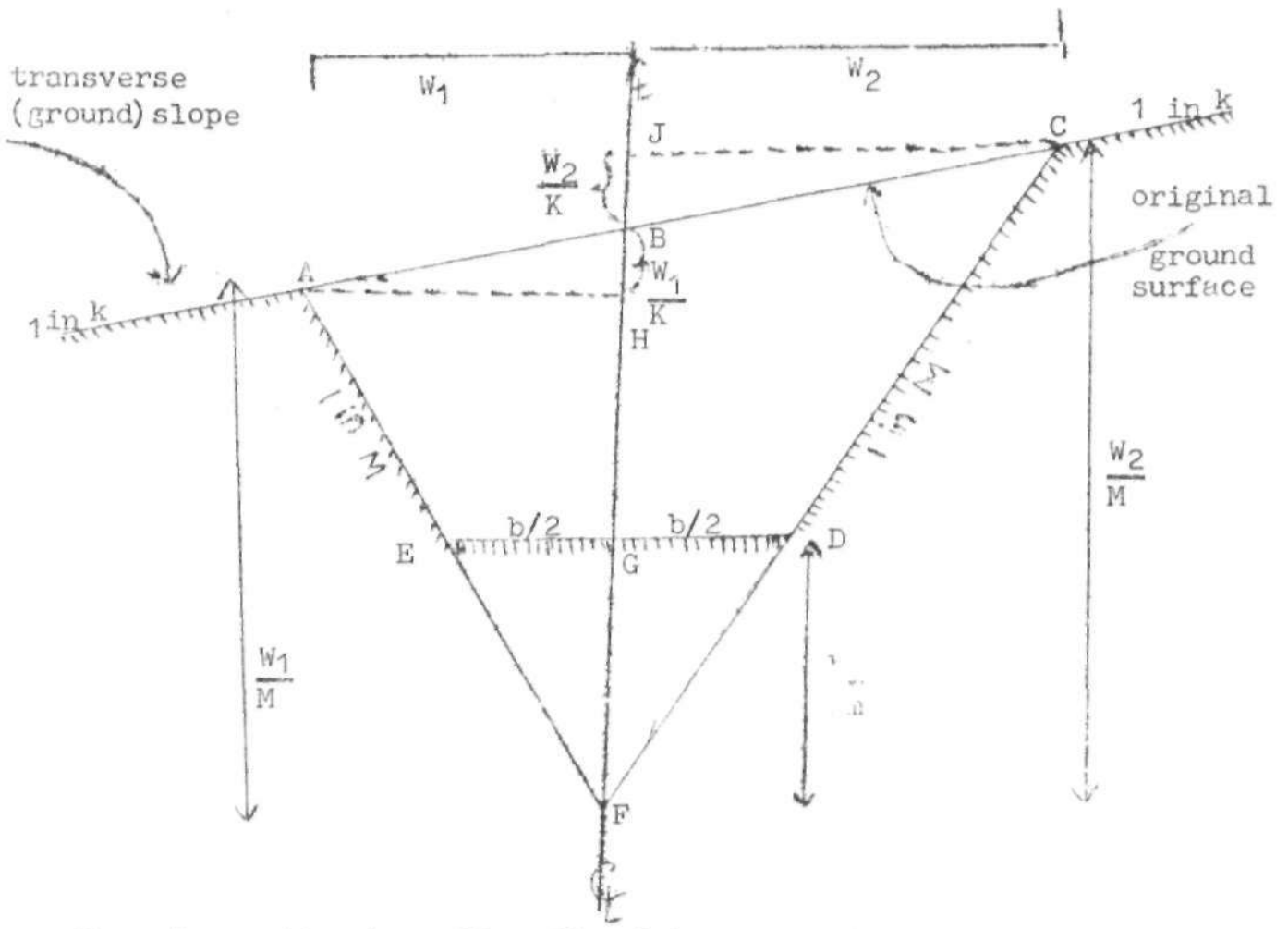
BG = h (centre height)

AH = W_1 (side Width on the left)

CJ = W_2 (side width on the right)

ED = b i.e. EG = DG = b/2.

Figure 2.8: Cross Section of excavation with existing ground Surface sloping.



Now since side slope AF = CF = 1 in m

$$GR = b/2m$$

Ground slope ABC = 1 in k

Since $AH = W_1$, it follows that $BH = \frac{W_1}{K}$

Similarly since $CJ = W_2$, then $JB = \frac{W_2}{K}$.

Considering vertical distances at the centre line gives

$$JF = \frac{W_2}{M} \quad \text{and} \quad HF = \frac{W_1}{M}$$

$$HF = \frac{b}{2m} + h - \frac{W_1}{k}$$

$$\text{i.e.} \quad \frac{W_1}{m} = \frac{b}{2m} + h - \frac{W_1}{K}$$

Multiplying by km gives

$$kw_1 = \frac{bk}{2} + hkm - mw_1 .$$

Rearranging,

$$W_1 (k + m) = k(b/2 + mh)$$

$$w_1 = k\frac{(b/2 + mh)}{k + m}$$

Thus we obtain the formula for the smaller side width.

In similar manner,

$$JF = b/2m + h + \frac{W_2}{K}$$

$$\frac{W_2}{M} = \frac{b}{2m} + h + \frac{W_2}{K}$$

Multiplying by KM gives

$$KW_2 = \frac{ok}{2} + hkm + mw_2$$

Collecting like terms gives

$$W_2 (k-m) = K(b/2 + mh)$$

$$W_2 = K(b/2 + mh)$$

We thus obtain the formula for computing the greater side Width of the cross section.

Now area of cut = Area ACDE

Area ACDE = Area ABF + Area BCR - Area EDF

Area ABF = $\frac{1}{2}$ (BF) x AH

= $\frac{1}{2}$ (b/2m + h) W_1

Area BCF = $\frac{1}{2}$ (BF) x CJ

= $\frac{1}{2}$ ($\frac{b}{2m}$ + h) W_2

$$\text{Area EDF} = \tfrac{1}{2} \times b \times \frac{b}{2m} = \frac{b^2}{4m}$$

$$\text{Area ACDE} = \text{Area of excavation}$$

$$= \tfrac{1}{2}\left(\frac{b}{2m} + h\right).\ W_1 +$$

$$\tfrac{1}{2}\left(\frac{b}{2m} + h\right).\ W_2$$

$$- \frac{b^2}{4m}$$

$$= \tfrac{1}{2}\left(\frac{b}{2m} + h\right)(W_1 + W_2) - \frac{b^2}{4m}$$

(ii) Like in the case of the preceding example, For embankments, figure 2.8 above is inverted and the same formula is similarly obtained.

Ex. Consider the following hypothetical case. We want to find the side widths and cross sectional area of the embankment with road width 22.5m, existing transverse slope 1 in 20, side slopes 1 in 5, centre height 10m.

Let W_1 , W_2 be the side widths

$$W_1 = K\,\frac{(b/2 + mh)}{k + m}$$

$K = 20,\ b = 22.5\text{m},\ m = 5,\ h = 10\text{m}$

$$W_1 = 20\,\frac{(11.25 + 50)}{25} = 49\text{m}$$

$$W_2 = K\,\frac{\left(\frac{b}{2} + mh\right)}{k - m}$$

$$= 20\,\frac{(11.25 + 50)}{15} = 81.67\text{m}$$

Therefore the side widths are 49m, 81.67m and the plan width = $W_1 + W_2$ = 130.67m.

Cross sectional area = $\frac{1}{2}\left(\frac{b}{2m} \cdot h\right)(W_1+W_2) \cdot \frac{b^2}{4m}$

$= \frac{1}{2}(12.25)(130.67) - 25.3125$

$= \underline{775.04m^2}.$

2.3.3 Existing ground surface having both negative and positive grades i.e. sections of variable levels.

An example of this type is a Cross section of Cut represented by ABCDEF in figure 2.9 below. The two ground slopes are AB, 1 in K and BC, 1 in n.

Side slopes AG = CG = 1 in m

Side Width AH = W_1 , CJ = W_2

Since AB slopes 1 in k, BH = $\frac{W_1}{K}$

Similarly, BC slopes 1 in n, JB = $\frac{W_2}{n}$

Also HG = $\frac{W_1}{m}$; and JG = $\frac{W_2}{m}$

Since FE = b/2

Considering vertical distances at the centre line gives

$$\frac{W_1}{m} = \frac{W_1}{k} + h + \frac{b}{2m}$$

where H = BE (centre height)

$$W_1\ (1/m - 1/k) = h = \frac{b}{2m}$$

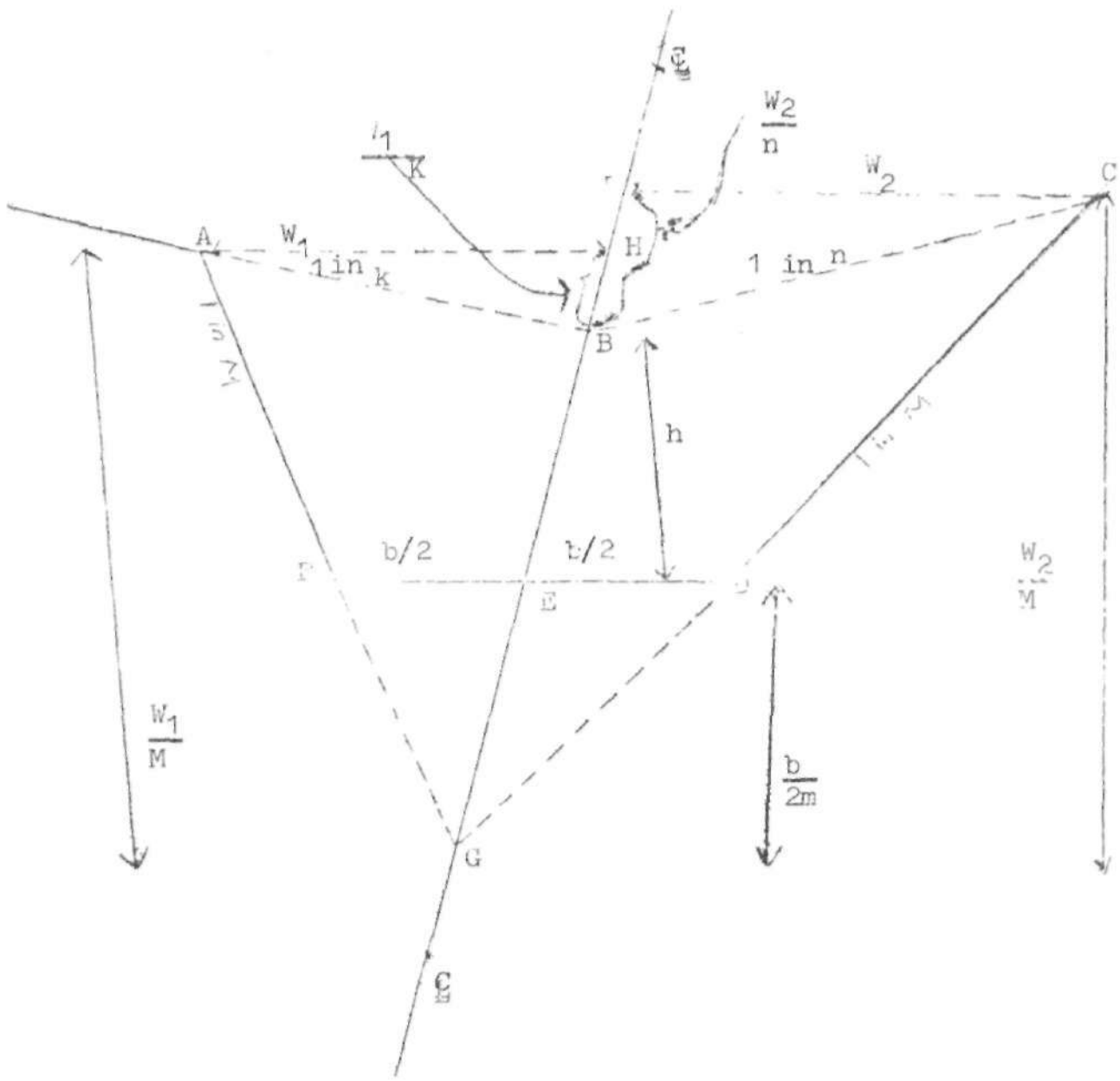
$$W_1\ \frac{(k-m)}{mk} = h + \frac{b}{2m}$$

$$W_1\ (k-m) = mhk + \frac{bk}{2}$$

$$= k\ (mh + b/2)$$

$$W_1 = k\frac{(mh + b/2)}{k-m}$$

Figure 2.9 : Cross Section of excavation having both positive and negative grades



Similary, $\frac{W_2}{m} = \frac{W_2}{n} + h + \frac{b}{2m}$

$$W_2 \frac{(n - m)}{mn} = h + \frac{b}{2m}$$

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$$W_2\,(n-m) = mnh + \frac{b}{2} \cdot n$$

$$= n(mh + b/2)$$

$$W_2 = \frac{n(mh + b/2)}{n-m}$$

Area ARCDF = the Cross sectional area of excavation.

Area ABCDF = Area ABG + Area BCG - Area FDG.

Area ABG = $\frac{1}{2}$. BG. W_1

Area BCG = $\frac{1}{2}$. BG. W_2

Area FDG = $\frac{1}{2}$. $b/2m = \frac{b^2}{4m}$

But BG = $h + \frac{b}{2m}$

$\therefore$ Area ABCDF =

$$\frac{1}{2}\,(H + b/2m)\,(W_1+W_2) - \frac{b^2}{4m}$$

Where W_1 & W_2 are as defined by the above formulae.

Ex. road width = 15m, existing ground slope

= 1 in 10 and 1 in 15 respectively, side slope

1 in 3, Centre height = 12m. Find the side widths and the cross sectional area of the excavation.

Let the side widths be W_1 , W_2.

$$W_1 = K\frac{(mh + b/2)}{k-m}$$

Where K = 10, m = 3, b = 15, h = 12

$$W_1 = \frac{10\,(36 + \frac{15}{2})}{7} = 62.14m$$

$$W_2 = \frac{n(mh + b/2)}{n-m}$$

$$= \frac{15\,(36 + \frac{15}{2})}{12} = 54.38m$$

$$\text{Area} = \tfrac{1}{2}\left(h + \frac{b}{2m}\right)(W_1 + W_2) - \frac{b^2}{4m}$$

Plan Width = $(W_1 + W_2)$ = 116.52

$$\text{Area} = \tfrac{1}{2}\left(12 + \frac{15}{6}\right) \times 116.52 -$$

$$= (844.77 - 18.75)\ m^2$$

$$= 826.02\ m^2.$$

(ii) Again, for embankments, figure 2.6 is inverted and the same formulae above apply.

2.4 Cross sectional areas in hillside involving cut and fill sections by formulae.

2.4.1 Centre line in cut i.e. greater area in cut.

The original ground surface (see figure 2.10) is represented by the transverse slope ACJD, 1 in k.

The formation level is represented by ABED.

As can be observed in the figure, Area ABC is in embankment; with side slope 1 in m. Area CDE is in cut (excavation) with the side slope ED being 1 in n.

The Centre line is represented by JG produced.

The centre height, h = JG.

The formation width is represented by BE = b.

BG = EG = b/2.

