INVESTIGATIONS IN THE METHODS OF IMPROVING THE STRENGTH OF NIGERIAN HORIZONTAL GEODETIC CONTROL NETWORK

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ABSTRACT

The effect of the approximate treatment of geodetic observations on the Nigerian network is described with a view to highlighting its adverse effect on the network. In the approximate treatment of observations, the geoidal undulations and the deviation of the vertical are neglected. Methods are suggested for determining these parameters. The correct procedures for carrying out the reduction of observations are also given.

And since unknown systematic, errors usually exist in a network, methods (both computational and observational) are given for the possible ways these errors can be eliminated or minimised. The effect of additional observations on the Nigerian network is evaluated by means of an optimisation technique based on computer simulated observations. A block of the network is used for this purpose and the method of variance-covariance analysis together with the related accuracy criteria is used.

Also a brief history of the development of the network is discussed.

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in evergreen memory

INTRODUCTION

Until recently geodetic triangulation had been accepted as the most accurate method of determining the "precise" (most accurate) co-ordinates of the stations in a planimetric framework. It is now well known that for extended triangulation systems, systematic errors such as lateral refraction, calibration errors, propagation of personal equation errors, residual polar motion effects on latitude, longitude and azimuth, whose effects are sometimes treated as non-significant, accumulate. In practice steps are usually taken to minimise the effects of such errors by adopting suitable observational techniques and by applying scale and azimuth (Laplace) checks at certain intervals within the network.

Modern technological advancement in the field of geodesy has made it possible to achieve a "zero order" control net (that is of the best attainable accuracy, better than the existing first order). This is usually provided by satellite triangulation and super-control traverse such as the zero-order geodimeter traverse. This super-control net is usually designed to constitute a modern geodetic super structure, within which the classical geodetic triangulation is supposed to provide a geodetic control densification.

Also it is an established concept that no adjustment technique can give an accurate answer from inacurrate observations. Thus any adjustment procedure, in which all possible sources of systematic errors have not been carefully evaluated and eliminated, should not be considered rigorous. Maladjustment, rather than a shortage of observations, constitutes the main factor affecting the strength of a large geodetic network and leads to distortions and inconsistencies. The elimination of all possible sources of systematic errors in any survey network is therefore essential for the proper and rigorous adjustment of a geodetic surveying network.

The present work is therefore aimed at considering the different possibilities of improving the strength of the Nigerian geodetic network. And in order to fulfil the objective, the work has been designed to fulfil the following functions:

(i) to investigate whether any significant improvement in strength can be achieved by surrounding the framework with a form of super-control traverse,

(ii) to investigate the effect of additional measurements on the accuracy of the network by using modern instrumentation and techniques, and thus make recommendations on the observations needed to strengthen it,

(iii) to investigate the effect of incorporating satellite Doppler observations on the existing network and thus make recommendations on the advantages/disadvantages of such a programme,

(iv) to determine the optimum number of azimuths and distances (observed using modern instrumentation and techniques) to be incorporated in the network (actually on the test network),

(v) to explore the possibility of using pure trilateration in conjunction with orientation control provided by

(a) azimuth observations at selected points

(b) azimuth and doppler satellite observations

for the establishment of geodetic control.

And in order to have a fair estimation of the geoid-spheroid separation in Nigeria, which in turn will pave the way for the proper and rigorous adjustment of the national framework, different methods of deriving the geoid and deflections of the vertical are examined. A suggestion, based on our economic restraints, both in manpower and instrumentation, is then made on the most suitable method.

For investigations (i) - (v) a network optimization procedure based on computer simulated observations has been adopted. A block of Nigeria's primary horizontal geodetic network is used for the simulation exercise, and the error analysis is based on variance-covariance matrix of the simulated network and related accuracy criteria.

In view of the improved observational techniques for Laplace azimuth determinations (Black's method), the improved accuracy and high range obtainable from the Geodolite, and the all weather geodetic fixes by satellite-Doppler observations, it appears reasonable to use the current field accuracies from these techniques/instrumentation for the simulation exercise. A total of 31 computer simulated models of the test network were carried out.

The strength of a network is defined as its ability to propagate scale and orientation within the network without undue deterioration.

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CHAPTER ONE

HISTORY OF THE FRAMEWORK OF NIGERIA

1.1 Historical background

The history of the Nigerian horizontal geodetic control network dates back to the early part of this century (about 1910) when it followed a pattern of self-contained ad-hoc schemes, each to serve the need of the area where it was located. These areas were mainly places of particular economic or strategic significance. However the network has since the year 1930, developed from these fragmentary origins to the present form of nationwide coverage and uniform standard. The stages of these developments are given in the subsequent/relevant sections of this chapter.

The present network (fig. 1a) consists essentially of classical triangulation chains and traverse stations for an area of over one million square kilometres. It is oriented roughly in the North-South and East-West direction. The triangulation scheme was observed mainly between 1930 and 1960. Important later additions were however made to the resulting network after 1960. These consisted mainly of

(a) Scale and azimuth checks at chain junctions using microwave EDM instruments and precise astronomical observations made with modern astronomical theodolites. These observations were solely undertaken by the Nigerian Federal Surveys.

(b) Several EDM traverses some of which are of first order standard. Most of these were made by the Directorate of Overseas Surveys (the DOS) and the Shell BP company, and were designed primarily as controls for topographical mapping. Though the angular measurements of the traverses executed by these organisations were not of the highest precision, the distances are of value in providing scale checks where these do not exist. The first order traverses have been designed for those areas where triangulation schemes are impracticable.

(c) The high order geodimeter traverse following the 12th parallel of latitude (North) across Africa. This was established by the United States Corps of Engineers.

The records of the network are thus compiled by several different organisations, and the observations have been made by several generations of observers. Some of these records (as one would expect), especially those compiled at the early stages of the network development have been lost. Consequently published material on the network is not extensive. The main sources of information for the present work are the papers written by Calder Wood, 1931-35-36, Stamers Smith and Sompola, 1955, and Field 1977-78. Though published material on the network is scarce, some amount of detail/information can be found in the records controlled by the Nigerian Federal Surveys. Efforts by the writer (through correspondence) to get more information from the Federal Surveys, especially on the latest development of the network, were unsuccessful.

The network has had a series of adjustments since its inception. The first adjustment was carried out about the year 1926. It was a semi-rigorous adjustment and the Clarke 1858 spheroid was used. The subsequent adjustment with the exception of the last two (1968 and 1977 adjustments) were performed piecemeal as observations were completed, and the Clarke 1880 (modified) spheroid was then used. Each chain was divided into convenient figures which were then adjusted by the method of condition equations to produce adjusted angles which gave a consistent solution for that figure. This piecemeal adjustment of parts of the network produced inconsistent co-ordinates, and computations between stations in different adjusted figures revealed large discrepancies between measured and computed values. In some instances it was difficult to isolate the source of the discrepancies. For example, the errors that existed in the main Minna-Udi chain could not be resolved until in 1963 when re-observation and re-arrangement of the junction figure at Lokoja solved the problem. Consequently in the 1955 adjustment any figures or chain (such as the Udi-Minna chain) which contained significant errors were excluded from the adjustment. The 1968 adjustment (usually referred to as "preliminary adjustment of the network") was made by the U.S. Corps of Engineers. This adjustment involved the whole triangulation network but excluded all the traverses, or any of the recent scale and azimuth check observations within the framework. The 1977 (also the latest) adjustment involved the whole of the triangulation plus some traverses and all the recent scale and azimuth check observations. This is the first adjustment so far that has produced consistent set of adjusted co-ordinates for the whole network.

The network statistics for the 1977 adjustments are as outlined below |Field 1977|:

(a) Stations

There were 515 stations in the redefined primary network. The breakdown of these stations is as follows:

Main triangulation	429
M-chain triangulation	12
12th parallel geodimeter traverse	33
EDM microwave traverse	41
total	515

The M-chain was observed between 1957-61. With the exception of the 12 stations stated above, the rest of the stations in this chain lie outside the borders of Nigeria. The twelve stations - four in the extreme South East, and the other eight further North, complete the connecting loop of the easternmost chain of the triangulation.

(b) Observations

A total of 2411 observations connect the stations of the network and are classified as follows:

Horizontal angles			2197
Measured distances			174
Invar bases	8		
Geodimeter lines	46	174	
Microwave lines	120		
Azimuths			40
Laplace azimuths	29		
Non-Laplace azimuths	11	40	
		total	2411

1.1.2 Origin of the Network and Datum

The Clarke 1880 (modified) figure is the spheroid of reference presently in use in Nigeria. The dimensions of the spheroid are as follows:

$$a = 6378\ 249.145\ m$$

$$f = 1/293.465$$

The origin of the network is located in Minna (a town situated near the centre of the country) at a station designated as L40. This station is the northern terminal of the Minna Base established in 1928. Unfortunately its height is not known. Instead the height of the southern terminal of the Minna Base is usually quoted. This height given by Calder Wood |1935| as 767.410 feet (233.96 metres) was the mean of two sets of spirit levelling between Lagos Sea Level datum and Minna, plus trigonometrical heighting from Oshogbo (where it presumably tied in to railway levelling). It is therefore worth mentioning here that there should be need for an urgent determination of the height of this datum, as the assumption of $N_o = 0$ i.e. spheroidal height = geoidal height at the origin (used in the 1977 adjustment) may not be valid. Also Field |1978| remarked that the spread of the three results of levelling used to determine the height of the southern terminal of the Minna base is 0.85 metres. Thus a check on this large discrepancy is also desirable.

The planimetric position of this datum (the origin) was established by taking the mean of astronomic values (latitude and longitude) propagated through four arms of the network. These astronomic values were obtained at four stations viz Kano, Naraguta, Lafia Beri-Beri, and Zaria. By computation these values were projected through the triangulation to give a mean value at Minna (the northern terminal). Of the above-mentioned points two lie to the east of Minna

and the other two to the north-north-east. Calder Wood |1935| observed this unfortunate situation in the following words: "a better mean value at Minna would have been obtained if it had been possible to carry up co-ordinates from the south and west also, but at the time the datum was adopted in 1928 no triangulation to the south and west of Minna was available". Thus one can conclude from the above statement that a redefinition of the Nigerian national datum is necessary.

However the effect of determining the position of the origin by computation through the triangulation was to give an origin at which assumed and astronomical values did not agree (and thus some deviation of the vertical is present - the geoid and spheroid are not coincident or parallel). The method on the other hand ensured that the definition of the origin was not dependent on only one set of observations (the conventional practice, also see § 3.1.3) which in 1928, for longitude, could not have been very reliable.

The results of the astronomical values at the five points are as follows |Calder Wood, 1935 p. 392|:

Station		Latitude (N)	Longitude (E)
N1	(North end of Naraguta Base)	09° 56' 38".0	08° 53' 38".10
N26	(Lafia Beri-Beri)	08° 47' 45".1	07° 23' 40".50
N144	(Zaria)	11° 05' 32".1	07° 39' 33".45
K2	(South end of Kano Base)	11° 59' 34".1	08° 32' 47".25
L40	(North end of Minna Base)	09° 38' 11".7	06° 30' 53".85

(Table 1a)

The values of Latitude and Longitude projected to the North terminal of the Minna base are as follows:

Station	Latitude (N)	Longitude (E)
Observed at Minna L40	9° 38'11".700	6° 30' 53".850
From Kano K2	07".278	31' 00".278
From Naraguta N1	12".768	31' 00.370
From Lafia Beri-Beri N26	03".940	30' 56".713
From Zaria N144	08".665	31' 02".580
Mean	9° 38'08".87	6° 30' 58".76
Adopted values	9° 38'09".0 N	6° 30' 59".0 E

(Table 1b)

Thus by comparison of the geodetic (assumed) and observed (1928) astronomical values of L40, one obtains the values for the components of the deviation of the vertical and azimuth correction (Laplace) at the origin as

ξ = +2".70 i.e. Astro lat (Φ) - Geod lat (ϕ)

η = -5".08 i.e. Astro long (Λ) - Geod long (λ)) cos ϕ

Laplace correction = -0".86 i.e. η tan ϕ

It is to be noted that the respective stations used to derive the position of the origin consisted of triangulation which had then not been "properly" adjusted, and so "provisional" values were used. After the adjustment of 1955, the following results were obtained for comparison between the observed astronomical values at the four stations and the values derived through the adjusted triangulation (presumably after applying the necessary corrections to the geodetic co-ordinates to make them astronomical) |Stamers Smith and Sonola, 1955|:

Stations	Latitude (N)		Longitude (E)	
	Observed	Derived	Observed	Derived
Naraguta Base N1	9° 56'38".0	33".68*	08° 53' 38".10	36".62
Lafia Beri-Beri N26	8° 49'45".1	49".56*	08° 23' 40".50	42".65*
Zaria N144	11° 05'32".1	31".10	07° 39' 33".45	30".57
Kano Base K2	11° 59'34".1	34".24	08° 32' 47".25	46".87

(Table 1c)

The above table shows fairly close agreement between the observed and the "corrected" derived co-ordinates. But in the cases marked with asterisks, the differences seem to be larger than one would expect to find either in the astronomical observations or to have been generated in the chains of triangulation (supposedly properly adjusted). However evidence provided by a number of astronomical stations (1953-1954) which had been fixed to an alleged accuracy of ±1 arc second, indicated a systematic difference of +2 arc seconds of longitude when compared with the values of the triangulation. Also the chains connecting Ukehe with Minna on the west, and Lafia on the east were not included in the 1955 adjustment because the cause of the difference in the order of 10 arc seconds between the observed azimuth at Ukehe base and those carried forward through the connecting chains had not been resolved. Consequently Smith and Sonola |1955| suggested that too much reliance should not be placed upon the above comparison. It was therefore recommended that the original values should stand and that the comparisons should be regarded only as indicative that the geodetic positions of the four stations were not in serious discordance with the observed astronomical positions elsewhere throughout the system.

Also two of the stations (namely Zaria N102 and Lafia Beri-Beri N26) used in deriving the Minna datum are no longer part of the primary framework. They have been re-classed as secondary stations.

Furthermore astronomical observations in 1967/68 at station N102 10 kms North of the Origin, throw doubt on the earlier observations, particularly the longitude. Computing from these, the components of the deviations of the vertical at origin are |Field 1978|:

$$\xi = +0''.74$$
$$\eta = -1''.43$$

Thus the definition of the National datum - the Minna Datum could be summarised as below:

Station:	L40 Minna Base (northern terminal)		
Latitude:	09^{o} 38' 09" N		
Longitude:	06^{o} 30' 59" E		
Geoid height N_o:	0		
Deflections:	in meridian ξ = +2".70	)	
	in prime vertical η = -5".08	)	1928
	in meridian ξ = +0.74	)	
	in prime vertical η = -1".43	)	1968

Unfortunately irrespective of the way the origin was defined and the discrepancies existing among all the checks made so far, for the co-ordinates at the origin, the 1928 definition of the Minna Datum still stands for the definition of the Nigerian national datum.

It is therefore necessary to make another independent check on the deflections of the vertical in order to resolve the above anomalies. Also an investigation of the fit of the Clarke 1880 spheroid is necessary with a view to replacing it with a more suitable spheroid and the accompanying transformation of the already adjusted co-ordinates. The more suitable spheroid to adopt should be the one that has a wider application.

1.2 Stages of the Network Development

The Nigerian horizontal framework, as pointed out earlier, has been built up to the present level of fairly good standard by a series of surveyors. The work to bring it to this level falls into three separate periods each with a different emphasis but also to a large extent making significant contributions to the whole. Brief outline of the stages of this development is given here. More information can be obtained in |Field, 1977|.

1.2.1 Network before 1931

Work on triangulation schemes started in 1910 and at the end of 1915 observations were made in five areas of the country viz:

Abeokuta)	These were (and are still) thickly
Kano)	populated agricultural areas.
Bauchi	This was very rich in tin mining.
Udi	It has since become the largest coalfield in Nigeria.
Udi/Port Harcourt	This was (and is still) an important railway route in Nigeria.

At this stage of development the survey work was highly localised and the concept of a national network was in no way visualised. The schemes were mainly designed to produce fixed points for the control of topographical maps. The work was not of a primary standard, but was geared to the needs of the planetablers for suitable points in the shortest possible time. Observations were made with the 6-inch micrometer theodolites made by Troughton and Simms. The targets were opaque beacons either in the form of single quadripod of squared timber or pole and dart signals. Triangular misclosures were high, averaging 5" but often very large values ranging between 8" - 12" featured |Calder Wood, 1931|.

The records of this work are very scanty, and although the work was not of primary standard two of the four bases (Eruwa and Naraguta) measured between 1910 and 1912 by the Royal Engineers Party still form part of the present network. However they were re-computed and checked in 1932 |Calder Wood, 1935, p. 391|. The other two bases Udi and Kano were discarded after later measurements, because of their poor accuracy. In all these base measurements, invar tapes or wires were used, and they were standardised at the NPL England or at Sevres, France. The measured lines were short, and therefore complex base extension nets had to be established to bring the scale up to the main triangulation. This resulted in poor accuracy of some of the bases. The Kano base which had a large and complex base net of doubtful strength was replaced by the Chafe base (only 180 km. away from Kano) established in 1935. The Udi base which was another complex figure created some computational problems and so had to give way for a nearby main triangulation side U12 - U34. This "side length" was measured by a precise traverse.

Between 1914 and 1923 work was stopped on the scheme because of the First World War. However after the war work was resumed and this time a decision was taken for the first time to establish a national triangulation system. The scheme was designed to form a grid over those parts of the country suitable for triangulation, and in particular it should be uninfluenced by local mapping requirements. Work started on this postulated framework in 1923 and progressed for four observing seasons until 1926 when the position was as follows |Calder Wood, 1931; Field, 1977|.

(A) Northern area

(i) 98 stations between Kano and Naraguta (Jos) were established and observed.

(ii) Bases at Kano and Naraguta were measured with invar tapes.

(iii) Azimuths were also observed along the base lines.

(B) Southern area

A total of 42 stations were established in two widely separated areas around the bases of Eruwa and Udi, with azimuths observed along the baselines.

(C) Station Marks

During this period the ground marks varied in design and size. Wherever possible, they were sited on solid rock in which a hole about six inches deep had been drilled and then filled with cement to hold a centre-mark consisting variously of a brass screw, cartridge case or sparklet bulb. Surrounding the centre mark was a broad ring of cement mortar of varied diameter forming a protection to the mark itself. On earth sites cement blocks two feet square and one foot deep were buried to a depth of eighteen inches. In every case reference marks were buried near the station and referred to the centre mark by bearing and distance. All stations had their names and numbers impressed into the cement with a die.

(D) Computation

The work was computed semi-rigorously and was based on Clarke 1858 spheroid.

In 1927 the methods of executing the work was made more stringent for improved precision, greater speed and permanency of the station marks. Owing to the weather conditions, the Schreiber method of observing horizontal angles was introduced to replace the method of rounds. During the observations signals were observed in groups of two, three, or four depending on conditions of visibility. The observations of more than two signals in a round were made to economize time.

This method of angular observation has significant advantages for primary work (triangulation) in Nigeria, particularly for long lines, because the weather changes rapidly during the observing season. The observing season falls between the months of February and May, and this period is usually characterised with heavy winds, dust and thunderstorms.

From 1927 onwards, better methods of monumenting were introduced. The stations on solid rock were much the same as those already described except that a brass rod with "centre-punch mark replaced the older form of centre mark. In soil, a pre-cast reinforced concrete pillar four feet long and one foot square was sunk to a depth of not less than three feet. Reference marks were established and station marks inscribed as before. An improved design of double quadripod beacon replaced the old single quadripod or pole and dart signals (the pole and dart signals are liable to give large errors if the pole is not vertical). Also luminous signals were introduced within this period, and were used for about 85% of the new work which was executed between 1927 and 1954 |Smith and Sonola, 1955|. Five inch heliotropes were employed and were attended by ex-signallers of the Nigeria Regiment or the more intelligent labourers drawn from the Survey labour force.

A common origin for the country was established in 1928 at Minna. The least squares adjustment method was introduced for all the computations and Clarke 1880 (modified) spheroid was adopted.

Work started on the measurement of five bases - Minna, Rijau, Chafe, Yola and Ilorin in 1928 and the Guillaume and Carpentier apparatus |Clarke Vol. II, 1963, pp. 191-207| was used for these measurements. Also astronomical observations were made at or near the baselines at the same time as the measurements of distances were made.

In 1930 it was possible to compute through the triangulation the positions of the stations between three of the bases. The misclosure after the computation is as follows |Calder Wood, 1931; Field, 1977|.

Line	Misclosure		Distance
	Azimuth	Distance	(approx)
Naraguta-Minna	2".27	0.63 m	342 km
Minna-Eruwa	10".01	1.07 m	441 km
Kano-Minna	5".74	0.01 m	405 km

(Table 1d)

At this time the Udi-Minna series had not been completed due to difficulties in running the chain through very heavily forested low-lying country between Udi and Lokoja. This problem was finally solved by establishing two stations on two very high trees (about 55 m and 36 m in height respectively) near Awka.

Practically the whole work accomplished up to 1930 was not of a quality and precision to be classed as primary triangulation. Much of it has now been downgraded to secondary, or in some cases

discarded. The old Minna-Pategi triangulation chain is still rated as primary, though it is of a lower standard than later work. Certain stations, and even figures, in the Minna-Udi series south of Lokoja are also retained without re-observation, though the tree stations no longer form part of the primary triangulation |Field, 1977|.

1.2.2 Network from 1931 - 1960

In 1931, the whole of the existing triangulation scheme was re-organised to create a truly national framework with sound accuracy and uniform standards. This wise and carefully thought-out venture originated from the then Surveyor General of Nigeria, Captain J. Calder Wood. He attributed the low standard of work inherent in the scheme to a number of factors viz old and worn instruments, asymmetrical and unsuitable signals, inexperienced personnel, and the piecemeal way the triangulation had been built up from local schemes designed mainly for topographical control.

The new plan made significant changes in some aspects of the work. And in order to standardise field techniques, some series of technical instructions (these were later published as technical manuals such as Anon |1936|) were introduced. An average triangular misclosure of 1".0, with a maximum of 3".67 was recorded for observations over the network. The "classical" methods of triangulation were used wherever topography permitted, with invar tape bases and azimuth control placed at infrequent intervals. Re-observation of the accepted old points was concurrently carried out with the reconnaissance, beaconing and observation of new chains. In areas where the low lying or heavily forested nature of the country made triangulation impracticable, taped traverses (standard traverses) were substituted.

The Wild (Geodetic models) theodolites were introduced for the angular and azimuth observations to supersede the Troughton and Simms 6 inch and 8 inch micrometer theodolites. With the exception of few figures around the Kano base where the 8 inch micrometer theodolite was used, all the subsequent observations were made with the Wild instruments. The new instrument brought about a remarkable progress on the work in the form of improved precision and speed. The use of the heliotropes as signals was intensified and much effort was made in training helio operators and other survey personnel.

The standard traverses were limited to those parts of southern Nigeria where triangulation was difficult. Some of the traverses formed closed loops while others were connected between triangulation stations. The equipment and methods used for these traverses were described in detail by Calder Wood |1936 pp. 5-12|. For most of the traverses the three-tripod system was adopted and the 1-second theodolites were used for the angular measurements. Two rounds of horizontal angles were observed, and bearings were controlled by astronomical azimuths observed every 20-25 stations. The distance measurements were made in catenary with steel tapes. These tapes were frequently compared with standard tapes whose length had been certified by the NPL England to a precision of 1/100 000. The linear misclosures of these traverses either on themselves or on triangulation network varied from 1/65 000 upwards over distances of 80 km. and more |Calder Wood, 1935|. The main drawback of these traverses was that the stations were close to the road and thus often presented problems of re-establishment. Also their connections to the main triangulation network are in several places via stations observed before 1931, which are no longer classified as primary. The standard traverses were therefore not included in the 1977 adjustment |Field, 1977|.

As was the practice before the main period of triangulation (1931 - 60), astronomical observations were made at or near the baselines concurrently with the distance measurement. The method of hour angle of East and West stars (near the Prime Vertical) was used for determining the azimuth. At least 10 pairs of stars were combined to give the accepted mean value. Latitudes were usually observed, but longitudes were observed at very few stations. The instrument used for the observations was either the 8 inch micrometer or the Wild T-3 theodolite |Field, 1977|.

Vertical angles were measured at every triangulation station either to Heliotropes or to particular points on the opaque signals. The observations were made during hours of minimum refraction (1200 - 1500 hours), and comprised not less than two rounds of vertical angles at each station.

The effect of substituting a properly designed system for the inherently defective scheme of the pre-1931 era can be seen from the summary of the work done within the periods 1931 - 1935 and 1935 - 1955.

(A) For the period 1931-35, the following work was accomplished (also see fig. 1b) |Calder Wood, 1935|

(a) Old chains re-arranged, re-observed and, where *necessary, re-pillared:*

(i) Minna-Naraguta

(ii) Naraguta-Kano (part only re-observed)

(iii) Kano-Chafe

(iv) Chafe-Minna

(v) Minna-Udi (North of junction with Ilorin-Eruwa chain)

The base nets at Minna, Naraguta and Kano were re-observed and strengthened by additional rays.

(b) New Chains

(i) Minna-Udi (south of junction with Ilorin-Eruwa chain)

(ii) Birnin Gwari-Naraguta

(iii) Ilorin-Rijau (awaits completion of Ilorin base for final closure)

(iv) Rijau-Chafe

(v) Rijau-Kwomgoma (junction with Minna-Chafe chain)

(vi) Bauchi-Yola (half observed only)

(vii) Rijau and Chafe base nets.

(c) New Bases

(i) Rijau, including azimuth determination

(ii) Chafe, also including azimuth determination.

Thus by 1935 there were seven bases within the network viz Eruwa, Kano, Naraguta, Udi, Minna, Rijau and Chafe.

(d) New azimuth determination

(i) Kano base

(ii) Udi base

Also an azimuth was observed on the line N100 - N111 on the junction of the chains from the Minna, Naraguta, Chafe and Rijau bases.

(B) For the period 1936-1955 the new work accomplished is listed below. Also see fig. 1c for the diagram of the completed work and the projected plan for the network |Smith and Sonola, 1955|.

(a) Bases

Two new bases were established : Yola and Ilorin bases. A new base at Rijau was established to replace the old one and another new base at Ukehe was substituted for the old Udi base. All these bases were measured in 1937, 1939 and 1950 respectively.

(b) Azimuths

Azimuths were observed (or re-observed as the case might be) at every base which was measured prior to 1935 and also at the bases at Eruwa, Ilorin, Yola, Rijau and on the side U12 - U13 of the old Udi base extension.

(c) Old chains re-arranged and re-observed

(i) Naraguta - Chafe

The shortness of the Kano-Chafe section of the Naraguta-Chafe chain, and the low standard of the Kano base resulted in the exclusion of the Kano base and its extension from the scheme. The series of stations between the Naraguta base and the Chafe base were made to constitute one chain.

(ii) Minna - Ilorin

The whole chain was re-arranged and re-observed after the completion of the Ilorin base in order to strengthen the triangulation frame-work of the southern part of Nigeria.

(iii) Ilorin - Eruwa

The old Ilorin chain was re-observed in 1937 to bring it to first order standard. Also the azimuth on the Eruwa base was re-observed in 1946.

(iv) New chains

New chains were reconnoitred, beaconed and observed between Yola-Rop, Yola-Bauchi and Udi-Takum, New chains between Yola-Takum were reconnoitred but not beaconed.

It is to be remarked that progress in the work was interrupted by the Second World War. Also after 1945 the completion of the network on schedule was delayed by shortage of trained staff and administrative changes which regionalised the Nigerian Surveys. However by 1960 the network had attained its present shape apart from minor modification of the junction figure at Lokoja, which was completed in 1964.

1.2.3 Network after 1960

By this period the desired framework has been achieved and it has been possible to explore means of improving its accuracy. This latter consideration has been made possible by the use of newly developed instruments. Some of the new work carried out in the network include :

(A) A programme of scale and orientation checks was planned for improving the strength of the network. These observations were mainly limited to chain junctions (see fig. 1d). The distances were to be measured along the sides of 19 triangles using microwave Tellurometers. One triangle had

already been measured near Nkanbe in 1957, so this would give 60 distances at strategic points. The scale check observations were completed on 17 triangles and the average misclosure between measurements in opposite directions was 2.32 ppm |Field, 1977|.

The azimuth checks were also planned for observation at 19 points sited, like the distances, at chain junctions. Observation pillars suitable for the Wild T-4 theodolites were constructed at these stations. A detailed specification for observation was drawn up. Azimuths and latitudes were to be observed at all the stations, but longitudes at only a selected few. However due to lack of suitable staff for observing, the number of stations was curtailed and by 1968 only nine of the stations had been observed of which five were Laplace stations. The deflection values computed from the five stations are as follows:

Station	Latitude(N) (approx)	Longitude(E) (approx)	ξ	η
R26	13^{o} 14'	5^{o} 09'	-2".49	- 1".00
D17	10^{o} 46'	4^{o} 34'	+3".20	- 6".39
N114	13^{o} 14'	5^{o} 09'	-1".27	+ 1".83
N10/N2	09^{o} 47'	8^{o} 53'	+1".24	+14".54
N102	09^{o} 38'	6^{o} 33'	+0".76	- 1".44

(Table 1e)

The above few results immediately suggest that the deviation of the vertical varies significantly over the country; and one would rightly imagine that the geoid in the country varies likewise. However this prediction can only be confirmed by accurate determination of the geoid.

(B) Primary traverses, known as Cadastral Framework (CF) traverses and designated CFA to CFO have been planned for inclusion in the network. Some of these traverses have actually been completed. They are generally run and adjusted in between the primary triangulation stations. The Wild T-3 (geodetic) theodolite and the Kern DKM3 have remained as the main instruments for angular observations in these traverses. Also in some of these traverses (the CFM for example) Bilby Towers are employed for all the measurements, and traverse legs range from 12 km. - 19 km.

(C) Tellurometer traverses have been established by various organisations engaged in mapping different parts of the country. Most of the stations are often sited between the main chains of the triangulation network. Although the angular measurements in these traverses are not of primary standard, their distances can act as useful checks for the network.

(D) Between 1968 and 1971 the U.S. Corps of Engineers in collaboration with the Nigerian Federal Surveys established the part of the African 12th Parallel traverse across the north of Nigeria. This first order geodimeter traverse is connected to the main triangulation network at eight points, all of which are west of longitude 10° 25' E. Laplace azimuths were observed on this traverse at alternate stations. The stations have been designated CFL, and there are altogether 100 of such stations (CFL1 - CFL100) in the country.

(E) The latest adjustment viz the 1977 adjustment produced the first consistent set of co-ordinates for the primary triangulation and traverse network of Nigeria. The adjustment which was the first adjustment of the scheme as one block, was based on the geoid, thus assuming it to be coincident with the spheroid. The effect of this geometric approach on the network is discussed in certain chapters of this thesis. The spheroid of reference is Clarke 1880 (modified) figure.

The variance-covariance analysis of the network after the adjustment gave an overall strength of 0".8 in azimuth and 4 ppm in distance [Field, 1977]. However systematic errors are not included in this form of analysis (see also chapter 4). The m.s.e. of a unit weight observation was computed as 1.367, indicating that the weight estimation used for the adjustment was substantially correct. Also calculation of the product of the weighted residuals and the coefficients of the observation equations (A^TWv) gave a maximum value of 1.02×10^{-9}, indicating that rounding off errors in the computation were insignificant.

1.3 Other Survey Work of Geodetic Application within the Network

Apart from the triangulation/traverse network, some other survey data of geodetic application are available within the framework viz levelling, gravity and satellite-Doppler surveying. And as is the case with the triangulation/traverse data, more work is still being done to improve on the amount of existing data information both in quality and quantity.

1.3.1 Levelling Surveys

The vertical framework of Nigeria comprises a dense network of primary spirit levelled heights, and also a large number of trigonometrical and barometric heights. The spirit levelling program started in the early 1930's, and has been based upon the Mean Sea Level datum at Lagos, usually referred to as "Lagos Survey Datum". This height datum was determined from observations made for an unstated period of years by an automatic tide-guage at the Eastmole Lagos. The tide-gauge itself was established about 1912 and maintained by the Port Engineers. And by 1935, though the results of the observations had not been fully analysed, there were strong indications that this datum was not exactly coincident with the mean sea level. An error in the range of a fraction of a foot was actually envisaged [Calder Wood, 1935]. The zero of the tide-gauge was adjusted to Low Water Ordinary Spring Tides and connected (or tied) to a nearby survey bench mark.

Some efforts were made recently to determine the position of the Lagos Survey Datum with respect to mean sea level [Fajemirokun and Udoffa, 1978]. Analysis of the hourly means of tidal observations obtained from the Eastmole Tide Guage for a short period of five years (1967-1972) enabled a new Mean Sea Level (provisional) in the coastal areas of Lagos to be established. This result together with some precise levelling data gave some indications of the relationship between the Lagos Datum and the Mean Sea Level. However the results should be regarded as provisional values since the period of the data information used in the analysis is much less than the required 18.6 years. More work is still being done to use longer periods of observations.

The levelling work done before 1955 in Nigeria was of a limited amount and of such a low order of precision that it should not be described as "precise levelling" [Smith and Sonola, 1955]. A large programme of precise levelling was started early in 1955, and has continued to date. Detailed specifications on the field techniques, instrumentation and accuracy standards have been carefully drawn up to provide a uniform standard.