Since the transverse slope AD is 1 in k and JG = h, it follows that CG = kh

i.e. BC = b/2 - kh

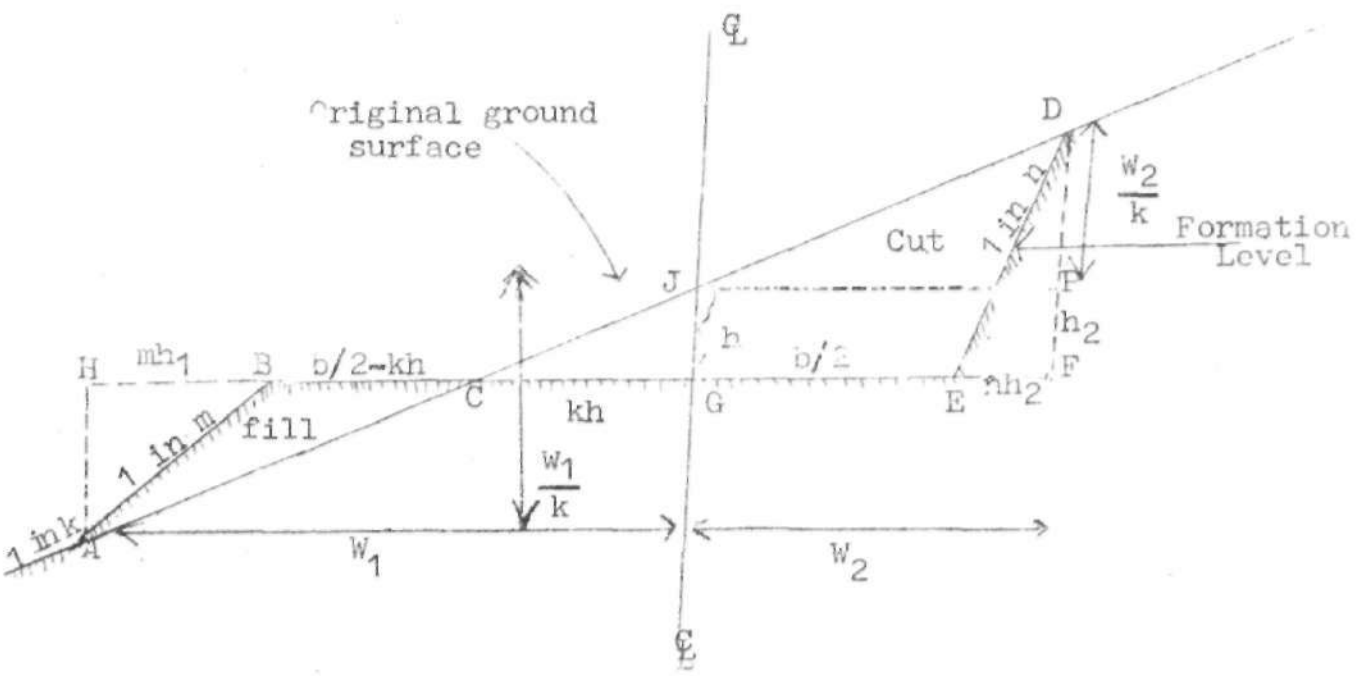
Let the side width on the embankement side be represented by W_1 (as indicated) and the side width on the cut section, W_2; then HG = W_1; and GF = W_2 .

Since the original ground surface slopes at 1 in k and AQ = HG, then it follows that JQ = $\frac{W_1}{k}$

Similarly, JP = GF; and DP = $\frac{W_2}{k}$.

Also , since slope of AB is 1 in m ; if the vertical height AH = h_1 , then HB = mh_1. Similarly, on the cut section the side slope ED is 1 in n ; if the vertical height DF = h_2 then the horizontal equivalent is EF = nh_2. Considering vertical distances at the centre line; from figure 2.10

Figure 2.10: Cross Section on a hillside



Considering the horizontal distances to the centre line

$$W_1 = b/2 + mh, \quad \ldots. \text{ (ii)}$$

$$\text{i.e. } \frac{W_1 - b/2}{m} = h_1 \text{ (i.e. from ii)}$$

or

$$\frac{2w_1 - b}{2m} = h_1$$

Substitute for h_1 in eqn. (i)

$$\frac{W_1}{k} = \frac{2w_1 - b}{2m} + h$$

$$2mw_1 = k(2w_1-b) + 2mhk$$

$$= 2kw_1 - kb + 2mhk$$

$$2w_1(m-k) = k(-b + 2mh)$$

$$W_1 = \frac{k(-b + 2mh)}{2(m-k)}$$

Dividing top and bottom by -2

$$W_1 \quad k\left(\frac{b/2 - mh}{k - m}\right)$$

This gives the width of the smaller area (fill).

From the figure again,

Considering the horizontal distance to the centre line, we have

$$W_2 = b/2 + nh_2 \ldots\ldots (iii)$$

and from the vertical distances at the centre line

$$h_2 = \frac{W_2}{k} + h \ldots\ldots (iv)$$

Substituting for h_2 in (iii)

$$W_2 = b/2 + n\left(\frac{W_2}{k} + h\right)$$

$$= \frac{b/2 + nw_2 + nhk}{k}$$

$$= \frac{bk + 2nw_2 + 2nhk}{2k}$$

$$2kw_2 = bk + 2nw_2 + 2nhk$$

$$2w_2 - 2nw_2 = bk + 2nhk$$

$$2w_2(k-n) = k(b + 2nh)$$

$$w_2 = k\left(\frac{b + 2nh}{2(k-n)}\right)$$

Dividing top and bottom by 2 gives $W_2 = k\left(\frac{b + 2nh}{k-n}\right)$

This gives the side width of the greater area, cut.

$$\text{Plan Width} = W_1 + W_2 .$$

Area of fill (Smaller Area).

Let this area be represented by A_1

Area of ABC is the area of embankment (A_1).

$$A_1 = \tfrac{1}{2}(b/2 - kh).\ h_1$$

We already had that $h_1 = \dfrac{2w_1 - b}{2m}$

Where $W_1 = \dfrac{k(b/2 - mh)}{k-m}$

$$h_1 = \frac{2k(\frac{b}{2} - mh) - b}{k-m} \Big/ 2m$$

$$= \frac{2k(b/2 - mh) - b\,(k - m)}{2m(k-m)}$$

$$= \frac{kb - 2mhk - kb + bm}{2m(k - m)}$$

$$= \frac{bm - 2mhk}{2m(k-m)}$$

$$= \frac{m(b - 2kh)}{2m(k-m)}$$

$$h_1 = \frac{b - 2kh}{2(k-m)}$$

$$A_1 = \frac{\frac{1}{2}(b/2 - kh)(b - 2kh)}{2(k-m)}$$

$$= \frac{\frac{1}{2}(\frac{b}{2} - kh)\,(b/2 - kh)}{k - m}$$

$$= \frac{(b/2 - kh)^2}{2(k-m)}$$

This is the smaller area i.e. area of fill since the centre line is in cut (where the greater area lies).

Area of cut (greater area).

Let this area be A_2.

This area of excavation is given by CDE.

Since area of triangle is

$$\tfrac{1}{2} \times \text{base} \times \text{height}$$

$$A_2 = \tfrac{1}{2}\,(b/2 + kh).\ h_2$$

Considering the vertical distances at the Centre line,

$$h_2 = \frac{W_2}{k} + h$$

and from the horizontal equivalent GF

$$W_2 = b/2 + nh_2$$

Substituting h_2, we have $W_2 = b/2 + n(\frac{W_2}{K} + h)$

$$= b/2 + \frac{nw_2+knh}{k}$$

$$KW_2 = b/2k + nw_2 + knh$$

$$W_2(k-n) = k(b/2 + nh)$$

$$W_2 = k(\frac{b/2 + nh}{k-n})$$

Substituting this value in h_2 above

$$h_2 = k(\frac{b/2 + nh}{k(k-n)})$$

$$= \frac{b/2 + nh}{k - n} + h$$

$$= \frac{b/2+nh+hk-nh}{k-n}$$

$$h_2 = \frac{b/2 + hk}{k-n}$$

$$= \frac{b + 2hk}{2(k-n)}$$

Substituting for h_2 in A_2

$$A_2 = \tfrac{1}{2}(b/2 + hk)\cdot \frac{b+2hk}{2(k-n)}$$

$$= \frac{\tfrac{1}{2}(b/2 + kh)(b/2 + kh)}{k-n}$$

$$= \frac{(b/2 + kh)^2}{2(k-n)}$$

This is the greater area.

Ex. Suppose we want to find the side Widths and cross sectional areas of a cut and fill section on a hillside with the following dimensions:

road width = 25m; existing ground slope = 1/5

side slope in cut = $\frac{1}{2}$, centre height in cut = 1.8m. Side slope in fill = $\frac{1}{3}$.

Using the formulae, if the side Widths are W_1 , W_2

$$W_1 = k\frac{(b/2 - mh)}{k-m}$$

Now, k = 5, b = 25, m = 3, h = 1.8, all in meters.

$$W_1 = \frac{5(\frac{25}{2} - 3 \times 1.8)}{5 - 3}$$

$$= \frac{35.5}{2}$$

$$= 17.75\text{m}$$

$$W_2 = k\frac{(b/2 + nh)}{k-n}$$

Where k = 5, b = 25, n = 2, h = 1.8 (all in meters)

$$W_2 = \frac{5(\frac{25}{2} + 2 \times 1.8)}{5-2}$$

$$= \frac{80.5}{3}$$

$$= \underline{26.83m}$$

Cross sectional area of fill $= \frac{(b/2 - kh)^2}{2(k-m)}$,

Where b = 25, k = 5, h = 1.8, m = 3 (in meters)

$$\text{Area of fill} = \frac{(\frac{25}{2} - 5 \times 1.8)^2}{2(5-3)}$$

$$= \frac{12.25}{4}$$

$$= 3.06m^2$$

Cross sectional area of cut

$$= \frac{(b/2 + kh)^2}{2(k-n)}$$

Where b = 25m, k = 5, h = 1.8, n = 2.

$$\text{Area of cut} = \frac{(\frac{25}{2} + 5 \times 1.8)^2}{2(5-2)}$$

$$= \frac{462.25}{6}$$

$$= \underline{77.04\ m^2}$$

The above simple example is used to illustrate the seeming ease and simplicity in computations of the various parameters through merely substituting in the formulae - especially in this age of manual desk calculators.

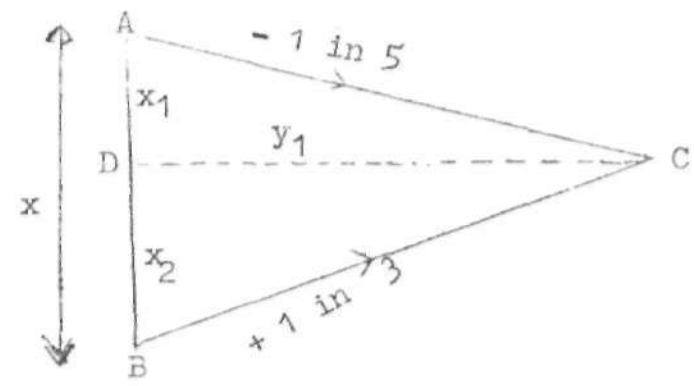
2.5 Calculation of Cross Sectional areas using the analytical or trigonometrical solution i.e. rate of approach method.

This method makes use of the grades of the ground surface and side slopes in evaluating all required dimensions. The rule is that all reference heights used in the calculations must be vertical.

The method is best illustrated by examples. The following shows the principles.

2.5.1 Two grades running in opposite directions

Figure 2.11 : Two grades running in opposite directions



We are given two grades AC, BC (see figure 2.9).

AC has a down - gradient of 1/5

BC has an up-gradient of $\frac{1}{3}$

The vertical height AB is given to be x, say.

The horizontal distance y, from C to AB is obtained from the following principle.

Since DC = Y_1 and AD = X_1 , then

$$x_1 = \frac{y_1}{5}$$

$$x_2 = \frac{y_1}{3} \quad \text{when} \quad DB = x_2$$

$$x_1 + x_2 = x = \frac{y_1}{5} + \frac{y_1}{3} = y_1 \left(\frac{5+3}{15}\right)$$

$$x = \frac{8y_1}{15} \quad \text{or} \quad y_1 = \frac{15}{8}x$$

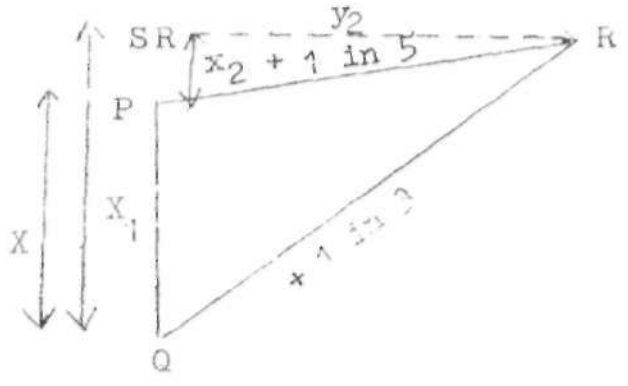
i.e to get y_1 add the two grades, invert the sum and multiply by x (the vertical height). In this example (figure 2.9)

$$y_1 = (1/5 + 1/3).\ x$$
$$= (8/15)^{-1}.\ x = \frac{15}{8}\ x \text{ or } 1.875\ x\ .$$

If in this example $x = 3m$; $y_1 = \underline{5.625m}$.

2.5.2 Two grades running in the same direction

Figure 2.12: Two grades running in the same direction



Let PQ = x, QS = x_1, PS = x_2, SR = y_2 (figure 2.12)

$$x_1 = \frac{y_2}{3}\ ,\ x_2 = \frac{y_2}{5}$$

$$x = x_1 - x_2 = \frac{y_2}{3} - \frac{y_2}{5}$$

$$x = y_2\ (\frac{1}{3} - \frac{1}{5})$$

$$= y_2 \cdot \frac{2}{15}$$

$$\text{or } y_2 = \frac{15}{2} \cdot x$$

∴ With reference to figure 2.12 we are give two grades PR and QR.

If PR has a positive up-gradient of 1 in 15 and QR also an upgradient 1 in 3.

The vertical height PQ is x.

The horizontal distance y_2 from R to QP produced is obtained using the following rule:

Subtract the two grades, invert them and multiply by x.

In the dimensions in figure 2.9 above

$$y_2 = (1/3 - 1/5)^{-1} \cdot x$$

$$= \left(\frac{4}{15}\right)^{-1} \cdot x$$

$$= \frac{15}{4} \cdot x$$

If in this example x = 2.3m, then

$$y_2 = \frac{15}{4} \times 2.3 \text{ in } = 3.75 \times 2.3\text{m}$$

$$= y_2 = \underline{8.625\text{m}}$$

We now illustrated the use of the analytical approach in evaluating the cross sectional area and side Withs when computing earthworks volume and costs.

Ex. Calculate the side widths and cross sectional area of an embankment having the following dimensions:

road width 21m; Existing ground slope =

1 in 15 ; side slopes = 1 in 3. Centre height

= 10m . (see fig. 2.11)

The ground slope AE is 1 in 15 (see fig. 2.10 below). The embankment is represented by ABDE whose area we shall find.

Side slope AB = DF = 1 in 3.

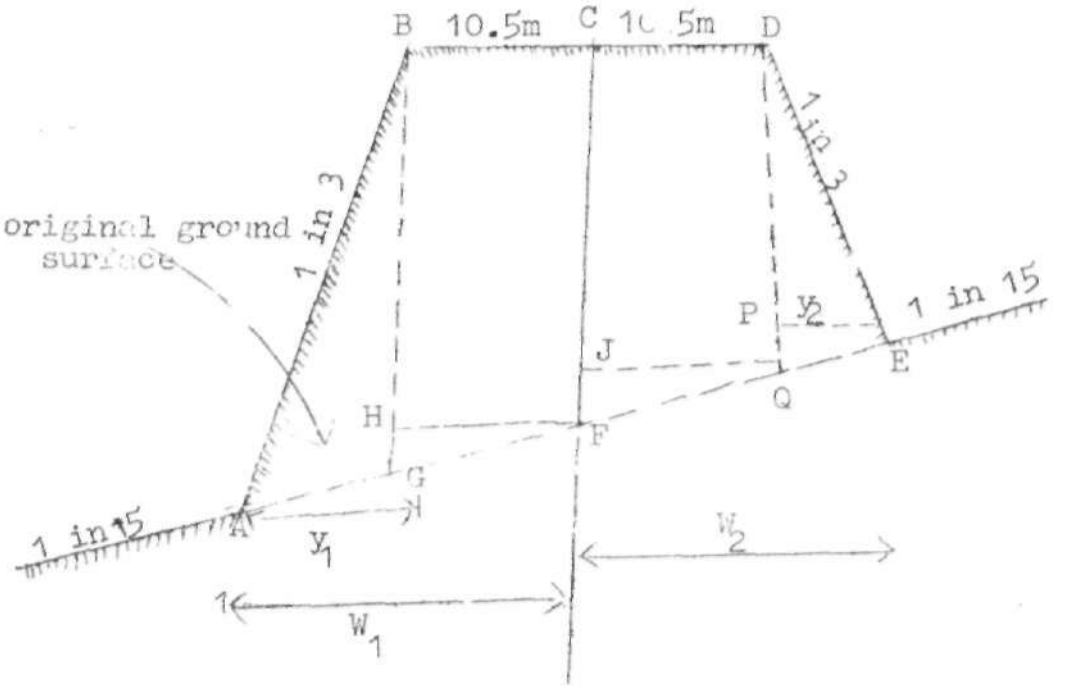
BD = the formation width = 21m

CF = the centre line

BC = 10.5m .

Figure 2.13 : Cross Section of embankment - example

(not drawn to scale please)



CF is the Centre height = 10m

BG is the Vertical height to slope AB, AG.

Any horizontal line from CF (Centre line) to BG shou equal 10.5m.

FH = 10.5m

But slope AG = 1 in 15 (or 1/15)

$$GH = \frac{10.5}{15} \text{ m}$$

But CF = BH = 10m (Center height)

$$BG = (10 + \frac{10.5}{15})m = 10.7m$$

Let y_1 be the horizontal distance from A to the vertical height BG.

$$y_1 = (\tfrac{1}{3} - 1/15)^{-1} \times 10.7\ m = \frac{15}{4} \times 10.7m$$

$$= 3.75 \times 10.7$$

$$= \underline{40.125m}$$

$$W_1 = y_1 + HF = 40.125 + 10.5$$

$$= \underline{50.625m}$$

Similarly $JF = \frac{10.5}{15} = 0.7m$

$$DQ = CF - JF$$

$$= (10-0.7)m = 9.3m$$

DQ is the vertical height to grades DF and QE.

$$y_2 = (\tfrac{1}{3} + 1/15)^{-1} \times 9.3$$

$$= \frac{5}{2} \times 9.3m = 23.25m$$

$$W_2 = JQ + y_2 = (10.5 + 23.25)m = 33.75m$$

We now find the area ABDE

= Area ABG + Area BDQG + Area DEG.

Area ABG = $\frac{1}{2}$. BG . y_1

$$= \tfrac{1}{2} \times 10.7 \times 40.125$$

$$= 214.67\ m^2$$

Area BDQG = $10.5 \times (10.7 + 9.3)\ m^2$

$$= 210\ m^2$$

Area DEQ = $\frac{1}{2}$. DQ . y_2

$$= \tfrac{1}{2} \times 9.3 \times 23.25$$

$$= 108.1125\ m^2$$

Area ABDE = Total area of embankment

$$= (214.67 + 210 + 108.1125)\ m^2$$

$$= \underline{532.78m^2}$$

The above example may tend to portray the analytical approach to be rather long. This is not so. With practice the computations go faster. The analytical method is still preferred because if one forgets the formula its derivation is rather long. For programming and adaptability to computer any of the methods can be used. For instance a practicing surveyor or engineer can always refer to the formulae.

2.6 Calculating the Cross Sectional areas of embankments and cuttings from first principles.

This method is often easier than the method of formulae. It is equally as elegant as the analytical method. The principle to be adopted in the calculation involves first obtaining the area of the trapezium ADGH and subtracting from it the area of triangles AHB and DCG. This is better explained by an example.

Let the formation width BC = b = 20 (fig. 2.11)

Height JK at centre line = h = 5m

Side slopes AB, CD = 1 in 3

Ground slope AD = 1 in 15

In Figure 2.14 the unknowns x and y are the projections of the side slopes needed for the area calculation.

From the data in the specimen question.

FD = 15. FJ (since ground slope is 1:15.

But FD = 10+ 3x

FJ = x-5

i.e. $10+3x = 15(x-5)$

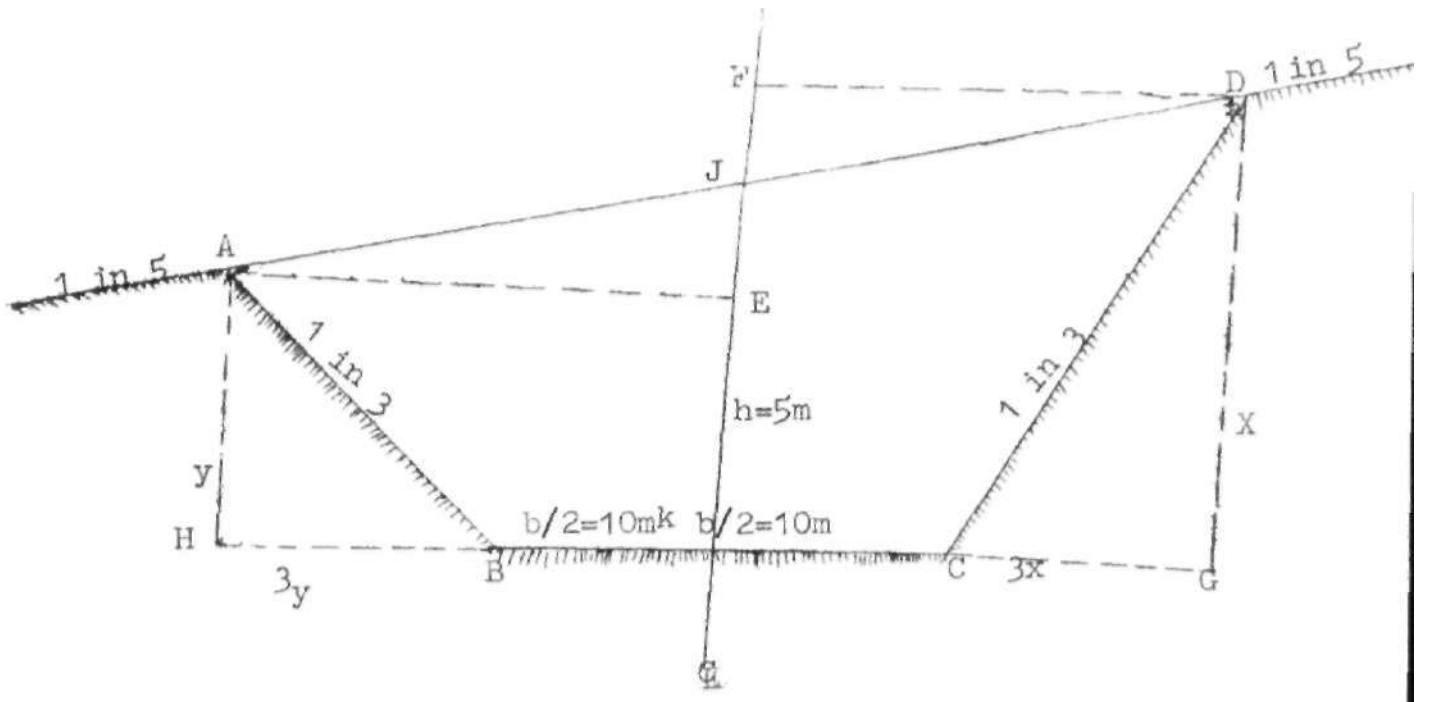
$10+3x = 15x-75$

$12x = 85$

$x = 7.083$

$3x$ = CG = 21.250m

Figure 2.14 : A Cutting ABCD



Similarly, AE = 15. JE

But AE = $10 + 3y$ and JE = $5 - y$

$\therefore 10 + 3y = 15(5 - y)$

$10 + 3y = 75 - 15y$

$18y = 65$

$y = 3.611m$

$HB = 3y = 10.833m$

Area ABCD = Area AHGD - Area AHB - Area DCG.

$$\text{Area ABCD} = \frac{(x + y)}{2} \times GH$$

$$= \frac{(7.083 + 3.611)}{2} (21.25 + 20 + 10.833) - \frac{(3.611 \times 10.833)}{2} - \frac{(7.083 \times 21.25)}{2}$$

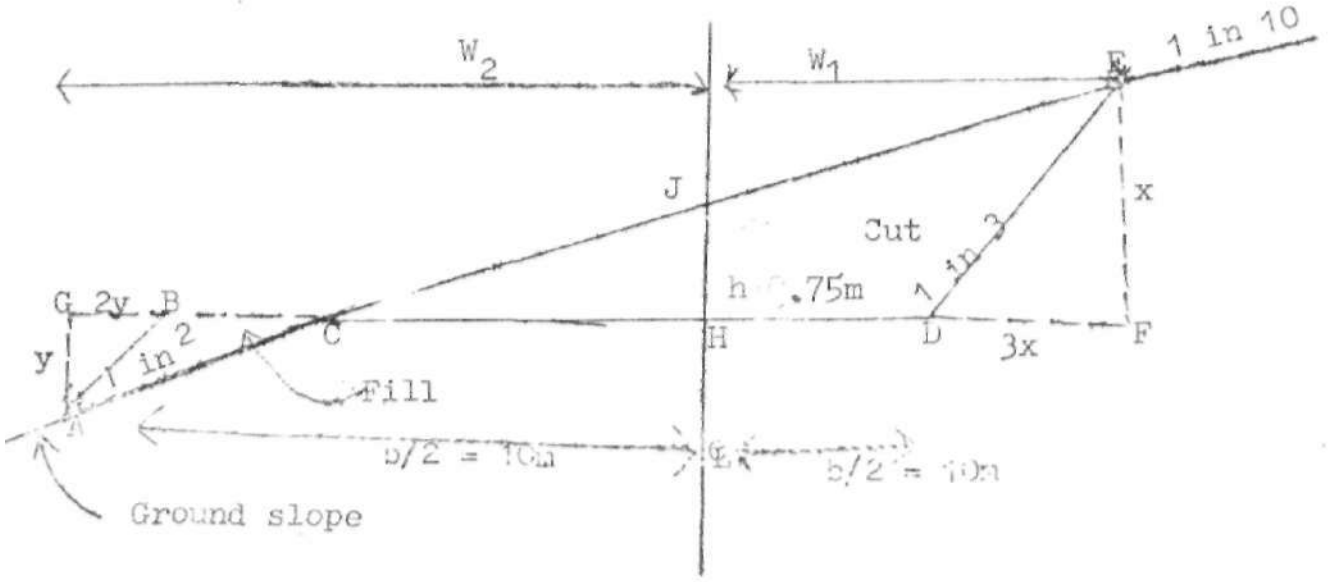
$= (278.488 - 19.559 - 75.257)\ m^2$

$= \underline{183.672\ m^2}$

2.7 Calculating the Cross sectional area of cut and fill sections from first principles.

As in the case of cuttings and embankments it is often easier to calculate the cross sectional area of cut and fill sections from first principles Referring to figure 2.15 the principle to be adopted will be illustrated by an example.

Figure 2.15 cut and fill section



Let the formation width BD = b = 20m

Height at centreline JH = 0.75m

Side slope in cut, DE = 1 in 3

Side slope in fill, AB = 1 in 2

Ground slopes, AE = 1 in 10

For the cut section, the principle is to find the area of triangle CFE and from it subtract the area of triangle DFE (see figure 2.15). For the fill section we calculate the area of triangle CGA and from it we subtract the area of triangle ABG.

From the above figure the unknowns x and y are the projections of the side slopes DE and AB respectively. The unknowns are obtained using the algebraic relations below:

Since JH = 0.75m, and ground slope

AE is 1 in 10;

$\therefore$ CH = 0.75 x 10m = 7.5m.

CD = (7.5 + 10)m = 17.5m

CF = 17.5 + 3x

But CF = 10. EF = 10x

$\therefore$ 17.5 + 3x = 10 x

7 x = 17.5

x = 2.5m

Side Width W_1 = HF = 10 + 3 x = 17.5m

Area CFE = $\frac{1}{2}$. CF. EF

= $\frac{1}{2}$ x 32.5 x2.5

= 40.625 m^2

Area EDF = $\frac{1}{2}$ x DF x EF = $\frac{1}{2}$ x 7.5 x 2.5

= 9.375 m^2

Area of cut = Area CFE - Area EDF

= 40.625 - 9.375) m^2

= 31.25 m^2

For the fill section

CB = 10 - CH = 10 - 7.5 = 2.5m

CG = 2.5 + 2y ; AG = y

But CG = 10.AG i.e. 2.5 + 2y = 10y

8y = 2.5 ; y = 0.3125m

Area of triangle CGA

$= \frac{1}{2} \cdot CG \cdot AG$

$= \frac{1}{2} \times 3.125 \times 0.3125$

$= 0.488m^2$

Area AGB $= \frac{1}{2} \cdot GB \cdot AG$

$= \frac{1}{2} \times 0.625 \times 0.3125$

$= 0.098m^2$

$\therefore$ Area of fill = Area CGA - Area AGB

$= (0.488 - 0.098)\ m^2$

$= \underline{0.391m^2}$

2.8 Cross sectional area using the planimater

For finding the area of any figure the planimeter is applied to any type of plan or drawing of whatever shape or nature of boundary - regular or irregular figure.

Mechanical integration using the planimeter achieves a most dependable precision of accuracy when compared with the alternative methods in finding the area of an irregular boundary especially.

Before a planimeter is used in finding an area it is advisable to investigate its accuracy by checking over a figure whose area is known previously and precisely. The discrepancy should be noted and the correction factor computed. This correction factor is then applied to all areas obtained from the readings of the planimeter. Usually, the planimeter has a testing bar for this particular purpose. In other words it is erroneous to assume, that the calibration of the planimeter is errorless.

In estimating areas by mechanical integration using the planimeter it is strongly advised that the pole should preferably be held outside the figure. This enables the computor to avoid bothering himself about the additive constant. However, when the pole of the planimeter is held inside the figure, a larger area of coverage is facilitated for a single setting. But this too can be guarded against by breaking the plan drawing to smaller figures each of which is mechanically integrated by pivotting the integrating unit with the pole held outside individual segments of the plan drawing. The total area is obtained by summing together the individual areas.

The planimeter is used mainly for estimating the cross sectional area between successive contours of a contoured area of a dam or reservoir so that the capacity can be evaluated; though the accuracy of the result depends to a large extent on the contour intervals. A 1 meter contour interval will produce better capacity estimate than 5 meter contour.

The planimeter does not directly produce the area of a figure. Usually the reading of the mechanical device is converted to the correct area using the plan scale of the drawing and the calibration constant of the Planimeter. An example below is designed to illustrated the conversion of the planimeter reading to the true area of the figure measured.