The adjustment of the levelling network is presently in progress. However due to lack of gravity data information along the levelling routes, the adjustment is being made with the value of standard gravity.

1.3.2 Gravity Surveys

There is not much published work on the gravity network of the country. A brief discussion on this subject is stated below. It is the result of private correspondence and discussion with Verheijen [1980].

A few gravity surveys are known to have been carried out in the country. They were made for various purposes such as geophysical and geological studies. Until 1978 there had not been a country wide first order or second order network to which all these previous surveys could be connected. This was due to many reasons mainly financial and instrumentation as such a network would involve considerable air and ground travel. Also only a few years ago was there a single Lacoste-Romberg gravimeter in the country [Ajakaiye and Verheigen, 1978].

The Physics Department, Ahmadu Bellow University, Zaria, in collaboration with the Federal Surveys, Lagos has recently embarked on the programme of establishing a nation-wide coverage of first and second order gravity network. Also the University of Lagos

The gravity observations campaign has been planned to be executed both by air and ground measurements. However due to limitations of airstrips, the number of stations to be linked by air-ties has been curtailed to forty-two. Three Lacoste-Romberg gravimeters model G have been used for the project.

The extent of coverage is not yet known as most of the records of observations and the results of the computations (if any) have not yet been released.

1.3.3 Satellite-Doppler Surveys

The Satellite-Doppler Positioning technique was introduced into the country in 1976. It is a joint venture between the Federal Surveys and the University of Lagos, under a program named UNIDOP. For the UNIDOP programme a total of 24 stations well distributed over the entire Nigerian network, have been chosen for Doppler observations (see fig. 1e). Five of these stations are stations of the CF traverses, three are located on the common points of CFL and triangulation stations, and the remaining sixteen are triangulation stations. The first observation campaign was made in 1976, and this was immediately followed by a second campaign in 1977 |Fajemirokun et al 1978|.

By the end of 1978 only 13 of the 24 stations had been occupied/observed. The observations were carried out in the translocation mode with 2 JMR1 instruments (Doppler receivers). It is to be remarked that in the translocation mode, the orbital and refraction errors are correlated for station separation less than the satellite height (the orbits of the five NNSS satellites are at altitudes of approximately 1100 km). Thus stations tracking the same satellite simultaneously are affected by the same error sources, which are transferred eventually to the position co-ordinates of these stations (assuming that these errors are not precisely modelled).

Consequently by determining the relative co-ordinates of the stations, a higher accuracy than the co-ordinates of each station alone is achieved. This is obviously the reason for the use of this method of data acquisition for the UNIDOP programme.

By the end of 1979, results of only five of the thirteen stations had been processed. Computations were made with the five doppler results assuming coincidence of the reference spheroid and the mean sea level of N102 (the station 10 km. north of Origin - Minna datum). The satellite-derived co-ordinates of the other four stations were then computed on the Minna datum using Precise Ephemeris data and then compared with their equivalent triangulation values |Obenson and Fajemirokun, 1979|. The results of these comparisons are given in Table 1f below. The Table shows the differences between the doppler-derived geodetic co-ordinates transformed to the Minna datum and the triangulation values of these stations.

Table 1f

Differences : Transformed Doppler minus Triangulation

Station	$\Delta\phi$	$\Delta\lambda$	Δh
N102	0	0	0
N10	-0".818(-24m)	-6".477(-195m)	17.12 m
U12	1".089(30m)	-0".143(- 4m)	4.97 m
PGS7	-	-	2.11 m
T4	-	-	-18.0 m

Obenson and Fajemirokun made the following remarks about the results of Table 1f "The differences between these two sets of values (Doppler and Triangulation) should generally be due to random errors in both doppler and triangulation values. N10 and U12 may not represent the situations in the whole network but the large differences do indicate that the triangulation co-ordinates of these two stations need to be re-checked."

Admittedly there is some amount of distortion in the triangulation network but the disturbing discrepancies shown in the Table are too large to have originated from the triangulation results alone. It would therefore be advisable to re-check the transformation and reduction of the Doppler results.

Also in the transformation of the CFL traverse between the Adindan and Minna datums, it was discovered that the latitude values of CFL44 to CFL100 were all in error by about 20" each |Obenson and Fajemirokun, 1979|. These discrepancies should be checked and the error remedied, since the CFM, N and O traverses are somehow tied to the CFL traverse. It is worth noting that the original computation of the CFL traverse was based on the Adindan datum in the Sudan and the spheroid was the Clarke 1880 (modified) figure |DMA, 1973|. The portion of the traverse within Nigeria was in 1977 re-computed on the Minna datum.

There have also been several other Doppler observations for other purposes such as for control extension and for location of telecommunications and navigation installations.

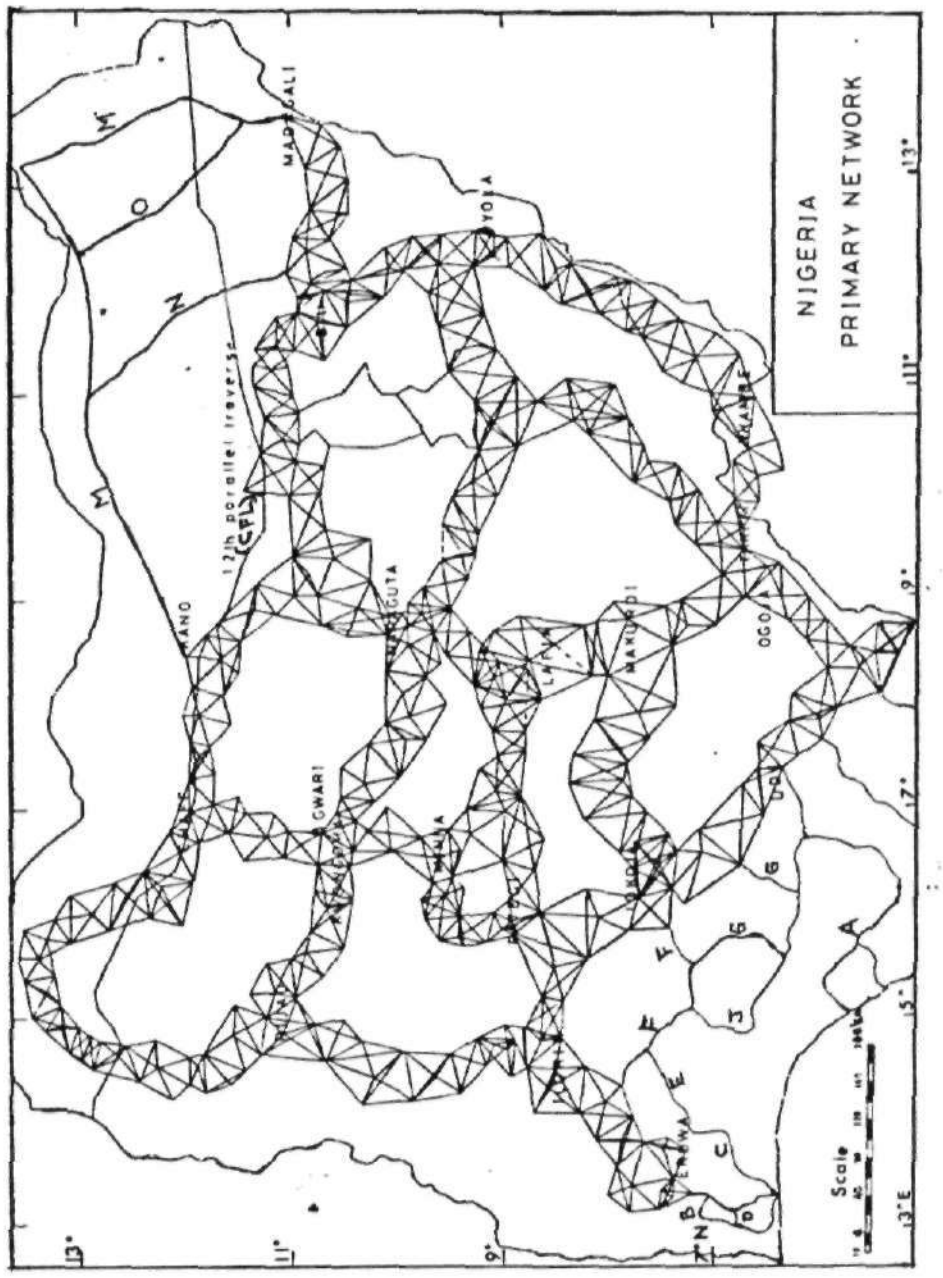

NIGERIA
PRIMARY NETWORK
MAIDUGURI
YOLA
KANO
ZARIA
ILORIN
ENUGU
OGOJA
Scale

FIGURE 1a



FIGURE 1b

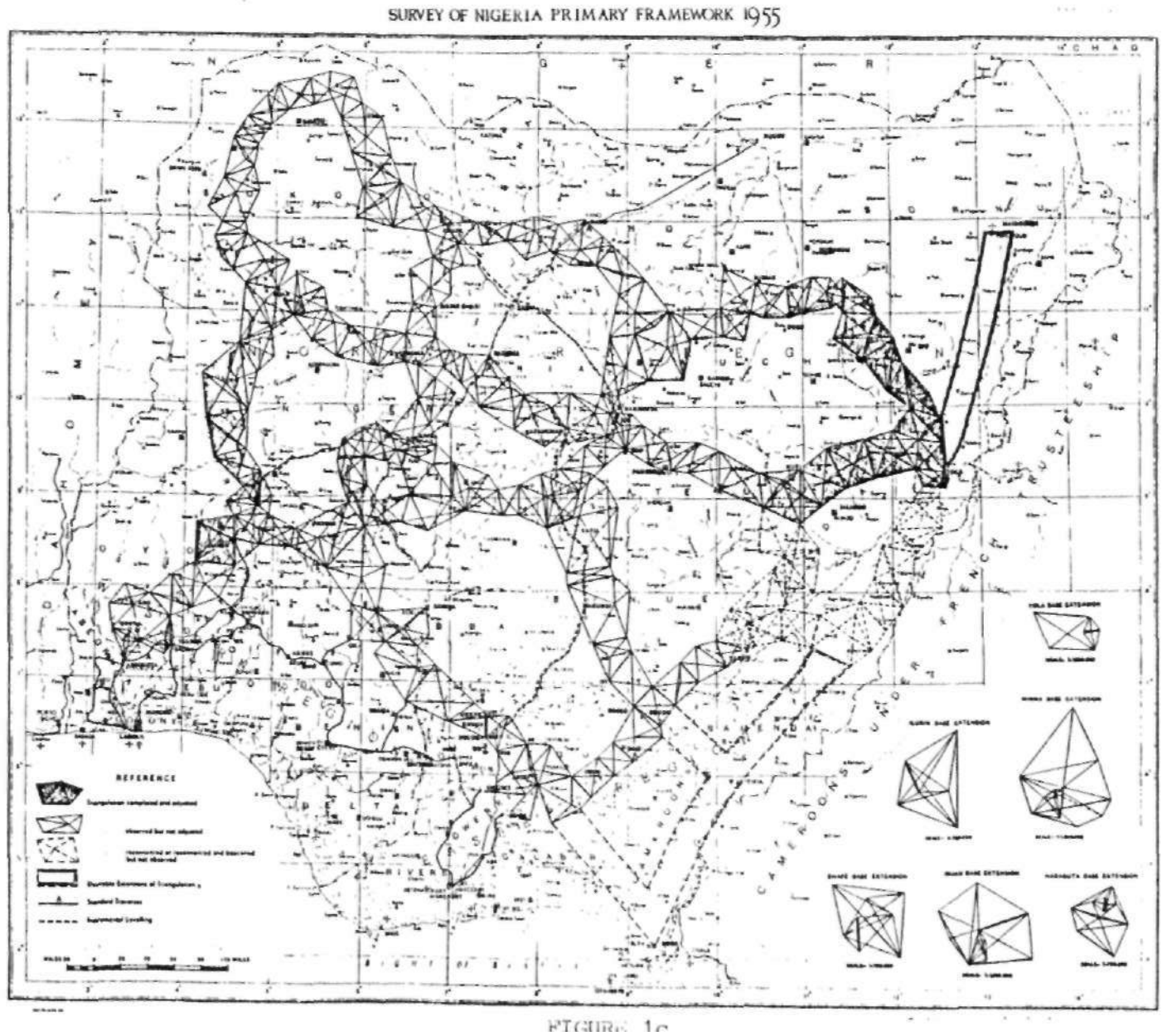


FIGURE 1c

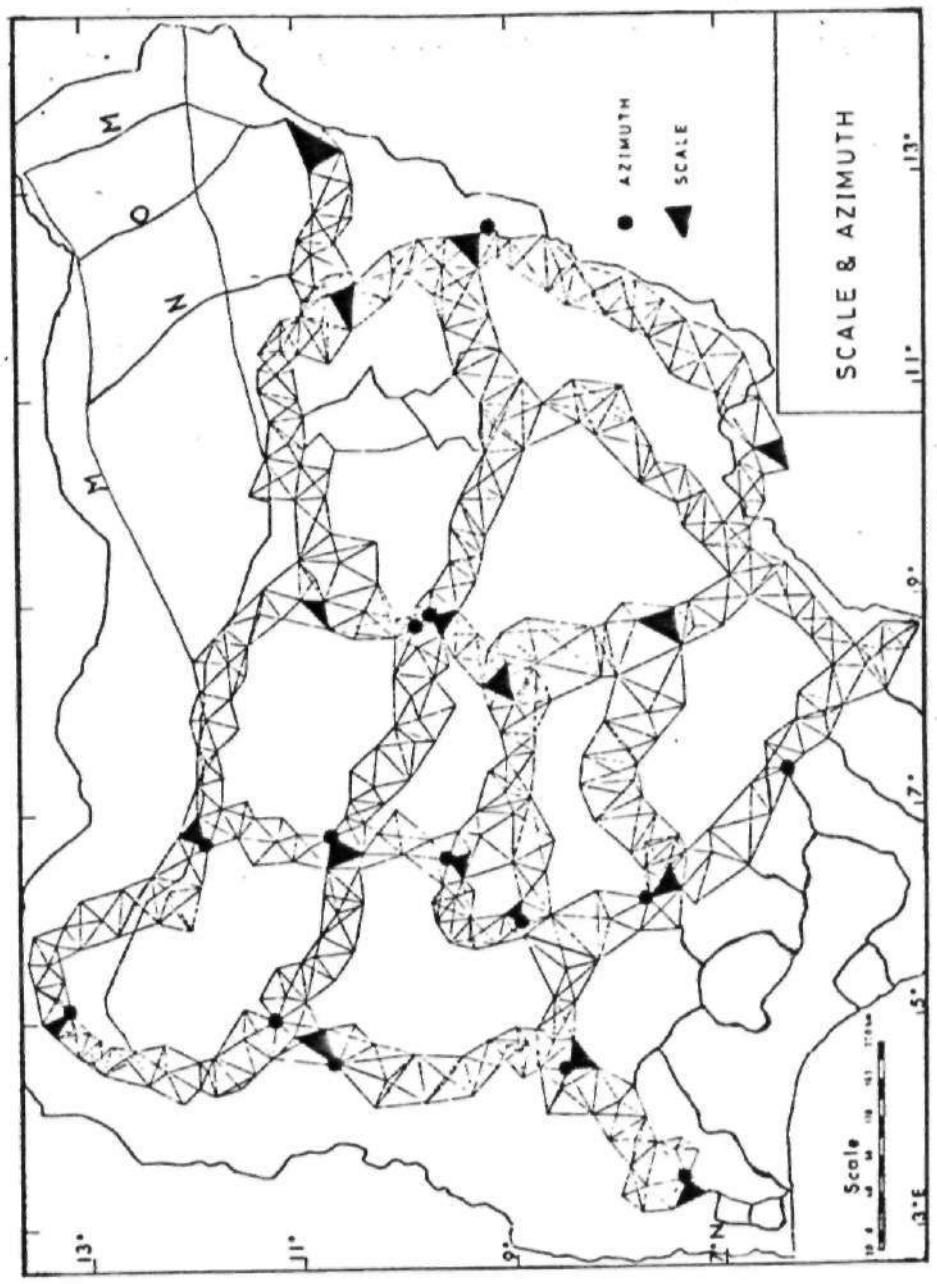


FIGURE 1d

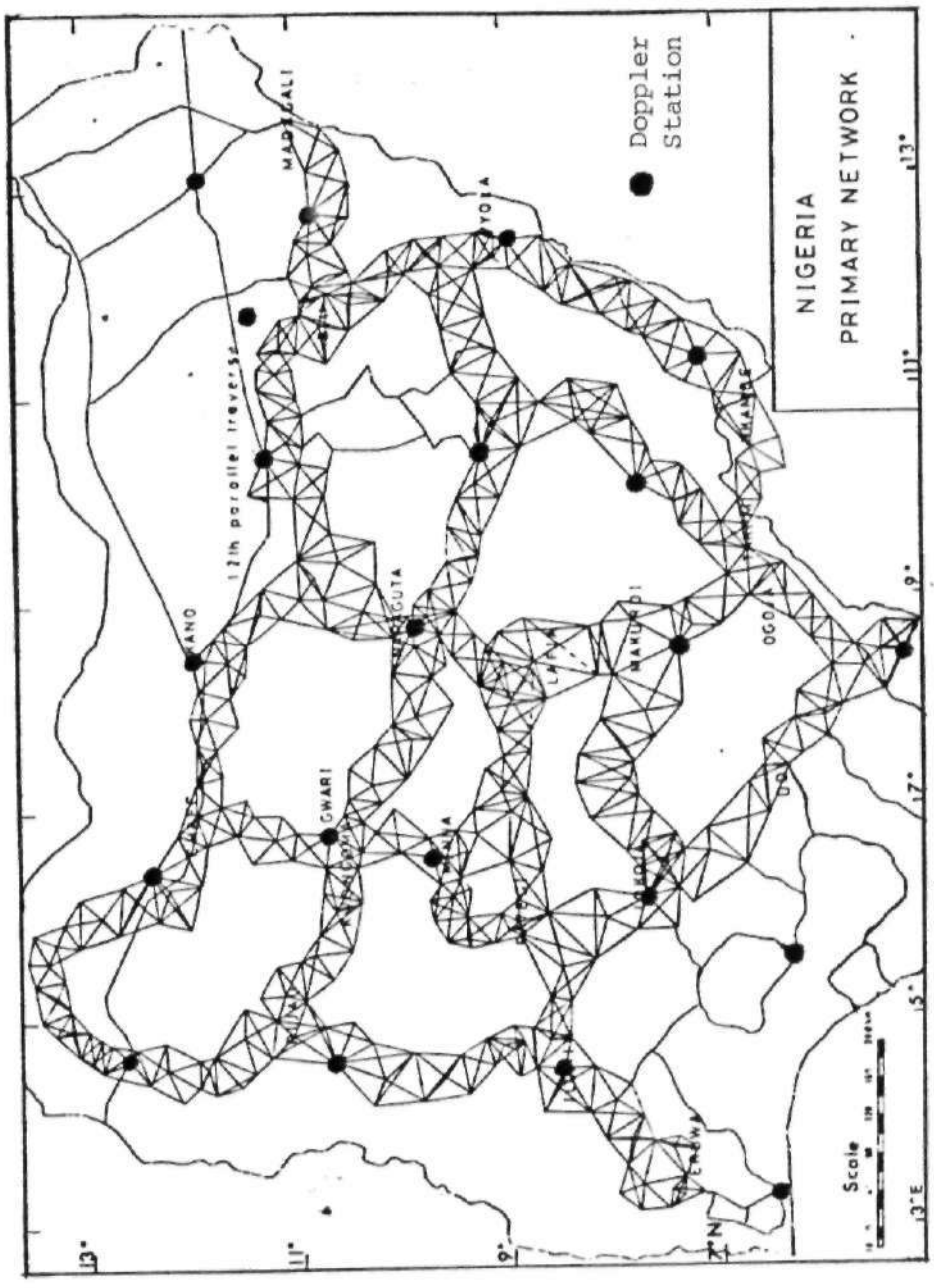

NIGERIA
PRIMARY NETWORK
Doppler Station
MAIDUGURI
YOLA
KANO
GWARI
MAKURDI
OGOJA
Scale
13°
11°
9°
7°N
13°E
15°
17°

FIGURE 1e

CHAPTER TWO

DEFECT OF THE NETWORK AND POSSIBLE REMEDIES

2.1 Introductory Remarks

On the basis of evidence available from the history of the Nigerian network, it is likely that the network is distorted by significant amounts both in scale and orientation. Consequently adequate steps should be taken to remedy such defects by providing the necessary information lacking in the network and by finding ways of constraining any other possible errors that are not immediately detectable.

The main information lacking in the network is the geoid figure in Nigeria and this has resulted in inaccurate reduction of measured distances. The other important information lacking in the network is the parameters for computing the deviation of the vertical. This in turn has led to the assumption that since the topography of the country is generally gentle, the deviations of the vertical over the country are too small to have any significant adverse effect on the network. The methods of providing all the above information, together with the methods for accurate reductions of observations, are described in chapter 3.

The other not easily detectable defect is the observational errors - the unknown systematic errors. The way the network has been built up might be one of the contributory factors to this defect. Also the rejection and repetition of observations (a practice which was common during the stages of the network development) invariably led to the over selection of the observations. Admittedly the over selection improved the triangular misclosures and minimised the scale and azimuth misclosures, but it is just

possible that this was done at the expense of biassing the sample of observations. It is worth noting that the Minna - Pategi chain which had been downgraded to secondary status during the 1955 adjustment, was accepted later to be of primary status and was therefore included in the 1977 adjustment |Ashkenazi and Field, 1979|. The effect of these observational errors on the network, could be minimised by providing adequate checks in the form of additional observations using modern instrumentation and techniques. And more recently it has been shown |Ashkenazi, 1980| that computational techniques can be used to eliminate these errors. Such techniques and instrumentation are briefly described in this section of the thesis. The effect of additional observations on the network is examined by an optimisation procedure based on simulating the projected observations in a block of the network, and by computing the resulting variance-covariance matrices and related accuracy criteria. This analysis is discussed in chapter 5 of the thesis.

2.1.1 The Observations

A geodetic horizontal network can be established by trilateration, triangulation, traverse or even by a combination of these methods. These techniques merely involve the measurement of angles/directions and/or distances. The main weakness of every triangulation network is its tendency to accumulate errors of scale and azimuth, since each side derives its scale and azimuth from the preceding side. These errors are usually controlled by extra distance measurements for the scale error, and Laplace azimuths for the azimuth error. Also this increase in the number of redundant observations leads to an improvement in the reliability of the network (i.e. its ability to detect mistakes). Another technique which has recently come into effect (since 1967) is the use of Doppler-satellite

positioning. Doppler-satellite observations have been found to take care of some sources of unknown systematic errors, and so its incorporation into a network helps to improve the accuracy of the framework |Ashkenazi, V., 1974|.

2.1.2 The Scale check

The instrument that is assumed in this thesis, for the distance measurements was the Geodolite. This is a commercially available electro-optical distance measuring instrument. The basic principles upon which the instrument operates are the same as those described by Bomford |1980| for the Geodimeter. The effect of refraction which is the main draw-back on the precision of EDM instruments is taken care of in the Geodolite in two ways |Savage and Prescott, 1973|:

(a) The high-frequency fluctuations in refractivity are eliminated by signal averaging (either 1 - or 10 - sec· averaging) built into the Geodolite.

(b) The very long-term fluctuations are eliminated by taking meteorological readings at the time the distance is measured.

Provided the zero error of the instrument and its modulation frequency are frequently checked and the meteorological readings carefully recorded, the instrument can give accuracy of 1 mm ± 2 ppm |Savage and Prescott, 1973; Chrzanowski, 1977|. Methods of recording the temperature measurement which is the critical factor in estimating the velocity of light are described in |Bomford, 1980, pp. 60, 61|.

Other features of this instrument are as follows |Robertson, K.D., 1971|:

(a) It can be read with a resolution of 0.3 mm by means of a digital display.

(b) The cyclic errors are small, the largest recorded in practice is 0.6 mm.

(c) The offset or zero error can be corrected by means of a small retroreflective prism built into an arm at the front of the instrument. This arm may be raised when desired to reflect the collimated laser beam back into the Geodolite. Because the prism is at a known distance from the plumb point of the instrument, it may be used to correct accurately for the offset.

2.1.3 Azimuth Check

The projected azimuth observations are assumed to be made by Black's method. This method of azimuth determination is the same as the method of hour angle, except that the computations are done using the geodetic latitude and longitude, and each observed direction is corrected by $\zeta \tan h$, where $\zeta = (\xi \sin A - \eta \cos A)$. Thus the corrected directions are what they would have been if the theodolite had been levelled on the spheroid; and the computed azimuth is therefore geodetic and not astronomic. The method has some significant advantages over the conventional method. These advantages include among others |Robbins, 1960|:

(a) Geodetic azimuth is observed directly and no observations for longitudes are required: it thus requires half of the time of conventional methods. All observational effort is put where it is required, namely into azimuth.

(b) Maximum efficiency is obtained from the observations. Except where circumstances make it impossible, stars of a low altitude as possible ($h = 5^{\circ}$ to 20°) are used to maximise the azimuth information obtained. The amount of azimuth information obtainable from an observation of unit weight is $\cos^2 h$.

(c) The effect of any error in the striding or suspension level on the observed azimuth is itanh (i = bubble sensitivity). In Black's method, this is at maximum about i/3 (for $h \simeq 20^{\circ}$). Levels with a sensitivity of 1" or 2" per mm may have surprisingly large errors (the calibrated value of one division may be wrong by as much as 40%).

(d) The effect of any error $\delta\eta$ in the observed prime vertical deflection on the deduced geodetic azimuth is $\delta\eta\tan\phi$ (in conventional methods). There is no comparable error in Black's method. The quantity η is determined weakly from the least squares solution and errors in it are multiplied by tanh (or sinh in a weighted solution) which is about 1/3 at altitude 20°.

(e) During bad observation conditions (cloudy nights, mist or fog) all stars that are visible may be utilized; observations are not restrained to, or to be close to, the meridian. The fact that stars are observed at altitudes greater than the recommended 20° does not invalidate the method - it merely means that less azimuth information is obtained from an observation of unit weight, than otherwise would be the case.

Admittedly observation to Polaris is adequate for azimuth determination in Nigeria, but such observation is usually hampered by poor visibility conditions during the observing season which unfortunately coincides with rains. Furthermore on the southern part of the country where Laplace azimuths are very sparse |Field, 1977|, the altitude of Polaris is about 8° or less. Experience has shown that Polaris is usually obscured by tall buildings and thick vegetation in these areas. Also azimuth from Polaris like every other conventional method, needs longitude observations for the Laplace azimuth correction.

Robbins |1976| quotes a value of $\pm 0''.7$ as the best estimate for the standard errors of geodetic azimuth (including errors due to personal equation) determined by Black's method.

2.1.4 Doppler satellite positioning

The principle of satellite Doppler tracking is well documented in many geodetic literatures |Bomford, 1980; Robbins, 1978; Ashkenazi and Gough, 1976; Krakiwsky et al, 1972; Black et al, 1975| and is therefore briefly described here:

A very stable crystal oscillator is carried in the satellite which controls the satellite clock and the continuous transmission of one or more frequencies to the receiver (or receivers) at the ground station (or stations). This ground station Doppler satellite receiver (also in motion with the Earth) measures the amount by which the stable frequencies have been changed owing to the Doppler frequency shift caused by the relative velocity between the satellite and ground station. The Doppler shift is integrated over a specified time interval within the receiver - this is the so-called Doppler count. The orbital parameters together with known satellite co-ordinates, are then incorporated into a least

squares adjustment to solve for the unknown ground station co-ordinates and the unknown frequency offset.

The observed integrated Doppler cycle count N is related to various quantities by the equation

$$N = \frac{f_o}{c} \Delta r + \Delta f . \Delta t \qquad 2.1$$

where the known quantities are the velocity of light in a vacuum c, the nominal reference frequency (400 MHz) of the ground receiver f_o, and the integration time Δt. Oscillators in both the ground receiver and satellite transmitter have rates of frequency drift that are usually not precisely known but they are of the order of about one part in 10^{10} per day |Wells, 1969|. Such drift rates introduce no significant error in the assumption that f_o is constant and equal to the nominal value of 400 MHz. However, the difference between the ground station and satellite transmitter frequencies Δf, cannot usually be treated as known. Thus for each satellite pass observed from a given ground receiver, there is a frequency offset unknown. The change in range Δr (i.e. change in distance between the ground station and satellite) during the integration time Δt embodies the known co-ordinates of the satellite positions and the unknown co-ordinates of the ground station.

The relation of satellite distance difference to the terrestrial Cartesian co-ordinates of the ground station and satellite positions is shown in the figure below:

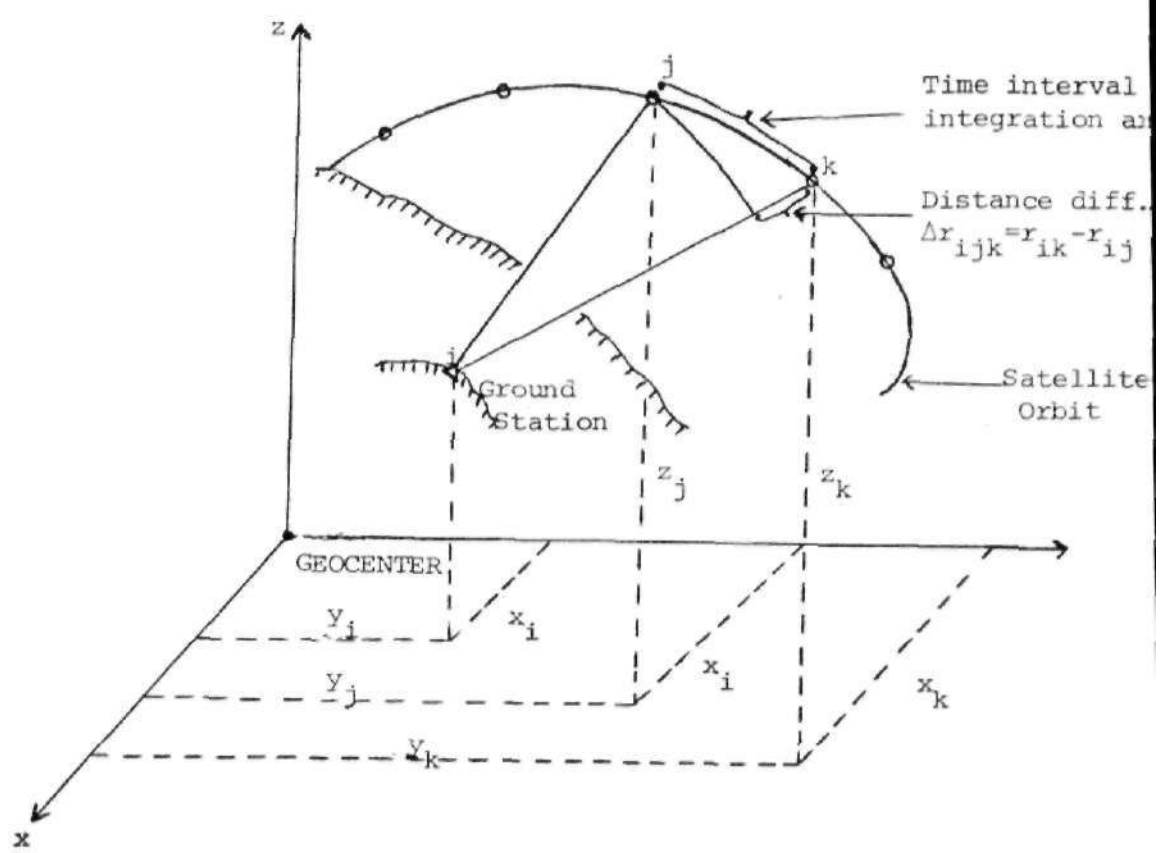


Figure 2a

The general non-linear mathematical model (Fig. 2a) which relates a *single satellite distance difference* Δr_{ijk} *to the co-*ordinates of a ground station i and the co-ordinates of two *successive positions of a moving satellite* j *and* k is |Krakiwsky and Wells, 1971|

$$\Delta\bar{r}_{ijk} = \bar{r}_{ik} - \bar{r}_{ij} \qquad 2.2$$

where in cartesian co-ordinates, the satellite distances r_{ij} and r_{ik} are:

$$r_{ij} = \left|(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2\right|^{\frac{1}{2}}$$
$$r_{ik} = \left|(x_k - x_i)^2 + (y_k - y_i)^2 + (z_k - z_i)^2\right|^{\frac{1}{2}} \qquad 2.3$$

and for Doppler satellite observations the distance difference Δr_{ijk} is, from equation 2.1

$$\Delta r_{ijk} = c/f_o \left| N_{ijk} - \Delta f . \Delta t \right| \qquad 2.4$$

By substituting equations 2.3 and 2.4 in equation 2.2 and solving for the observed integrated Doppler count N_{ijk}, the following non-linear observation equation is obtained

$$N_{ijk} = \frac{f_o}{c}\{ \left| (x_k - x_i)^2 + (y_k - y_i)^2 + (z_k - z_i)^2 - \right.$$

$$\left. \left| (x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2 \right|^{\frac{1}{2}} \right\} + \Delta f . \Delta t \qquad 2.5$$

There is one equation such as (2.5) for each Doppler count observation. Thus for many passes, tracked at a single or at many ground stations (simultaneously or otherwise), there will be many such equations. In general, many different combinations of ground and satellite co-ordinates may be considered as unknown, provided the total number of unknowns, which includes the frequency offsets, does not exceed the number of equations.

The resulting equations are usually solved by linearization using Taylor's series, and then by applying the techniques of the method of least squares to compute the unknowns. However due to the smoothing effect of the method of least squares adjustment any errors in the satellite co-ordinates are absorbed in the station co-ordinates estimates. Analysis of the effect of these errors on the derived ground station co-ordinates and the methods (computational and observational) of minimising them, are well detailed in Abbas Elhag (1981).

The determination of the ground station position within the Doppler System (NSWC 9Z-2) for the Precise Ephemeris, and WGS 72

for the Broadcast (predicted) Ephemeris is affected by several sources of errors, which can be grouped as follows [Kouba, 1976]:

(a) instrument noise (including both receiver and antenna noise)

(b) atmospheric refractions (ionospheric and tropospheric)

(c) orbit determination uncertainties

(d) method of data acquisition and reduction plus number of acceptable passes observed [Brown, 1976]

However Doppler satellite observation can at present give station positions to a sub-metre accuracy when the Precise Ephemeris is used. This can easily be testified from the remarks made by some geodesists in analysis of their observations.

Anderle [1976] noted that analysis of observations of about 20 satellite passes yielded geodetic solutions with a repeatability of 70 cm. in each co-ordinate, and these were in agreement with terrestrial measurements of inter-station distances.

Tanenbaum [1976] demonstrated that standard errors of 61 cm. and 53 cm. in ϕ (latitude) and λ (longitude) are consistent with the Doppler residuals when the azimuth and distance standard errors adopted for the terrestrial network are 0".8 and 1 ppm respectively.

Hothem et al [1978] in their work on comparison of satellite Doppler results with space systems, estimated that the resultant uncertainty for non-simultaneous Doppler satellite positioning was at the 70 cm. level if 40-pass solutions were used.

2.1.5 Scale and Azimuth Errors

All EDM instruments require regular and accurate calibrations

(zero error, cyclic error, and frequency error) before being used for field measurements (scale checks for example). Experience in Nigeria has shown that these calibrations, particularly the frequency errors, are not usually carried out; the manufacturers' specifications are often assumed to be adequate. The same practice might have been applicable to all the existing EDM distances in the national network. Also errors of systematic nature could result from difficulties in modelling atmospheric refraction, especially over very long lines. Thus these distances (apart from the systematic errors due to the non-availability of geoidal height information for reducing the measured distances to the reference spheroid) suffer from other systematic errors, and in particular, a proportional scale bias, which cannot be detected by internal consistency tests such as the Variance-Covariance Matrix (see § 4.2.3). It has also been realised that most national triangulation networks suffer also from systematic orientation errors |Ashkenazi, 1980b|.

These systematic scale and orientation errors can only be controlled and removed by using a more accurate (over long ranges) system of observations, such as satellite-Doppler positioning. However, test adjustment in which the Doppler derived positions were assigned appropriate a priori (observational) standard errors and incorporated into terrestrial observations (horizontal angles, distances and Laplace azimuths) did not provide a satisfactory solution to the problem |Ashkenazi et al 1980a|. It was therefore concluded that the overwhelming number of terrestrial observations were not being affected by the few Doppler positions, and would not be unless extremely large weights were assigned to the latter. Hence a method of modelling the unknown systematic scale and orientation parameters in a terrestrial network has been designed so that Doppler observations can effectively contribute their share in improving the strength of a network.

The method involves making some suitable modifications to the standard format of the "Variation-of-Coordinates" observation equations [Ashkenazi, 1980b]. The observation equations modified to include the unknown scale and orientation parameters, have the following format

$$\left.\begin{aligned} \delta\ell_{ij} + \ell^{c}_{ij} &= \ell^{o}_{ij}(1 + 10^{-6}s) + v \\ \delta\alpha_{ij} + \alpha^{c}_{ij} &= \alpha^{c}_{ij} + \beta + v \end{aligned}\right\} \qquad 2.6$$

where

$\delta\ell_{ij}$ and $\delta\alpha_{ij}$ are small changes in distance (metres) and direction (seconds of arc) of the line joining stations i and j, s is the scale parameter in parts-per-million, and β is the orientation parameter in seconds-of-arc.

More details on "Variation of Co-ordinates" are given in Chapter 4. Ashkenazi [1980] remarked that numerous simulation tests conducted with simple geometrical figures by using equations (2.6) showed that, in every case, scale and orientation errors which were artificially introduced into the network, were being recovered by this method, regardless of the a priori standard errors assigned to the various observed quantities.

2.1.6 The Test Network

The block of the network used for the simulation exercise lies in the north-west part of the country, and is between latitudes 10° 30' N - 13° 30' N, and longitudes 4° 15' E - 7° 15' E (see Fig. 2b). It is situated in that part of the network that has been considered relatively very accurate [Field and Ashkenazi, 1979]. The data used in the adjustment/analysis programme are in the conventional sign conventions i.e. longitude positive eastwards,

and azimuth reckoned clockwise from the north. It has also been assumed that the a priori standard errors that were assigned to the observed data in the 1977 adjustment are substantially correct since the mean standard error (m.s.e.) of a unit weight observation is 1.367.

The approximate co-ordinates used are from the 1977 adjustment. The details of the Test Network of the triangulation/traverse framework are as follows:

Number of stations	Traverse	13	
	Triangulation	80	
Number of bases (Triangulation)	Taped	1	
	Tellurometer	12	
Geodimeter distances (for CFL traverse only)		12	
Laplace stations	Traverse	6	
	Triangulation	3	
Distance between two adjacent stations	Traverse	Min. 10 km	
		Max. 43 km	
	Triangulation	Min. 11 km	
		Max. 72 km	

Observed angles (Traverse and Triangulation)

THE TEST NETWORK

(BLOCK OF NIGERIA'S HORIZONTAL GEODETIC NETWORK)

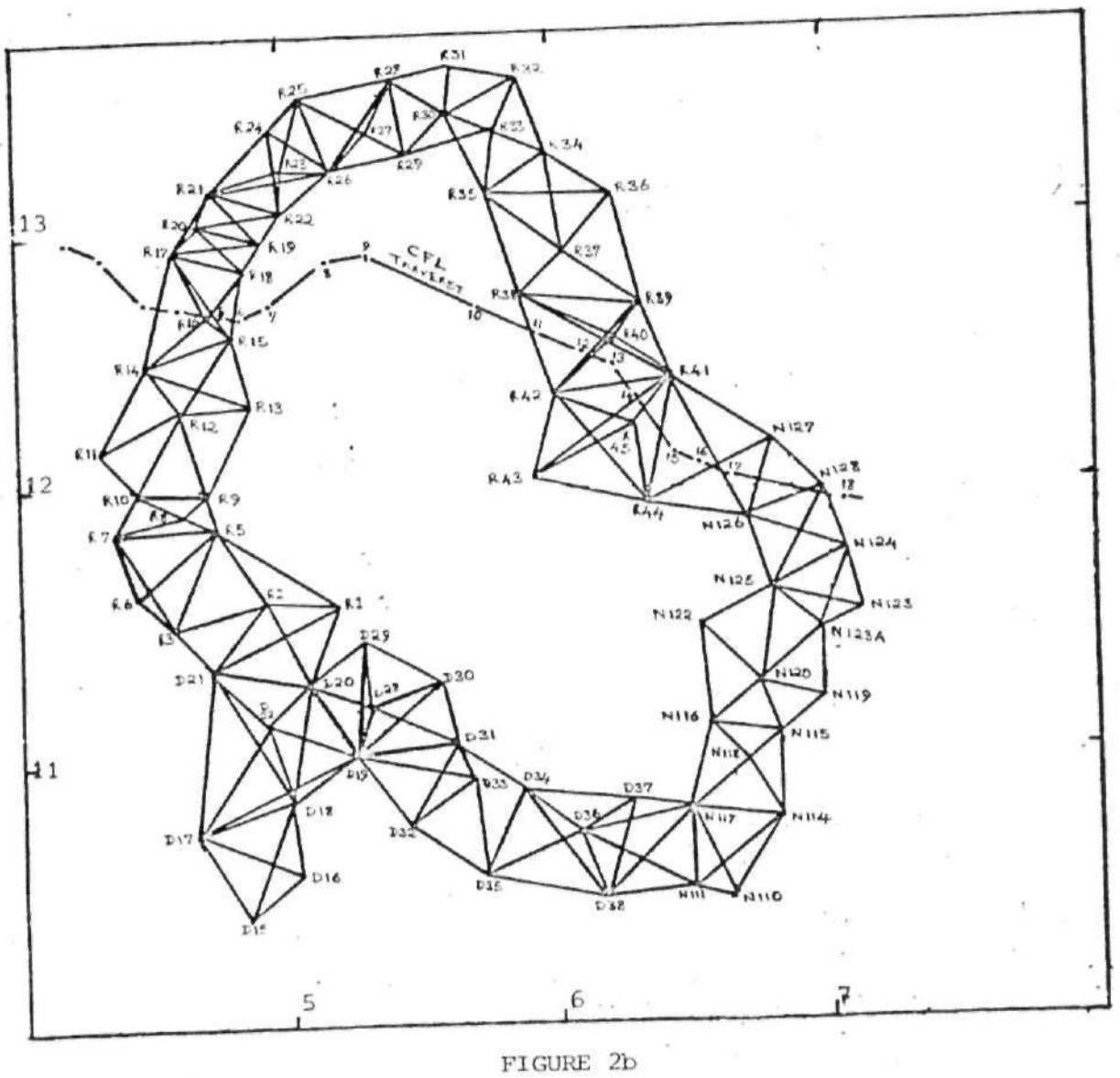
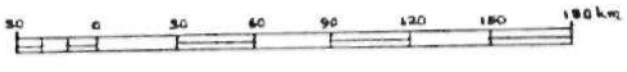


FIGURE 2b

Scale 1 : 3000 000



Existing Laplace Azimuth

Additional Laplace Azimuths (proposed)

Existing Scale Checks

Additional Scale Checks (proposed)

CHAPTER THREE

THE GEOID/REFERENCE SPHEROID IN NIGERIA

3.1 The Reference Surfaces

The object of every field survey is the establishment of the positional relationships of the surveyed points. A computing surface is therefore required for the establishment of this relationship. The earth's topographical surface on which the survey is carried out, with its mountains and valleys, its lakes, seas and oceans, and its unequal distribution of density, cannot be defined geometrically. This surface is therefore not useful either as a computing or as a reference surface. Consequently the positions of points on the surface of the earth are conventionally referred either to the Geoid or Spheroid.

(a) When these positions are defined, with respect to the geoid, they are referred to as natural co-ordinates and are denoted by the astronomical latitudes Φ, astronomical longitude Λ, and geopotential number C. These co-ordinates are gravity dependent and are usually determined by direct (natural) observations viz astronomic, gravimetric and spirit levelling. The geopotential number C (the natural co-ordinate for height) represents height above sea level. Unfortunately it is not directly measurable and so does not have the dimension of a length. It is measured in geopotential unit (g.p.u.). And since $C \approx gH \approx 0.98H$, the geopotential numbers in g.p.u. are almost equal to the height above sea level in metres. Consequently the geopotential number C is usually replaced by the geometrically more significant, though less directly measurable parameter - the orthometric height H, which is the elevation above the geoid.

(b) When the positions are defined with respect to the spheroid, they are referred to as geometric (geodetic) co-ordinates. These co-ordinates - geodetic latitude ϕ, geodetic longitude λ, and spheroidal height h, are determined from geometric (distances and/or direction) observations. However for topographical purposes the height co-ordinates are recorded above the geoid (mean sea level). Until the advent of artificial earth satellites, which have made the computations of geodetic positions in cartesian co-ordinates possible, the usual practice had been (and is still for most geodetic networks) the use of the spheroidal co-ordinate system for expressing the positions of survey points.

3.1.1 The Geoid

This is a natural surface, determined by the physical properties of the earth. It may be defined as that equipotential surface in the gravity field of the earth which on the average coincides with the undisturbed mean sea level. If the mean sea level were established everywhere on the earth's surface, it would define the shape of the geoid. The geoid is thus a physical reality : at sea-level the direction of gravity and the axis of a level theodolite are perpendicular to it; and the process of spirit levelling, together with gravity observations along the levelling routes, measures heights above it.

The geoid has the following properties :

(i) It is the only equipotential surface which can be connected in practice with the earth's surface.

(ii) It is the assumed reference level from which all astronomical, angular and height measurements are made.

Though measurements made on the very irregular surface of the earth, can be referred to the geoid (a less irregular surface), without appreciable error, it is not possible, however, to carry out any computations on it. This is because irregularities in shape and density make it impossible for the geoid to be described by any known geometrical figure. Furthermore astronomical co-ordinates Φ and Λ could be used to define "parallels" and "meridians" upon it, but these would not be a suitable basis for computing a geodetic network, because the irregular form of the geoid would cause them to be irregularly spaced.

3.1.2 The Reference Spheroid

Since the geoid is unsuitable for geodetic computations, the positions of points on the earth's surface are expressed by co-ordinates on a geometrically defined figure - the reference spheroid. This is an ellipsoid of revolution, formed by rotating the ellipse about its minor axis. Its axes are defined parallel to those of the geoid, and its meridians and parallels constitute the reference system. There are two varieties of the reference spheroids used in geodesy. One is a regional spheroid while the other is concentric with the earth, that is geocentric.

The regional spheroids are traditionally used for depicting continental and smaller scale horizontal geodetic networks. The method of defining this spheroid is described in § 3.1.3.

The geocentric spheroid is used for solving world-wide geodetic and other related problems. In this system the conventionally right-handed orthogonal system (the cartesian system) is adopted. The z-axis is oriented parallel to the earth's mean polar axis - the CIO (the Conventional International Origin) as defined by the International Polar Motion Service (the IPMS). The x-axis is

oriented parallel to the Greenwich Mean Astronomical Meridian (the CZM - Conventional Zero Meridian) as defined by the Bureau International de l'Heure [Robbins, 1976]. Also the x- and y-axes rotate with the earth. Since modern survey methods can now directly provide cartesian co-ordinates, there is a growing need for the adoption of this type of geodetic datum for local geodetic network. This has largely been brought about by the advent and use of artificial earth satellites positioning techniques for the determination of positions of local geodetic datums*. The geocentric system in current use for geodetic computations is the GRS 1967 - the Geodetic Reference System 1967 (see § 3.1.4). Other geocentric datums are also available for various types of observations and computations - example of this is the NSWC-9Z-2 system. This system is mainly used for the prediction of Doppler satellite precise ephemeris and related computations from Doppler observations.

3.1.3 Datum Definition

In order to compute the positions of points in a geodetic network on a reference spheroid, the relationship between the adopted reference spheroid and the geoid has to (and can only) be specified

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*The term datum is traditionally used in geodesy in two slightly different contexts : one more general, one more specific. The former meaning refers to any conventional framework toward which we relate our observations, be it a conventional zero of a scale or a system of fixed points in space. The latter meaning refers both to the two types of reference surfaces used in geodesy : the reference spheroid and the geoid [Merry and Vanicek, 1974].

at one point of the survey, usually the origin. This relationship can be established by the choice of certain parameters usually called datum parameters. The definition of the geodetic datum, as the operation is sometimes called, involves the choice of eight independent parameters (constants) as follows |Bomford, 1980 p.95|:

(a) Two parameters to define the size and shape of the spheroid. This is accomplished by assigning lengths a and b to the major and minor axes respectively; or (the common practice) by defining the major axis a and the flattening $f = (a - b)/a$.

(b) Two parameters to define the orientation of the semi-minor axis. This is generally and implicitly oriented to the earth's mean polar axis, the CIO, through the use of Laplace Azimuth equations in geodetic network adjustment.

(c) Three parameters to define the position of the centre of the spheroid relative to the geocentre. This can be done in either of the two ways:

(i) by defining the geoid-spheroid separation N_o, and ξ_o, η_o the components of the deviation of the vertical.

(ii) by defining N_o, ξ_o, and geodetic azimuth α.

(d) One parameter to define an arbitrary zero-meridian for longitudes. This is usually taken as the Conventional Zero Meridian (CZM) of the BIH zero of longitude.

Also it may be shown that the parallelism of the axes (condition b above) is assured if at one point the following conditions (known as the "extended" Laplace equations) are fulfilled [Hotine, 1969 : 134, Heiskanen and Moritz, 1967 : 186, 190]:

$$\begin{aligned} A - \alpha &= \eta\tan\phi + (\xi\sin\alpha - \eta\cos\alpha)\cot z \\ Z - z &= -(\xi\cos + \eta\sin\alpha) \end{aligned} \qquad 3.1$$

where : A and Z are the astronomic azimuth and zenith distance to another station, α and z are their geodetic (spheroidal) equivalent.

However in conventional work, largely because vertical angles cannot be measured accurately, and also because $z \simeq 90^{o}$, (since for first-order triangulation, all lines of sight are usually almost horizontal) only a simplified version of the above set, known as the (short) Laplace equation (see equation 3.2 below) is enforced as in 3.1.3(b).

$$A - \alpha = \eta\tan\phi \qquad 3.2$$

Thus the use of equation 3.2 throughout the network (i.e. at several stations) *from the point of view of parallelism, is equivalent with* the use of equation 3.1 at a single station.

Also for condition c, there is always an arbitrary choice between the two categories of the specified parameters and never both. That this provision holds true can easily be shown from the relations below [Bomford, 1980 : 99 and 100]:

$$\eta = (\Lambda - \lambda)\cos\phi \qquad 3.3$$

$$\eta = (A - \alpha)\cot\phi \qquad 3.4$$

The above two equations show that η can be deduced from the difference of either astronomical and geodetic longitudes or of astronomical and geodetic azimuths. The two values of η, must therefore be the same, hence

$$(\text{Astro} - \text{Geod longitude})\cos\phi = (\text{Astro} - \text{Geod azimuth})\cot\phi \qquad 3.5$$

Whence

$$\text{Geod azimuth} = \text{Astro azimuth} - \eta\tan\phi = \text{Astro azimuth} - (\text{Astro longitude} - \text{Geod longitude})\sin\phi \qquad 3.6$$

and this is the Laplace Equation (see equation 3.2). And since Laplace's equation must be satisfied and the two values of η must be the same, only one of the two categories of the parameters is needed in datum definition.

The result of specifying all the above parameters is that the minor axes of the spheroid and the geoid (the CIO) are parallel, and the relationship between the centre of the spheroid and the centre of mass of the earth is fixed, but unknown, and cannot be determined except by external means, such as artificial earth satellites.

If the datum parameters are such that the centre of the reference spheroid coincides with the centre of mass of the earth and the parameters defining its size and shape, correspond to those of the equipotential (level) spheroid defining the normal gravity field, the datum is said to be "absolute" (geocentric or gravimetric) as opposed to "astrogeodetic" (relative or regional). Similar terms are used for the deflections and undulations.

Presently opinions differ amongst some geodesists notably [Fischer, 1974; Mueller, 1974; Moritz, 1978] on the choice of the type of reference spheroid for local networks. Admittedly the regional spheroid has the great advantage of being a best fit spheroid for the local geoid. This feature, undoubtedly reduces the amount of error that may occur in reducing the measured distances to the spheroid. However if it is accepted that any such reduction of distances should be carried out accurately to eliminate all possible scale errors from this source, one rightly wonders if there are any disadvantages in choosing a geocentric spheroid with its wider application for a local network than the conventional regional spheroid with its limited application.

The main problem that confronts a local surveyor, when a geocentric spheroid is adopted is the possible great discrepancy between his measured distance on the ground and the corresponding published distance on the spheroid (geocentric). However if tables for distance corrections are provided for such a surveyor, then there is no reason he should not obtain the distance on the surface accurately.

3.1.4 The Geodetic Reference System, 1967 (and 1980)
(A complete detail of this geodetic datum can be found in Olliver (1979) and IAG Special Publication No. 3)

This is a complete and homogenous geometrical and gravimetric reference system adopted by the 14th General Assembly of the IAG at Lucerne in 1967. The system specified to be an equipotential spheroid is defined by the following set of constants :

(i) Its size and shape, given by its major axis a_e and flattening f. The latter is however not explicitly defined. Instead the quantity J_2 which is the coefficient of the second degree zonal harmonic of the potential field of the spheroid, and known as the dynamic form factor, is specified. The eccentricity e and thence the flattening can be unambiguously derived from J_2. The major axis a_e is defined to be equal to the equatorial radius of the earth.

(ii) Its mass M (this is the same mass as the earth or rather the best known value of this quantity) usually associated with the Newtonian gravitational constant G in the form GM.

(iii) Its angular velocity ω, which is the same as the angular velocity of the earth or the best known value of the quantity.

By assigning to the spheroid the same mass and angular velocity as the earth - or rather the best known values of those quantities - the values of potential on its surface and the geoid are taken to be equal. The resulting gravity field of the spheroid, the so called normal gravity, provides a standard field to which gravity measurements can be referred. The theoretical value of gravity on the surface of the spheroid is referred to as normal gravity or standard gravity. Thus the GRS 1967 serves as a reference both for the geometry of the earth's surface and for the terrestrial gravity field.

Also, as it is the convention, the minor axis of the reference equipotential spheroid is oriented parallel to the direction defined by the mean rotational axis of the earth (CIO) and the primary meridian is oriented parallel to the zero meridian (the CZM) of the BIH adopted zero of longitudes.

The effect of the atmosphere is excluded in the definition of this reference ellipsoid. The mass of the atmosphere is included in the mass of the earth. And the effect of atmospheric influences on gravity measurements is taken care of by suitable corrections. By convention (IAG, 1971) the observed gravity is usually increased by 0.87 mgal in order to relate the gravity anomalies to GRS 1967.

The numerical values of the parameters defining GRS 1967 are :

$$a_e = 6378\ 160\ m$$

$$J_2 = 0.001\ 0827$$

$$GM = 398\ 603 \times 10^9 m^3 s^{-2}$$

$$\omega = 7.292\ 115\ 1467 \times 10^{-5} \text{ radians/second}$$

The flattening f can be derived using the iterative relationship

$$e^2 = 3J_2 + \frac{4\omega^2 a^3}{15GM} \cdot \frac{e^3}{2q_o} \qquad 3.7$$

where

$$2q_o = (1 + \frac{3}{e'^2}) \arctan e' - \frac{3}{e'} \qquad 3.8$$

and

e' = ae/b is the complementary eccentricity

The formula for normal gravity γ is :

$$\gamma = 978\ 031.8(1 + 0.005\ 3024\sin^2\phi - 5.9 \times 10^{-6}\sin^2 2\phi)\text{mgal} \qquad 3.9$$

where ϕ is the spheroidal latitude; the numerical values of the coefficients being consistent with the parameters given above.

It should be remarked that although the GRS 1967 is only about 14 years old, the rapid improvement in the knowledge of the fundamental constants brought about by advances in Very Long Base Interferometry (VLBI), lunar and satellite laser ranging, and other space probes, has brought about the adoption of a new reference system - the GRS 1980.

The Geodetic Reference System, 1980 (GRS 80)

The GRS 80 was adopted at the XVII General Assembly of the IUGG in Canberra, December 1979 to replace the GRS 67. The GRS 67 was considered to be no longer accurate enough to represent the size, shape and gravity field of the earth to an accuracy adequate for many geodetic, geophysical, astronomical and hydrographic applications. The new GRS 80 is also based on the theory of the geocentric equipotential ellipsoid. It is defined by the following conventional constants |Moritz, 1980|:

Equatorial radius of the Earth, a	=	6378 137 m
Geocentric gravitational constant of the Earth (including the atmosphere), GM	=	$3986\ 005 \times 10^{8} m^{3} s^{-2}$
Dynamical form factor of the Earth (excluding the permanent tidal deformation), J_2	=	$108\ 263 \times 10^{-8}$
Angular velocity of the Earth, ω	=	$7292\ 115 \times 10^{-11}$ rads^{-1}

The same computational procedure and criterion for definition applicable to GRS 67 are also applicable to this. The system - the GRS 80 (at the time of writing) has not come into general use yet.

It is to be remarked that tidal effects are no longer included in all geodetic observations of gravity |Uotilla, 1980|, And in order to be consistent with a simple and unambiguous way of treating the permanent tidal deformation due to the attraction of sun and moon, the following correction is to be added to the published IGSN 71 values

$$c = 0.037(1 - 3\sin^2\phi)\text{mgal}$$

where ϕ is the geodetic latitude of the station.

The purpose of the above correction is to remove the so-called Honkassalo correction included in the IGSN 71 adjustment.

3.1.5 Reduction of Observations

All terrestrial geodetic network should be computed by the Projection method, but because of lack of certain necessary data in some countries like Nigeria, they are generally computed by the Development method. In the Projection method, all geometric observations are reduced to the reference spheroid and all natural observations reduced to the geoid. This implies that the true deviations of the vertical ε, and the undulations of the geoid N, (height of geoid above or below the spheroid) are known at some points of the network. However where they are not known, the Development method is used. In this method all measurements are reduced to the geoid and the computations are done using the specified parameters of the reference spheroid; thus in effect assuming that the geoid and spheroid are the same surface. This can result in distortions of the adjusted geodetic network, since the adjusted quantities do not only depend on errors of measurements but also on the deviations of the geoid from the adopted reference spheroid (see § 3.3).

A. For the reduction of geometric observations, the necessary corrections (and the quantities most significantly affecting them) are outlined below |Bomford, 1980, Heiskanen and Moritz, 1967|.

(i) Distance Correction (at the terminal points : orthometric heights, and undulations) : the correction for geoid undulation is usually called scale correction. This correction depends on the height of the line above the spheroid and amounts to 1 ppm for each 6.4 metres of neglected geoid-spheroid separation.

(ii) Direction Corrections

(a) For the deflection of the vertical (deflection components at the point of observation); this is because the measuring instruments are aligned with respect to the plumbline and not with respect to the spheroidal normal. This correction, depending on the size of the deflection, may amount to several seconds of arc.

(b) For the non-coplanarity of the ellipsoidal normals at the observation and observed points, commonly known as the correction for the "skewness of the normals" (spheroidal height of the observed point - height of the target above the spheroid); the value of this correction is about 0".1 for every 2000 m in height.

(c) For the angle between the normal section on the spheroid and the geodesic between the observation and observed points, usually called "Geodesic correction" (distance on the spheroid between the points).

B. For the natural quantities, the necessary corrections (and the quantities most significantly affecting them) are as follows:

(i) Observed astronomic latitude, longitude and azimuth need to be corrected [Robbins, 1976; Mueller, 1969]

(a) for polar motion (co-ordinates of the instantaneous pole with respect to the CIO)

(b) to UT1, for longitudes only (UTC - UT1 corrections)

(c) for the curvature of the plumbline between the observation station and the geoid (average horizontal gradient of gravity along the plumbline section)

(d) for the curvature of the plumbline between the observed and the geoid, for azimuths only (same as in (e))

(ii) Orthometric correction to spirit-levelled height (average gravity along the plumbline between the station and the geoid, gravity observations along the levelling lines) [Heiskanen and Moritz, 1967].

In practice, difficulties usually arise for the accurate evaluation of (i)c, (i)d and (ii) above. This is largely because, for their accurate calculations, the distribution of densities in the earth's crust between the topography and the geoid needs to be considered. Such information is not readily available and assumptions and approximations are usually made. Similar problems arise

in connection with the reduction of observed gravity values to the geoid. Problems regarding the accurate knowledge of the distribution of earth's densities have led to the development of new theories - Molodensky's formula. These theories which have not been adopted for general use yet because of practical difficulties avoid the use of the geoid (and the reduction thereto).

3.1.6 Relative Advantages of the Geocentric and Regional datums

The relative advantages of the two types of geodetic reference systems are outlined below.

REGIONAL DATUMS

(i) Advantages

(a) Better local fit to the geoid (if accurately determined), thus the errors of the development method may not be overpowering.

(b) Also map requirements for scale may be easier to meet.

(ii) Disadvantages

(a) Relationships with respect to the geocentre and the geoid is uncertain, resulting in the need for separating the horizontal from the vertical control and also in inaccurate 3-D cartesian co-ordinates.

(b) Unsuitability for global applications in geodesy, astronomy, oceanography etc.

(c) Inconsistency with the gravity reference (normal) field.

(d) Since it is defined through parameters valid only at the initial point, distortions arise throughout the network. (Practitioners therefore have to use sets of distorted station co-ordinates as their "definition" of the datum, while the datum is accurately defined only at the initial point).

GEOCENTRIC DATUMS

(i) Advantages

(a) Convenient use for world wide applications in geodesy, astronomy etc.

(b) Can be made consistent with the gravity reference field.

(c) Can be conveniently used in conjunction with future three-dimensional (possibly time variant) geodetic co-ordinate systems.

(ii) Disadvantages

(a) Local fit to the geoid in a given country may be unsatisfactory, thus accurate (rigorous) reductions of observations (the correct application of the projection method) is a requirement.

(b) Same as (ii)d for regional datums.

3.2 Datum Transformation

3.2.1 Introduction

It may be necessary in Nigeria to effect a change of spheroid

from the current one in use to another more suitable regional spheroid (an undesirable venture), if the former is proved solely by scientific consideration, as not being the best fitting spheroid for the geoid in the country. It is to be remarked that the adoption of Clarke 1880 (modified) figure of the earth for use in Nigeria, was not based on any scientific consideration but rather it was considered then to be more handy and manageable. Evidently such a choice would result in inaccurate geodetic co-ordinates for the network.

Also, with the advent and increase in the use of satellite positioning techniques, the determination of the position of local geodetic datums in a geocentric system is becoming important. Thus it may be necessary to relate the co-ordinates computed on local datums to a global geodetic datum (geocentric datum).

It would be possible to accomplish the above operations by re-computing the whole triangulation and its adjustment using new values of a and f, new deviations of the vertical, new Laplace azimuths and new reductions of bases to spheroid level. But this method would be laborious and costly, and the operations can be done more easily by the methods of datum transformation. The method of datum transformation is merely a transformation of co-ordinates, since every geodetic datum corresponds to a different set of datum parameters.

3.2.2 Transformation between two local datums

For transformation between two regional spheroids, it is necessary to deduce the differences between the parameters defining the old and new spheroids, S_2 and S_1 respectively. However the following assumptions are usually made:

(a) The minor axes of the two spheroids are parallel since both have been defined as in § 3.1.3(b) by the direction of the CIO. But if there is an error of more than 1" in astronomical longitude at the origin, leading to non-satisfaction of the Laplace equation in one of the spheroids, say, S_1, then there is need for a constant correction of the value of the error, to be applied to all points which are to be considered |Bomford, 1980 : 177|.

(b) The sizes and shapes of the two spheroids are known.

(c) The CZM of the BIH is accepted as the zero longitude for both spheroids.

Also allowances are made for any possible differences in scale standards such as feet/metres.

In order to effect transformation of co-ordinates from system S_2 to system S_1, it is essential to know the position of at least one common point in both systems, such as perhaps the origin. Then by using the parameters of the datum a_2, f_2, ξ_2, η_2 and N_2 for S_2, together with the corresponding values a, f, ξ_1, η_1, N_1 for S_1, the transformation can easily be effected by using the transformation equation in Heiskanen and Moritz, 1967 (- equations 5-59 -) to compute the changes $\delta\xi$, $\delta\eta$ and δN. These formulae give variations of ξ, η and N due to the effect of a shift of the geodetic datum. Also the formulae are given in terms of spheroidal co-ordinates viz latitude and longitude and all data and computations must be in this mode. The mathematical model for the transformation $S_2 \rightarrow S_1$ is as detailed below :

$$\delta\xi_1 = \xi_1 - \xi_2$$

$$\delta\eta_1 = \eta_1 - \eta_2$$

$$\delta N_1 = N_1 - N_2$$

$$\delta a = a - a_2$$

$$\delta f = f - f_2 \qquad 3.10$$

The derived values in the above equations are substituted in the relevant equations 5-59 to obtain the changes $\delta\xi$, $\delta\eta$ and δN. Then the ξ, η and N in the S_1 system are given by:

$$\xi = \xi_2 + \delta\xi$$

$$\eta = \eta_2 + \delta\eta$$

$$N = N_2 + \delta N \qquad 3.11$$

The geodetic co-ordinates in the system S_1 are then obtained from [Heiskanen and Moritz, 1967].

$$\phi = \phi_2 - \delta\xi$$

$$\lambda = \lambda_2 - \delta\eta\sec\phi$$

$$h = h_2 + \delta N \qquad 3.12$$

If there are several common points $\delta\xi$, $\delta\eta$ and δN can be got for each, and a suitably weighted mean accepted. But if the discrepancies are unacceptably large, the only remedy is to readjust one or both of the two nets in the systems, so as to secure equal changes. The method of the adjustment will depend on the size of the discrepancies and may be anything from a semi-rigorous to a complete rigorous adjustment (see Chapter 4 of this thesis).

An alternative method to the above is to convert the spheroidal co-ordinates of the common point (or points) to three-dimensional cartesian co-ordinates within their own individual systems by means of the relations [Heiskanen and Moritz, 1967]:

$$x = (\nu + h)\cos\phi\cos\lambda$$

$$y = (\nu + h)\cos\phi\sin\lambda$$

$$z = \{(1 - e^2)\nu + h\}\sin\phi \qquad 3.13$$

where

ν is the transverse radius of curvature of the spheroid, and
e is its eccentricity.
h is the height of the point above the spheroid.