Ex. Using a fixed - arm planimeter the plan area of a piece of land is tracedout and read as 1819mm. If the plan scale is 1/2500 we estimate the land area thus:

From the scale 1mm represents 2500mm or $1mm^2$

$= (2500 \times 2500)\ mm^2$ on the ground.

Plan area $1819mm^2$

$= 1819 \times 2500 \times 2500\ mm^2$ on ground

$= \frac{1819 \times 2500 \times 2500}{1000 \times 1000 \times 10,000}$ ha.

$= 1.137$ ha.

CHAPTER THREE

3.1 VOLUME OF EARTHWORKS

There are many methods of calculating volumes of earthworks. In this Chapter, we shall restrict ourselves to the three most popular methods viz: estimation of volumes from cross sections, volumes from spot heights and volumes from contours.

The various methods of calculating the cross sectional areas were treated in Chapter two. The intention was to give a good background to methods of determining the volumes contained between the cross sections.

Under volumes from cross sections, three methods are considered, two of which are comparable to the trapezoidal rule and Simpson's rule for areas (popularly called prismoidal formula).

3.1.1 Volumes from Mean Areas

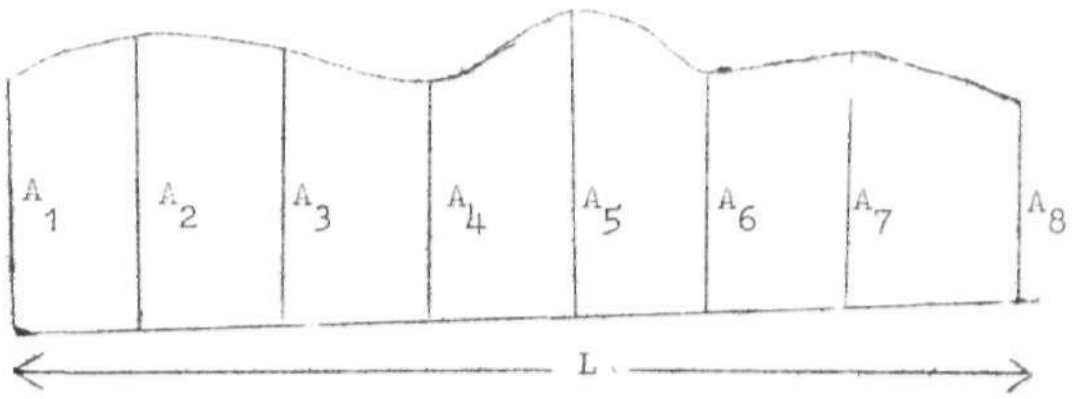
This method is not often used because it is inherently inaccurate due to faulty assumptions.

Using this method the volume obtained could be fairly dependable if the successive areas (A^S) are almost equal in value or if the areas are alternating between two definite values.

The volume is determined by finding the mean of the cross sectional areas and multiplying the result by the distance between the two extreme end cross sections.

From figure 3.7 if the areas of the cross sections are:

Figure 3.1 : Mean Areas in Diagram



A_1 , A_2 , ... A_7 , A_8 , say

the mean area

$$= \frac{A_1 + A_2 + A_3 + ... + A_8}{8}$$

Let the distance between cross section A_1 and A_8 be L , then volume of the figure is

$$V = \frac{(A_1 + A_2 + ... + A_7 + A_8)}{8} . L$$

This gives the volume between the 8 sections.

The device can be generalized to n cross sections.

i.e. If the areas are A_1 , A_2 ,... A_{n-1} , A_n

then volume by mean area is given by

$$V = \frac{(A_1 + A_2 + ... + A_n).W}{n}$$

Where W is the distance between sections A_1 and A_n.

As earlier mentioned, this method could be reasonable if the successive areas are almost the same.

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3.1.2 End Areas Method

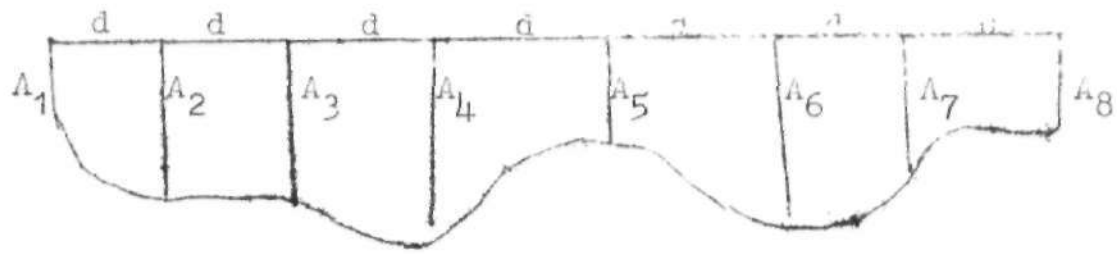
This is comparable in similarity to the trapezoidal rule for cross sectional areas.

The end areas method will give accurate results if the cross sectional areas have the same order of magnitude. The prismoidal rule may be more accurate but the end areas method is by far preferred to the mean areas method.

Like in trapezoidal rule (see figure 3.2) if two cross sectional areas A_1 and A_2 are a horizontal distance d apart, the volume between the cross sections (V_1 , say) is give by

$$V_1 = d\frac{(A_1 + A_2)}{2}$$

Figure 3.2: End Areas in diagram



The volume between A_2 and A_3 (V_2 , say) is given by

$$V_2 = d\frac{(A_1 + A_3)}{2} ,$$

and similarly volume between cross sectional areas A_3 and A_4 (i.e V_3) is given by

$$V_3 = d\frac{(A_3 + A_4)}{2}.$$

Let V be the total volume for the eight cross sections in figure 3.2.

$$V = V_1 + V_2 + V_3 + \ldots + V_7$$

$$= d\left(\frac{A_1+A_2}{2}\right) + d\left(\frac{A_2+A_3}{2}\right) + d\left(\frac{A_3+A_4}{2}\right)$$

$$+ \ldots + d\left(\frac{A_7+A_8}{2}\right)$$

$$= d\left(\frac{A_1+A_8}{2} + A_2 + A_3 + A_4 + A_5 + A_6 + A_7\right).$$

We can now generalize for n cross sectional areas.

$$V = d\left(\frac{A_1+A_n}{2} + A_2 + A_3 + \ldots + A_{n-1}\right)$$

In the case of the end areas method (as in the trapezoidal rule for areas) the cross sections can be odd or even in number, the formula accommodates either.

3.1.3 Prismoidal Formula

This method is comparable to the Simposon's rule for areas. It is more accurate than the end areas method (trapezoidal rule for volumes).

A prismoid is a solid consisting of two ends which form plane parallel figures. The number of sides may not be the same but they can be measured as cross sections. The faces between the parallel ends form plane surfaces between straight lines which join all the corners of the two end faces. A prismoid is made up of a series of prisms, wedges and pyramids, all having a length equal to the perpendicular distance between the parallel ends. The geometrical solids forming the prismoid are usually three:

(a) Prism. The end polygons are equal and the side faces are parallelograms.

(b) Wedge. One end is a line, the other a parallelogram, and the sides triangles and parallelograms.

(c) Pryramid. One end is a point, the other a polygon and the side faces are triangles.

If a series of cross sections are separated by the same constant distance apart the volume contained between the cross sections can be approximated to the volume of a prismoid. Because of its relative accuracy, the prismoidal formula is used for earthworks calculations of excavations and embankments. The prismoidal formula can give an exact volume of earthworks calculation if the transverse (ground) slopes are parabolic (as indicated in Simpson's rule for cross sectional area) and the longitudinal profile is a straight line; or if the ground slopes at right angles to the centre line are straight and the longitudinal profile on the centre line is parabolic.

The above indicates that both the transverse slope and the ground profile must be regular for the prismoidal formula to give a true volume. Generally, in earthworks calculation of volumes the prismoidal rule is more often applied with the assumption in mind that the relevant ideals hold. Even where the ideal situation does not hold due to irregularity in the ground profile and transverse slopes the error introduced is negligible relative to the earthworks volume involved. The prismoidal formula is, therefore, more accurate and preferred to the end areas method.

In general, let

d = the perpendicular distance between the parallel end planes

A_1 and A_2 = the areas of these end planes

M = the mid-area, i.e. the area of the plane parallel to the end planes and midway between them.

M does not necessarily represent the mean of A_1 and A_2 .

V = the volume of the prismoid.

(i) In a prism,

$$A_1 = A_2 = M \text{ and}$$

$$V = d.\ A_1$$

$$= \frac{d}{6}(6.A_1)$$

$$= \frac{d}{6}(A_1 + 4M + A_2)$$

(ii) In a Wedge,

$$A_2 = 0,\ M = \tfrac{1}{2} A_1 \text{ and}$$

$$V = \tfrac{1}{2}d.\ A_1$$

$$= \frac{d}{6}(3.A_1)$$

$$= \frac{d}{6}(A_1 + 4M + A_2)$$

(iii) In a pyramid, $A_2 = 0$ and $M = \frac{1}{4}A_1$ and

$$V = \tfrac{1}{3}.d.\ A_1$$

$$= \frac{d}{6}(2.A_1)$$

$$= \frac{d}{6}(A_1 + 4M + A_2)$$

This implies that the volume of each solid can be expressed in the same terms.

Thus the prismoidal formula is expressed as

$$V = \frac{d}{6}(A_1 + 4M + A_2)$$

Therefore for any three cross sections of areas A_1 , A_2 , A_3 is given by

$$V_1 = \frac{d}{3}(A_1 + 4A_2 + A_3)$$

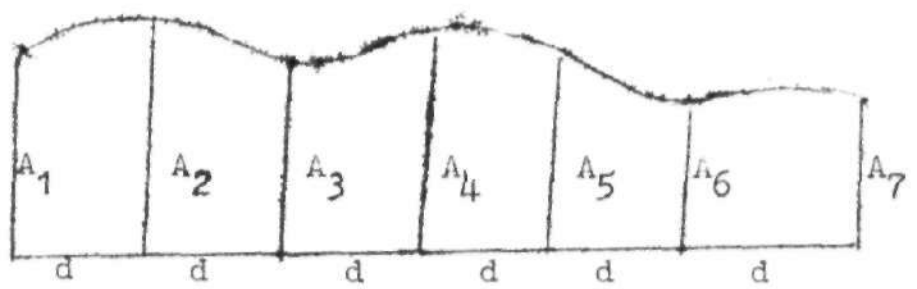
Also the volume between the next three cross sections of areas A_3 , A_4 , and A_5 is

$$V_2 = \frac{d}{3}(A_3 + 4A_4 + A_5).$$

Similarly volume V_3 between cross sections of areas ar A_5 , A_6 , A_7 is

$$V_3 = \frac{d}{3}(A_5 + 4A_6 + A_7).$$

Figure 3.3 : Prismoidal rule in Diagram.



The total volume contained between the first cross section (area A_1) and 7th cross section (area A_7) is V where

$$v = v_1 + v_2 + v_3 \text{ - fig. 3.3}$$

$$v = \frac{d}{3}(A_1 + 4A_2 + A_3) + \frac{d}{3}(A_3 + 4A_4 + A_5) + \frac{d}{3}(A_5 + 4A_6 + A_7)$$

$$= \frac{d}{3}((A_1 + A_7) + 4(A_2 + A_4 + A_6) + 2(A_3 + A_5))$$

This leads to a general formula for any n successive cross sections of areas

A_1 , A_2 , A_3 , ... , A_{n-1} , A_n for which the volume v is

$$v = \frac{d}{3}(A_1 + A_n + 4(\text{Sum of even areas}) + 2(\text{Sum of the remaining odd areas}).$$

Like in the Simpson's rule for areas the Prismoidal formula works for ODD number of cross sections.

3.2.1 Prismoidal Correction:- This is adopted where greater accuracy is required. The Prismoidal correction is deduced by merely subtracting the volume obtained by the Prismoidal method from that obtained by the end areas method.

Prismoidal Correction = V(by end areas method)

- V(by Prismoidal formula).

This correction can be deduced using the appropriate formula for either cutting or embankment or the cut and fill section.

If h_1 and h_2 are the differences in height between ground level and formation level at A_1 and A_2 respectively, the prismoidal correction for cuttings and embankments is

$$\frac{d}{6} \cdot \frac{k^2}{k^2-m^2}$$

Wilson J. P; 1971

Where transverse slope is 1 in k and side slopes 1 in m; d is the length between sections.

If the ground slope is level i.e. k $= \infty$ the prismoidal correction becomes

$$\frac{d}{6} \cdot m\,(h_1-h_2)^2 .$$

The correction is always deducted from the volume obtained from the end areas method.

For cut and fill section the prismoidal correction formula is $\frac{d}{12(k-m)} \cdot k^2\,(h_1-h_2)^2$.

This correction must also be deducted from the volume obtained by the end areas formula. The appropriate value of m must be used if it differs between the cut and fill section.

The Prismoidal formula is not frequently used in practice because it is only accurate for true prismoids or where the distance between cross sections is kept to minimum.

3.2.2 Curvature Correction

The previous discussions assumed that the cross sections are taken on a straight centre line or longitudinal section. However, where horizontal curve occurs the cross sections will no longer be parallel to each other and errors will result in calculating volumes using the conventional methods of end areas or the prismoidal formulae.

Pappus Theorem:

(i) "States that the correct volume is where the distance between the cross sections is taken along the path of the centroid"

Schofield W; 1979

(ii) "States that a volume swept out by a plane constant area revolving about a fixed axis is given by the product of the cross sectional area and the distance moved by the centre of gravity of the section".

Uren and Price ; 1983

(iii) "States that if a plane area rotates about an axis in its plane, but outside its area, the volume of the solid generated is given by the product of the area and the length of the path traced by the centroid of the area".

Wilson J. P., 1971

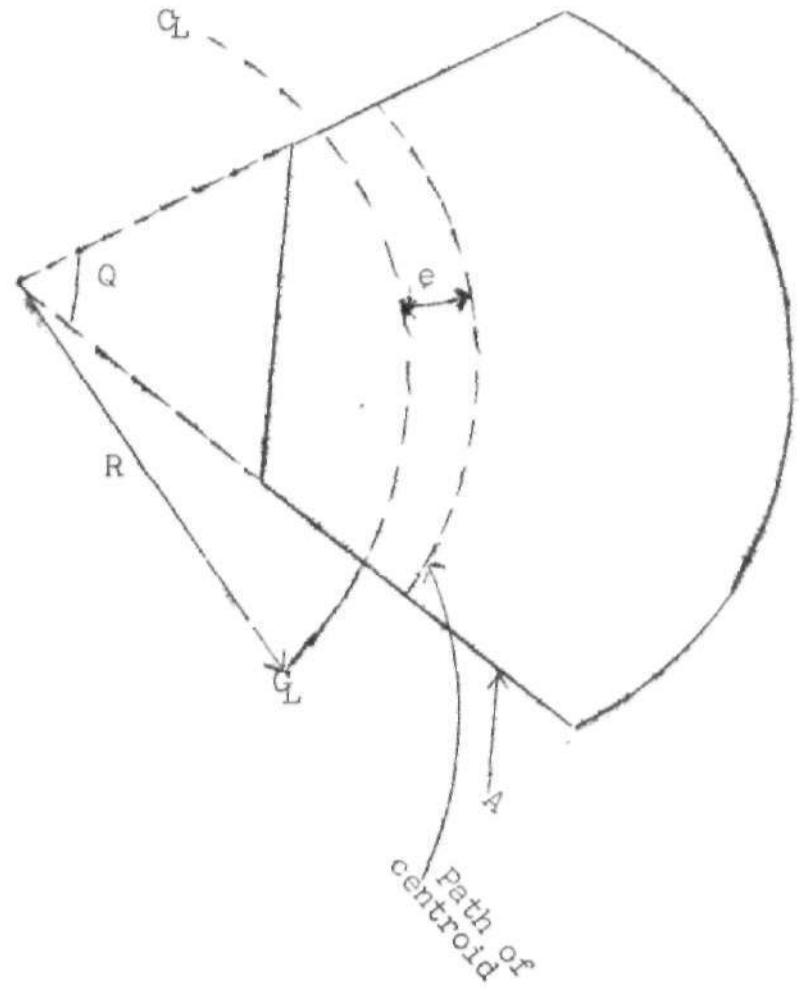
All the above definitions border on the illustration in figure 3.4 and the following explanation.

d = length of area along centre line.

If two cross sectional areas A_1 and A_2 are separated by an arc distance d apart along the centre line the volume could be assumed as

$$\frac{A_1 + A_2}{2} \times d$$

Figure 3.4: A Cross Sectional area A Sweeping through an arc R.



This is the situation if the path traced by the centvoid is displaced distances e_1 and e_2 from the centre line at either end of the cross sections respectively.

From figure 3.4, if R is the radius of curvature then

$$RQ = d$$

∴ The arc distance on the path of the centroid is

$$Q\left(R + \frac{e_1 + e_2}{2}\right)$$

where e_1 , e_2 are called eccentricities and $\frac{e_1+e_2}{2}$ is the mean eccentricity of the centroid path at cross sections A_1 , A_2 .

Since $RQ = d$,

$$Q = \frac{d}{R}$$

∴ Length of the centoid path is

$$= \frac{d}{R}\left(R + \frac{e_1+e_2}{2}\right)$$

$$= d\left(1 + \frac{e_1+e_2}{2R}\right)$$

i.e. the correction to be applied for curvature to d is therefore

$$d\ \frac{(e_1+e_2)}{2R}$$

where as before,

d = the distance between cross sections along the centre line.

e_1 , e_2 = eccentric distances obtained at either end of cross sections measured from the centre line

R = radius of curvature.

The curvature correction is either added or subtracted depending on which side of the centreline the centroid path falls.

The curvature correction is seldom applied in practice because the error produced by its omission is generally small.

"Indeed it can be shown that the effect of curvature is cancelled out on long earthwork projects"

Schofield W; 1979.

End Areas Method - Example

The following cross sectional areas, 30m apart were obtained along a longitudinal section of a read for an embankment. All measurements in m^2.

502, 436, 829, 530, 322, 641, 391, 702, 915, 223.

To calculate the volume of earthworks involved using the trapezoidal rule.

Let V be the volume

$$V = d\left(\frac{A_1+A_{10}}{2} + A_2+A_3+ \ldots+A_9\right)$$

$$= 30\left(\frac{502+223}{2}+436+829+530+322+641\right.$$

$$\left.+391+702+915\right).$$

$$= (30 \times 5128.5)\ m^3$$

$$= 153855\ m^3$$

or = 154000 m^3 approximately.

Prismoidal Rule - Example

The centre heights of the ground above formation level at three sections 50m apart are 12m, 15m, 18m and the cross fall at these sections respectively 1 in 20, 1 in 30 and 1 in 40.

If the formation Width is 30m and side slopes 1 in 3.

Calculate the volume of excavation in the 100m length.

For the first cross section (see figure 3.5)

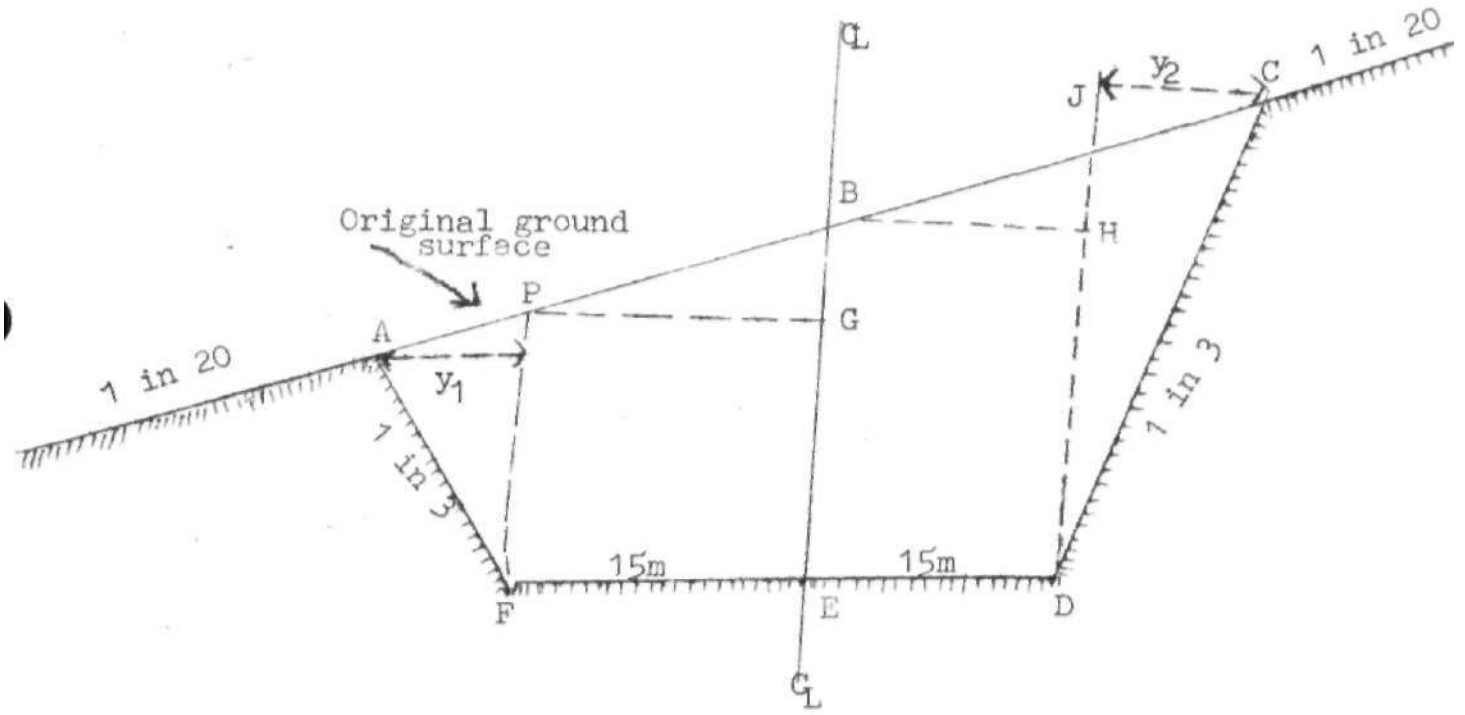
h = 12m

Side slope AF = CD = 1 in 3

Transverse slope AC = 1 in 20

FE = DE = 15m

Figure 3.5 : Cross Section of cut (1)



BE = 12m = h

Side slope AF = CD = 1 in 3

Transverse slope + 1 in 20 i.e. AC

FE = DE = 15m

PG = 15m

BG = $\frac{15}{20}$ = 3/4m

PF = (12 - 3/4)m = $11\frac{1}{4}$m

Similarly JH = (12 + 3/4)m = $12\frac{3}{4}$m

From figure 3.5, by the analytical method

$$y_1 = (\tfrac{1}{3} + 1/20)^{-1} \times 11\tfrac{1}{4}$$

$$= \frac{60}{23} \times 11\tfrac{1}{4}$$

$$= 29.35\text{m}$$

Area APF = $\frac{1}{2} \times 11\frac{1}{4} \times 29.35$

= $\underline{165.10\text{m}^2}$

Area of trap. PJDF = $15\text{x}(11\frac{1}{4}+12\frac{3}{4}) = \underline{360\text{m}^2}$

By analytical method also

$$y_2 = (\tfrac{1}{3} - 1/20)^{-1} \times 12\tfrac{3}{4} = \frac{60}{17} \times 12\tfrac{3}{4}$$

y_2 = 45m

Area of CDJ = $\frac{1}{2} \times 12\frac{3}{4} \times 45$

= 286.875m^2

Cross sectional area of excavation

= Area APF + Area PJDF + Area CDJ

= $(165.10 + 360 + 286.875)\ \text{m}^2$

= $\underline{811.975\ \text{m}^2}$

For the second Cross section (see Figure 3.6

BE = h = 15m

Side slope AF = CD = 1 in 3

Transverse slope = 1 in 30 i.e. AC

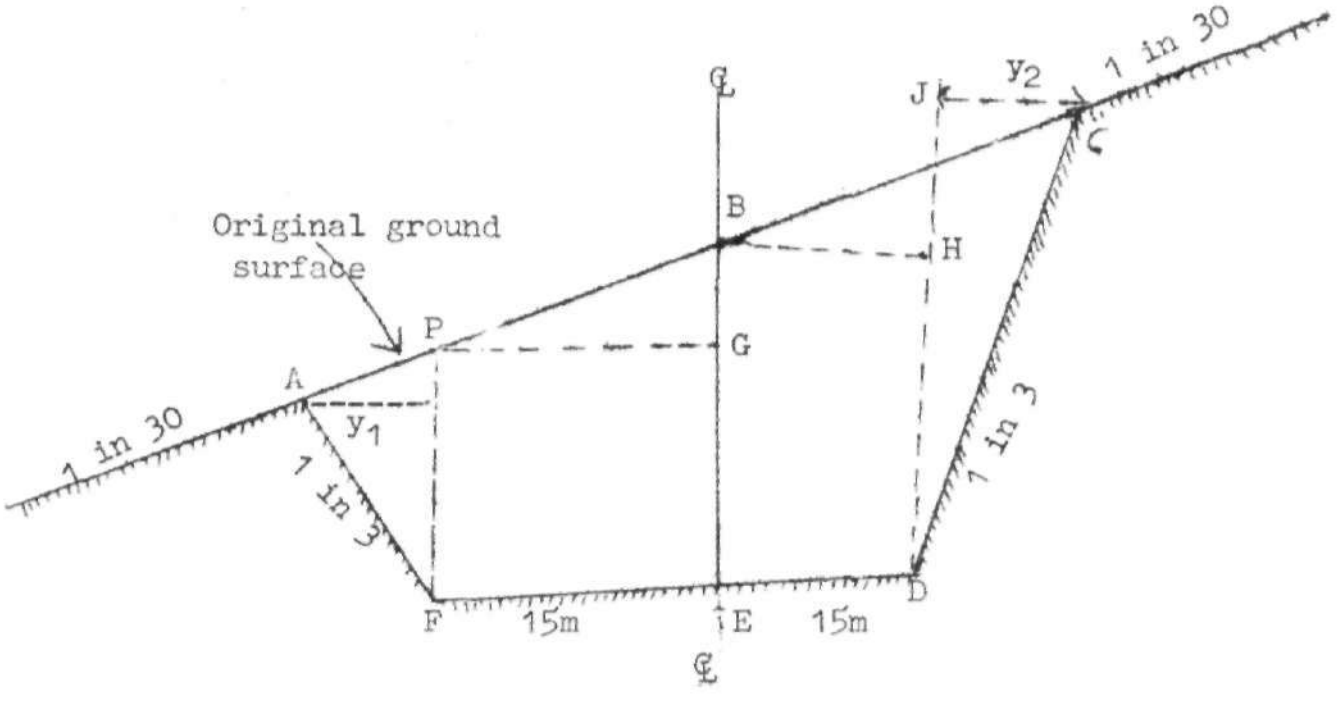
FE = DE = 15m

PG = FE = 15m

BG = $\frac{15}{30}$ = 0.5m

PF = (15 - 0.5) m = 14.5m

Figure 3.6 : Cross Section of cut (2)



Similarly JH = (15 + 0.5)m = 15.5

From the figure, by analytical method

$$y_1 = (\tfrac{1}{3} + 1/30)^{-1} \times 14.5 = \frac{30}{11} \times 14.5 = 39.55\text{m}$$

Area of APF = $(\frac{1}{2} \times 14.5 \times 39.55)\text{m}^2 = \underline{286.70\text{m}^2}$

Area trap. PJDF = $15 \times (14.5 + 15.5) = \underline{450\text{m}^2}$

By analytical method also

$$y_2 = (\tfrac{1}{3} - 1/30)^{-1} \times 15.5 = \frac{10}{3} \times 15.5 = 51.67\text{m}$$

Area of CDJ = $\frac{1}{2} \times 15.5 \times 51.67 = \underline{400.42\text{m}^2}$

Cross sectional area of excavation

= Area APF + Area PJDF + Area CDJ

= $(286,70 + 450 + 400.42)\text{m}^2$

= $\underline{1137.12\text{m}^2}$

For the third section (see figure 3.7)

BE = h = 18m

Side slope AF = CD = 1 in 3

Transverse slope = 1 in 40 i.e. AC

FE = DE = 15m

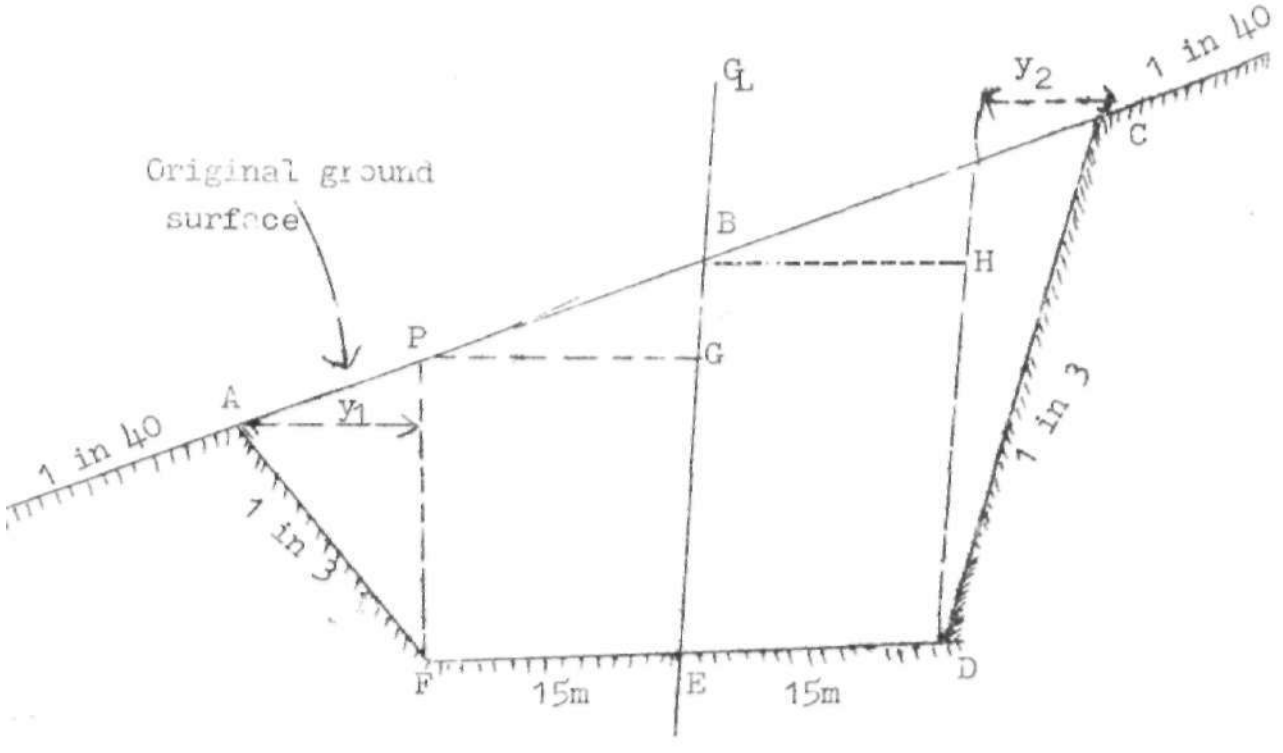
PG = FE = 15m

BG = $\frac{15m}{40}$ = 3/8m

PF = 18 - 3/8 = $17\frac{5}{8}$

Similarly JH = 18 + 3/8 = 18 3/8

Figure 3.7 : Cross section of cut (3)



From figure 3.7, by analytical method

$y_1 = (\frac{1}{3} + 1/40)^{-1} \times 175/8 = \frac{120}{43} \times 17\frac{5}{8} = 49.19m$

Area of APE = $\frac{1}{2}$ x $17\frac{5}{8}$ x 49.19 m^2 = $\underline{433.45m^2}$

Area of trap. PJDF = 15 x ($17\frac{5}{8}$ + 183/8) = $\underline{540m^2}$

By analytical method also

$$y_2 = (\tfrac{1}{3} - 1/40)^{-1} \times 183/8 = \frac{120}{37} \times 183/8 = 59.60m$$

Area of CDJ = $\frac{1}{2}$ x 183/8 x 59.60 = $\underline{547.575\ m^2}$

Cross sectional area of excavation

= Area APF + Area PJDF + Area CDJ

= (433.15 + 540 + 547.575)m^2

= $\underline{1521.025m^2}$

In this problem there are three (ODD NUMBER) Cross sectional areas. We can now use the prismoidal rule to compute the volume of excavation within the 100m length. Let V be the volume required.

$$V = \frac{d}{3}(A_1 + 4A_2 + A_3)$$

Where A_1 = 811.975, A_2 = 1137.12, A_3 = 1521.025

and d = 50m

$$V = \frac{50}{3}((811.975 + (4x1137.12) + 1521.025))$$

$$= \frac{50}{3} \times 6881.48m^3$$

$$= 114691.33m^3$$

V = 115, 000 m^3 approximately.