Also in the above equations, the origin of the cartesian system coincides with the centre of the spheroid, the x-axis passes through the point ($\phi = 0, \lambda = 0$) and the z-axis coincides with the semi-minor axis of the spheroid. The differences $x_1 - x_2$, $y_1 - y_2$ and $z_1 - z_2$ between the two sets of co-ordinates, give the datum shifts (the separation of the two centres), δx, δy and δz respectively. These are the corrections to be applied to all S_2 co-ordinates, transformed by means of the above equation. The results are converted back to spheroidal co-ordinates on S_1 using [Olliver, 1979]:

$$\tan\phi = \frac{z + e^2\nu\sin\phi}{(x^2 + y^2)^{\frac{1}{2}}}$$

$$\tan\lambda = y/x$$

$$h = (x\sec\phi\sec\lambda - \nu)$$

$$= (y\sec\phi\operatorname{cosec}\lambda - \nu) \qquad 3.14$$

The first relationship is an iterative process. Subtracting the spheroidal latitude and longitude from the astronomical latitude and longitude gives the components of the deviation in terms of S_1. This method can therefore be used to check the results obtained by the first method.

Also if several common points are known, then the specifications stated in the last paragraph of the first method should be applied.

3.2.3 Transformation between two systems

Since a geodetic datum is defined by the co-ordinates adopted at the origin, the assumptions are again made that -

(a) the minor axes of the two reference systems are parallel to the CIO, provided Laplace's condition at the origin and elsewhere has been satisfied.

(b) the adopted zero longitudes for the two systems are the same - CZM.

(c) the sizes and shapes of the two systems are known.

Also for the rigorous computation of the co-ordinate values of the physical points in a geodetic net, an accurate knowledge of the geoidal heights at all points together with the reductions of all observations to a consistent pole are required.

In practice all these conditions are not accurately satisfied due to the propagation of unavoidable systematic or model errors through the net. Thus some degree of distortions are introduced and these may manifest themselves as small rotations of the geodetic system with respect to the earth's rotation axis. Consequently rotation elements should be introduced into the transformation process. Also allowances are made for any scale errors.

The relationship between the two systems can be established as follows:
Let the co-ordinates of a ground station referred to the centre of its local datum or spheroid as origin be x, y, z and let X, Y, Z be its co-ordinates with reference to the geocentre, as given by satellite observations. The conversion to cartesian co-ordinates x, y, z can easily be effected through equation (3.13).

Then for any geocentric system, the axes of which are not necessarily parallel to those of the geodetic system, the relationship is given by |Vanicek and Merry, 1974|:

$$\begin{vmatrix} X \\ Y \\ Z \end{vmatrix} = \begin{vmatrix} X_o \\ Y_o \\ Z_o \end{vmatrix} + R \begin{vmatrix} x \\ y \\ z \end{vmatrix} \qquad 3.15$$

where $\begin{vmatrix} X_o \\ Y_o \\ Z_o \end{vmatrix}$ are the co-ordinates of the centre of the reference spheroid with respect to the geocentre; and is usually called "shift parameters", and R is a rotation matrix composed as follows:

$$R = \begin{vmatrix} \cos\omega & \sin\omega & 0 \\ -\sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} \cos\psi & 0 & -\sin\psi \\ 0 & 1 & 0 \\ \sin\psi & 0 & \cos\psi \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\varepsilon & \sin\varepsilon \\ 0 & -\sin\varepsilon & \cos\varepsilon \end{vmatrix} \qquad 3.16$$

and $(\varepsilon, \psi, \omega)$ are rotations about the (x, y, z) axes respectively, to bring them parallel to the (X, Y, Z) axes. It is to be remarked that $(\varepsilon, \psi, \omega)$ are positive when rotations from geodetic to the geocentric systems are anti-clockwise.

In equation (3.15), it has been assumed that the scale errors have been accounted for. Where this has not been the case, then the equation is modified as follows:

$$\begin{vmatrix} X \\ Y \\ Z \end{vmatrix} = \begin{vmatrix} X_o \\ Y_o \\ Z_o \end{vmatrix} + (1+S)R \begin{vmatrix} x \\ y \\ z \end{vmatrix} \qquad 3.15b$$

Where S is the scale error, and the other parameters have the same meaning as before.

Differentiating equations (3.15) one gets, after some manipulation, and neglecting second-order terms:

$$\delta X = \delta X_o - a\sin\phi\cos\lambda\delta\phi - a\cos\phi\sin\lambda\delta\lambda + \cos\phi\cos\lambda(\delta h + \delta a + a\sin^2\phi\delta f) + a\cos\phi\sin\lambda\delta\omega - a\sin\phi\delta\omega$$

$$\delta Y = \delta Y_o - a\sin\phi\sin\lambda\delta\phi + a\cos\phi\cos\lambda\delta\lambda + \cos\phi\sin\lambda(\delta h + \delta a + a\sin^2\phi\delta f) - a\cos\phi\cos\lambda\delta\omega + a\sin\phi\delta\varepsilon$$

$$\delta Z = \delta Z_o + a\cos\phi\delta\phi + \sin\phi(\delta h + \delta a + a\sin^2\phi\delta f) - 2a\sin\phi\delta f + a\cos\phi\cos\lambda\delta\psi - a\cos\phi\sin\lambda\delta\varepsilon \qquad 3.17$$

But if it is now considered that the geocentric co-ordinates of a point do not change (i.e. the position of a ground point relative to the centre of mass of the earth remains unchanged) then it follows that $\delta X = \delta Y = \delta Z = 0$. Consequently equations (3.17) represent the difference in co-ordinates (i.e. the change in co-ordinates between two ellipsoids of different size, shape, orientation and position. This method of transformation is usually called the "7-parameter solutions" (if the scale errors have been included as in equation 3.15b) viz three datum shifts, one over-all scale error for the survey, and three rotations.

Also equations (3.17) can be rearranged to solve for the differences (or changes) in spheroidal co-ordinates, $\delta\phi$, $\delta\lambda$, δh thus:

$$\delta\phi = \sin\phi\cos\lambda\frac{\delta X_o}{a} + \sin\phi\sin\lambda\frac{\delta X_o}{a} - \cos\phi\frac{\delta Z_o}{a} - \cos\lambda\delta\psi + \sin\lambda\delta\varepsilon$$

$$+ 2a\sin\phi\cos\phi\delta f$$

$$\delta\lambda = \frac{\sin\lambda\delta X_o}{a\cos\phi} - \frac{\cos\lambda\delta Y_o}{a\cos\phi} + \delta\omega - \tan\phi\sin\lambda\delta\omega - \tan\phi\cos\lambda\delta\varepsilon$$

$$\delta h = -\cos\phi\cos\lambda\delta X_o - \cos\phi\sin\lambda\delta Y_o - \sin\phi\delta Z_o - \delta a + a\sin^2\phi\delta f \qquad 3.18$$

But since the natural co-ordinates - astronomical latitude Φ, astronomical longitude Λ, and orthometric height H - are not changed by datum shift, and since by definition

$$\xi = \Phi - \phi$$

$$\eta = (\Lambda - \lambda)\cos\phi$$

$$h = N + H \qquad 3.19$$

then

$$\delta\phi = -\delta\xi$$

$$\cos\phi\delta\lambda = -\delta\eta$$

$$\delta h = \delta N$$

The quantities δN, and $\delta\xi$, $\delta\eta$ are changes in geoidal height and deflections of the vertical respectively. Hence, if these are known for a number of points in a geodetic network, the values of δX_o, δY_o, δZ_o (the translation components) and $\delta\varepsilon$, $\delta\psi$, $\delta\omega$ (rotation elements) can be found. The quantities δa and δf are generally considered known, since one knows the shapes and sizes of the two ellipsoids one deals with.

On the other hand if δX_o, δY_o, δZ_o and $\delta\varepsilon$, $\delta\psi$, $\delta\omega$ are known, one can compute the values of $\delta\xi$, $\delta\eta$ and δN for all the points in a geodetic network.

Sometimes, in practice, it is found that many astrogeodetic deflections referred to a local datum are available, but without a corresponding set of gravimetric (geocentric) deflections. Consequently the determination of δX_o, δY_o and δZ_o is limited to the use of last equation in equations (3.18). Then by using the last equation in (3.20) and substituting for δh in (3.18) one obtains a system of observation equations of the type:

$$\delta N = -\cos\phi_i \cos\lambda_i \delta X_o - \cos\phi_i \sin\lambda_i \delta Y_o - \sin\phi_i \delta Z_o - \delta a + a\sin^2\phi_i \delta f \qquad 3.21$$

$$i = 1, \ldots\ldots\ldots\ldots, n$$

where n is the number of points at which comparisons of the two systems of geoidal heights are made. Thus if the geoidal height differences are known at a number of suitable points, n say, the observation equations can be solved by least squares adjustment to yield the values of the unknowns δX_o, δY_o and δZ_o.

3.3 Analysis of the Effects of Errors due to Inaccurate Reduction of Observations

Introduction

For the adjustment of the Nigerian horizontal control network in 1977 [Field, 1977] the geodetic observations were not rigorously reduced to the reference spheroid. The effects of such reductions on the adjusted network are outlined (see §§ 3.3.1 - 3.3.3) so that accurate reduction procedure should be adopted in future adjustment of the framework. The analysis is purely theoretical, as no practical investigations in this regard has been performed in this work. However the analysis has been based on work done by some geodesists, with some existing networks, and it is very unlikely that Nigeria's network differs significantly from such networks.

3.3.1 Errors due to lack of Direction/Angle Reductions

There are three corrections to directions that should be applied to reduce them to the spheroid (see also § 3.1.5 A(ii)). Two of these are due to the geometry of computing on a spheroidal surface rather than a spherical surface. The third correction is due to non-parallelism of the spheroidal surface and the local equipotential surface.

In general, the plumbline, perpendicular to the equipotential surface passing through the observing station, does not coincide with the normal to the spheroid. This results in the observing instrument being disleveled with respect to the normal to the spheroid, the computing surface (fig. 3.3a below). The correction is given by [Vanicek, 1972]:

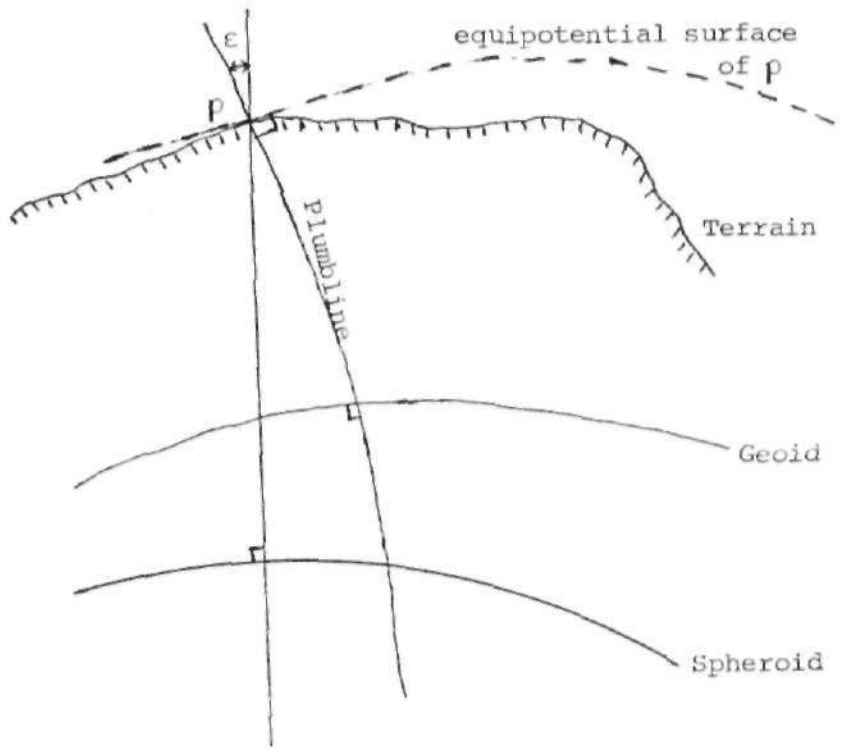


Fig. 3.3a

For a direction, α:

$$\Delta\alpha = -(\xi\sin\alpha - \eta\cos\alpha)\cot z$$

$$= -\varepsilon\sin(\alpha - \theta)\cot z \qquad 3.22$$

For angle β $= \alpha_2 - \alpha_1$

$$\Delta\beta = \varepsilon\left|\sin(\alpha_1 - \theta)\cot z_1 - \sin(\alpha_2 - \theta)\cot z_2\right| \qquad 3.23$$

where α is the azimuth of the line, measured from north clockwise : ξ, η are the meridional and prime vertical components of the surface deflection of the vertical; ε, θ are the magnitude of the deflection of the vertical and its azimuth; z is the zenith distance of the target.

The application of the above corrections reduces the observed direction (angle) to lie in the plane (planes) containing both the normals to the spheroid at the observing station and the target (targets).

The other two corrections which should be applied are the skew normal correction and the correction for geodesic. They are respectively given by [Bomford, 1980]:

$$\text{(i)} \quad \Delta\alpha \propto + \frac{h_2 e'^2}{2R}\sin 2\alpha\cos^2\phi \qquad 3.24$$

where e' is the second eccentricity of the spheroid, given by

$$e' \propto (a^2 - b^2)^{\frac{1}{2}}/b$$

for a, b being the axes of the spheroid, and

$$\text{(ii)} \quad \Delta\alpha = -0.028(\ell/100)^2\sin 2\alpha\cos^2\phi \qquad 3.25$$

where ℓ is in kilometres.

The value h_2, the height of the target station above the spheroid, requires the knowledge of the geoid-spheroid separation, but the replacement of h_2 by H_2 (the orthometric height) has a negligible effect.

All the above corrections were not applied in the 1977 adjustment of Nigeria's geodetic network because the parameters for computing the corrections were generally not known, and in the few cases where they were known, the computed corrections were small. Computing from the scanty evidence available (Table 1e) it is seen that the maximum value of the correction $\Delta\alpha$ for the deviation of the vertical is -0".2 for an assumed maximum zenith distance of 88°. (This figure is for the direction N10 - N12 where azimuth is approximately 238° 59'). Field [1977] stated that the maximum value of the correction for the skewness of the normals is 0".2, and this is at station C19.

It has been demonstrated by some geodesists [Gregerson, 1964, Merry and Vanicek, 1973, and Bomford, 1980] that the neglect of the effect of the correction for the deflection of the vertical can cause some systematic distortions in horizontal control networks. Furthermore since the deflection correction depends on several parameters, it is rather difficult to predict the magnitude. Experience in Australia [Fryer, 1971] has shown that changes of deflection of 25" may occur in a comparatively short distance of 60 km. even in relatively flat topography. Bomford [1980] also remarked that a deflection correction of 4½" has been obtained for an angle in the primary triangulation of India. Furthermore it has been found that the distortion due to non-application of the deflection correction can be, only to a limited extent, compensated for by strengthening the network with astronomically determined azimuths and subsequent application of Laplace corrections (or alternatively geodetic azimuths determined by Black's method could be used.) Also Moritz [1978] demonstrated that

the Laplace equation is a condition, not for fixing the geometry of a network, but for adjusting measuring errors incurred in the process of fixing the geometry. In other words, the error due to deflection of the vertical is systematic, and cannot therefore be eliminated by adjustment technique. And as the correction for the deviation cannot be applied unless the deviation has been observed, it follows that it may be necessary to observe astronomical latitude and longitude (or azimuth) at all the primary stations, or more practicably at a large number of the stations, particularly those in mountainous areas. Bomford [1980] suggested that the deviation of the vertical at each station can be determined from reciprocal vertical angles.

3.3.2 Errors due to incomplete reduction of distances

The reduction of a terrain spatial distance to its corresponding value on the reference spheroid is given by [Heiskanen and Moritz, 1967]:

$$S_o = 2R \arcsin(\ell_o/2R) \qquad 3.26$$

where

$$\ell_o = \left\{ \frac{\ell^2 - (h_j - h_i)^2}{(1 + h_i/R)(1 + h_j/R)} \right\}^{\frac{1}{2}}$$

in which S_o is the spheroidal distance; R is the mean radius of curvature of the spheroid for the measured line; ℓ is the terrain spatial distance, and h_i and h_j are the heights of the end points of the line above the reference spheroid. These heights (h_i and h_j) are given by

$$h_i = H_i + N_i \qquad 3.27$$

and $$h_j = H_j + N_j$$

where H_i and H_j are the orthometric heights, obtained from levelling, and N_i and N_j are the heights of the geoid above/below the reference

spheroid. For the adjustment of Nigeria's network, N_i and N_j were considered to be zero, and their effect on the distance reduction was neglected. In other words, the distance ℓ has been incompletely reduced, that is, to the geoid as S'_o and not to the spheroid as S_o. The correction δs that needs to be applied to the incompletely reduced distance S'_o to obtain its corresponding spheroidal distance S_o is given as [Merry and Vanicek, 1973]:

$$\delta s = S_o - S'_o$$

$$= -\frac{N}{R} S'_o \qquad 3.28$$

where $N = (N_i + N_j)/2$. An alternative and preferable approach is to recompute the spheroidal distance S'_o, using the corrected values of h_i and h_j.

Owing to variations in the geoidal height, the scale error is non-linear and the distortions introduced into the geodetic network become a function not only of this varying scale, but also of the shape of the network, and the positions of the measured lines within the framework. Furthermore, the error in neglecting the geoidal height is a systematic one and its sign is unlikely to change within even a large region; the error will therefore have the tendency to build up.

3.3.3 Remarks

From the brief comments made in the preceding sections, it is apparent that the approximate treatment of geodetic observations by neglecting the geoid-spheroid separation and the deviations of the vertical, might have caused appreciable distortions (scale and orientation) on Nigeria's horizontal geodetic network. The magnitude of such distortions depends on the variations in the earth's gravity field

in the vicinity of the network. Also since the errors introduced into the network are systematic, they would accumulate to have a significant effect on the geodetic positions obtained from the adjustment. This effect will become more pronounced if a geocentric datum is adopted for the redefinition of Nigerian geodetic datum (where the geoidal heights will be magnified). Furthermore, attempts to calibrate distances from satellite observations with distances derived from first order geodetic control become meaningless when the error in terrestrial distances exceeds the error in satellite observations.

Hence it is therefore necessary that rigorous observation reduction techniques, through accurate knowledge of the geoid heights, should be performed for the national geodetic framework. Also attention should be devoted to the development of a viable method of deflection densification for reduction of angular observations. This can be achieved by either astrogeodetic method or by the use of geodetic and gravity data information within the framework.

3.3.4 Reasons for Geoid Figure

From all the remarks made in the previous sections, it is obvious that the determination of geoid figure is very important. The following summarizes the reasons for the accurate knowledge of such a figure.

(i) For scientific purposes; scientists require the shape of the earth for their studies.

(ii) For the reduction of angular and linear measurements to the reference spheroid on which the geodetic network is computed and to which the final co-ordinates of the stations are to be referred.

(iii) A knowledge of geoid undulations is needed in connection with satellite observations (Doppler satellite positioning for instance) where they are required in the transformation of co-ordinates from a mass-centred 3-D inertial cartesian framework into a geodetic co-ordinate system of spheroidal latitude, longitude and orthometric height.

3.4 GEOID Determination - Methods

Because the geoid is a complicated surface, it does not exactly conform to any known geometric figure. Its shape (figure) within an area can only be specified by its physical departures N (height above or below) from the reference spheroid. The departures N otherwise known as the geoidal undulations (or geoid height or geoid-spheroid separation) are basically functions of density distribution of matter inside the earth, and of the parameters and position (with respect to the geocentre) of the reference spheroid. There are currently many different methods of determining the geoid. Full details of some of these methods, including their practical computations and analysis of error, can be found in Olliver (1979); also Fubera et al, 1971 and Torres, 1976 give outlines of the methods with their shortcomings. Examples of these methods include the following which are briefly described below:

(i) astrogeodetic

(ii) gravimetric

(iii) satellites

(iv) astrogravimetric methods

3.4.1 Astrogeodetic method

This method depends on a knowledge of the angular difference between the direction of gravity vector and the direction of the normal to the reference spheroid. The relationship between this small angle (the deflection of the vertical) and the deviations of the geoid from the spheroid (geoidal undulation) is shown below:

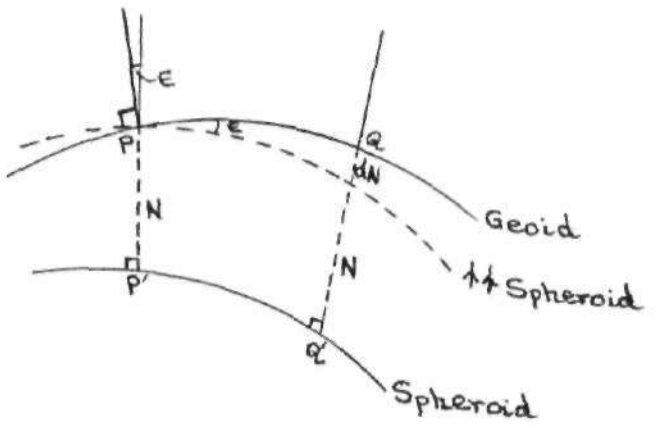


Fig. 3.3b

P and P' are points on the geoid and spheroid respectively, Q and Q' are also points on the geoid and spheroid respectively and very close to P and P' such that the distance P'Q' = ds. ε is the deviation of the vertical and is identically the angle between the spheroid tangent plane and the horizontal. This angle (expressed in radians) multiplied by the distance ds, gives a small change in geoid height dN between points P and Q. Thus the basic equation for geoid height is:

$$dN = \varepsilon ds \qquad 3.29$$

Between two points A and B in azimuth α, and distance s, we have on integrating equation (3.29):

$$N_B - N_A = \int_A^B \varepsilon \, ds \qquad 3.30$$

where $\varepsilon = -(\eta\sin\alpha + \xi\cos\alpha)$, the component of the deviation of the vertical in azimuth α, along AB, positive if the geoid is rising towards the south and west. ξ and η are the components of the deviation of the vertical.

In practice AB is made short enough to assume with sufficient accuracy that the mean ε along the line is given by the mean of its values at the two ends. Thus:

$$N_B - N_A = dN = \tfrac{1}{2}(\varepsilon''_A + \varepsilon''_B)S\sin 1'' \qquad 3.31$$

Robbins (1976) states that except in mountainous countries, S is usually less than 30 km.

If any "section" passes through the origin of a Datum (whose N is defined), this method gives a direct measure of the form of the geoid. The astronomical latitude and longitude from which the components of the deviation of the vertical are derived are observed with respect to the ground level equipotential surface. This involves the application to the latitude only, of the correction $-0''.00017h\sin 2\phi$, where h is the height of the station above sea-level in metres. The reason for this correction is that the ground-level equipotential surfaces (to which the vertical is perpendicular) are not in general parallel to the geoid below them. This is due to the variation of gravity along a level surface, causing the distance between adjacent level surfaces not to be constant. And theory, assuming a homogeneous earth, indicates an inclination in meridian only of the amount of correction stated above [Bomford, 1980].

All the formulae derived or quoted in the preceding paragraphs give only differences in geoid heights. This means that geoid undulations derived from astrogeodetic levelling are relative, and

depend entirely on the parameters (accurately chosen or otherwise) adopted for the geodetic datum. The classical configurations for lines of astrogeodetic levelling is along E - W and N - S sections. Greater accuracy is usually obtained for north-south (N - S) lines since the latitude observations are not burdened with the uncertainties in the determination of personal equation errors in longitude. For the purposes of providing a check and improved accuracy, the geoidal profiles are arranged to form circuits, which can be adjusted by the method of least squares.

The astrogeodetic method gives detailed information of geoid heights over land areas only, because of the difficulties in determining usable geodetic and astronomic co-ordinates at sea. Over great distances it is liable to accumulate significant errors, which can be controlled by other methods such as by satellite doppler. The astrogeodetic geoid has excellent local scale.

3.4.2 Gravimetric Method

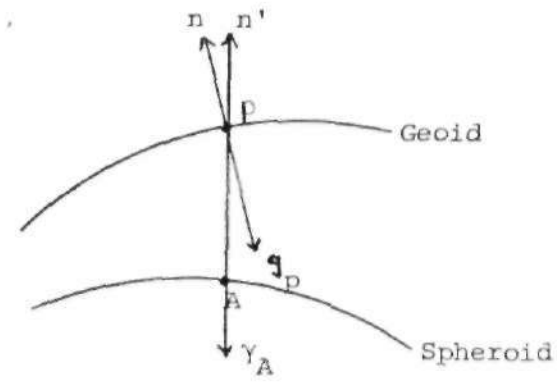


Fig 3.3c

In the above figure, g_p is the gravity vector at point P on the geoid and γ_A is the normal gravity vector at A on the spheroid. A vector is characterised by magnitude and direction. The difference

in direction between the two vectors is the deflection of the vertical and the difference in magnitude Δg is termed the gravity anomaly. Thus

$$\Delta g = g_p - \gamma_A \qquad 3.32$$

The quantity Δg is related to the geoidal undulation N, according to Stokes's integral. Stokes (1849) derived the integral as the solution to the geodetic boundary-value problem. The evaluation for N implies, in principle, integrating the following equations (Stokes' integral)

$$N = \frac{R}{4\pi\bar{g}_m} \int_\sigma \int \Delta g \; S(\psi) \, d\sigma \qquad 3.33$$

where

N = geoidal undulation (actually it is in this case, the height of free-air co-geoid above the reference spheroid).

R = mean radius of the earth, 6371 km.

$\bar{g}_m$ = mean value of gravity over the earth, 980.6 gal.

Δg = free-air gravity anomaly.

$d\sigma$ = element of area on the unit sphere.

σ = surface of the sphere of radius R with centre at the center of gravity.

$S(\psi)$ = Stokes function

= $\operatorname{cosec} \psi/2 - 6\sin\psi/2 + 1 - 5\cos\psi - 3\cos\psi \ln(\sin\psi/2 + \sin^2\psi/2)$

Stokes's integral is not an entirely rigorous solution for determining N, because it depends on several assumptions and approximations. For instance the utilisation of equation (3.33) implies among other things that

(a) Δg is known everywhere on the earth.

(b) g_p is measured on the geoid or its equivalent is deducible.

(c) the reference spheroid should have the same potential as the geoid and enclose the same mass as the actual earth.

Consequently the geoid heights derived from Stokes's integral are in error to the extent that these conditions are not met in practice. Unlike the astrogeodetic geoid (if relative geodetic datum is adopted) the gravimetric geoid and the reference spheroid, by implication of the mathematical structure and the field measurements involved, are in absolute position. Also it has "true" shape but lacks proper scale.

A detailed exposition of the methods for scaling the gravimetric geoid is given in Heiskanen and Moritz (1967). In brief it involves generalizing Stokes's integral for geoid height, N_a, to hold for any arbitrary reference spheroid whose centre coincides with the center of the earth. The generalised formula is of the form:

$$N_a = \frac{G\delta M}{R\bar{g}_m} - \frac{\delta W}{\bar{g}_m} + \frac{R}{4\pi\bar{g}_m}\iint \Delta g S(\psi)\, d\sigma \qquad 3.34$$

or

$$N_a = N_o + \frac{R}{4\pi\bar{g}_m}\iint \Delta g S(\psi)\, d\sigma$$

where

$$N_o = \frac{G\delta M}{R\bar{g}_m} - \frac{\delta W}{\bar{g}_m}$$

δM = exact mass of the earth minus the mass of the spheroid in use.

δW = potential of the geoid minus that of the spheroid.

G = Newtonian gravitational constant.

Also Olliver (1979 .) states that N_o (usually termed zero-order geoid height) can only be determined by reference to external sources. This involves direct comparison with externally derived geoid heights from, for example, satellite-derived potential coefficients and Doppler-satellite observations.

An alternative to the use of Stokes's integral is to compute, from gravity anomalies all over the earth, the quantities ξ and η - the meridian and prime vertical components of the deviation of the vertical respectively through the use of Vening Meinesz formulas. The formulas can be stated in abbreviated version as follows

$$\xi = \frac{1}{4\pi\bar{g}_m} \iint_\sigma \Delta g \frac{ds}{d\psi} \cos\alpha d\sigma$$

$$\eta = \frac{1}{4\pi\bar{g}_m} \iint_\sigma \Delta g \frac{ds}{d\psi} \sin\alpha d\sigma \qquad 3.35$$

where all the terms stated have the same meaning as before. Details of the theory and computations of the formulas can be found in Heiskanen and Moritz (1967). The ξ and η so obtained are absolute. that is to say, they are referenced to the earth's centre of mass. The deviation of the vertical ε, can then be computed, using the relation $\varepsilon = -(\eta\sin\alpha + \xi\cos\alpha)$, and hence the geoidal undulation N, according to equation (3.31).

Unlike Stokes' integral, Vening Meinesz formulas are valid for any arbitrary reference spheroid. However, they also require the use of gravity anomalies all over the earth, and in particular a dense gravity net around the computation points. The main source of systematic scale error in gravimetric geoid deduced through the use of Vening-Meinesz formulas is on the value of geoid height at the initial point N_A, which may not be obtained with very high accuracies.

In view of the massive computations involved in gravimetrically computed geoid heights and deflections of the vertical, and the deficiencies in theory, data quality and quantity, applicable to the method, a purely theoretical determination of the estimates of the errors involved has been difficult.

3.4.3 Satellite Methods

The information available from artificial satellites may be used in three different ways to determine geoid heights.

(a) Satellite Derived Potential Coefficients

Satellite orbits are influenced by irregularities of the earth's gravity field, and these irregularities are usually expressed in terms of a development in spherical harmonics. Analysis of the perturbations of the satellite orbits enables the coefficients of the spherical harmonics to be determined.

Thus the coefficients J_n, J_{nm} and K_{nm} in the harmonic expansion of the geopotential

$$W = \frac{GM}{r}\left[1 - \sum_{n=2}^{\infty}\left(\frac{R}{r}\right)^n J_n P_n - \sum_{n=2}^{\infty}\sum_{m=2}^{n}\left(\frac{R}{r}\right)^n \{J_{nm}R_{nm} + K_{nm}S_{nm}\}\right] + \tfrac{1}{2}\omega^2 r^2 \cos^2\phi \qquad 3.36$$

can be determined, up to a certain degree n. By subtracting an expression for U (the spheropotential) from W, the disturbing potential T is obtained. The desired geoid heights are derived by means of Brun's equation

$$N = \frac{T}{\gamma} \qquad 3.37$$

where γ is the value of normal gravity.

In equation (3.36) above -

G = gravitational constant

M = the mass of the earth

R = the Earth's equatorial radius

r = distance from the earth's centre of mass

ϕ, λ = geocentric latitude and longitude

ω = the earth's angular velocity

$R_{nm} = P_{nm} \cos m\lambda$

$S_{nm} = P_{nm} \sin m\lambda$

P_{nm} and P_n are the Associated Legendre functions and Legendre Polynomials respectively.

Also the summation begins with degree 2 because the zero degree and the first degree terms are assumed to be zero. First degree terms are zero, because the undulations refer to a geocentric system. The assumption that the zero degree term is zero is equivalent to supposing that the reference spheroid has the same potential and mass as the earth. This is not generally true as the mass of the earth is not exactly known. Hence the computed geoid height is in error by a certain amount. This error can be evaluated by comparison with external sources, such as Doppler derived geoid heights.

The geoid figure derived by this method is in absolute position but is very highly generalised. This is due to the fact that at satellite heights, this technique cannot detect small-scale features of the geoid but only a general outline. The technique thus excludes all undulations of wavelength less than $2\pi/n$, where n is the highest degree of coefficients which have been determined. Thus if $n = 36$, the shortest wavelength to be determined is 10° or about 1000 km.

Also since there is the theoretical problem about the convergence of the series in spherical harmonic expansion |Heiskanen and Moritz, 1967|, the geoid figure by this method should also suffer from this inherent problem. In particular the poor quality of these coefficients makes the method seem unsatisfactory (for example the value $C_{6,2} = 0.0283$ with a standard error of ± 0.0396 |Rapp, 1969|).

The main advantage of this method is that the geoid figure gives some idea of the magnitude of the geoid heights that are expected from the gravimetric methods. In Canada this method has been used to estimate geoid heights required for reduction of the observations for the national geodetic network. By combining the satellite and terrestrial data such as gravity, it is possible to obtain a much detailed geoid for accurate reduction of distance and direction/angular measurements |Lachapelle, 1978|. However this method is very complex because of the fundamental problem in establishing efficient mathematical and statistical models that easily give stable solutions in the least squares adjustments of these hybrid data. It is uneconomical, requires high expertise, very time consuming and better suited for broad features of global geoid mapping.

(b) Satellite Doppler Positioning

The Doppler-derived three-dimensional co-ordinates of a point can be used to determine the height h, of the point above any specified spheroid. If the orthometric height H, of the point is known, then the geoid height is given by the equation

$$N = h - H \qquad 3.38$$

The accuracy of Doppler undulations is functions of the accuracies of Doppler ellipsoid heights and orthometric heights. The accuracy of ellipsoid heights on the other hand, is dependent on the ephemeris

used and the method of data acquisition - point positioning or translocation. The current estimate of absolute fix from Doppler observations using Precise Ephemeris is about 0.7 m in each co-ordinate. The accuracy of orthometric heights on the other hand, depends on whether the height is spirit (1 m or better), trigonometric (2 m) or barometric (3 m) levelled. Hence the accuracy of Doppler undulation varies between 1 and 3 m.

As with the astrogeodetic method, only point values are obtained by this method. And due to current distances, about 250 km, required for Doppler observations, this method provides geoid-heights that are too sparsely distributed to be used for deriving detail geoid within an area. However they are suitable (and in fact usually used) for comparison with geoid-heights derived from other methods.

In order to use Doppler results for comparison with other methods, the 3-D cartesian co-ordinates obtained in the Precise Ephemeris system have to be transformed into a properly defined geodetic reference system - a system whose axis of rotation is parallel to the CIO, and whose origin of longitude is the BIH zero meridian. The resulting co-ordinates can be transformed (if required) into spheroidal co-ordinates in the local geodetic system. These can easily be accomplished by using the transformation formulae in §§ 3.22 and 3.23.

The reference system which was used between January 1, 1973 and June 15, 1977 for the computation of Precise Ephemeris was based on the tracking station co-ordinates set, known as NWL 9D (Naval Weapons Laboratory), the gravity field model NWL-10E and the spheroid NWL-8E. The spheroid parameters are -

$$a = 6378\ 145\ m$$

$$f = 1/298.25$$

The three-dimensional cartesian co-ordinates in the Precise Ephemeris system are generally referred to as NWL 9D co-ordinates.

The NWL 9D Doppler datum was unique and independent. Its orientation was based on the longitude values adopted for the tracking stations, and the scale was dependent on terrestrial and camera-derived distances. Since the longitude origin of the system was arbitrary, a rotation (about the Z-axis) was applied to bring it into agreement with the gravimetric deflection in longitude of a properly defined datum. A correction was also applied to correct for a discrepancy in scale with respect to independent determinations, where the cause of the scale discrepancy is unknown. The recommended values of these corrections are : a longitude correction of 0".26 and a scale correction of -0.8 ppm (-5.27 m). The rotated and scaled system is termed the NWL 10F system. The accuracy of the scale is still doubtful to the extent of about 1 ppm [Bomford 1980]. The spheroid parameters are:

$$a = 6378\ 135\ m$$

$$f = 1/298.26$$

Since June 16, 1977 ~the Doppler positions derived from Precise Ephemeris are referred to the NSWC 9Z-2 co-ordinate system and the NSWC 10E-1 gravity model. However this change does not affect users. Also in use is the Doppler 78 system. This is a properly defined version of NWL 9D system. The system is obtained by applying a longitude correction of +0".80 and a scale correction of 0.38×10^{-6} to the NWL 9D system [Anderle, 1976, Ashkenazi, 1979, Hothem et al, 1978 and Olliver, 1979].

(c) Satellite Altimetry

A satellite carrying a vertically oriented radar altimeter can be used to measure the height of the satellite above the water surface, directly and continuously. These values can be used together with details of the satellite orbit to determine geoid heights relative to any specified spheroid.

Subject to the accuracies of the computed satellite positions and altimeter calibration, the detailed geoid so deduced is in absolute position, and has proper scale and shape. The present accuracy of the method is about ±0.3 m for blocks of 100 km, and its use is limited only to ocean areas.

3.4.4 Astrogravimetric Method

This method is basically a combination of all the desirable features of the astrogeodetic and gravimetric methods of computing geoid heights. In addition to the fact that it is not affected by any of the shortcomings of either methods, it does not require complete global coverage of gravity data as the influence of distant zones is not important.

In principle the method involves integrating Vening Meinesz formulas only over the neighbourhood of the point considered. The computed gravimetric deflections are used for interpolation between astrogeodetic deflections. If the absolute geoidal undulation is known at the datum, then in accordance with equation (3.31) the geoid of the area can be found by using the various values of ε obtained from the "astrogravimetric levelling" (the combination of astrogeodetic deflections with gravimetrically interpolated values). With this method, the astrogeodetic stations may be as far apart as 100 to 200 km. in countries (such as Nigeria) with gentle relief.

The theory of the astrogravimetric levelling is as follows: Since the integration of the Vening Meinesz formulas is not extended over the whole earth, then an error is introduced because the distant zones are neglected. For distances not too far apart, this error is the same, and varies only slowly over points of a short profile. Hence the reason for using it for interpolation between astro-geodetic deflections [Heiskanen and Moritz, 1967].

Suppose, ξ' and η' are the gravimetrically computed values, then the gravimetric deflections ε' (approximate) can be obtained from the usual relation

$$\varepsilon' = \xi'\cos\alpha + \eta'\sin\alpha \qquad 3.39$$

The differences $\delta\varepsilon$ between the "correct" astrogeodetic deflections and the approximate gravimetric values ' vary only slowly and may therefore be assumed to change linearly with distance. Hence these changes can be computed by a linear interpolation thus:

$$\delta\varepsilon = \varepsilon - \varepsilon'$$

$$\delta\varepsilon_P = \delta\varepsilon_A + \frac{\delta\varepsilon_B - \delta\varepsilon_A}{S_{AB}} S_{AP} \qquad 3.40$$

where P is any point on the profile between the astronomical stations A and B; and S is the distance between the points corresponding to the subscripts.

The practical computation of these deflections is as follows: At stations A and B, the astronomical deflections ε_A and ε_B are obtained by direct observations and subsequent computations. At these points and at other intermediate points P_1, P_2 .....P_n, the gravimetric deflections ε'_A, ε'_B, ε'_1, ε'_2, ..., ε'_n are evaluated using Vening Meinesz formulas. The differences $\delta\varepsilon_i$

at the intermediate points are interpolated by the last equation of (3.40). The desired deflections of the vertical ε at the intermediate points, referred to the astrogeodetic datum are computed by

$$\varepsilon_i = \varepsilon'_i + \delta\varepsilon_i \qquad 3.41$$

The geoid by this method acquires correct shape and absolute orientation from the gravity data employed. It obtains correct scale from the astrogeodetic parameters. It is highly suitable and accurate for mapping local details of the geoid. In addition, it is speedy and economical because it requires only a dense local gravity net in the mapping area alone.

3.5 Choice of the Geoid/Spheroid in Nigeria

The need for a geocentric datum for all survey work cannot be overemphasised. Some of the advantages of this datum have been outlined in § 3.1.6. Apparent advantages of non-geocentric datums, especially possibly slightly better local fit of the geoid, certainly do not outweigh the advantages of a geocentric system. The geocentric system can be rightly considered as a physically, meaningful and unambiguous definition of the co-ordinate origin, since its center is the same as the geocenter - a natural position. In practice, the geocentric datum has an immediate relation to global reference systems such as provided by satellite observations, for example Doppler satellite positioning. In particular, the geocentric datum satisfies most (if not all) of the needs of geodesy, oceanography and earth gravity modelling.

In the highly desirable case of a geocentric datum being acceptable to Nigeria as a reference spheroid, it is then logical that the method to be adopted for determining the geoid figure in the country, is the one that gives the geoid in absolute position. Also for the accurate reduction of distance and direction/angular measurements, the technique

should, in addition to meeting the requirements of scale and shape, be capable of giving detailed geoid figure.

The above requirements, coupled with the country's economy, available data and instrumentation, and the urgent need to improve the strength of the network, rule out the applicability of any of the techniques described except for the astrogravimetric method.

CHAPTER FOUR

SURVEY ADJUSTMENTS AND ERROR ANALYSIS

4.1 SURVEY ADJUSTMENTS

All survey measurements contain some errors which cannot be eliminated no matter the degree of precision of the observations. These are the so-called random errors and are caused by imperfections in the observer and the instrument, and the varying conditions at the time of observations. Observations which contain only these random errors can be adjusted to improve the geometrical consistency of the results. If the adjustment is properly carried out, the results give the most probable homogenous answer arising from the observed evidence. Three categories of adjustment are used in field survey, namely :

Arbitrary	)	
	)	
Semi-rigorous	)	adjustments
	)	
Rigorous	)	

Apart from brief explanations of the first two types of adjustment no further consideration of them is made in this thesis, because of their limitations in producing the "best fit" results of the observed quantities.

4.1.1 Arbitrary adjustments

These are based on assumptions chosen more for convenience than absolute correctness. Equal weights are usually given to good and poor observations. The observations are not "actually" corrected but the resultant errors are merely distributed round the figure being adjusted. The purpose of this type of adjustment is to make

the results consistent and not necessarily to produce the true nor the most probable answer. They are therefore employed only where the more elaborate adjustments are not justified. Examples of this type of adjustment are :

(a) Adjustment of a triangle

(b) Adjustment of a braced quadrilateral

(c) Adjustment of a centre point polygon

4.1.2 Semi-rigorous adjustments

These do not allow for the simultaneous adjustment of all the observations. The adjustment is carried out progressively so that a particular solution is influenced by the results of previous parts of the adjustment. In general this type of adjustment relies on semi-graphic techniques. Examples of this type of adjustment are :

(a) Semi-graphic intersection or resection (these are generally regarded as methods of fixation)

(b) The position line method used in field astronomy

(c) The Directions method - this is a semi-graphic method of computing co-ordinates and it produces a solution consistent with the existing control and the observations taken. Complicated weighting and mathematical adjustment of closed figures are not required. Though the method is designed to deal with angular measurements, it can be adapted for use with either distance measurements or mixed observations.

4.1.3 Rigorous adjustments

This type of adjustment allows for the simultaneous adjustment

of all the observations and is carried out according to the principle of least squares. The least squares principles states that the most probable values of measured quantities are those which make the sum of the squares of the residuals (or weighted squares of the residuals) a minimum. The object of the least squares adjustment is to change the values of a set of observations, whilst keeping to a minimum the sum of the squares of the differences between the observed and adjusted values.

Consequently any rigorous adjustment should give the most probable homogeneous results (best fit results) from a set of observations unless there are blunders (gross errors) or systematic errors that have not been eliminated before the adjustment. However if there are any systematic errors present in the observations, this method of adjustment still gives consistent results but these are not really the most probable values of the observations. Also this adjustment technique permits for the analysis of the results.

There are two distinct methods of deriving the most probable values from observations by the application of the principle of least squares - method of Condition equations, and method of Observation equations.

4.1.4 Condition Equations

This can be used when direct measures have been made on a number of unknown quantities which must exactly satisfy certain conditions. This method is applicable only when the number of unknowns is greater than the number of conditions. For adjustment by this method, condition equations are set up with the corrections to the observed quantities as unknowns. If a normal distribution of errors is assumed, the most probable set of corrections or

residuals is given according to the "least squares" principle by that set which minimizes the sum of the weighted squares of the residuals while satisfying the condition equations.

4.1.5 Observation Equations

These can be defined as those equations set up amongst a series of unknowns which are independent of each other and are subject to no restrictions other than those of the observations themselves. They can only be used when the number of observations is greater than the number of unknowns. There are two different adjustment techniques under this method - Classical Observation equations and Differential Displacements equations.

(A) Classical Observation Equations : In this case the unknowns in the equations are the unknowns themselves.

(B) Differential Displacements : In this case, initial values i.e. provisional values are assumed for all the unknown quantities in the network. Observation equations are then constructed to determine by least squares the amounts by which these provisional values are in error. If the errors (i.e. displacement) resulting from the adjustment are too large, the corrected values may be used as the next provisional values, and the process repeated (iterated) to further improve these estimates until the desired precision is achieved.

When the unknowns are the geographical or the cartesian co-ordinates of the stations in a network, the differential displacements method is known under the name of "variation of co-ordinates" [Ashkenazi, 1967]. The advantages of the variation of co-ordinates method in network adjustments are

well detailed in|Bomford, 1980|. In particular the computational process involved is very suitable for computer programming; and in addition to the most probable set of station co-ordinates, the variation of co-ordinates method leads to some very useful by-products. These include estimates of the accuracies of these co-ordinates and a set of residuals which can be used to detect possible blunders in the observation.

4.1.6 Relative merits of the observation and condition equations methods

The advantages and disadvantages of these two methods of adjustments are well documented in different geodetic literature. Prominent among them are (for the observation equation method) |Military Engineering Vol. XIII part III|:

The Advantages

(a) Separate station adjustment is unnecessary and no account need be taken of specific geometric conditions because they are taken care of in the adjustment which produces accepted co-ordinates of all stations.

(b) The method is flexible and can accept azimuth, angular, distance and Doppler fixes observations or any combination of them.

(c) It avoids the difficulties that can arise in the correct formation of the conditions and overcomes the problem of selecting the optimum route for the adjustment in large networks.

(d) Additional observations made subsequent to the adjustment can be incorporated relatively easily to produce a revised adjustment.

The Disadvantages

Considerable care has to be taken in the formation of the observation equations. This is particularly true of angle equations where it is quite easy to produce an incorrect sign for the absolute term.

4.2 Selection of Adjustment Method

The rigorous adjustment method is invariably used for the adjustment of geodetic networks. The unknown quantities to be determined are often the positions of the stations of the network or any other property related to these stations. The observations required to determine these unknowns usually include a large number of combinations of observations.

The very nature of survey work is such that there are invariably larger number of observations than the minimum required for the unique determination of the unknown quantities. The redundant (i.e. extra) observations serve both to guard against blunders and to obtain a (statistically) better estimate of the unknowns.

The method of observation equations using the variation of co-ordinates technique has been preferred for the present investigation due to reasons of simplicity, clarity and its better adaptability to programmed computations.

4.2.1 Mathematical Model (Method of Observation Equations)

Let L_i be the m independent observed quantities, and x, y, z,... the n unknown parameters to be determined. If $m > n$, as is usually the case with survey work, discrepancies in determining n appear. The discrepancies are eliminated by adding the residuals

v_i to the observation equations. Each observation gives an observation equation whose general form is

$$L_i + v_i = f(x, y, z \ldots\ldots) \qquad 4.1$$

where i = 1, 2, 3, ...m, and f represents a linear or non-linear function.

The method of least squares however demands that:

(i) f should be linear i.e. a linear relationship between the observations and the number of unknowns.

(ii) the number of equations (m) should be greater than those of unknowns (n)

Therefore equation (4.1) has to be linearized unless it is already linear. This can be done by the use of Taylor series and introducing such good approximate values of the unknown parameters x_o, y_o, z_o, ... such that the second and higher order terms can be neglected.

Thus equation (4.1) can be written as

$$L_i + v_i = f_i(x_o + dx, y_o + dy, z_o + dz, ...) \qquad 4.2$$

where

$$x = x_o + dx; \quad y_o + dy; \quad z_o + dz; \ldots\ldots$$

If the above equation is linearized by a Taylor series, one obtains

$$L_i + v_i = f_i(x_o, y_o, z_o, \ldots] + a_i dx + b_i dy + c_i dz + \ldots \qquad 4.3$$

Hence

$$v_i = a_i dx + b_i dy + c_i dz + \ldots\ldots\ldots + \ell_i \qquad 4.4$$

where

$$a_i = \frac{\partial f_i}{\partial x} \qquad b_i = \frac{\partial f_i}{\partial y} \qquad c = \frac{\partial f_i}{\partial z}$$

$$\ell_i = f_i(x_o, y_o, z_o \ldots\ldots) - L_i$$

Thus the general expression for an observation equation such as (4.4) can be written in the matrix form as

$$v = Ax + \ell \qquad 4.5$$

where

A is the m × n matrix of the (known) coefficients, also known as the design matrix

x is the n × 1 matrix of unknowns

v is the m × 1 matrix of (unknown) residuals (i.e. the amount by which the equation does not equal zero)

ℓ is the m × 1 matrix of the constant terms derived from the measurements.

If all the observations were perfect, the least squares process would make all the residuals zero.

Details of the observation equations for angle, distance, Laplace azimuth and Doppler fix, together with rules for their solution, are given in section 4.3 of this chapter.

4.2.2 Normal Equations

The least squares solution for finding the vector matrix x (of unknowns) is obtained by applying the condition V^TV = minimum. If the observations are not of the same accuracy and type the observational weight matrix W is introduced, and the condition is then (in matrix form)

$$V^TWV = \text{minimum}$$

The above condition if applied to equation (4.5) gives the so-called normal equations:

$$(A^T WA)x + A^T WL = 0 \qquad 4.6$$

Equation (4.6) is often written as

$$Nx = d \qquad 4.7$$

where N is the coefficient matrix of the normal equations.

The normal equation matrix N, has certain properties:

(i) It is symmetrical about the principal diagonal, and of size n × n where n is the number of unknowns.

(ii) It is positive definite.

(iii) It is non-singular. This ensures that there is always a (unique) solution of the normal equations.

4.2.3 Variance-Covariance Matrix

The observation weight matrix W is a diagonal matrix if the observations are uncorrelated with each other. Otherwise and in general W is given as |Ashkenazi, 1967|:

$$W = k|\sigma_{bb}|^{-1} \qquad 4.8$$

where k is an arbitrary scalar and $|\sigma_{bb}|$ the estimated or computed Variance-Covariance matrix of the observed quantities.

In geodetic literature the Variance-Covariance matrix σ_{xx} *of the vector of unknowns*

$$x^T = x_1\ x_2\ x_3\ \ldots\ldots\ x_n$$

is a symmetrical matrix of the form

$$\sigma_{xx} = \begin{bmatrix} \sigma^2_{x_1} & \sigma_{x_1x_2} & \sigma_{x_1x_n} \\ \sigma_{x_2x_1} & \sigma^2_{x_2} & \sigma_{x_2x_n} \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ \sigma_{x_nx_1} & \sigma_{x_nx_2} & \sigma^2_{x_n} \end{bmatrix} \qquad 4.9$$

Thus for two points i and j in a geodetic network, σ_{xx} is a 4 × 4 matrix. The diagonal elements are the two variances $\sigma^2_{x_i}$ and $\sigma^2_{x_j}$, and the non-diagonal elements $\sigma_{x_ix_j}$ and $\sigma_{x_jx_i}$ (at the intersections of the i-th and j-th rows and columns) are the covariances. The variance (also called mean square error, m s e) is usually denoted by σ^2 and is a statistical measure of the reliability of the co-ordinate values of a point in a network. The covariance usually denoted by $\sigma_{x_ix_j}$ (for a pair of points i and j) is a measure of statistical dependence of the co-ordinate or observation values of the two points. Therefore if the co-ordinates or observation values of a pair of stations in a network are uncorrelated, their covariance equals zero.

Two variances and their common covariance satisfy the relationship

$$\sigma^2_{(x_i \pm x_j)} = \sigma^2_{x_i} + \sigma^2_{x_j} \pm 2\sigma_{x_ix_j} \qquad 4.10$$

and in statistical language is read as [Ashkenazi, 1967]:

$$\text{var}(x_i \pm x_j) = \text{var } x_i + \text{var } x_j \pm 2\text{cov}(x_ix_j) \qquad 4.11$$

Also it is shown in |Ashkenazi, 1965| that if a geodetic network is adjusted by differential displacements resulting in the set of normal equations of the form stated in equation (4.6), then the variance-covariance matrix of the vector of unknowns x can be expressed as (in matrix form)

$$\sigma_{xx} = \sigma_o^2 (A^T W A)^{-1} \qquad 4.12$$

where σ_o^2 is the unit variance, better known as the "unbiased" estimate of the m.s.e. of an observation of unit weight (see § 4.4.2) and $(A^T W A)^{-1}$ is the inverse of the normal equations, N^{-1}.

In a simulated adjustment (see chapter five), ℓ and v of equation (4.5) are usually zero (assumed), but equation (4.12) is always available and its analysis is the only means of studying what is expected of the derived parameters. Such analysis is usually based on computations involving the elements of the variance-covariance matrix viz the variances and their related covariances. In matrix terms, this requires the computation of a number of the diagonal and non-diagonal elements of N^{-1}.

The variance-covariance matrix is therefore used for assessing the strength of an adjusted network as well as the strength of any planned or hypothetical network. The "matrix" allows the calculation of the different quantities (see section 4.5) frequently used in describing the positional accuracy of a station in a network.

However this form of analysis has an important drawback viz it is based entirely on the propagation of random errors. Consequently any unmodelled systematic errors existing in a network cannot be detected by such an internal consistency check - variance covariance analysis. For example, in the 1977 adjustment of the Nigerian network, the mean values of the a posteriori standard

errors in length and bearing were respectively 4 ppm and 0.8 arc seconds. These error estimates seem reasonably good but they do not however include such systematic errors as EDM scale (frequency or crystal calibration) errors, and reductions (to spheroid level) errors. An inspection of the few geoidal contours (above/below the Minna datum) has revealed that the geoidal undulations of the country range from +15 m to - 25 m. Any failure to account for these will lead to substantial scale errors (see § 3.3.2). It is worth noting that the geoidal contours mentioned above have been derived from the CFL traverse and the five Doppler results |Obenson and Fajemirokun, 1979|.

4.2.4 *Solving for the inverse of the Normal Equations*

Since the normal equation matrix N is non-singular, its inverse exists. Thus both sides of equation (4.7) can be multiplied by N^{-1} to give

$$N^{-1}Nx = N^{-1}d \qquad 4.13$$

But by definition

$$N^{-1}N = I \text{ (the identity or unit matrix)}$$

and hence the solution for x is obtained.

Any inverse of a non-singular matrix (such as N) with given elements can always be computed from the definition

$$N^{-1} = N^*/|N| \qquad 4.14$$

where

N^* is the adjugate matrix of the square matrix N

$|N|$ is the determinant of the matrix

It is usually laborious to obtain the inverse by using the above formula, especially when the matrix is of order greater than 3 × 3 [Thompson, 1969]. An alternative method which saves labour in computation is to solve the linear equations

$$Nx_j = e_j \qquad 4.15$$

for j = 1, 2, 3, ... , n and e_j is the jth column of the identity matrix of order n.

By using the same procedure as for equation (4.13), one obtains

$$N^{-1}Nx_j = N^{-1}e_j \qquad 4.16$$

And since $N^{-1}N = I$, it is easily verified that $N^{-1}e_j$ is the jth column of the inverse. The NAG library routine (see chapter six) used for calculating the inverse in this thesis makes use of the above principles. The routine also incorporates Cholesky's triangular decomposition technique in solving the equations.

4.2.5 Choleski's symmetric decomposition method (Principles)

The principle of this method is that any square matrix such as the normal equation matrix N can be expressed as the product of an upper and a lower triangular matrices. In the particular case of a symmetric matrix, these triangular matrices are the transpose of each other and therefore N can be written as

$$N = L^T L \qquad 4.17$$

where L is an upper triangular matrix. If the equations to be solved are

$$Nx = d$$

they can be re-written, using the above decomposition process, as

$$\left.\begin{array}{rcl} L^T L x & = & d \\ & & \\ L^T f & = & d \end{array}\right\} \qquad 4.18$$

or

where $Lx = f$

The "triangular decomposition" method has therefore three distinct stages:

(i) decomposition of the matrix N into the product of a lower triangular matrix and its transpose i.e. $N = L^T L$.

(ii) a forward substitution process, $L^T f = d$ to determine the values of the vector f.

(iii) a back substitution process, $Lx = f$ to obtain the vector x.

The necessary and sufficient condition for the decomposition of a matrix into upper and lower triangular matrices is |Thompson, 1963| that the matrix should be non-singular and positive definite.

4.2.6 Positive definiteness

The matrix of coefficients of geodetic normal equations resulting from the least squares procedure is (in the case of observation equations) of the form $A^T A$. And because the normal equation matrix has the property of positive definiteness, then the following expression holds |Thompson, 1969|:

$$y^T (A^T A) y = (Ay)^T (Ay) > 0$$

for any real and non-zero y. Also to prevent the vector (Ay) from being a null vector (because this causes algebraical singularity and makes it impossible to obtain a unique solution) a proviso is made to the above condition viz for the vector (Ay) to be always a non-zero vector, the matrix A must have a rank equal to its smaller dimension.

In survey adjustments, the smaller dimension of the matrix A is equal to the order of the normal equation matrix. In general the coefficient-matrix N of the normal equations may be said to be singular and of rank n-k, where n is the order of N and k is the defect of the network.

For a two-dimensional triangulation network, with measured distances and azimuths, the only defect is position, and hence $k = 2$. This corresponds to saying that the network has 2 degrees of freedom. Therefore to prevent the algebraical singularity caused by this rank deficiency (thus satisfying the above proviso), the value of k should be made zero. In practice this is achieved by defining the position of any station in the network i.e. by holding it "fixed".

However, in a special case, when the network includes several absolute or relative terrestrial co-ordinate determinations (obtained by satellite-Doppler techniques) these could be regarded and treated as "position observation" equations whose inclusion in the adjustment, together with the rest of the observations, would automatically remove all likely types of network singularity |Ashkenazi, 1975|.

In this thesis, a station was chosen as the origin (i.e. a fixed station). The choice of the station is completely arbitrary as the criteria used for the error analysis are invariant with the origin |see § 4.5.3|.

4.3 Structure of the Observation Equations - Variation of Co-ordinates technique

The Variation of Co-ordinate method involves basically the computation of provisional (approximate) co-ordinates for all the stations of the network, and the setting up of observation equations,

one for each observed quantity. The resulting equations are then solved simultaneously by least squares procedure to produce the most probable values of the desired quantities. Each observation equation describes mathematically the relationship between an individual observation of any type (i.e. angle, distance, azimuth and doppler fix) and its related station. The unknowns in the equations (for a 2-dimensional network) are the shifts in co-ordinates $\delta\phi$ and $\delta\lambda$ from the provisional values. Thus the general form of an observation equation connecting stations i and j may be expressed in the form:

$$E\delta\phi_i + F\delta\lambda_i + G\delta\phi_j + H\delta\lambda_i + T_{ij} = v \qquad 4.19$$

where

E, F, G, H are the coefficients which depend upon the type of observations.

T_{ij} is the absolute term and also depends upon the type of observation

v is the residual or the amount by which the equation fails to be satisfied.

In an adjustment of a control network, the set of equations of the form (4.19) describe different and physically dissimilar observed quantities which are usually in angular or linear units (i.e. seconds of arc, and metres). An in order to solve the resulting normal equations simultaneously, the different measurements should be compatible as far as the units of measure are concerned. This condition is usually fulfilled by the use of dimensional weight factors (see § 4.3.5 for discussion on weights).

In theory normal equations are formed by first multiplying each observation equation by the square root of the corresponding

weight and then by using the Gaussian summation algorithm. The weight of an observation is usually defined by the reciprocal of its mean square error (i.e. its variance including physical dimensions). Therefore the multiplication of an observation equation by the square root of its weight, turns it into one of unit weight and with dimensionless elements. Furthermore, failure to assign dimensional weights to any group of observations would amount to discarding the contribution of that group of observations from the simultaneous adjustment [Ashkenazi, 1970].

4.3.1 Rules for the formation of the observation equations

In order to avoid placing too much "weight" on any particular observation and also to avoid confusion in selection, the following rules were applied in the formation of the observation equations used in this thesis:

(a) "Observed" Laplace azimuth

At any Laplace station, all the rays (directions) emanating from the station are treated as observed directions and an azimuth equation was formed for each ray. The δZ term was omitted in each equation (as is often the case) because the structure of the azimuth equation accounts for it.

It is worth noting that the above procedure is the common practice (assuming Laplace azimuth is treated as fallible) even if the method of angles are used |Bomford, 1980|. Also it is stated in |Military Engineering, Vol. XIII, part VI, 1978| that at any azimuth station, all the angles at the station should be adjusted to close the horizon, and an azimuth equation generated for each radial line from that station.

The condition that a station adjustment should be performed before the formation of the azimuth equation appears to contradict the provision in § 4.1.6(a). However this criticism seems to be the only possible way of eliminating the inconsistencies that may occur in forming two types of angular observation equation (i.e. one for angle and one for azimuth) for the same unknowns.

(b) Angle equations

For a station situated "inside" a network (centre point figures) angle equations were formulated for each angle within the full 360° horizon (hence the use of EXTERNAL statement in the computer program § 6.1.7). And for the stations on the edges of the network, angle equations were formed for the angles which were internal to the network.

(c) Known or Fixed stations

In practice, the effect of a fixed station is to provide fixed opening co-ordinates and azimuth for other stations in the network. For this reason, azimuth equations were formed for all rays leaving the fixed station. Also it is to be remarked that the unknowns $\delta\phi$ and $\delta\lambda$ corresponding to a fixed station are omitted from all observation equations which would otherwise include them.

(d) Doppler Fixes

Doppler observations are usually regarded as "absolute position observations". Thus the observation equations are limited to only those stations they define. Consequently two observation equations were formed for each station where the Doppler observations were projected.

4.3.2 Differential formulae

Small changes $\delta\alpha_{ij}$ (seconds) in azimuth and $\delta\ell_{ij}$ (metres) in length that occur due to variations in the provisional co-ordinates of the two end stations of a line ij, are given by the following differential formulae |Ashkenazi et al, 1972|:

$$\left.\begin{aligned}\delta\alpha_{ij} &= K_{ij}\delta\phi_i + L_{ij}\delta\lambda_i + M_{ij}\delta\phi_i + N_{ij}\delta\lambda_j \\ \text{and}\qquad \delta\ell_{ij} &= P_{ij}\delta\phi_i + Q_{ij}\delta\lambda_i + R_{ij}\delta\phi_i + S_{ij}\delta\lambda_j\end{aligned}\right\} \qquad 4.20$$

where $\delta\phi_i$, $\delta\lambda_i$, $\delta\phi_j$ and $\delta\lambda_j$ are the variations in the provisional co-ordinates.

The numerical coefficients K, L, M, N, P, Q, R and S are computed from the following formulae:

$$K_{ij} = \left(\frac{\rho_i}{\ell_{ij}}\right)\sin\alpha_{ij}$$

$$L_{ij} = -N_{ij} = \left(\frac{\nu_j}{\ell_{ij}}\right)\cos\phi_j\cos\alpha_{ji}$$

$$M_{ij} = \left(\frac{\rho_j}{\ell_{ij}}\right)\sin\alpha_{ji}$$

$$P_{ij} = \sin1''(-\rho_j\cos\alpha_{ij})$$

$$Q_{ij} = -S_{ij} = \sin1''(\nu_j\cos\phi_j\sin\alpha_{ji})$$

$$R_{ij} = \sin1''(-\rho_j\cos\alpha_{ji})$$

Details of the proof of the expressions in (4.20) and the practical assessment of their precision are given in |Olliver, 1977|. The

resulting maximum error limits to geodetic positions when these expressions are used in an adjustment are [Olliver, 1977]:

0.05" for lines of 20 km

0.15" for lines of 50 km

0.50" for lines of 200 km

The above limits represent 1.5, 5 and 15 metres approximately at latitude 0°.

4.3.3 The Observation Equations

Using the differential formulae of (4.20), the observation equations may be as follows :

(a) Observed direction ij

$$K\delta\phi_i + L\delta\lambda_i + M\delta\phi_j + N\delta\lambda_j - \delta Z_i = (0 - c)_{ij} + V_{ij} \quad 4.21$$

where

δZ_i is the correction to the orientation parameter Z_i (the orientation parameter accounts for the possible instability of the station marks which may occur if the observations are made at different dates); $(0 - c)_{ij}$ is the difference between the observed theodolite direction 0, and the computed azimuth of the line c, using provisional co-ordinates and V_{ij} is the correction to the observation. It is worth noting that the unknown δZ_i is usually multiplied by a factor in order to make its coefficient of the same order of magnitude as the other coefficients in the formula. If this factor is 100, then the computed value of δZ is in units of 0.01" instead of the whole seconds. Also the orientation correction term

is retained in all the observation equations which correspond to the theodolite direction observations taken at the fixed station. However this type of observation equation was not considered in this thesis because the adjustment was made by the "angle method".

(b) Clockwise angle jik

$$(K_{ij} - K_{ij})\delta\phi_i + (L_{ik} - L_{ij})\delta\lambda_i - M_{ij}\delta\phi_j - N_{ij}\delta\lambda_j + M_{ik}\delta\phi_k + N_{ik}\delta\lambda_k = (0 - c)_{jik} + V_{jik} \qquad 4.22$$

where $(0 - c)_{jik}$ is the difference between the observed angle 0, and the corresponding angle c, computed from the provisional co-ordinates; V_{jik} is the correction to the observed angle.

(c) Measured distance ij

$$P_{ij}\delta\phi_i + Q_{ij}\delta\lambda_i + R_{ij}\delta\phi_j + S_{ij}\delta\lambda_j = (0 - c)_{ij} + V_{ij} \qquad 4.23$$

where $(0 - c)_{ij}$ is the difference between the measured length 0, and the corresponding length c, computed from the provisional co-ordinates.

(d) Laplace Azimuth ij

$$K_{ij}\delta\phi_i + (L_{ij} - \sin\phi_i)\delta\lambda_i + M_{ij}\delta\phi_j + N_{ij}\delta\lambda_j = (0^G_{ij} - C^G_{ij}) + V_{ij} \qquad 4.24$$

where

C^G_{ij} = azimuth computed from provisional co-ordinates, and

0^G_{ij} = observed geodetic azimuth

This geodetic azimuth can be obtained directly by Black's method or deduced from Laplace's equation :

$$\alpha^{G}_{ij} = \alpha^{A}_{ij} - (\lambda^{A}_{i} - \lambda^{G}_{i})\sin\phi_{i} \qquad 4.25$$

where

α^{A}_{ij} = observed astronomical azimuth

λ^{A}_{i} = observed astronomical longitude, and

$\lambda^{G}_{i} = \lambda_{i}$ = geodetic (provisional) longitude, positive east.