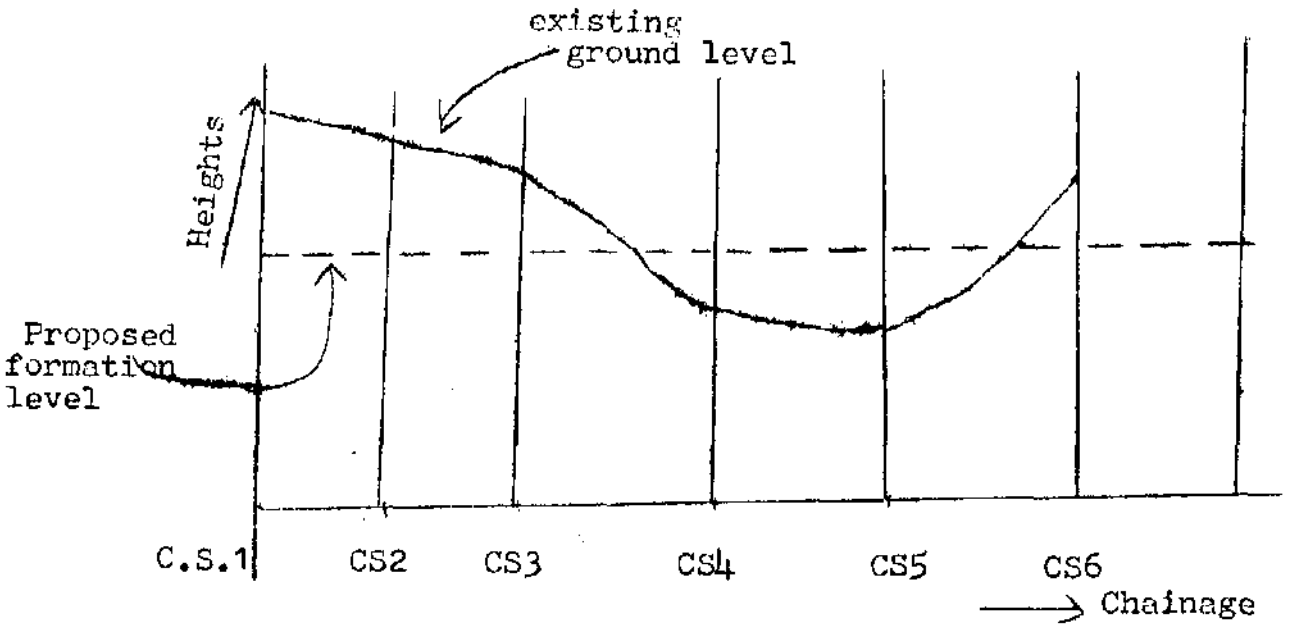
The above method of calculating cross sectional areas also obtains for embankments. Where the number of cross sections is even we either use the end areas method to find the volume; or use the prismoidal rule for the first odd number of cross sectional areas and the end areas method for the last two.

3.3 Volume Calculation by extrapolation.

There could be occasional or intermitent occurrence of both cut and fill between cross sections. The exact beginning of cut or fill may not be definitely straight forward. This means this point has to be determined by estimation; preferably using method of extrapolation as done in the following example.

Figure 3.8 shows a longitudinal section for the series of six cross sections shown in figure 3.9 taken at 30m intervals along the proposed centre line of a road. We want to evaluate the earthworks volume contained between cross section one (s.s.1) and cross section six (c.s.6).

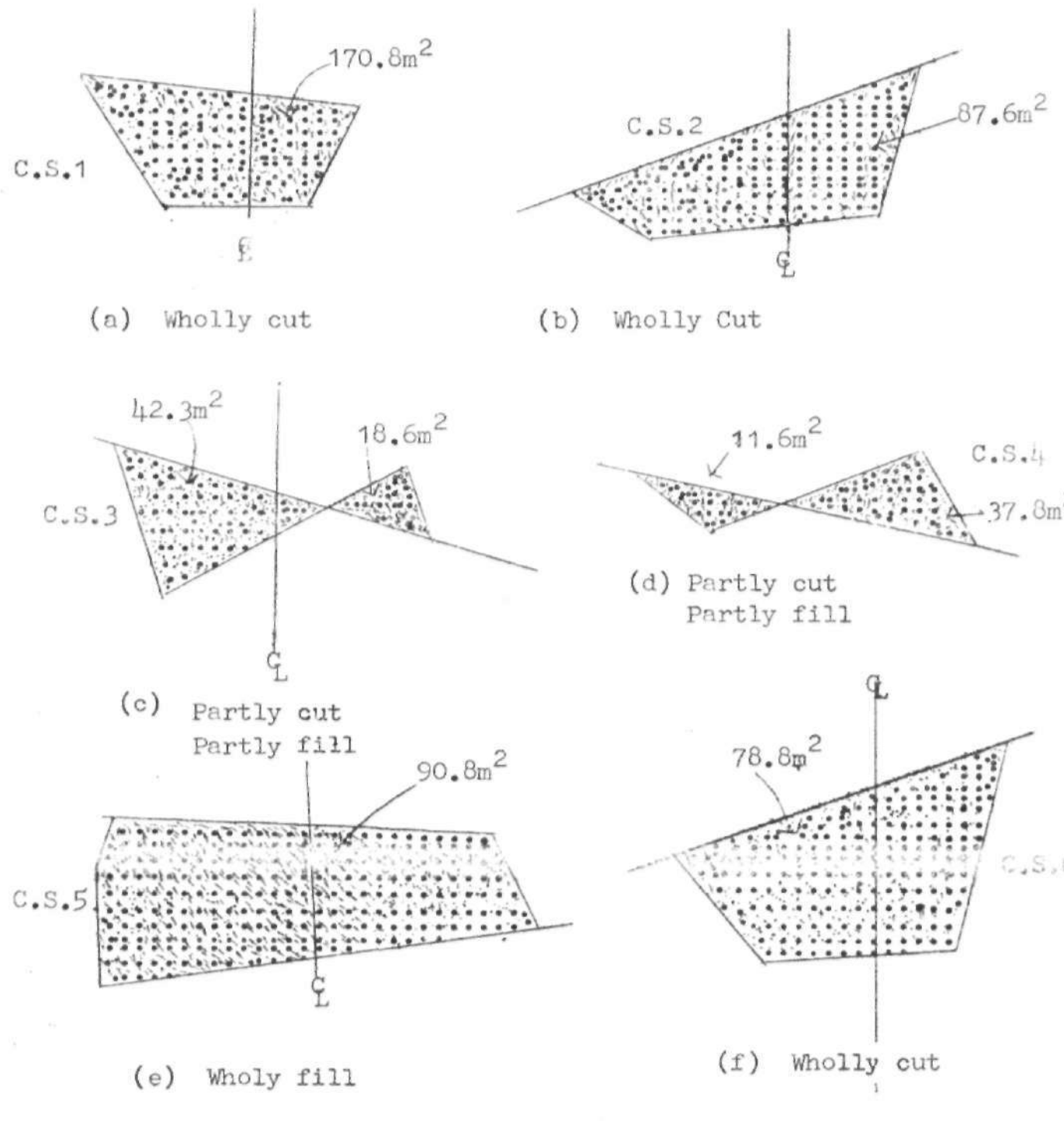
Figure 3.8: Longitudinal section for a series of cross sections involving extrapolations



The rule is that if the cross sections are of the same type we can use either the end areas formula or the prismoidal rule,

preferably the prismoidal; or both. If, however, the cross sections are changing as in the figure 3.8 it is advisable to work from one cross section to the next using the end area method. The following volume calculations are based on figure 3.9

Figure 3.9 : Cross sectional areas corresponding to the cross sections in the profile in figure 3.8



(a) C.S.1 to C.S. 2. From figure 3.9 the Cross Section one (C.S.1) is wholly in cut. So also cross section two (C.S.2)

$$\text{Volume of cut} = \frac{30}{2}(170.8 + 87.6)$$

$$= 15 \times 258.4 = \underline{3876m^3}$$

Volume of fill = Zero

(b) C.S.2 to C.S.3

Cross section two is wholly in cut while cross section three is partly in cut and partly fill.

$$\text{Volume of cut} = \frac{30}{3}(87.6 + 42.3)$$

$$= 15 \times 129.9 = \underline{1948.5m^3}$$

For the volume of fill in this interval we have to extrapolate the point at which the fill begins in order to obtain accurate volume figures. To do this we assume that the rate of increase of fill between C.S 2 and C.S.3 is the same, for all practical purposes, as that between C.S. 3 and C.S. 4.

Let d_1 be the length of fill from C.S.2 to C.S.3.

Between C.S.3 and C.S.4 the area of fill increases from $18.6m^2$ to $37.8m^2$ within a distance of 30m.

By simple proportion (extrapolation)

$$\frac{d_1}{18.6} = \frac{30}{(37.8-18.6)}$$

$$d_1 = \frac{30 \times 18.6}{19.2} = \frac{558}{19.2} = 29.06m$$

Therefore volume of fill $= \frac{29.06}{2}(0+18.6)$

$$= \underline{270.26m^2}$$

..

(c) C.S. 4 to C.S. 4

Cross section three is partly in cut and partly in fill while cross section four is in the same manner.

$$\text{Volume of cut} = \frac{30}{2}(42.3+11.6)$$

$$= 15\text{x}53.9\text{m}^3$$

$$= \underline{808.5\text{m}^3}$$

$$\text{Volume of fill} = \frac{30}{2}(18.6+37.8)$$

$$= 15\text{x}56.4\text{m}^3 = \underline{846\text{m}^3}$$

(d) C.S.4 to C.S.5

Cross section four is partly in cut and partly fill. Like in (b) above we must also extropolate the interval of the area of cut since cross section five is wholly fill.

$$\frac{d_2}{11.6} = \frac{30}{42.3-11.6} \qquad d_2 = \frac{30 \times 11.6}{30.7}$$

$$= 11.34\text{m}$$

$$\text{Volume of cut} = \frac{11.34}{2}(11.6+0) = 5.67\text{x}11.6\text{m}^3$$

$$= \underline{65.77\text{m}^3}$$

$$\text{Volume of fill} = \frac{30}{2}(37.8 + 90.8) = 128.6 \text{ x } 15\text{m}^3$$

$$= \underline{1929\text{m}^3}$$

(e) C.S. 5 to C.S. 6

Cross section five is wholly in fill and cross section 6 is wholly in cut. Since the cross section changes from fill to cut in this interval we have to extrapolate where the fill section ends and the cut section begins. We normally use a linear

relationship involving the cross sectional area thus:

$$\frac{d_3}{A_5} = \frac{d}{A_5+A_6}$$

where d = 30m and d_3 is to be estimated from the above relation.

$$d_3 = \frac{d.\ A_5}{A_5+A_6}$$ which in this case

$$= \frac{30 \times 90.8}{90.8+78.8} = \frac{2724}{169.6} = \underline{16.06m}$$

Therefore volume of fill

$$= \frac{16.06}{2}(90.8 + 0) = 8.03 \times 90.8m^3$$

$$= \underline{729.12m^3}$$

Volume of cut = $\frac{(30-16.06)}{2}(0+78.8)m^3$

$$= 6.97 \times 78.8m^3$$

$$= \underline{549.24\ m^3}$$

Now, total volume of cut from C.S.1 to C.S.6

$$= (2876+1948.5+808.5+65.77+549.24)m^3$$

$$= \underline{7248.01m^3}$$

Total volume of fill from C.S.1 to C.S.6

$$= (0+270.26+846+1929+729.12)$$

$$= \underline{3774.38m^3}$$

3.4 Calculation of volumes from Contours

This method is used for calculating volumes from various earthworks projects such as earth dams,

reservoirs, spoil heaps (e.g. in road construction, pipeline trenching, railway etc. for estimating volume of tips of wastes and dumps).

The method is better explained from an example. It involves the calculation of the plan area enclosed by each contour. Each area is then treated as a cross sectional area. The contour interval is regarded as the constant distance between the cross sections; and the volume is estimated using either the trapezoidal rule or the prismoidal formula.

The prismoidal rule is applicable provided the number of contours is odd. At times both methods are combined in calculating the volume depending on the nature of the data. The plan area contained by individual contour is usually calculated using the planimeter or the graphical method. For very accurate work the contour intervals should be small. A one-half meter contour interval will provide better accuracy than a 2 meter contour interval.

Ex. Figure 3.10 shows the contour plan of a proposed agricultural irrigation dam with the dam wall on the left side. The contour vertical interval is 5m and the full capacity water level is to be 143m. The lowest point in the dam is at a reduced level of 108m. We require the capacity of the reservoir.