Formulae (4.24) and (4.25) allow for the change in the value of the provisional geodetic longitude of the Laplace station as a result of the adjustment, and the corresponding change in the final geodetic azimuth of the line.

(e) Doppler observations

$$\delta\phi_{i} = v$$
$$\delta\lambda_{i} = v \qquad 4.26$$

The above equations indicate that the Doppler observations are used as provisional co-ordinates. The reason for this is that satellite-Doppler observations are absolute position measurements and are therefore used as constraints for geodetic networks.

4.3.4 Accuracy of Provisional Co-ordinates

The required provisional geographical co-ordinates may be obtained as follows:

(a) By forward computation - this involves computing the co-ordinates from a known station using observed or deduced values of azimuth and distance, and the spheroidal parameters. Any approximate formulae such as the mid-latitude formulae, are adequate for the computations.

(b) From maps - this involves direct measurement of the co-ordinates of the required stations from a map, preferably large scale maps.

The accuracy of these co-ordinates does not matter greatly because the variation of co-ordinates method easily allows for any iterative process viz if the computed corrections are too large, the corrected co-ordinates are used as new provisional co-ordinates and the whole process repeated until the required precision is achieved.

However, if the computed (or scaled), approximate co-ordinates are such that the errors in the computed lengths and azimuths do not exceed 1 part in 4000 and 1 minute respectively, the resulting co-ordinates are of sufficient accuracy for most geodetic purposes and no iteration is necessary |Bomford, 1980|.

In this thesis, the provisional co-ordinates used for all the "adjustments" are the values obtained from the 1977 adjustment. They therefore satisfy the requirements in the preceding paragraph.

4.4 Dimensional Weights (Weight Matrix) and Unit Variance.

It has been stated in (§ 4.3) that dimensional weights are usually introduced into a network adjustment in order to make the different observations having unidentical dimensions compatible for a simultaneous solution. If on the other hand, these weights are merely numerical factors without any physical dimensions

(including the case when the weights are ignored altogether) then the Gaussian summation process for forming the normal equations will consist (in the case of triangulation network with measured angles and distances) of the absurd task of adding together numerical coefficients with different physical dimensions - seconds of arc and metres |Ashkenazi, et al, 1972|. Moreover the dimensional weight factors express the relative accuracy of the different observations within a group of observations with identical dimensions and also between one group and another.

However there are two special cases in which the implicit use of dimensional weights can be avoided |Ashkenazi et al, 1972|:

(i) When the standard error of all the angular measurements, in seconds, is equal to the standard error of all measured distances, expressed in the same units as the distances themselves. This is an unlikely situation and is rarely obtainable in practice.

(ii) When the standard error of all the angular observations, is ±1", then the distance equations can be (with sufficient accuracy) multiplied by the factor.

$$\frac{4.848}{p.D.\sin 1''}$$

where D is the length of the line and p is the number of ppm of the distance in which the corresponding standard error is expressed.

The above factor is easily seen as the exact arithmetic equivalent of the square root of the ordinary weight factor for distances viz

$$\frac{1}{(p.D.10^{-6})}$$

Again this second alternative has no significant advantage over the use of dimensional weight factor; instead it could lead (through misinterpretation) to mistakes.

Thus there is virtually no adequate substitute for dimensional weights in a network adjustment.

4.4.1 The Weight Matrix

The weight matrix W (§ 4.2.3) is generally accepted as the inverse of the variance-covariance matrix of the observations |Thompson, 1969; Cross, 1972|. In other words the weight matrix is the inverse of a matrix whose diagonal elements are the variances of the observations and whose non-diagonal elements are the observational covariances. And because errors in the estimation of weights cause errors in the final adjusted co-ordinates of the stations in a network |Cross, 1972|, then it is necessary that the elements of the weight matrix should be accurately estimated. Presently it is known that the covariance elements in this weight matrix are not generally zero, thus indicating some measure of correlation within the elements. Unfortunately it has not been possible to represent this correlation mathematically. Consequently it is usually assumed that these elements (or the observations) are uncorrelated i.e. the covariance elements are zero. The weight matrix therefore becomes a diagonal matrix, each diagonal element being the reciprocal of the variance of the observation to which it corresponds. And since variance is also defined as the square of the standard error σ, the problem of weighting reduces (in practice) to one of estimating the standard errors of the observations.

4.4.2 Unit Variance

Usually it is not possible to make precise estimate of the standard errors of observations (and hence the weight) because all

the physical factors affecting geodetic observations cannot be accounted for in practice. Hence an assessment of the precision of the estimated weights is often made after every adjustment. The method usually used is based on Gauss's formula for the unbiased estimate of the variance of an observation of unit weight, viz

$$\sigma_o^2 = \frac{\Sigma(v^T W v)}{(m - n)} \qquad 4.27$$

where $(v^T W v)$ is the weighted square of an individual residual and $(m - n)$ is the number of redundant observation equation.

Theoretically a value of σ_o^2 equal to unity should result if the a priori (observational) standard errors of the various quantities used in the network adjustment have been assessed on average correctly. In practice this objective is never attained because of the limitations (stated above) in assessing the various weights precisely. However a small departure of the derived σ_o^2 from unity is usually considered adequate for most geodetic networks.

In a simulated adjustment, the residuals v are not computed and hence it is impossible to compute σ_o^2. But since the adopted standard errors in simulated observations are usually assumed to be correct, then the value of σ_o^2 can be considered to equal unity. Ashkenazi |1970| confirmed the above assumption in the following words: "When dealing with computer simulated hypothetical observations and estimates of their variances and covariances, one can assume σ_o^2 to equal unity and drop it altogether from the equation $\sigma_{xx} = \sigma_o^2 (A^T W A)^{-1}$".

4.4.3 Criteria for Weight Estimation

There is as yet no unique method for estimating the actual numerical values of the a priori standard errors of observations.

Different methods therefore exist for estimating these errors, and they include :

(a) Ferrero's formula, based on the average triangular misclosure.

(b) Side equation misclosures

(c) A method based on Gauss's formula for σ_o^2

(d) Realistic assessment of the standard errors, based on past experience with other similar cases and other vital considerations.

(A) Ferrero's formula

This is probably the best known and the most satisfactory existing classical method for estimating the actual numerical value of the m s e of a theodolite bearing or angle |Ashkenazi, 1970|. It is based on the average triangular misclosure given by :

$$\sigma_o^2 = \frac{\Sigma(\Delta^2)}{3n} \qquad 4.28$$

where

σ_o^2 is the unbiased estimate of the mean square error of an observed angle

Δ is the triangular misclosure after correcting for spherical excess.

n the number of triangles considered.

Provided each angle is used in only one triangle, the above formula generally leads to an unbiased estimate of the variance *of an observed angle.*

The method has the following fundamental drawbacks :

(a) The ease with which triangular misclosures can be calculated could easily lead to biassing the results of the observations in the field in order to achieve triangular misclosures considered to be within the "permissible limit"

(b) The formula does not take into account the side equation discrepancies which, although less numerous than the number of triangular misclosures, can nevertheless be expected to contribute their share to the average angular standard error.

(c) There is no Ferrero type formula for calculating estimates of the standard error of distance.

(B) Side Equation misclosures

This method is based purely on side equations excluding the measured distances. The unbiased estimate of the variance of an observed angle, σ_o^2 is given by

$$\sigma_o^2 = \frac{1}{n}\sum\frac{(\varepsilon.206265)}{\Sigma\cot^2\alpha} \qquad 4.29$$

where

ε is the (dimensionless) misclosure of an individual side equation.

$\Sigma\cot^2\alpha$ is the sum of the squares of the cotangents of all the angles taking part in that side equation.

n is the number of side equations in the formula.

The disadvantages of the method are [Ashkenazi, et al, 1972]:

(a) It involves extra computations - the data on the side equations, when the method of variation of co-ordinates is used for the network adjustment.

(b) The computed variance is generally larger than the variance derived by using Ferrero's formula. This is probably due to the fact that the number of side equations in a network is too small to give a sufficiently accurate estimate of the variance of an observed angle.

(c) Again the formula has no corresponding expression for estimating the variance for distance.

(C) Gauss's formula for σ_o^2 method

The method is based on the principles of Gauss's formula that if the correct weights are assigned to all observed quantities, the resulting variance of unit weight is unity. Thus a result larger than unity would suggest an under-estimation of the initial variances and a corresponding over-estimation of the quality of the observations. For example if the value larger than unity were p, this would mean that the initial variances (m.s.e.) have been under-estimated p times. If then the adjustment were re-done with the new estimates, the new σ_o^2 would equal unity.

For a triangulation network adjustment, computations using this method are usually carried out in the following iterative steps |Ashkenazi, 1970; Ashkenazi and Dodson, 1978|:

(a) One station (the origin) is held fixed, and assuming unit weight throughout, the network is adjusted simultaneously with all the angular observation equations but only one distance and one azimuth equation. The resulting value for σ_o^2 is the variance of an observed angle (or direction, if the direction method is used for the adjustment). This should be very close to the value obtained by using Ferrero's formula (as a check).

(b) With a station held fixed, the network is readjusted with all the angular observations and their new variances (derived from above), all the distance equations and their respective variances, but only one azimuth equation. Any departure of the resulting value for σ_o^2 from unity will now entirely be due to the poor estimation of the standard errors of the distances. The adjustment is repeated with successively "improved estimates" until the value for σ_o^2 is sufficiently close to unity. This is the variance of an observed distance.

(c) Lastly, the Laplace azimuths are introduced with their initial estimated standard errors (usually obtained from internal assessment only), and holding only one station fixed, the iterative cycle is repeated as with distances until the value for σ_o^2 is again sufficiently close to unity.

The main disadvantage of this method is that it is correct to the extent to which the observations are normally distributed. Small values of v (and hence a unit variance close to unity) indicate that the observations are normally distributed. Thus the method gives good estimates for angles and distances which are usually large in number in a triangulation network but not for the very limited number of Laplace azimuths.

Also the amount of computation involved is very large and therefore the only answer to the problem is an electronic computer.

(D) Weight estimate based on experience

This is basically an "intelligent guess" technique and is mainly based on past experience in assessing various types of observations. It usually depends on consideration of many factors such as the manufacturer's estimates and internal discrepancies of the observed quantities.

4.4.4 Remarks

Methods (D) and (C) in the preceding section were used by the Ordnance Survey and the Nottingham University respectively in estimating the weights of observations for the Readjustment of the Retriangulation of Great Britain - the Ordnance Survey Great Britain Scientific Network 1970 (OSGB 1970(SN)). The results show close agreement in terms of adjusted co-ordinates and adjusted lengths but not in the values of the adjusted azimuths where they are different for some lines. Probably the observation of more Laplace azimuths would remedy the discrepancy |Ashkenazi et al, 1972|. Ferrero's formula was used in estimating the standard errors of angles used for the 1977 adjustment of the Nigerian network |Ashkenazi and Field, 1977|.

4.4.5 The a priori standard errors of observation used in this thesis

For the simulation tests carried out in this thesis, all the existing observations in the Test Network were assigned the a priori standard errors that were used for the 1977 adjustment (Nigerian network). These weights were considered appropriate for this investigation because -

(a) The computed σ_o^2 has the value 1.367 which indicates that the weights were on the average correctly estimated.

(b) It was the effect of additional observations on the current strength of the network that was being investigated. Any changes in the way in which the various observation equations were weighted would lead to a different set of results. In effect, a new network of different strength is created.

The additional observations (assumed) were weighted according to the projected instrument/technique to be used for the observations.

The weights therefore used were in accordance with the current estimates obtainable in practice from such instruments/techniques.

The instruments/techniques together with their respective standard errors are described in chapter two of this thesis.

Thus the estimated standard errors (dimensional weights) for the investigation are as follows:

(i) Existing Observations

Angles, σ_α = 0.7"

Laplace azimuths, σ_α = 1".0 (All azimuth stations in the Test Network are Laplace stations)

Distances - the following relationship was used to calculate the standard errors of "measured" distance ℓ:

$$\sigma_\ell = p.D.10^{-6}$$

where p = estimated s.e. in parts per million of a measurement.

D = length of the line.

The values of p for the different measurements considered are :

Invar tape	:	2.0 ppm
Geodimeter	:	2.0 ppm
Tellurometer	:	3.5 ppm

(ii) Additional Observations

Geodetic azimuth (Laplace azimuth) by Black's method = 0".7

Distances (measured with Geodolite) = 2.0 ppm

Doppler observations = 0.7 metres in each of the two co-ordinates ϕ and λ. And expressed in angular units this is equivalent to 0.0227"

It is to be remarked that the estimates of the r.m.s. of lines measured with EDM instruments consist of a constant part and a systematic component depending on the length of the line. For short distances of up to 10 km, the constant component appears to be overwhelmingly dominant, and should therefore be used as the estimated weight, but for long distances (greater than 10 km), the systematic part dominates. In this work and also in most geodetic networks, the measured distances are often longer (or at least equal to) 10 km, and hence the systematic component is usually used |Chrzanowski, 1977|.

4.5 Criteria for Strength Analysis

The current method for assessing and analysing the strength of a network is based on the variance-covariance matrix. It serves as a basis for calculating the absolute and relative positional errors of adjusted quantities. From the matrix, it is possible to calculate the following quantities frequently used in describing the positional accuracies :

(a) Standard errors of co-ordinates or differences in co-ordinates.

(b) Semi-major and semi-minor axes of standard error ellipses (absolute or relative).

(c) Standard errors of directions (azimuths), and/or angles, and distances calculated from adjusted co-ordinates (i.e. the a posteriori standard errors).

4.5.1 Standard Error of a co-ordinate value

This is the positive square root of the corresponding variance. It is also called standard deviation or root mean square error (rms). Standard error, usually denoted by σ, represents a probability

(confidence level) of 68% that the difference between the calculated (given) co-ordinate and its true value is within $+\sigma$ and $-\sigma$ interval. The confidence level is increased to 95% or 99%, if the standard error is multiplied by 1.96 or 2.58 respectively.

Standard errors of co-ordinates (ϕ and λ for spheroidal co-ordinates, and x and y for rectangular co-ordinates) of a point describe the positional accuracy of the point only in directions of the areas of the co-ordinate system. Usually it is the maximum and minimum standard errors (σ_{max} and σ_{min}) for the point and their respective azimuths (directions) ψ and $(\psi + 90^{o})$ that are of interest for any analysis. These values can be computed directly from the two variances and their joint covariance. The variances and their joint covariances for any adjusted point can be obtained from the inverse of the normal equations (i.e. the variance-covariance matrix). If the adjusted co-ordinates are in spheroidal co-ordinates (as in this thesis), the following expressions are used for the computations |Ashkenazi and Cross, 1972|:

$$\tan 2\psi = 2\sigma_{\phi\lambda}/(\sigma_{\phi}^{2} - \sigma_{\lambda}^{2}) \qquad 4.30$$

where the azimuth ψ of the σ_{max} lies between 0^{o} and 180^{o}.

and

$$\left.\begin{aligned} \sigma_{max}^{2} &= \sigma_{\lambda}^{2} + \sigma_{\phi\lambda}\cot\psi \\ \sigma_{min}^{2} &= \sigma_{\lambda}^{2} + \sigma_{\phi\lambda}\cot(\psi + 90^{o}) \end{aligned}\right\} \qquad 4.31$$

Also in an adjustment on the spheroid, the variance-covariance elements are in seconds of arc, and therefore the above values have the same units. For convenience the azimuth is expressed in degrees and fractions of a degree, and the variances (and hence standard errors) converted into metres. Before converting to metres, σ_{λ}^{2} and $\sigma_{\phi\lambda}$ should be multiplied by $\cos^{2}\phi$ and $\cos\phi$ respectively in order to relate them to the variation of scale of a unit of meridian with latitude.

The positional accuracy of any adjusted point computed from the above formulae is usually given in a graphical form (visual representation) by the absolute (point) error ellipse or sometimes relative/line error ellipse.

4.5.2 Standard Error Ellipse (absolute or relative) of a point

(A) This is an ellipse described by σ_{max} and σ_{min} as the semi-major axis a, ($a = \sigma_{max}$) and the semi-minor axis b. If this ellipse is drawn around a point, it is usually interpreted as depicting the region in which one has 39% confidence that it contains the position of the corresponding point determined from errorless observation. If an error ellipse is calculated on a basis of variances and covariances of differences of co-ordinates of two adjusted points, it is referred to as relative standard error ellipse. The relative error ellipse expresses the uncertainty in the relative position of one of the two stations with respect to the other.

After the two extreme variances of an error ellipse are determined, the variance along any other direction, subtending a clockwise angle θ with the direction of σ_{max} is obtained from the following expressions |Ashkenazi and Cross, 1972|:

$$\sigma_{\theta}^2 = \sigma_{max}^2 \cos^2\theta + \sigma_{min}^2 \sin^2\phi \qquad 4.32$$

The curve whose polar co-ordinates are θ and σ_{θ} (radius vector) is known as the Error curve (also called pedal curve). Thus the error curve describes the positional standard deviations σ_{θ} in any desired direction (Fig. 4a). Graphically the error curve is found by determining its points as an apex P of a right angle between the direction line of θ and the tangent line PR to the error ellipse.

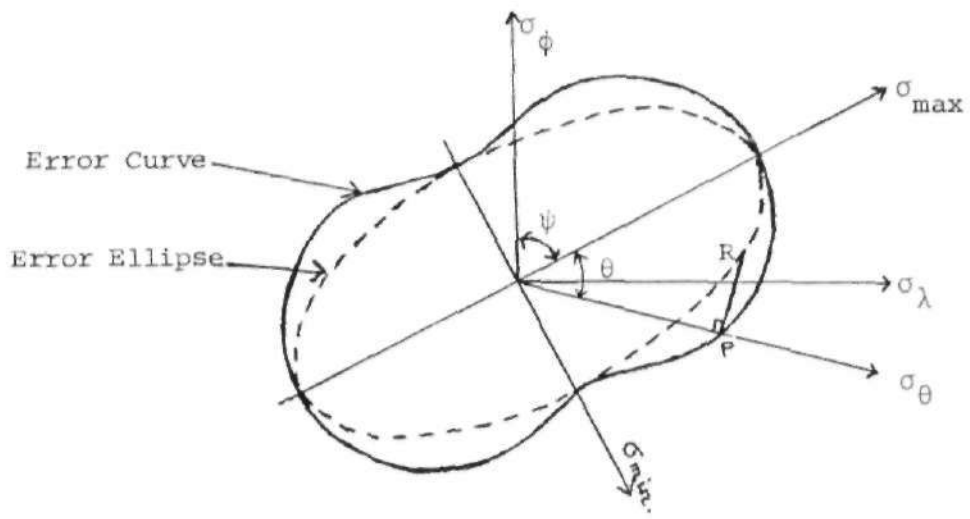


Fig 4a

(B) Properties of the standard Error Ellipse

It has been demonstrated in some geodetic literature |Ashkenazi, 1974; Ashkenazi and Cross, 1972; Ashkenazi and Dodson, 1978|, that the absolute error ellipse of a station (point) expresses the positional uncertainty (or strength) of the co-ordinates of that station with respect to a reference system. This reference system may be defined in terms of one or more fixed stations (the origin). Any different set of parameters chosen for defining the origin results to a different reference system (see §§ 3.1.3, 3.1.4), and hence the variation in sizes of the error ellipses. It has been found that the sizes of these ellipses tend to increase in proportion to their distances from the fixed origin. Thus the absolute error ellipses provide no information about the covariances or correlations between the co-ordinates of different stations and consequently do not provide data for computing the inter-station errors.

Except for the case of absolute position measurements (such as those resulting from satellite-Doppler observations with respect to an earth's mass centred reference system) the absolute error ellipse (and hence the standard error of position) cannot therefore be used to give any quantitative definition of the strength of a network.

In spite of the above disadvantages the shapes and orientations of error ellipses in a network serve a useful purpose in a qualitative treatment of the strength of a network. For example, if the axes of the ellipses are extremely unproportional, it indicates an overwhelming weakness in either scale or orientation, depending on the alignment of the major axes. A direct alignment towards the origin indicates scale weakness, whereas perpendicular alignment to the origin would indicate orientation weakness. On the other hand, near-circular ellipses indicate a balance in scale and orientation errors. Because of this characteristic feature of an error ellipse, it has been used in this thesis to provide only a visual presentation of the strength of some simulated models.

The relative error ellipse, on the other hand, does not depend on the origin (i.e. it is invariant with respect to changes in the reference system used). Its two components, along the line joining the two stations and normal to it, are directly related to the standard errors in length and in bearing of that line [Ashkenazi, 1974]. However the relative error ellipse has no direct significance to the observed quantities (angles, azimuth, and distance) and is therefore not particularly useful as a practical criterion which would help in pin-pointing the precise sources of local geometrical weaknesses in a network. For this reason, the relative error ellipse has not been considered in this thesis for assessing the strength of the test models.

4.5.3 The a posteriori standard errors - angles, azimuth and distance

The standard errors of observed quantities (adjusted) - directions or angles, azimuths, and distances, are known to be invariant with the origin. In other words they involve only the stations which define the network and have no connections to the "arbitrarily" defined reference system. These standard errors are usually deduced from the variance-covariance matrix (4.12) of the adjusted network.

Obviously a change in origin of a reference system would affect the resulting absolute co-ordinates of the stations and hence the elements of the variance-covariance matrix. However this change does not alter the relative geometry of the stations and hence does not affect their relative accuracies. Therefore a practical and most useful/meaningful criterion for assessing the strength of a network is that which involves only the relative positions of the stations of a network - the a posteriori standard errors of the observed quantities, Ashkenazi |1975| observed the above assertion with the following remarks "..... these quantities remain invariant even when they are deduced from an entirely different variance-covariance matrix resulting from the adoption of a different co-ordinate reference system".

Furthermore a classical geodetic network may be considered as a network made up of a system of relative co-ordinates. All the different observations (excluding Doppler position measurements) used in establishing the stations of a network therefore serve as a means of defining only the internal features of that network viz the shape, the scale and the (relative) orientation of the network. Consequently any practical/meaningful criterion for assessing the strength of the network should be based on these internal features only. Again only the a posteriori standard errors of the observed quantities fulfil this objective.

For any line ij, the a posteriori standard errors of azimuth and distance $\sigma_{\alpha ij}$ and $\sigma_{\ell ij}$ respectively can be deduced from the combination of the coefficients of the observation equations linking the two stations, and the variance-covariance elements of these stations. The elements of the variance-covariance matrix can be extracted either from the fully computed inverse matrix, or be computed specifically for those stations.

Thus from equation (4.20) we have in matrix notation

$$\left.\begin{aligned} \delta\alpha_{ij} &= K^T e \\ \delta\ell_{ij} &= P^T e \end{aligned}\right) \qquad 4.33$$

It can be shown |Ashkenazi, 1970| that the variances of α_{ij} (azimuth of the line ij) and ℓ_{ij} (distance between the points i and j) are given by

$$\left.\begin{aligned} \sigma^2_{\alpha ij} &= K^T \sigma_{xx} K \\ \sigma^2_{\ell ij} &= P^T \sigma_{xx} P \end{aligned}\right) \qquad 4.34$$

and σ_{xx} is a 4 × 4 matrix of the elements of the Variance-Covariance matrix.

Equation (4.34) can therefore be expressed symbolically as follows:

For adjusted distance, $\sigma^2_{\ell ij} = |P\ Q\ R\ S| \begin{vmatrix} & & & \\ & 4 \times 4 & & \\ & & & \end{vmatrix} \begin{vmatrix} P \\ Q \\ R \\ S \end{vmatrix}$

For adjusted azimuth (bearing)

$$\sigma^2_{\alpha ij} = \begin{vmatrix} K & L & M & N \end{vmatrix} \begin{vmatrix} & & & \\ & 4 \times 4 & & \\ & & & \end{vmatrix} \begin{vmatrix} K \\ L \\ M \\ N \end{vmatrix}$$

The above computations can be done for a number of lines well distributed throughout the network, and average values deduced.

This criterion (standard errors of azimuth and distance) therefore forms the basis for the strength analysis carried out in this thesis.

CHAPTER FIVE

OPTIMISATION OF THE NETWORK BY THE USE OF SIMULATED OBSERVATIONS

5.1 Introductory Remarks

In the analysis of the positional accuracy of stations in a geodetic network, the following computational/operational routines are often used. Strength analysis, Optimisation and Simulation. These terms are defined here to the extent they are related to this work.

(a) Strength analysis - this leads to a series of absolute and relative measures of accuracy. This accuracy expresses the statistical strength of the network as defined by the "adjusted" co-ordinates.

(b) Optimisation - a computational procedure used for determining the best procedure to correct by additional observations of various types, the specific deficiencies of an already existing network. The deficiences can be revealed either by a strength analysis or from some independent field measurements.

(c) Simulation - this consists of imposing some hypothetical observations with known/assumed precision on an existing/planned network. The effect of such observations on the network is determined by a strength analysis. And in order to determine the most economical method of achieving the desired accuracy of the stations in the network prior to field measurements, the hypothetical observations together with their locations are successively altered. In other words, the scheme has been "optimised".

5.1.1 Simulation technique

It has been mentioned in § 4.2.3 that equation (4.12) i.e. $\sigma_{xx} = \sigma_o^2(A^TWA)^{-1}$ is always present in a simulated adjustment. If σ_o^2 is assumed to be unity or to be reasonably close to unity (suggesting the correctness of the estimates of the observational accuracies), then its inclusion in the equation has little or no effect. This is the basic assumption in every computer simulation of observations. Thus equation (4.12) can be expressed as

$$\sigma_{xx} = (A^TWA)^{-1} \qquad 5.1$$

In an adjustment by the variation of co-ordinates methods, the terms in the right-hand side of the above equation do not depend on the observed quantities. Matrix A can be determined (and is actually generated by the computer) with only a knowledge of the approximate co-ordinates of the stations and the location and type (but not the actual values) of the observations. In the same way, the weight matrix W can be extablished by estimation based on previous experience with observations of similar type made with similar instruments and possibly under identical weather conditions. Thus the variance-covariance matrix of an adjusted network can be computed not only for an actually observed network,but also for any planned or hypothetical network.

In this thesis the procedure outlined below was adopted for the computation of the variance-covariance matrix, and the related accuracy criteria used for the strength analysis of the various simulated models.

(a) The matrix A was established for each model by defining the following three components -
  (i) the approximate co-ordinates of the stations
  (ii) the type of the proposed observations (angles, distances, azimuths, and Doppler fixes) and their positions in the network.

(iii) the weight (precision) of each type of proposed/existing observations.

(b) The inverse of the resulting normal equations (i.e. the variance-covariance matrix) was then computed using a NAG library routine (see § 6.1.4)

(c) The values of σ_{max} and σ_{min} for the error ellipses together with the standard errors in distance and azimuth for some selected points and lines respectively were then computed using the elements of the variance-covariance matrix from (b). The mean of the standard errors in distance and azimuth were respectively computed.

5.1.2 Distribution of the stations used for the error analysis

The stations or pairs of stations selected for computing the relevant accuracy criteria were distributed randomly over the whole network. However the following factors were taken into consideration for their distribution:

(a) the various geographical sections of the network were represented.

(b) areas of possible weaknesses, such as those of poor geometry were given more weight in the distribution of the points.

(c) in order to provide a fair estimation of precision, areas with good geometry were also covered.

Altogether ten stations were selected for the computation of the parameters of the error ellipses and fourteen pairs of stations (lines) were selected for the computation of the standard errors of adjusted distance and azimuth. The two lines that were chosen to

span the open area between the arcs (i.e. sections of the chains of the triangulation network) were used to observe the movement of one arc relative to another. Thus each simulated model has a line oriented in the north-south and east-west directions. The same set of stations or lines were used for all the test models generated in this work. This was done in order to observe any signifcant changes occurring at the same locations of the network between the various test models.

5.1.3 Outline of the tests performed in this work

The groups of "observed" data listed below were "adjusted" simultaneously and the resulting parameters for the error ellipses and the standard errors in azimuth and distance were obtained. The same geometrical model was used for all (except the case of trilateration) the different versions of the simulated network. Also for all the tests (except for the trilateration tests where station no. 80 was held fixed) the co-ordinates of station 93 were held fixed.

TEST 1 (models 1 and 2)

The Test Network (mainly) which acted as the basic model in most of the tests conducted in this thesis

Model 1 was the Test Network and was made up of the following groups of data:- 93 geodetic co-ordinates, 361 angles, 25 distances and 9 Laplace azimuths.

Model 2 was the same as model 1 except that Doppler derived positions at four stations were introduced. The Doppler stations were the same as those chosen for the UNIDOP program.

TEST 2 (models 3 and 4)

Investigations on the effect of high order control traverse around the network

Model 3 consisted of model 1 enveloped with a high order control traverse.

Model 4 was the same as model 2 but enveloped with a high order control traverse.

TEST 3 (models 5-12 and models 13-21)

Investigations on azimuth and scale control requirements

Models 5-12 were made up of all the angles in the triangulation scheme in model 1 together with all the observed data in the portion of the CFL (12th parallel) traverse within the model. Azimuths and distances were introduced gradually as detailed in Table 5a.

For the scale control requirements (models 13-21) the same number of angles and the CFL traverse data were used, and distance and azimuths were introduced as given in Table 5b.

It is to be remarked that the a priori standard errors assigned to the azimuths and distances up to the stage of 3 azimuths and 13 distances were the same as for the existing network. From here on, new variances were assigned to the additional measurements.

TEST 4 (models 22 and 23)

Investigation on the effect of optimum number of scale and azimuth control in the Test Network

Model 22 was made up of the usual number of angles and the CFL traverse data together with the optimum number of scale and azimuth control derived from Test 3.

Model 23 was the same as model 22 except that four Doppler derived positions were introduced.

TEST 5 (models 24 and 25)

Effect of high order control traverse around models 22 and 23.

The same type of experiment as in Test 2 was repeated here with models 22 and 23.

TEST 6 (models 26-31)

Requirements for trilateration as a method of establishing geodetic controls.

In all the models established in this test, the CFL traverse together with all the angles of the triangulation scheme were excluded. For the different models, the groups of observed data adjusted simultaneously were as follows:

Model 26 involved only the "measured" distances (side lengths)

Model 27 was the same as model 26 except that four satellite-Doppler fixes were introduced.

Model 28 was the same as model 26 except that three Laplace azimuths were introduced.

Model 29 involved all the data of model 28 plus four Doppler fixes.

Model 30 was made up of the data of model 26 plus four Laplace azimuths.

Model 21 was the same as model 30 plus four Doppler fixes.

The data composition of the different 31 simulated models are shown in Table 5a.

DATA FOR DIFFERENT **MODELS**

(Table 5a)

Test	Model	EXISTING OBSERVATIONS			ADDITIONAL OBSERVATIONS			
		No. of Angles σ_ϕ = 0".7	No. of Distances σ_ℓ=2 or 3.5 ppm	No. of Laplace Azimuths σ_α = 1".0	No. of Distances σ_ℓ = 2ppm	No. of Laplace Azimuths σ_α = 0".7	No. of Doppler Fixes σ_D = 0.7m	No. of Angles σ_θ = 0".7
1	1	361	(13 of σ_ℓ= 2ppm (12 of σ_ℓ= 3.5ppm	3	-	-	-	-
	2	361	" "	"	-	-	4	-
2	3	361	(13 of σ_ℓ= 2ppm (12 of σ_ℓ= 3.5ppm	3	36	5	-	36
	4	361	" "	"	"	"	4	"
3	5	361	(13 of σ_ℓ= 2ppm (1 of σ_ℓ= 3.5ppm	1	-	-	-	-
	6	361	13 of σ_ℓ= 2ppm) 12 of σ_ℓ= 3.5ppm)	1	-	-	-	-
	7	361	" "	2	-	-	-	-
	8	361	" "	3	-	1	-	-
	9	361	" "	3	-	3	-	-
	10	361	" "	3	-	5	-	-
	11	361	" "	3	-	7	-	-
	12	361	" "	3	-	9	-	-
	13	361	(13 of σ_ℓ= 2ppm (1 of σ_ℓ= 3.5ppm	1	-	-	-	-
	14	361	13 of σ_ℓ= 2ppm) 1 of σ_ℓ= 3.5ppm)	2	-	-	-	-
	15	361	(13 of σ_ℓ= 2ppm (6 of σ_ℓ= 3.5ppm	3	-	-	-	-
	16	361	" "	3	12	-	-	-
	17	361	" "	3	18	-	-	-
	18	361	" "	3	24	-	-	-
	19	361	" "	3	30	-	-	-
	20	361	" "	3	36	-	-	-
	21	361	" "	3	42	-	-	-

(Table 5a continued)

Test	Model	EXISTING OBSERVATIONS			ADDITIONAL OBSERVATIONS			
		No. of Angles σ_ϕ = 0".7	No. of Distances σ_ℓ = 2 or 3.5ppm	No. of Laplace Azimuths σ_α = 1".0	No. of Distances σ_ℓ = 2ppm	No. of Laplace Azimuths σ_α = 0".7	No. of Doppler Fixes σ_D = 0.7m	No. of Angles σ_θ = 0".7
4	22	361	(13 of σ_ℓ= 2ppm (12 of σ_ℓ=3.5ppm	3	6	2	-	-
	23	361	" "	3	6	2	4	-
5	24	361	(13 of σ_ℓ= 2ppm (12 of σ_ℓ=3.5ppm	3	42	7	-	36
	25	361	13 of σ_ℓ= 2ppm) 12 of σ_ℓ=3.5ppm)	3	42	7	4	36
6	26	-	-	-	205	-	-	-
	27	-	-	-	205	-	4	-
	28	-	-	-	205	3	-	-
	29	-	-	-	205	3	4	-
	30	-	-	-	205	4	-	-
	31	-	-	-	205	4	4	-

5.1.4 Analyses of the Results of the Tests

Table 5d gives the mean a posteriori standard errors of distances and azimuths of the different simulated models. The detailed values of the adopted accuracy criteria for the different models are given in the Annex. For the geodetic co-ordinates of each point, the corresponding 2 × 2 sub-matrix of the N^{-1} was used to compute from equations (4.30) and (4.31) the orientations and the lengths of the major and minor axes of the error ellipses. The axes of the error ellipses of some models are schematically shown in pages 158-167. The analyses of the different tests performed are given below.

TEST 1

In order to determine the current strength of the Test Network and also to ascertain whether there was any deficiency in scale and orientation, the Test network was "adjusted" and a strength analysis of the simulated model (model 1) was carried out.

Results of the computed accuracy criteria (the error ellipses and the standard error in azimuth and distances) revealed no particular trend in the orientation of the major axes. However the lengths of the major axes were on the average longer than those of the minor axes, indicating a deficiency in both scale and azimuth observations. Furthermore as it is to be expected, the lengths increased the further away the point was from the fixed station.

The mean a posteriori standard error in azimuth was 0.82" with an upper bound of 1".03 along line 49-56; and that in distance was 4.3 ppm with an upper bound of 6.3 ppm along line 39-38. A comparison of this network with some other geodetic horizontal networks of reputed high standard, such as Block VI of European

Triangulation, σ_α = 0.52", σ_ℓ = 2.44 ppm, and OSGB 70 (SN) with σ_α = 0.52" and σ_ℓ = 2.44 ppm, indicates that the present network (the Test Network) is not of geodetic standard. Thus further improvement in strength of the network is needed and this can be done by incorporating additional observations into the network. However it is to be remarked that the a posteriori standard errors (0".82 and 4.3 ppm) are compatible with each other, thus indicating the compatibility of the current strength with the data used.

When satellite-Doppler derived positions were incorporated into the network adjustment (model 2) the axes of the error ellipses were equal on the average, indicating a balance in scale and azimuth controls.

The improvement in the mean a posteriori standard error in distance and azimuth was insignificant, approximately 4% and 6% respectively.

TEST 2

It has been demonstrated by |Saxena, 1972| that the strength of geodetic horizontal network could be improved by means of precision traverse around the network. To determine the effect of this on the present network, a high order traverse network was assumed to envelope the Test Network (model 3). The traverse stations were the same as those of the triangulation network, and Laplace azimuths were "observed" at every fifth station.

Inspection of the error ellipses revealed the same trend as model 1 except that the lengths of the axes were halved on the average, thus indicating an improvement in the positional accuracy of the points.

A significant improvement was observed in the mean a posteriori standard errors in azimuth and distance. The mean adjusted standard error in azimuth was 0.50" with an upper bound of 0.60" along line 39-38. That of distance was 2.7 ppm with an upper bound of 5.0 ppm along line 39-38. These results indicate an average improvement in azimuth and scale errors of about 39 per cent and 37 per cent respectively.

Again with the introduction of the four Doppler derived positions into the adjustment (model 4), an insignificant improvement in azimuth and scale errors were observed. This indicates that there is a reduction in the usefulness of Doppler observations as a means of improving the standard errors, when the number of constraints in a system is increased. These results agree with those of Ashkenazi and Cross |1975| when a simulated network was also used. Quoting the conclusions of Ashkenazi and Cross, "For every well connected network, there is a limit to the number of base lines and azimuths that serve any useful purpose in constraining the system. Base lines and azimuths added to the system beyond this sufficient number serve only to slowly reduce the standard errors".

Also the lengths of the major axes of the error ellipses decreased by about 0.1 m, with no appreciable effect on the lengths of the minor axes. These results indicate a better balance in scale and azimuth controls than in the case of model 3. However the cost both in labour and time to produce such a small effect does not justify the incorporation of this programme in this situation.

TEST 3

Theoretically in a triangulation scheme one measured length and azimuth should be sufficient to furnish the scale and the orientation for the framework, provided enough horizontal angles or directions are observed in the network to enable lengths and azimuths of any

side to be computed at any part of the network. However due to errors in the observed angles/directions, errors of scale and azimuth accumulate. To provide sufficient check on the accumulation of errors, it has now been shown |Ashkenazi and Cross, 1972| that there is an optimum number of scale and azimuth that can be introduced in the network.

The present test (models 5 to 12 and models 13 to 21) was designed to indicate the trends in the requirements for azimuth and scale controls, and thence to determine the optimum number of scale and azimuth that can be introduced in the Test Network. By measuring the three sides of a triangle, a reasonable proof is obtained on the stability of the stations forming the triangle. Therefore it was decided in the present investigation to adopt this principle in simulating distance measurements. The trends in azimuth and scale controls requirements are shown graphically in Figures 5b and 5a. Also Tables 5c and 5b give the mean a posteriori standard errors in azimuth and distance for the different simulated models generated in this test.

For the scale control requirement (Table 5b and Figure 5a), it was observed that with only one extra distance (model 13 compared to model 5) there was improvement of about 16% in the a posteriori standard error in distance but there was none in azimuth. With a further increase of 12 distances (model 6 compared to model 5) there was a remarkable improvement of about 12 per cent in azimuth error. However when the number of azimuths was increased to three and the number of distances kept constant (models 1 and 6), there was an improvement in azimuth but none in scale. With more increase in the number of distances, there was improvement in scale until after network model 16 (with 19 distances) when further increase produced only gradual changes in scale error, and again no noticeable effect in azimuth error.

For the azimuth control requirement (Table 5c and Figure 5b) it was observed that with one extra azimuth and distances maintained constant (models 6 and 7), there was an improvement of 12 per cent in azimuth error but an insignificant improvement of 2 per cent in scale error. Further increase in the number of Laplace azimuths produced significant improvement in the a posteriori standard error in azimuth, but negligible effects in the standard errors in distance were observed. This significant improvement in the orientation of the network continued until after model 9 (with 6 Laplace azimuths) when further increases produced a steady but small decrease in the average value of standard errors in azimuth. Again there was no significant improvement in the scale accuracy.

Obviously the effect of scale measurements on azimuth error, and that of azimuth observations on scale error, clearly demonstrates that each of these observational checks simply plays its rightful role in the strength of the network. Any attempt to use either of these checks for a different role, is fruitless and therefore illogical.

Inspection of the graphs in Figures 5b and 5a indicates that the optimum number of Laplace azimuths and distance measurements required in the Test network are respectively six and nineteen. In other words, three additional azimuths and six additional distances are to be incorporated into the existing network (Test networks).

TEST 4

In order to find the effect of optimum number of scale and azimuth control in the Test network (model 22) three Laplace azimuths with a priori standard error of 0.7 seconds and six distances with a priori standard error of 2 ppm were simulated into the existing network. The mean standard errors of adjusted azimuth and distance were respectively 0.68" and 3.7 ppm.

CHANGE IN THE NUMBER OF MEASURED DISTANCES

(excluding CFL traverse distances)

Network Model	No. of Azimuths	No. of Distances	Mean Adjusted	
			σ_ℓ (ppm)	σ_α (secs)
5	1	1	7.9	1.23
13	1	2	6.6	1.23
6	1	13	4.5	1.06
14	3	2	6.2	0.88
15	3	7	4.9	0.86
1	3	13	4.3	0.82
16	3	19	3.9	0.80
17	3	25	3.8	0.78
18	3	31	3.5	0.77
19	3	37	3.2	0.75
20	3	43	3.0	0.73
21	3	49	2.7	0.72

Table 5b

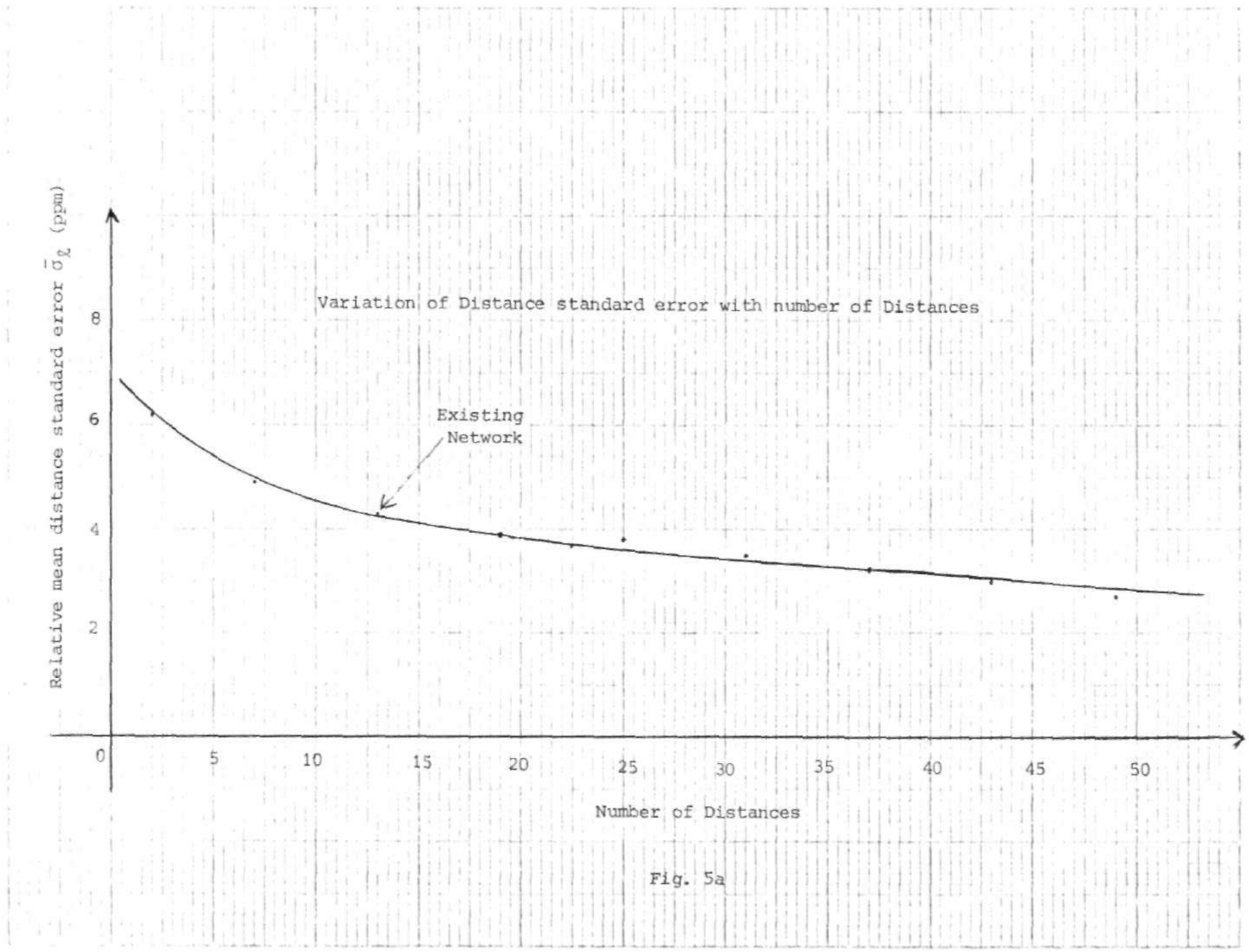

Variation of Distance standard error with number of Distances
Relative mean distance standard error $\bar{\sigma}_{\ell}$ (ppm)
Existing Network
0
2
4
6
8
5
10
15
20
25
30
35
40
45
50
Number of Distances

Fig. 5a

CHANGE IN THE NUMBER OF OBSERVED AZIMUTHS

(excluding CFL traverse azimuths)

Network Model	No. of Distances	No. of Azimuths	Mean Adjusted	
			σ_α (secs)	σ_ℓ (ppm)
5	1	1	1.23	7.9
13	2	1	1.23	6.6
6	13	1	1.06	4.5
7	13	2	0.93	4.4
1	13	3	0.82	4.3
8	13	4	0.74	4.2
9	13	6	0.65	4.1
10	13	8	0.60	4.1
11	13	10	0.56	4.1
12	13	12	0.52	4.0

Table 5c

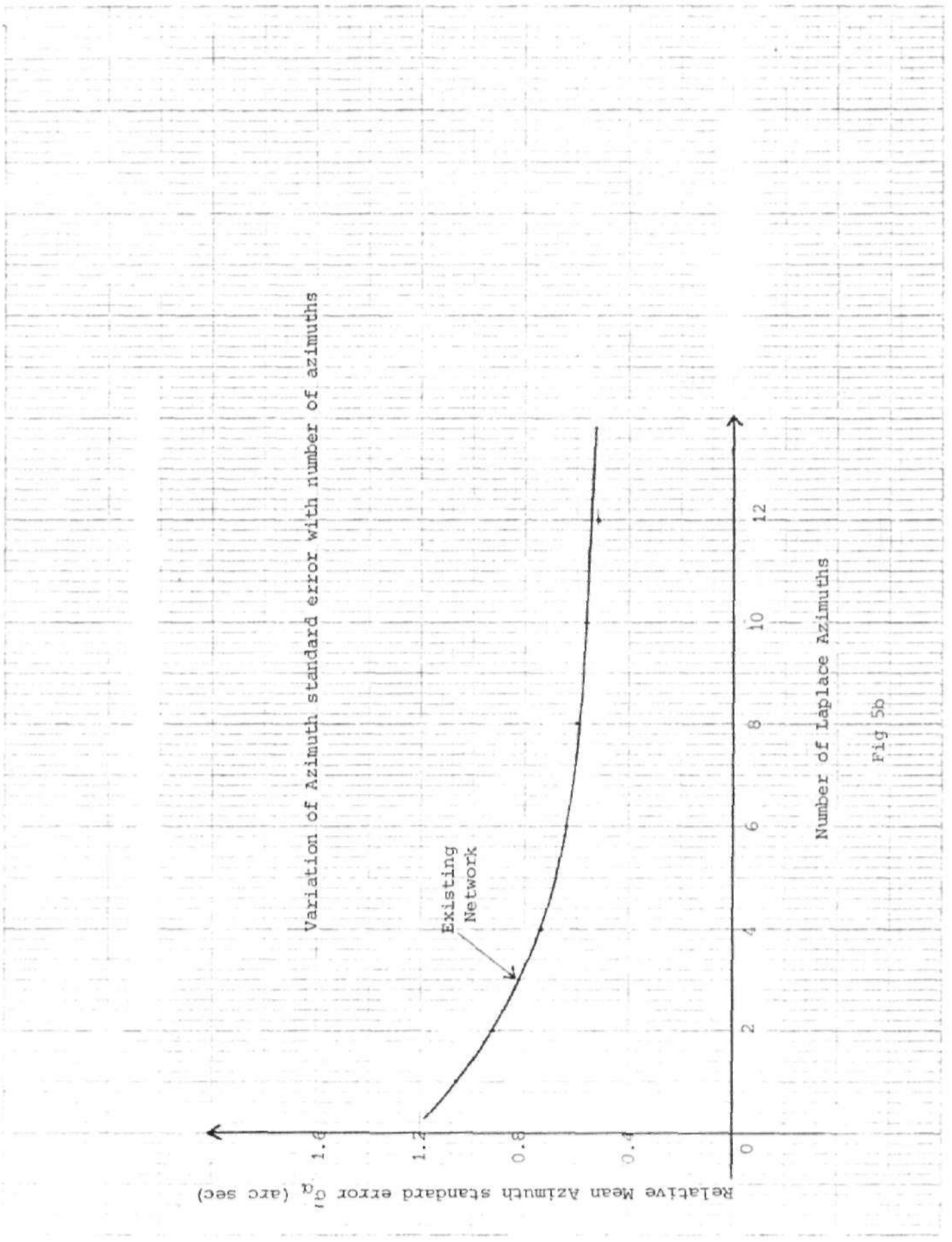

Variation of Azimuth standard error with number of azimuths
Relative Mean Azimuth standard error σ_α (arc sec)
1.6
1.2
0.8
0.4
0
2
4
6
8
10
12
Existing Network
Number of Laplace Azimuths

Fig 5b

The lengths of the error ellipse axes are approximately equal, indicating a balance in scale and azimuth controls.

When satellite-Doppler fixes were incorporated into the network (model 23), the improvement in the a posteriori standard errors of both azimuth and distances was negligible. They were respectively 0.67" and 3.6 ppm. These results confirmed the conclusions made in Test 2 about the optimality of constraints required in a network. Also the dimensions of the error ellipse axes were exactly equal, and of smaller lengths than those in model 22. This indicates a better balance in scale and azimuth controls.

TEST 5

The above two models were enveloped with a high order control traverse as in Test 2. There was significant improvement in the mean standard errors of azimuth and distance. These were respectively 0.48" and 2.6 ppm, representing a percentage increase of 29 per cent and 30 per cent over the situation in model 22.

The dimensions of the error ellipse axes decreased appreciably when compared with the situation in all the previous models. Also the axes were approximately equal.

With the incorporation of the four Doppler fixes into the above model (i.e. model 24) the resulting a posteriori standard errors for this model (i.e. model 25) were 2.5 ppm for distance and 0.47" for azimuth. These results do not show any significant improvement of accuracy in both the orientation and scale of the network. Moreover the dimensions and orientations of the error ellipses were on the average the same as those in model 24.

TEST 6

With the advent of EDM instruments, it has now become possible to increase the frequency of measured distances in a network. And because EDM distances can be measured with ease and with very high accuracy speculations are that a sufficiently large number of distances can be measured to provide a pure trilateration solution of the whole network, thus eliminating completely the need for triangulation. However, this does not imply that no azimuths (or any other method of orientation control) need be measured since these are still required for the provision of orientation in the network.

The present test was therefore designed to find the suitability or otherwise of using trilateration as a substitute for triangulation, in the establishment of a network made up of a system of chains (such as the Test Network excluding the 12th parallel traverse). Also the need to use satellite-Doppler controls instead of Laplace azimuths, for the orientation of a whole network was investigated.

When the whole network was assumed to be composed of only measured distances, σ_{ℓ} = 2 ppm (i.e. pure trilateration) with orientation provided by the fixed station (model 26), it was observed that the average standard error of adjusted distance was 1.8 ppm with an upper bound of 2.0 ppm; the mean standard error of adjusted azimuth was 0.99 seconds, with an upper bound of 1.24 seconds. These results indicate that the orientation of the network was incompatible with the distances. For example, for a side length of 30 km, the relative positional accuracy of the adjusted azimuths was about 5.1 ppm and the corresponding relative positional accuracy of the adjusted distance was 2.0 ppm (see Annex Model 26 stations 36/43). Furthermore, inspection of the error ellipses (model 26), reveals extremely disproportionate axes, thus indicating an overwhelming imbalance in scale and orientation of the network.

However with the addition of the four satellite-Doppler fixes (model 27), the major axes of the error ellipses were almost halved, thus indicating a better balance in scale and orientation than in the previous model (i.e. model 26). Also the mean a *posteriori standard error in azimuth improved by about 15 per cent* (i.e. from 0.99 seconds to 0.84 seconds), with no improvement in scale error indicating that Doppler fixes can improve azimuth controls better than scale.

On the other hand, when three Laplace azimuths (model 28) were simulated into model 26 to replace the Doppler observations, the mean a posteriori standard error in azimuth dropped to 0.70 seconds, i.e. an improvement of about 29 per cent. Also the dimensions of the error ellipses decreased. This indicates that the Laplace azimuths can improve orientation control better than the Doppler observation at its present level of accuracy.

Again four satellite-Doppler derived positions were simulated into the preceeding model. The results showed very small improvement in the mean a posteriori standard error in azimuth - 0.68 seconds, but no improvement at all in the scale error. Also the sizes of the error ellipses decreased very slightly. With more addition of Laplace azimuths (model 30) and Doppler fixes (model 31) there were *no significant improvement in both scale and orientation accuracy.* This indicates that the optimum number of constraints required for this network, was obtained between models 28 and 29.

5.1.5 Remarks

All the results from the above tests taken together indicate that :

(i) High order control traverse around a network can serve as a very useful constraint for the network, thus improving its strength.

(ii) The positional strength of the stations in a network improves as the strength of the network improves. This is easily revealed by the corresponding decrease in the sizes of the error ellipses.

(iii) The a posteriori standard error in azimuth and distance of a line decreases with increase in the length of the line. The standard error for the north-south and east-west lines in all the models reveal this clearly. This remark agrees with those of Ashkenazi and Cross |1976| when they made the following observations "Both scale error (expressed in ppm) and orientation error, in a conventional terrestrial triangulation network, diminish with increasing length of line, at a rate which is significantly large at first, but smaller and steadier beyond a medium range of about 400 km".

(iv) The effect of satellite-Doppler positions in most of the simulation tests carried out here, indicates that as the number of base line and azimuth observations increases in a network, there is a reduction in the usefulness of Doppler observations as a means of reducing the distance and azimuth standard errors. This same observation was made by Moose and Henriksen |1976|.

(v) At present Doppler stations at every 250 km or so in a terrestrial control system can provide a fairly satisfactory substitute for Laplace azimuth observations at those stations. In the future, provided that the equipment is reliable in field conditions and assuming that its accuracy increases Doppler observations will probably completely supplant astronomical azimuth in control for mapping, particularly because such observations require less operator training and skill. This remark agrees with those of Robbins |1977|.

(vi) Trilateration (in conjunction with azimuths or Doppler fixes or both) can be used as a substitute for triangulation because of the ease and accuracy with which EDM distances can be measured. However Bomford |1980| remarked that *trilateration cannot be used as a satisfactory substitute* for triangulation if a system of chains is required. This is because a narrow trilateration chain rapidly loses direction and requires close azimuth control. See results of models 26 and 28 in Table 5d where the mean a posteriori standard error improved by about 29% because of addition of three Laplace azimuths.

On the other hand Ashkenazi et al |1974| remarked that if distances can be measured with a considerably higher degree of accuracy ($\sigma_\ell << \sigma_\alpha$) then pure trilateration obviously provides the best shape and scale control for a network.

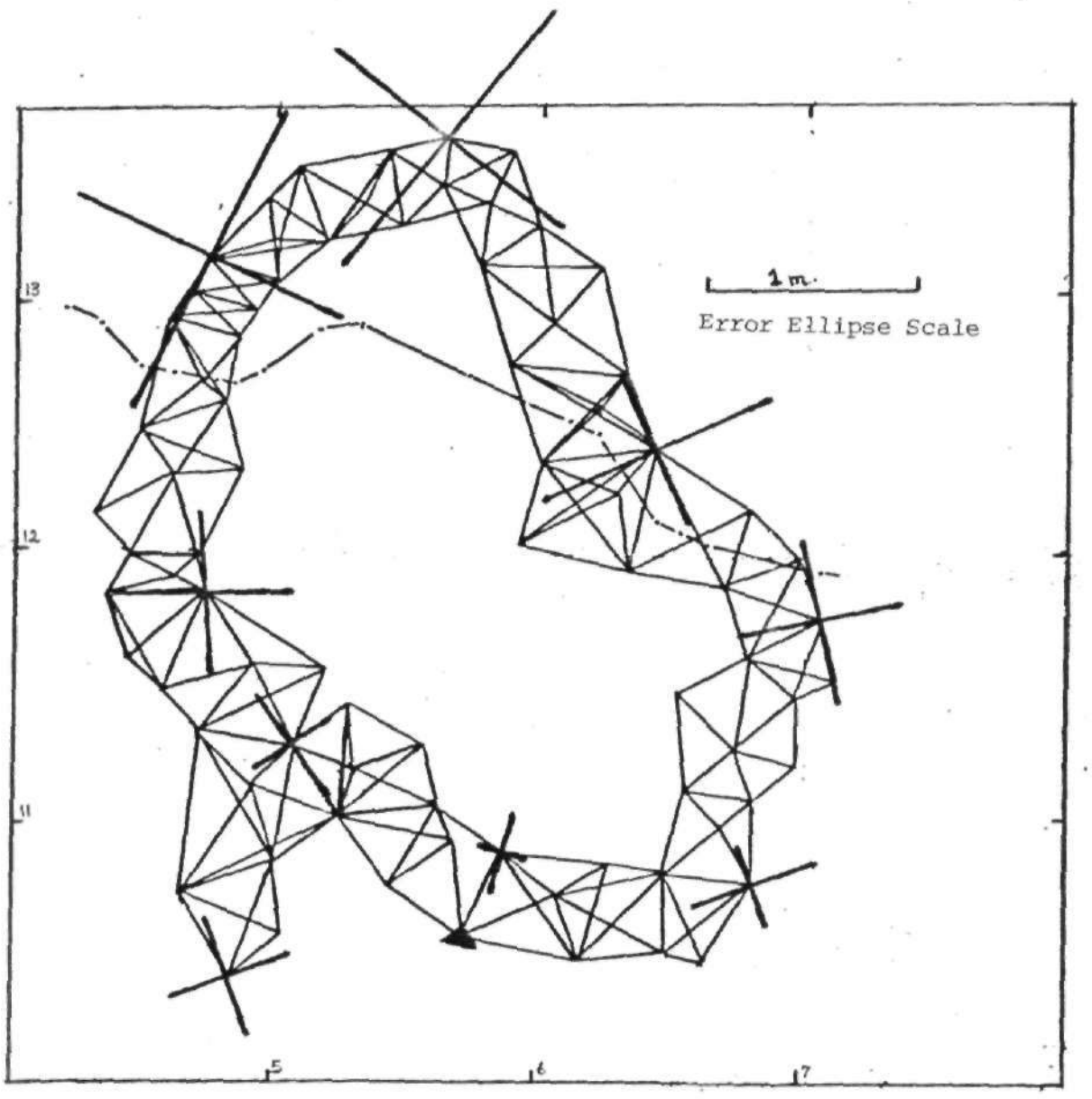
A POSTERIORI STANDARD ERRORS

Network Model	Origin	Mean		N-S Line		E-W Line	
		σ_ℓ (ppm)	σ_α (secs)	σ_ℓ (ppm)	σ_α (secs)	σ_ρ (ppm)	σ_α (secs)
1	93	4.3	0.82	2.2	0.45	1.9	0.41
2	93	4.1	0.77	1.6	0.35	1.6	0.36
3	93	2.7	0.50	0.6	0.24	0.8	0.21
4	93	2.7	0.49	0.6	0.21	0.8	0.19
22	93	3.7	0.68	1.3	0.29	1.6	0.31
23	93	3.6	0.67	1.1	0.26	1.4	0.29
24	93	2.6	0.48	0.6	0.19	0.8	0.19
25	93	2.5	0.47	0.6	0.17	0.8	0.18
26	80	1.8	0.99	0.9	0.57	1.2	0.52
27	80	1.8	0.84	0.8	0.32	1.1	0.31
28	80	1.8	0.70	0.7	0.30	1.0	0.25
29	80	1.8	0.68	0.6	0.24	0.9	0.21
30	80	1.8	0.67	0.6	0.22	0.8	0.21
31	80	1.8	0.67	0.6	0.21	0.9	0.20

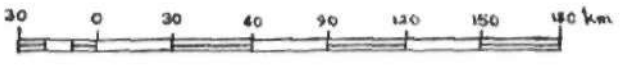
Table 5d

THE TEST NETWORK

(BLOCK OF NIGERIA'S HORIZONTAL GEODETIC NETWORK)



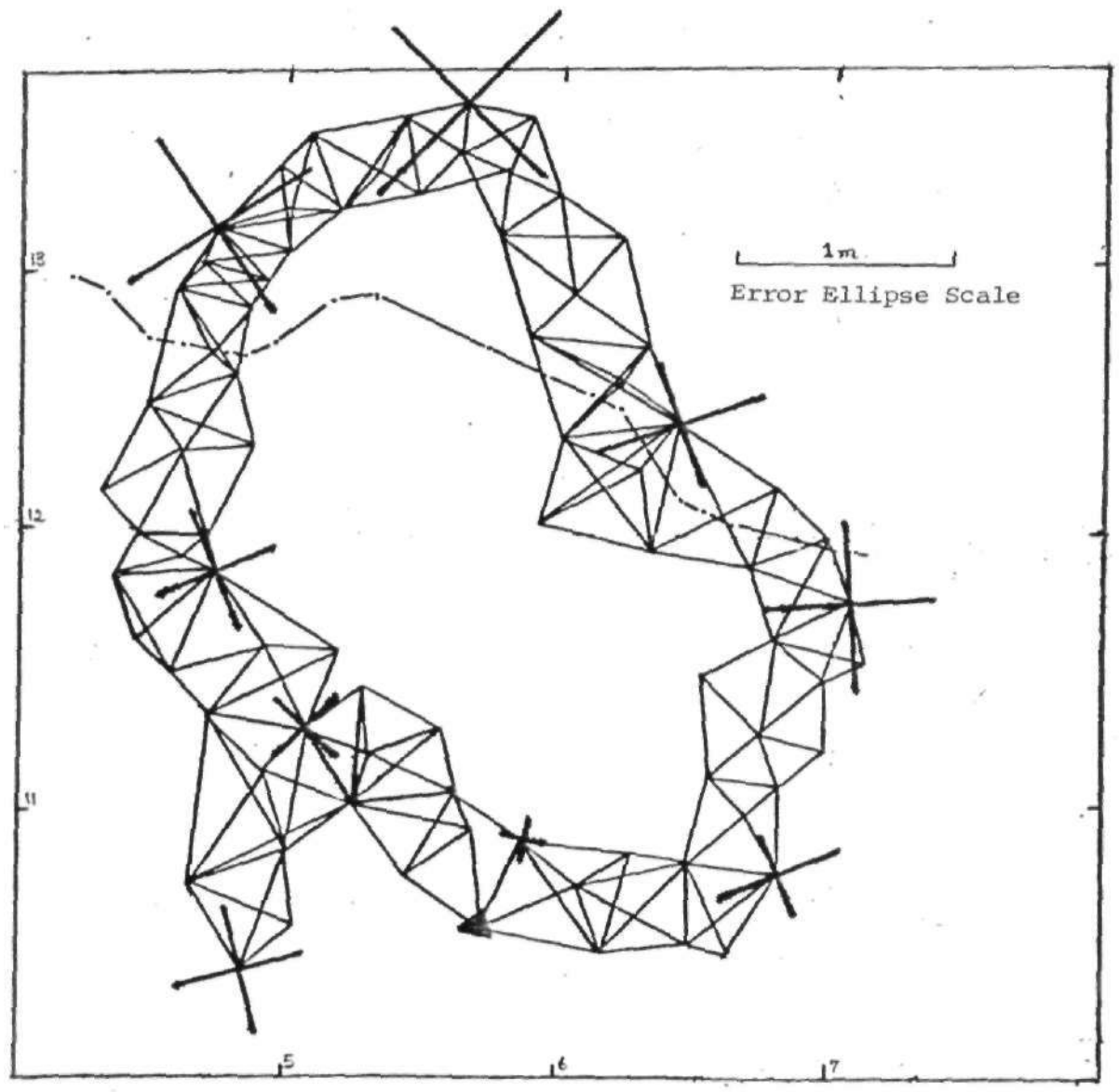
Scale 1 : 3000 000



Error Ellipse (Model 1)

THE TEST NETWORK

(BLOCK OF NIGERIA'S HORIZONTAL GEODETIC NETWORK)