The Cross sectional area of each contour was calculated using the planimeter and the following enclosed area corresponding to each contour was obtained:

Figure 3.10: Cross sectional areas from contours

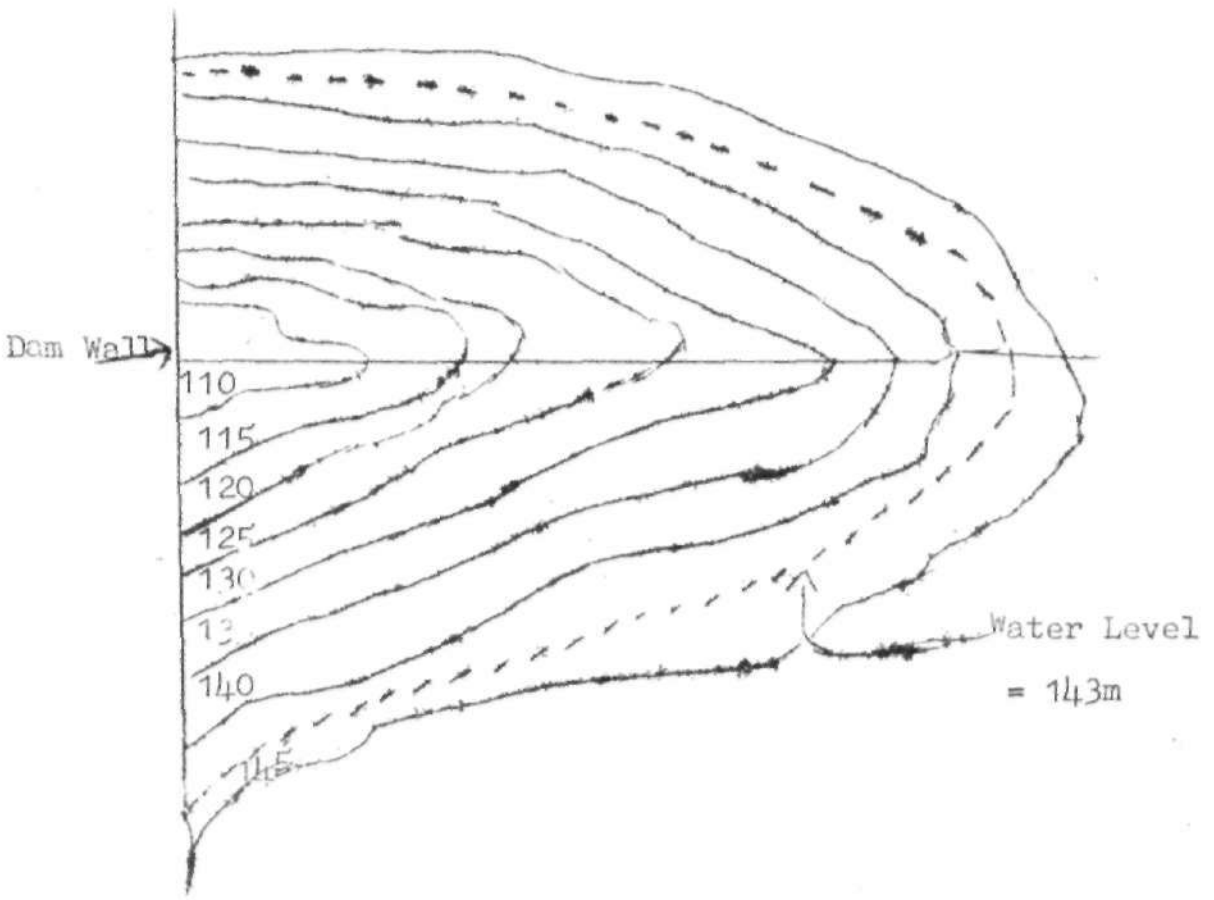


Table 3.1: Cross sectional areas and contours

Contour (m)	108	110	115	120	125	130	135	140	145
Enclosed Area(m^2)	0	2200	5600	7800	8900	9900	11500	13400	17100

Let V_1 represent the volume between contour 108m and 110m,

V_2 , the volume between 110m and 140m ; and

V_3, the volume between 140m and 143m water level using the end areas method

$$V_1 = \frac{(0 + 2200)}{2} \times 2 = 2200m^3$$

$$V_2 = 5((2200 + 13400) + 5600 + 7800 + 8900 + 9900 + 11500))m^3$$

$= 5(7800+5600+7800+8900+9900+11500)m^3$

$= 5 \times 51500\ m^3$

$= \underline{257500}\ m^3$

Similarly $V_3 = \frac{(13400 + 17100)}{2} \times 3\ m^3$

$= 15250 \times 3\ m^3$

$V_3 = \underline{45750\ m^3}$

Total capacity V is given by

$V = V_1 + V_2 + V_3$

$= (2200 + 257500 + 45750)m^3$

$V = 305450\ m^3$

$V = 30500m^3$ approximately.

On the other hand we can use both the end areas method and the prismoidal formula to obtain an estimate of the capacity of the dam.

Let V_1 (again), represent the volume between contour 108m and 110m ; V_2 , the volume between 110m and 140m; and V_3 , the volume between 140m and 143m.

For V_1, we use the trapezoidal rule i.e.

$$V_1 = \frac{(0+2200)}{2} \times 2 = \underline{2200\ m^3}$$

For V_2 we can use the prismoidal rule since the number of cross sectional areas is 7 i.e. odd number.

$$V_2 = \frac{5}{3}((2200+13400)+4(5600+8900+11500) + 2(7800 + 9900))\ m^3$$

$$V_2 = \frac{775000}{3}\ m^3 = \underline{258,33\ m^3}$$

For V_3, we use the trapezoidal rule

i.e. $V_3 = \frac{(13400 + 17100)}{2} \times 3\ m^3 = 15250 \times 3\ m^3$

$$V_3 = \underline{45750\ m^3}$$

If the total capacity of the dam is represented by V , then

$$V = V_1 + V_2 + V_3$$

$$V = (2200 + 258,333 + 45750)\ m^3$$

$$V = 306,\ 283m^3$$

The capacity, using the combination of trapezoidal and prismoidal rules is approximately 306, 000m^3.

The slight difference in capacities obtained by the two approaches is remarkable. The latter method should be more dependable because the contours are usually curved imaginary lines as the concept of prismoidal rule assumes a parabolic curved surface.

3.5 Calculation of volumes from Spot Heights

Volumes can be determined from spot heights in large open excavations such as underground tanks, borrow pits, basements and reservoirs and also in ground grading (or levelling) operations in building sites and construction of sports fields and lawns.

The volume estimations from spot heights is irrespective of the terrain.

The whole area is gridded into squares, rectangles or triangles. The Corner points (junction points) are levelled and their reduced levels are obtained. Having known the designed formation level, the depth of excavation at each corner point is calculated.

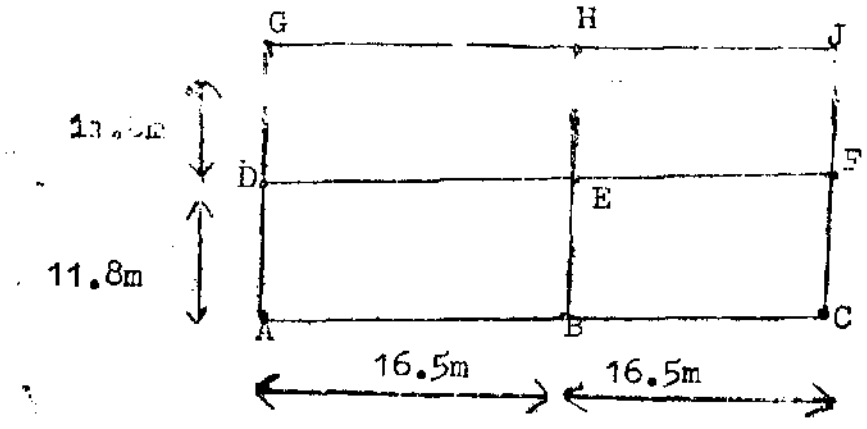
The plan area i.e. cross sectional area is evaluated from the grid scheme. So also the mean height of the entire excavation area is obtained. The volume V is given by

V = mean height x Cross sectional area.

The accuracy obtained depends on the size of the grid. The smaller the grid the greater the accuracy of the volume calculated. The small grids, however, result in greater amount of field work.

The method of calculating volumes from spot heights is illustrated by the example below. Consider the rectangular area ABCDEFGHJ. This is gridded into rectangles as in figure 3.11

Figure 3.11 : Rectangular ground showing Spot-heights at Junction points of rectangles



For greater accuracy it is further gridded into triangles as shown in figure 3.12 and the volume also computed from the formula. The depths of excavation are as indicated in the tables i.e. tables 3.2 and 3.3.

$$\text{Mean Height} = \frac{\text{Sum}}{4} = \frac{80.85}{4} = 20.21 \text{ m}$$

Table 3.2 : Computation of Volume from spot Heights (1) See Figure 3.11

Station	Depth of excavation (h) m	No. of rectangles in which levels occur (n)	Product (hxn)
A	4.26	1	4.26
B	5.62	2	11.24
C	2.91	1	2.91
D	3.86	2	7.72
E	4.89	4	19.56
F	5.21	2	10.42
G	3.50	1	3.50
H	7.23	2	14.46
J	6.78	1	6.78
		Sum	80.85

Cross sectional area = 11.8 x 16.5 = 194.7m^2

$$V = 20.21 \times 194.7 \ m^3$$

$$V = 3935 \ m^3$$

Figure 3.12 illustrates the situation when the area is regridded into triangles.

Figure 3.12: Rectangular ground showing spot heights at junction of triangles

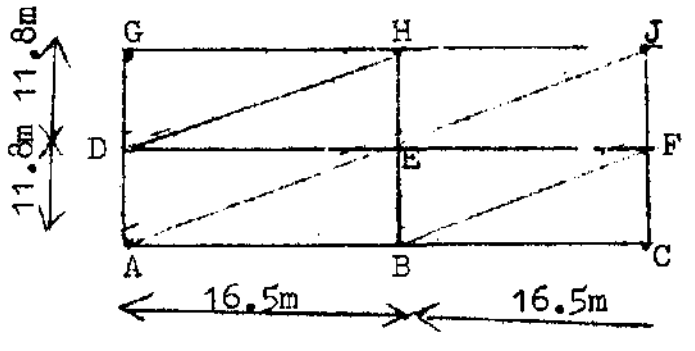


Table 3.3 : Computation of Volume from Spot heights (2)

Station	Depth of excavation (h) m	No of triangles in which levels occur (n)	Product (h x n)
A	4.26	1	4.26
B	5.62	3	16.86
C	2.91	2	5.82
D	3.86	3	11.58
E	4.89	6	29.34
F	5.21	3	15.63
G	3.50	2	7.00
H	7.23	3	21.69
J	6.78	1	6.78
		Sum	118.96

$$\text{Mean Height} = \frac{118.96}{3}$$

$$= 39.65\text{m}$$

$$\text{Cross sectional area} = \tfrac{1}{2} \times 11.8 \times 16.5$$

$$= 97.35\ \text{m}^2$$

Volume of excavation

= Cross sectional Area x Mean Height

$$= (97.35 \times 39.65)\ \text{m}^3$$

$$= 3859.93\ \text{m}^3$$

$$= \underline{3860\ \text{m}^3}$$

CHAPTER FOUR

MASS HAUL DIAGRAMS: ANALYSIS OF EARTWORKS VOLUMES

4.1 Introduction:

For effective economy and efficient use of plant and other materials the Mass Haul Diagram (M.H.D.) is of great use in analysing earthworks volumes in any construction scheme. In such projects such as roads, pipelines, railways and canals the earthmoving involved is of such considerable quantity that effective planning measures are necessary in order to achieve the utmost objective expeditiously and effectively with minimum cost.

Earth materials have to be brought to site from some distance to form embankments; while some quantity of earth is removed from the site in tips either as spoil heaps for use in fills later on or as wastes during cuttings. Earth movement on the site can, therefore, be extremely expensive and so needs careful planning. For effective and efficient handling of materials during any construction planning the Mass Haul Diagram (M.H.D.) is an essential and inevitable tool.

The M.H.D. is a graph of chainage against aggregate volume. The x- axis is usually representing the chainage along the project from the Zero chainage to the total chainage length being considered. The aggregate volume of earth materials corresponding to the successive chainage along the stretch to be covered by the M.H.D. is represented by the y- axis.

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In the plotting of the M.H.D., volume of cut are considered as positive and volume of fill, negative. To facilitate its use, the axes of the M.H.D. are exaggerated in scale. The M.H.D. assumes, correctly and reasonably, that earth is moved longitudinally in the direction of the Centre line of the project. Normally, earth movement at directions at right angles to the longitudinal section is considered of extremely small quantity and is rightly ignored.

4.2 Use of the Mass Haul Diagrams

The M.H.D. is used to find solutions to the following questions:

(a) the distances over which cut and fill will balance,

(b) the planning for a most expedient design of formation levels for any construction scheme in order to insure that distances over which earth is moved is juditiously controlled (without prejudice to geological, geomorphological and other relevant civil engineering considerations);

(c) planning the most economical earth moving pattern around the entire project so that the best estimate is obtained.

(d) Since volume of earth materials to be moved and the movement pattern on the project site is evaluated using the M.H.D. the correct and most suitable plant machineries can be chosen before hand.

(e) the choice of sites for borrow pits (quarries) and spoil heaps could be made; and the direction of haul established before construction commences.

(f) the quantity of earthworks involved being known enables the most economical choice of the number and type of labour force required at particular sections of the project site.

4.3 Some Definitions:

(a) Haul: Is the volume of excavation multiplied by the average haul distance. Or, it is the volume of materials multiplied by the distance moved. It is expressed either as cubic meter meter ($m^3.m$) or station meters i.e. 1 m^3 moved 100m. For example, if $30m^3$ of earth materials is moved over a distance 1000m this will give a haul of

$30 \times 1000\ m^3.m = 30{,}000\ m^3.m$

or $\frac{30 \times 1000}{100}$ stn. m = 300 stn. m

On the Mass Haul Diagram, the haul is represented by the area contained between the curve and the balancing line. The haul at every section of the M.H.D. can thus be scaled and calculated.

(b) Haul Distance: This is the distance from the point of excavation (of the volume) to the point where the material is tipped.

(c) Average Haul Distance: Is the distance from the centroid (or centre of gravity in scientific parlance) of the excavation to the centroid of the tip (e.g. for embankment purposes).

(d) Freehaul Distance: This distance is usually incorporated in the contract agreements. It is the distance over which charge is calculated on the volume of earth material excavated and not its distance of tips. It is the permissible distance for freehaul.

(e) Overhaul Distance: This is the distance beyond the freehaul distance over which extra charge is calculated separately. It is, thus, the distance of movement of any overhaul.

(f) Freehaul: This is the haul within the specified freehaul distance. It is normally expressed as volume i.e. m^3.

(g) Overhaul: This is the haul remaining after the freehaul has been removed. It is expressed in the same unit as HAUL i.e. either as m^3,m or station meters.

(h) Waste: This is the material that is completely hauled and removed from the project site and dumped in another locality because of unsuitability or surplusity. Usually, the wastes could be spread along the "shoulders" of the road to create verges ; or else they are regarded as useless. Waste is expressed as a volume i.e. m^3.

(i) Borrow: This is the earth material which is imported from a borrow - pit, quarry or any suitable locality away from the project site because of deficiency of suitable material. Borrow is expressed as volume i.e. m^3.

..

4.4 Properties of the Mass Haul Diagram

Figure 4.1 is a Mass Haul Diagram of chainage against corresponding aggregate volumes.

In any Mass Haul Diagram, any section where the curve rises i.e. in the direction of positive grade, the project is in cut. In figure 4.1, Sections fbg and kdm are in cut. On the other hand when the curve falls the projedt is in fill, for example gck and men in the figure. It is observed, for instance, that in sections fbg and kdm the aggregate volumes are increasing while they are decreasing in sections gck and men.

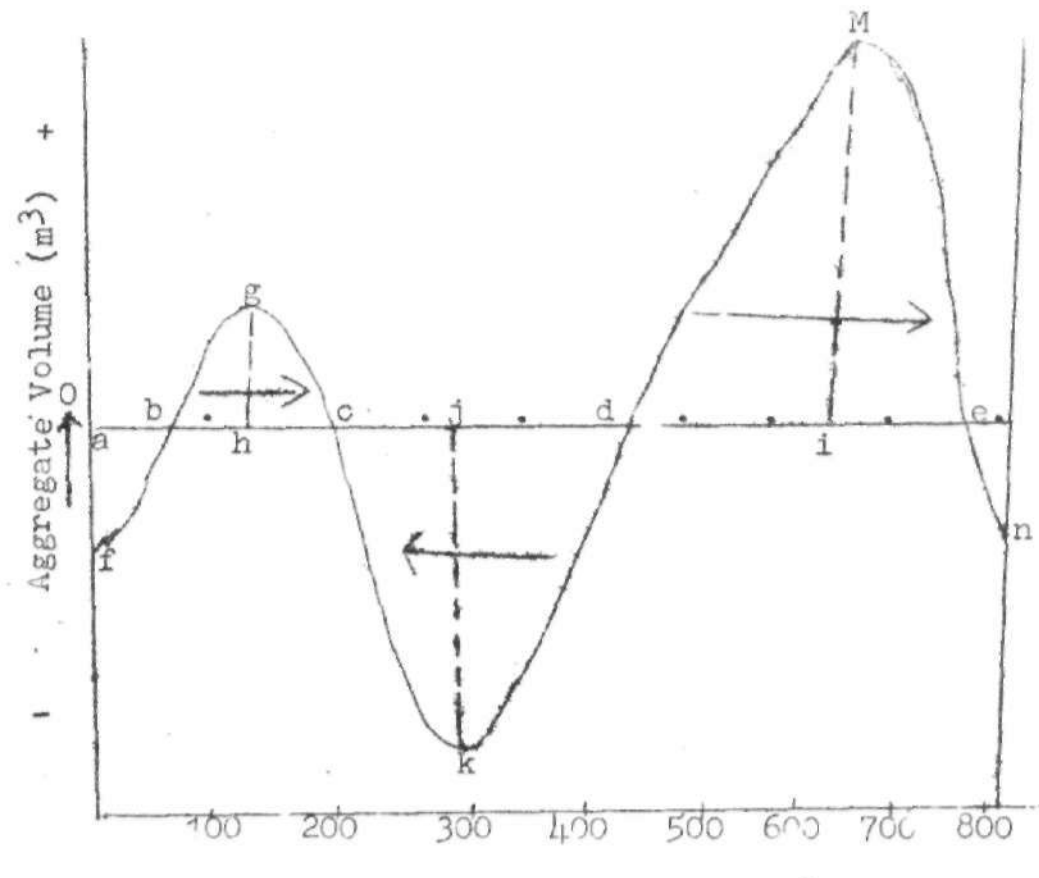
The vertical distance between a maximum point and the next forward minimum represents the volume of fill, for example (gh + JK). The vertical distance between a minimum point and the next forward maximum represents the volume of exavation, for example (jk + mi).

"Any horizontal line which cuts the Mass Haul Diagram at two or more points balance cut and fill between those points and the line is referred to as a balancing line" - Uren and Price, 1983.

In figure 4.1 abedep is a balancing line to the Mass Haul Curve. This is interpreted to mean that, with the availability of suitable material all earth excavation between b and e will produce all the fill requirements.

Any section of the Mass Haul Diagram lying above the balancing line indicates the earth movement is

Figure 4.1. : A typical Mass Haul Diagram



from left to right. On the contrary, any area of the curve lying below the balancing line signifies that material must be moved from right to the left. These directions of movement are shown by bold arrows on the M.H.D. in figure 4.1

The length of balancing line between intersection points is the maximum haul distance in that section. For example the maximum haul distance in section ed is chainage J - Chainage C.

The area of the Mars Haul Diagram contained between the curve and the balancing line is equal to the haul in that section, for example bgchb, ckdc and dmed.

In figure 4.1, there is waste at chainage zero where the curve rises from f to b and the volume of waste is represented byaf. At chainage 800m, there is borrow where the curve is falling from e t o n; and the volume of borrow is represented by pn.

All required quantities are, normally, scaled from the Mass Haul Diagram.

It should be emphasized, at this juncture, that the x-axis is used here as the balancing line to ease convenience of illustration. Any balancing line can be chosen. In fact, when investigating the most expedient earth moving plans etc. so many balancing lines could be tried until finally, the most satisfactory results are arrived at.

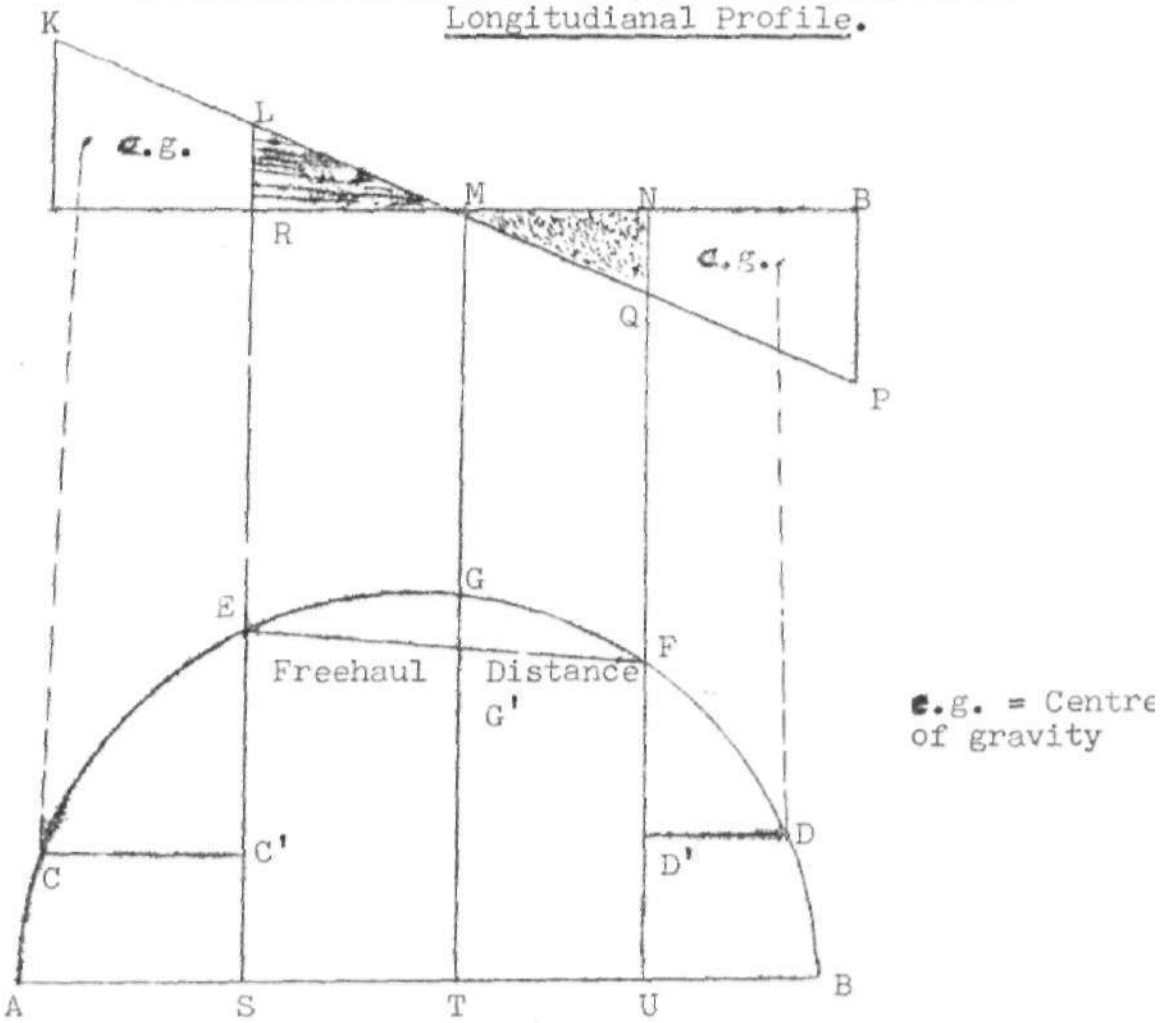
4.5 Interpretations of the M.H.D. using the Balancing Line(s) and the Freehaul Distance

The following refer to figure 4.2.

The original ground surface is represented by KMP and this is to be levelled down to the horizontal grade line AB. We shall assume that the cut volumes equal (or balance) the fill volumes. The Mass Haul Diagram plots below the longitudinal profile as shown.

Let us assume the freehaul distance is 150m. By use of the set squares this freehaul distance EF parallel to AB (the balancing line) is scaled to be exactly 150m intersecting the M.H. curve at E, F.

Figure 4.2: Mass Haul Curve and Dorresponding Longitudianal Profile.



From figure 4.2 it means the movement of earth materials LMR to fill up section NMQ falls within the Freehaul distance. The Freehaul volume involved in the earthmoving is represented by the ordinate GG^1.

It therefore follows that the remaining cut volume KLRA is represented by the ordinate ES. This volume KLRA is the volume of excavation that will fill NBPQ the average distance being from centroid to centroid.

The positions of the centroids are found by bisecting ES and FV, giving the horizontal distance between centroids i.e. CD.

Suppose CD = 280m. This overhaul volume has to be moved through this distance. Obviously , this movement through 280m must also include the 150m distance the freehaul volume too is moved. This then means that the overhaul distance is (280-150) m = 130m.

From the explanation above it is also patently obvious that the total volume (GG^1 + ES) = GT is a volume that falls within the freehaul contract.

The overhaull =

$$ES(CD - EF)\ m^3.m$$

$$\text{or} = ES\ \frac{(CD - EF)}{100}\ \text{st}\ m.$$

In this example, suppose the scaled overhaul ordinate (volume) ES = $3700m^3$ and the overhaul distance, 130m

$$\therefore \text{Overhaul} = \frac{3700 \times 130}{100}\ \text{stn. m.}$$

$$= 4810\ \text{stn. m.}$$

From the foregring, the following inferences are pertinent:

(a) All dimensions required, with respect to earthworks movement are scaled from the Mass Haul Diagram,

(b) All calculations are done using the balancing line and the Freehaul Distance as pertinent basis.

(c) The balancing line, in particular, is designed to give a most definite satisfactory answer to all questions pertaining to finandal rationalisation of any construction project and the most economic use of plant.

(d) The purpose of plotting the freehaul distance on the mas Haul Curve is to assess the overhaul.

(e) If there is no Freehaul, it implies, therefore, that all the volume of earth materials is transported regardless of distance. The contract is thus predicated purely on the volume alone.

(f) The forward Freehaul distance, say of 350m, is the Freehaul distance of 350m in any section of the Mass Haul Diagram above the balancing line. Similarly, the backward freehaul distance, say of 350m, is the freehaul distance in any section of the M.H.D. below the balancing line.

(g) When more than one balancing line is used there will be waste and borrow at intermediate points along the project which consequently involves extra volume of excavation and transportation of material tips.

(h) Usually, different balancing lines are tried and costed with regards to wasting, borrowing and hauling. The most economically expedient solution is then adopted. With regards to this phenomenon, due attention needs be paid to the direction of haul since this aspect may not attract special credence in the contract. Generally, it is advisable to haul downhill to economisepower. Where haul uphill is circumstantially inevitable it may be better to waste at the lower points and borrow at the higher points.

(i) The Freehaul limit, as a matter of economy, should be exceeded as little as possible so as to minimise the amount of overhaul. Short, unconnected balancing lines are, therefore, more economical than one long continuous balancing line, more especially where the balancing lines are shorter than the freehaul distance since no overhaul costs will be involved. This further implies that the most economical solution from the haul aspect of the earth materials is to minimise the area between the Mass Haul Curve and the balancing line. As mentioned earlier, this suggestion may result in large amounts of waste and borrow at intermediate points along the project. The most reasonable economics can only be arrived at by considering all probable points; that means those of hauling, wasting and borrowing. With practice the best approach naturally occurs to the mind.

CHAPTER FIVE

5. CONCLUSION

It has been shown in this thesis that the method of estimating any earthworks volume depends on the situation and prevailing circumstance. If, for example, we were to compute the volume of earthworks involved in a borrow-pit during a civil engineering construction work a rectangular area containing the suitable materials could be selected. The whole area could then be redivided into triangles (as in 2.1.1), squares (as in 2.1.2), or smaller rectangles (as in 2.1.3). Spot heights are then taken at the junction points and the mean height calculated. The volume of excavation is then obtained as in 3.5 (pp. 79).

To buttress the above inference, six different methods of areas from coordinates (2.1.5, pp. 3) are treated. The prevailing circumstance will dictate which method to adopt on every situation. If the coordinates are in large figures (e.g. six digits or more) and the calculator being used is of only ten (or below) digits and the area is very large, it won't be reasonable nor even feasible for the computor to resort to the double area method in 2.1.5 (ii), pp. 5. The method of cross multiplication as in 2.1.5 (iii) pp. 6 may not be advisable. A method that will bring down the number of digits so that the available calculator can accomodate the reduced figures automatically recommends itself. The method of departures and total latitudes is most appropriate (see 2.1.5 (iv)), pp. 7 - 10; or Sanyaolu's analytical methods and automatic checks for area computation (2.1.5 (vi)) pp. 14 - 22.

In estimating the volume of earthworks for a large dam the method of contouring as elucidated in the example in 3.5. pp. 75 could be used.

On road works, cross sectional areas could be calculated using formulae as in 2.3 pp. 26 - 41. If one is not armed with the text book, the formulae are complicated, confusing; or can be forgotten outright. For this very important consideration the computor/engineer should be very conversant with the use of the analytical or trigonometrical solution for earthworks cross sectional areas as enunciated adequately in 2.5, pp. 41 - 47. Similarly, in such a situation cited above, the approach from first principles as demonstrated in 2.6 pp. 47 - 51 will give dependable results.

Of course for irregular figures any of the methods in 2.2 pp. 22 - 26 can be used to obtain the area. However, most experts prefer the Simpsons rule to the trapezoidal method. Where the plan of the area is available the graphical method (viz: counting of small squares) can give a most reliable result. Experience normally dictates which method should be adopted on the situation.

As earlier mentioned in the text, volume from Mean Areas Method is a very rough estimate and could not be used where accurate figure is required. The End Areas Method is better; but the prismoidal formula can give a most dependable result (refer to pp. 54 - 61). In road works or any other mass civil engineering concern, the error attributable to curvature is negligible relative to the amount of volume involved at every section of the project. As advised in the

text, this correction for curvature could be discarded without any tangible damage to our estimates. However, where greater accuracy is required the prismoidal correction should be applied as explained in 3.2.1 pp. 61 - 62.

Since every large scale civil engineering works e.g. modern highways or railways involved painstaking and meticulous detailed planning before the commencement of construction execution, it is imperative that the engineer should resort to extensive use of the Mass Haul Diagrams for a most rational analysis of earthworks volumes envisaged on the project. A neglect of this recommendation could result in colossal loss of money through wrong expenditures that could have been avoided. To this end all the considerations that must, of necessity, enter the planning strategies as enunciated on pp. 91 - 93 should be rigorously investigated in order to reach a most acceptable conclusion.

From this thesis, therefore, it is easily inferred that, on any construction engineering, there should be adequate cooperation between the Engineering Surveyor and the Civil Engineer. Though the real construction is done by the Civil Engineer, the levelling and the massive data on the levels are provided by the Surveyor. In order to obtain a most favourable result, therefore, the planning for the scheme should bear from the team work of the Civil Engineers and the Surveyors. To ascertain that the planned levels and the correct earthworks volumes are obtained for surplus materials, wastes, cut, and fill the Surveyors should be on site throughout the duration of the construction scheme.

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