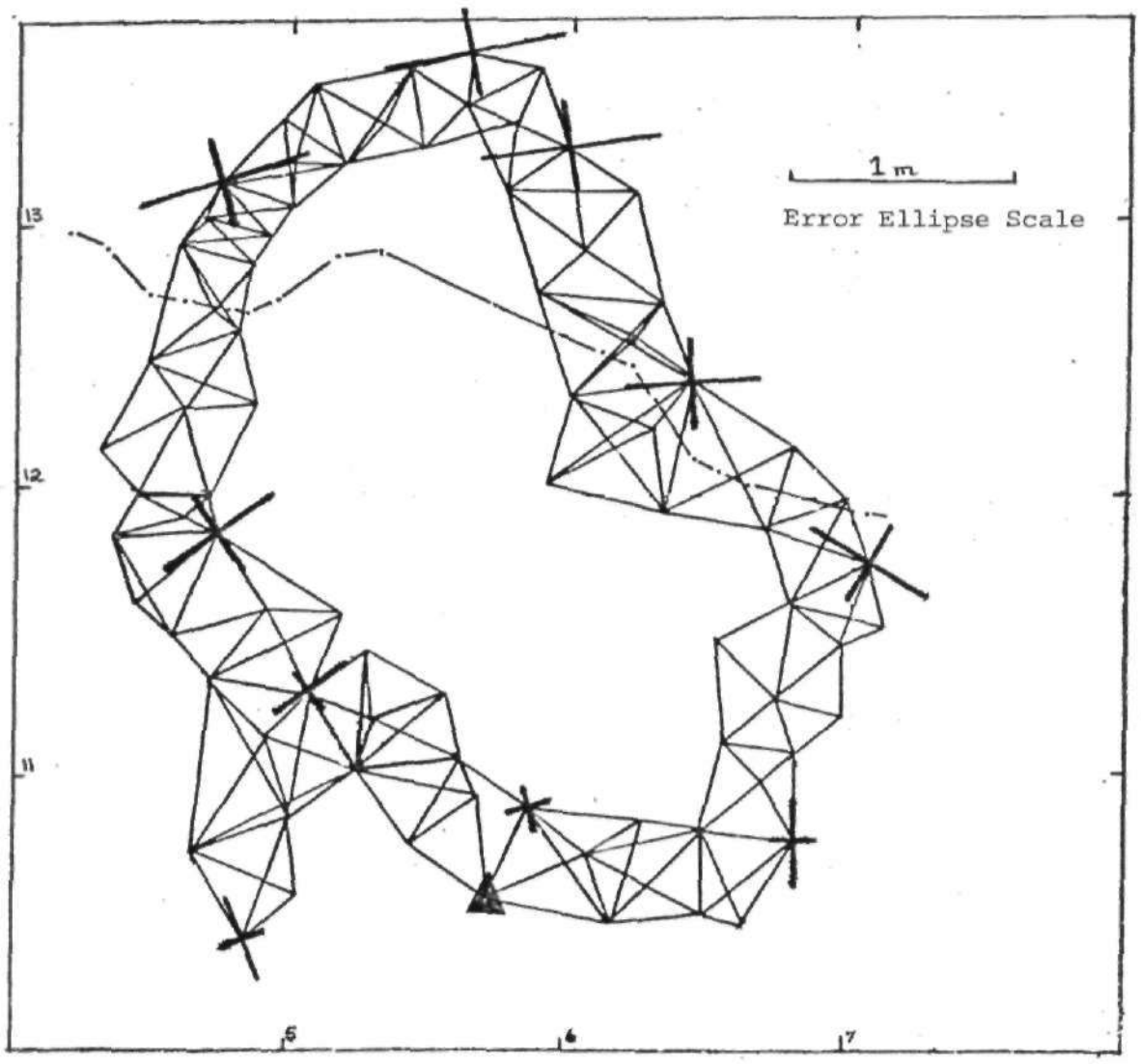
Scale 1 : 3000 000
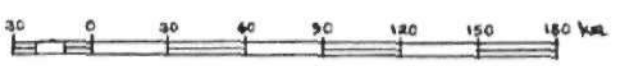


Error Ellipse (Model 2)

THE TEST NETWORK

(BLOCK OF NIGERIA'S HORIZONTAL GEODETIC NETWORK)



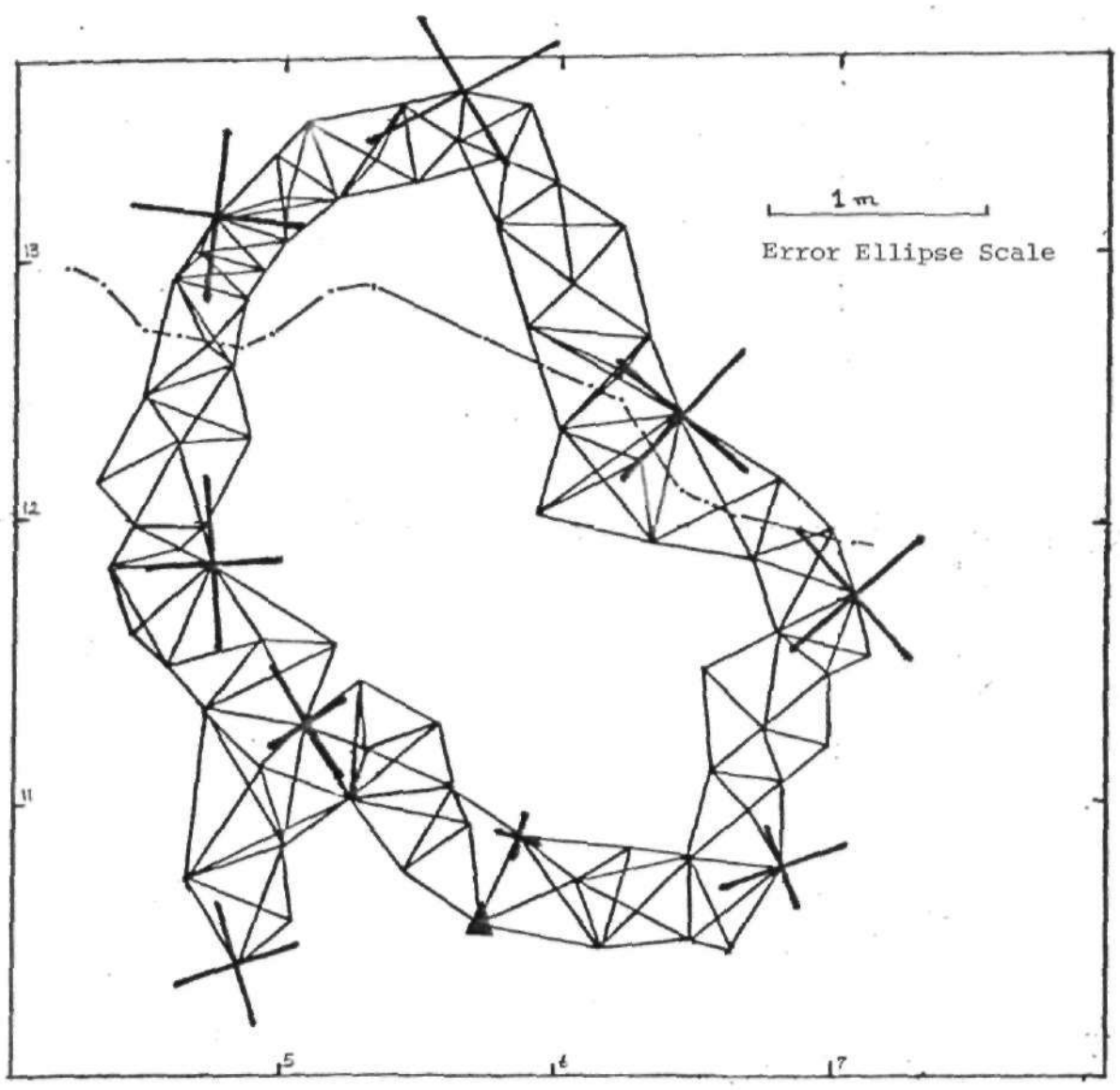
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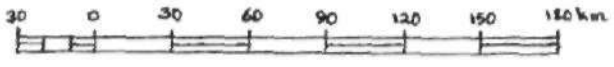
Error Ellipse (Model 3)

THE TEST NETWORK

(BLOCK OF NIGERIA'S HORIZONTAL GEODETIC NETWORK)



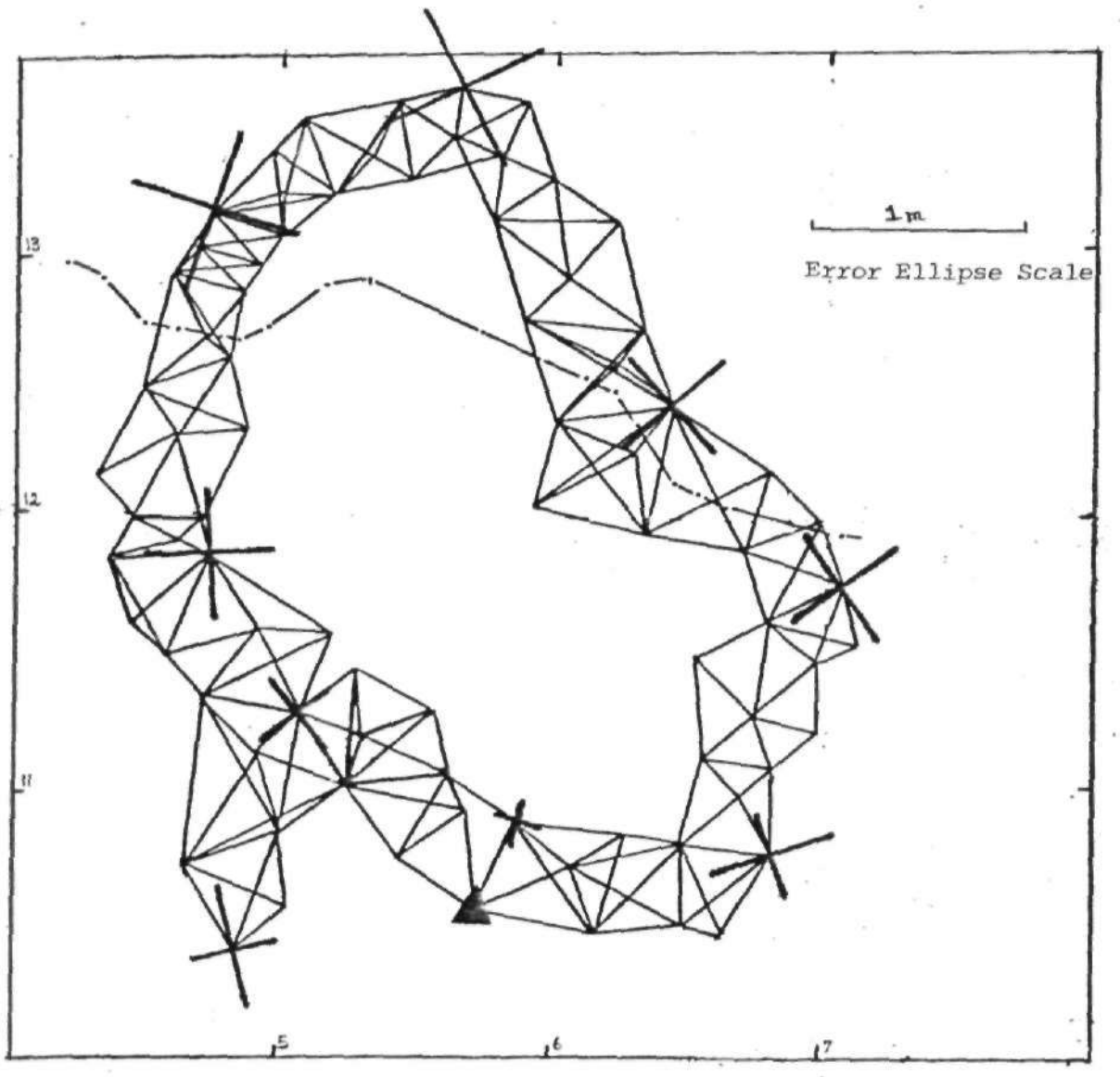
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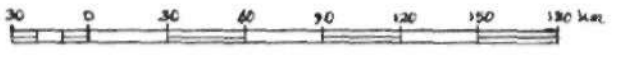
Error Ellipse (Model 22)

THE TEST NETWORK

(BLOCK OF NIGERIA'S HORIZONTAL GEODETIC NETWORK)



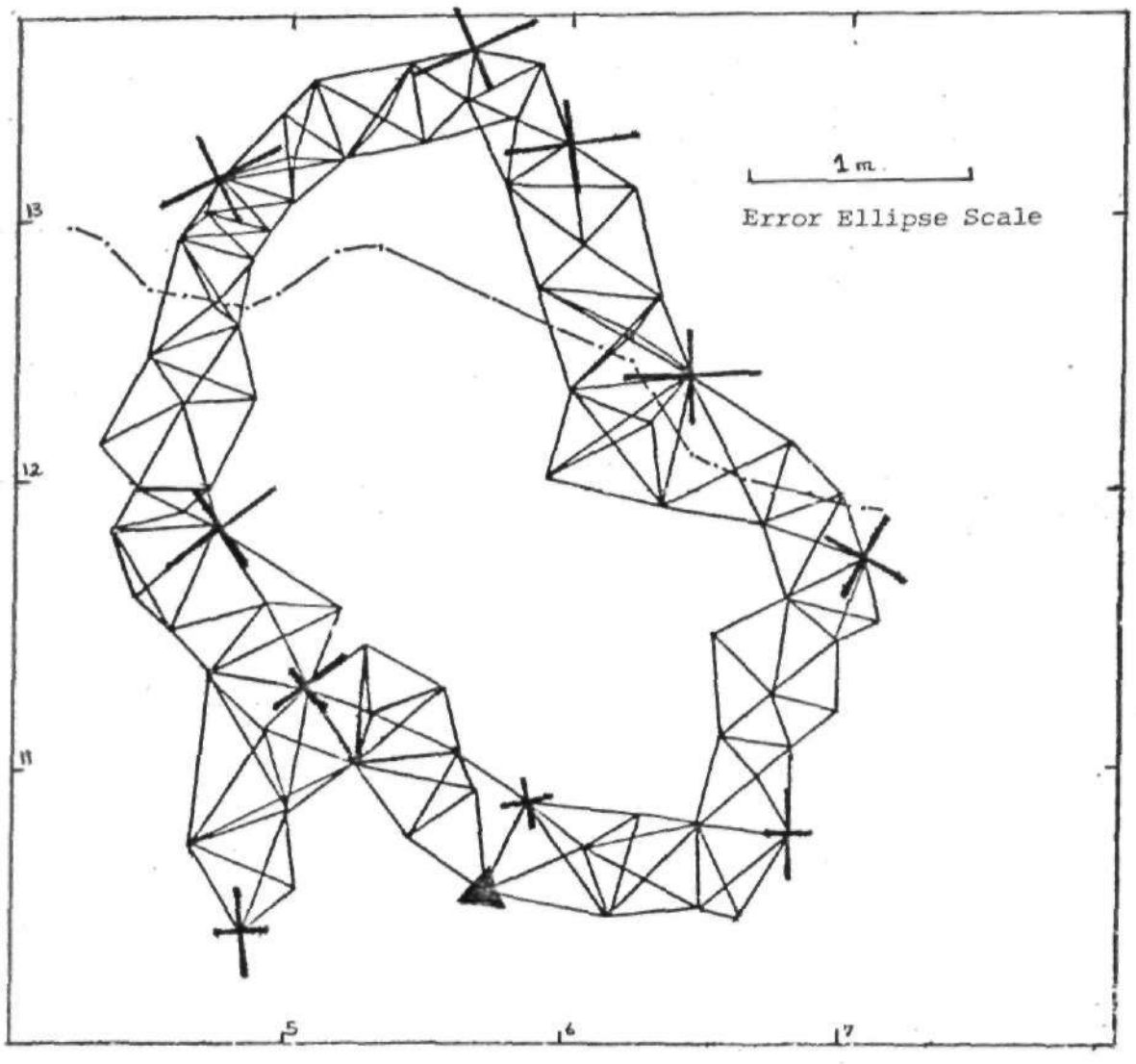
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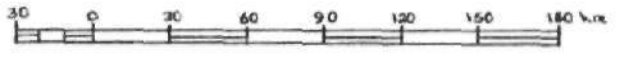
Error Ellipse (Model 23)

THE TEST NETWORK

(BLOCK OF NIGERIA'S HORIZONTAL GEODETIC NETWORK)



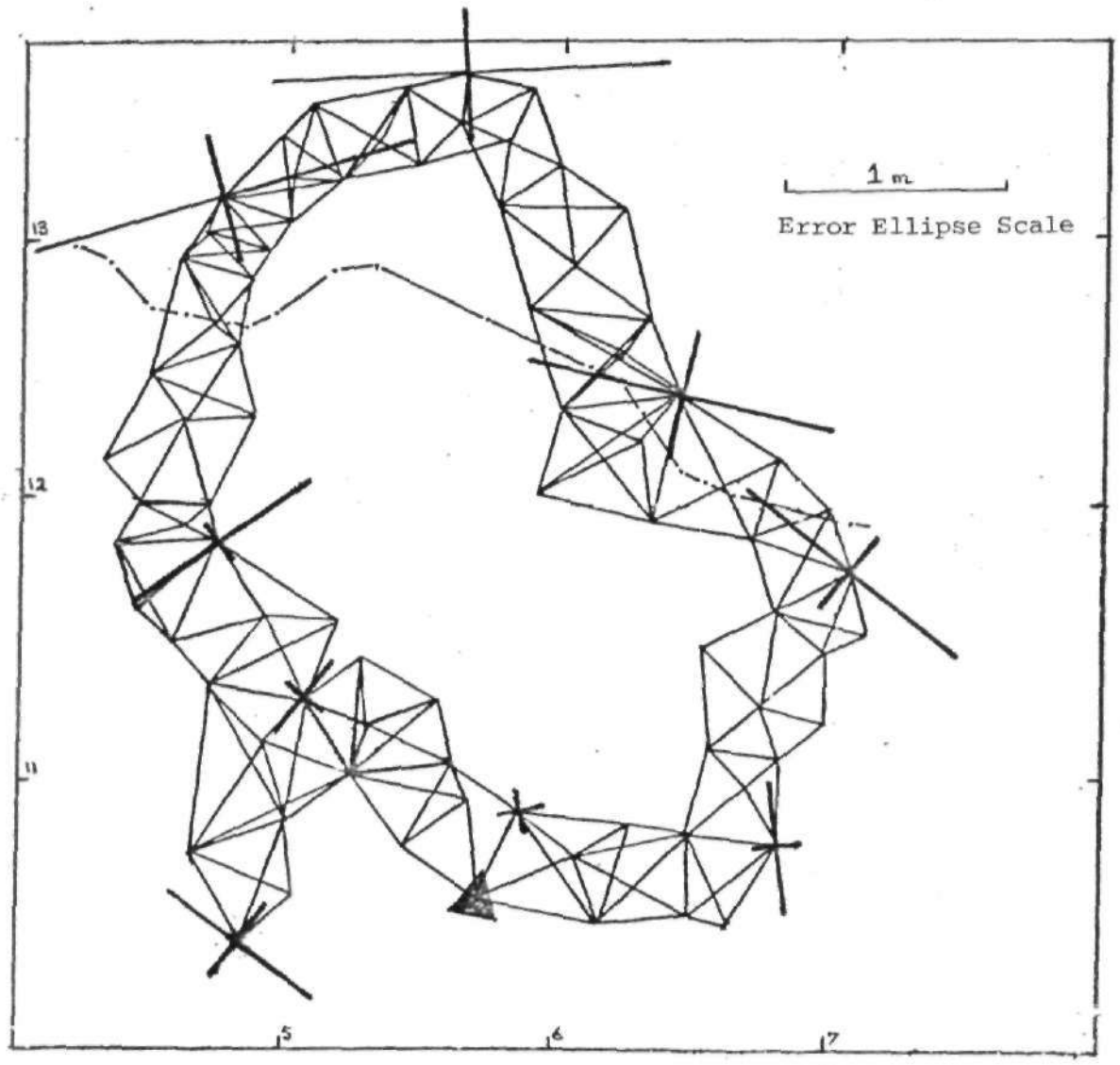
Scale 1 : 3000 000



Error Ellipse (Model 24)

THE TEST NETWORK

(BLOCK OF NIGERIA'S HORIZONTAL GEODETIC NETWORK)



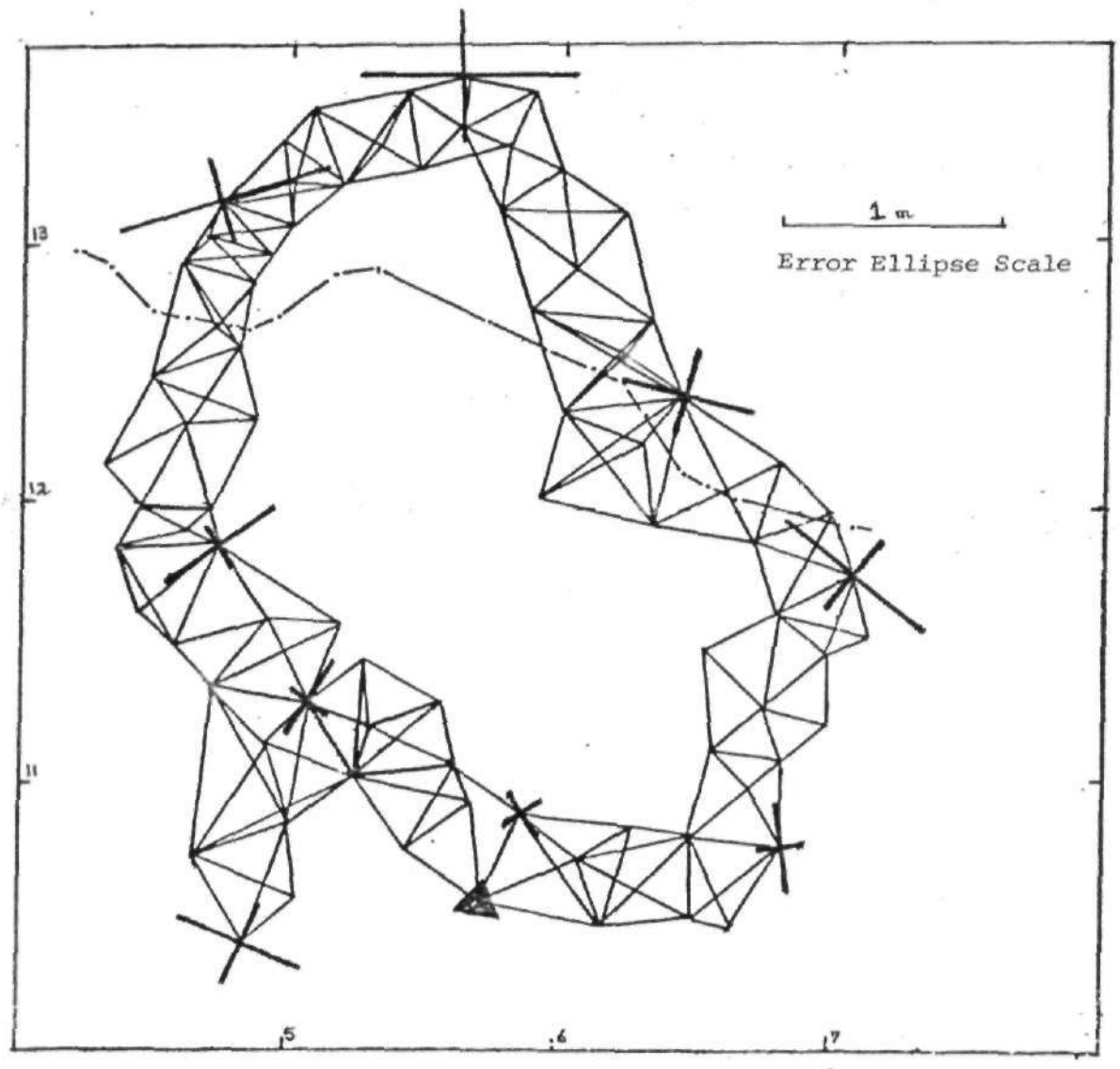
Scale 1 : 3000 000



Error Ellipse (Model 26)

THE TEST NETWORK

(BLOCK OF NIGERIA'S HORIZONTAL GEODETIC NETWORK)



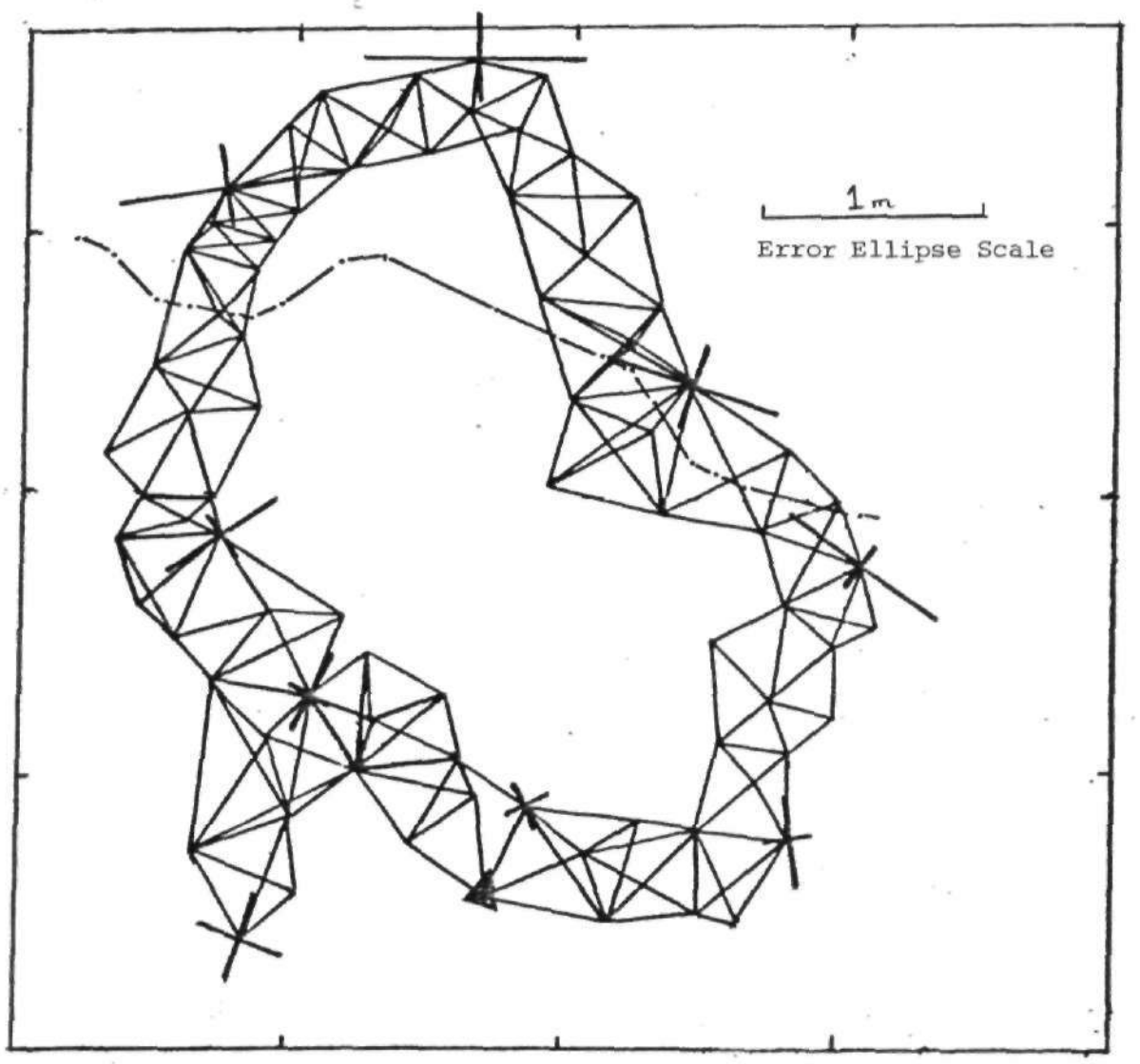
Scale 1 : 3000 000



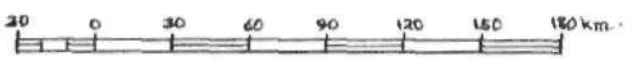
Error Ellipse (Model 27)

THE TEST NETWORK

(BLOCK OF NIGERIA'S HORIZONTAL GEODETIC NETWORK)



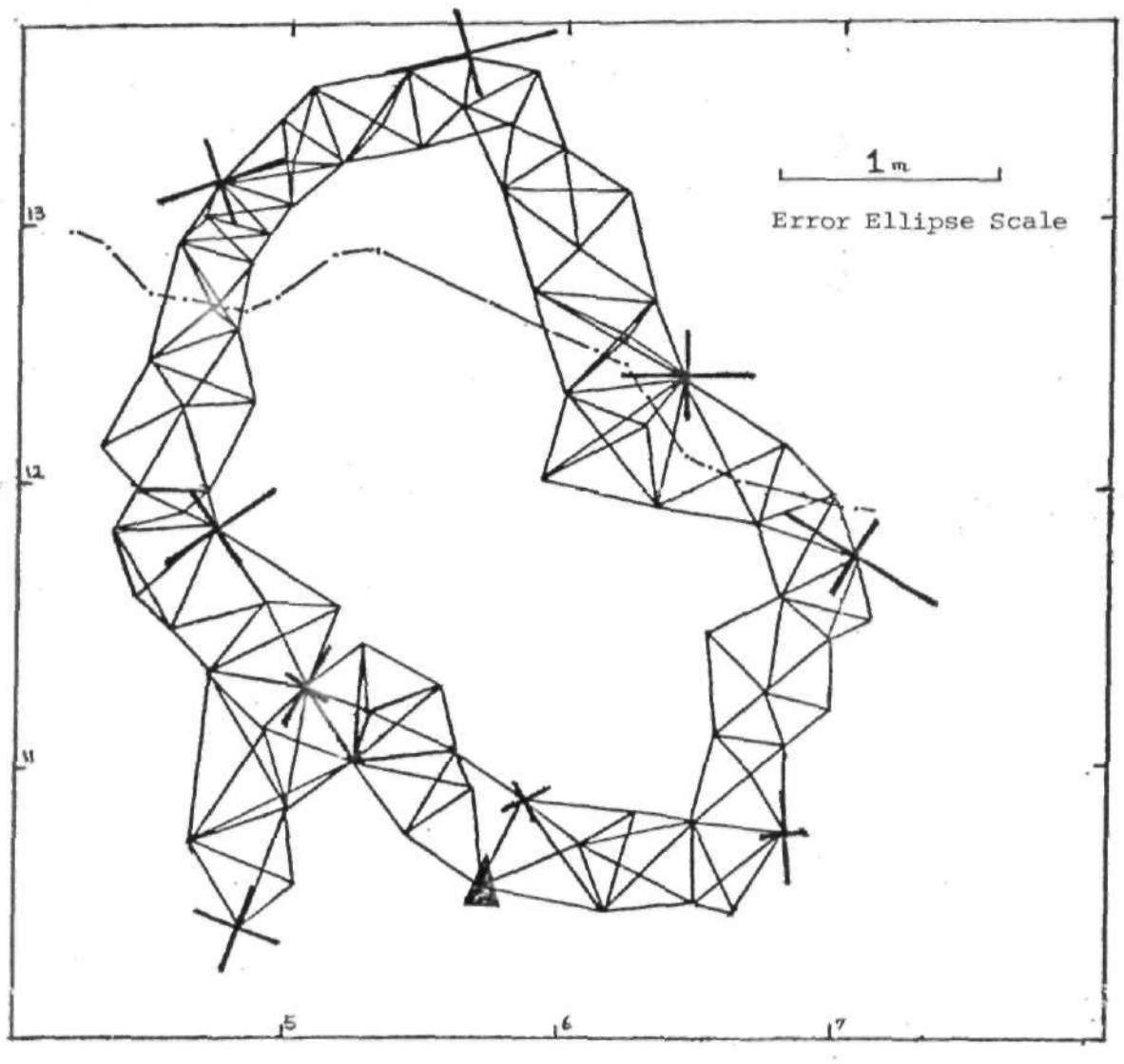
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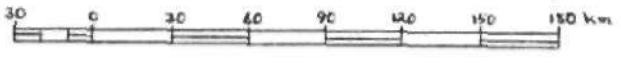


Error Ellipse (Model 28)

THE TEST NETWORK

(BLOCK OF NIGERIA'S HORIZONTAL GEODETIC NETWORK)



Scale 1 : 3000 000



Error Ellipse (Model 30)

CHAPTER SIX

THE COMPUTER PROGRAM

6.1.1 Introductory Remarks

The FORTRAN programs developed for the error analysis were run in the ICL 2980 computer of the University of Oxford. Access to the computer is normally by MAC terminals, and also via a card reader. All the jobs for this thesis were run as batch jobs (i.e. with cards). The program falls into five segments which are described in § 6.1.3. As the allowable maximum size of any one file in the computer is about 1 megabyte (1Mb), and a total of 93 stations, giving 184 unknowns were involved in the adjustment, the program was constructed to fall within this storage requirement. Besides the necessary use of Common Blocks, equivalence association was employed so that every array was used for more than just its initial purpose. Also very careful numbering of the stations was adopted to economise storage and time (see § 6.1.5). The amount of storage required for each of the tests performed was about 0.76 Mb and each job took about 443 seconds of computer time to run (see the Annex). This time was for the following operations:

(a) the formation of observation equations.

(b) the formation of the normal equations of order 184 × 184 (except the case of trilateration where the normal equation matrix is of the order 158 × 158 and the storage requirement and time differ from the above also).

(c) the inversion of the normal equation matrix.

(d) the computation of the (point) error ellipses at the ten stations, and the computation of the standard errors in azimuth and distance between the 14 pairs of stations.

Except for the computation of the inverse of the normal equations which was carried out by a NAG library routine of the Oxford University Computing Service (see § 6.1.4), the rest of the programs are self-explanatory especially with the aid of the comment statements included at each relevant stage. However a brief description of the functions of the different program segments are given in § 6.1.3.

The normal equations and the inverse share the same storage locations as the observation equations. The normal equations are formed by transposing the observation equations one column at a time, and replacing it with the resultant column of the normal.

6.1.2 Program Validation and Data Checking

Independent checks are usually required in all survey computations to ensure that the computed results are free of all arithmetic errors. For this reason and also to ensure that the program is functioning properly throughout its different stages, some data sets were used to validate the program. The data sets were abstracted from C.P.Soufleris' M.Sc. thesis, 1976. Standard errors in azimuth and distance of 33 pairs of stations each for two different data sets were computed and compared with the results obtained by Soufleris. The 66 pairs of stations compared were found to be in perfect agreement (see the Annex).

Also for each test model, the number of equations formed by the computer was printed out and this was checked against the number of equations derived from the corresponding observation equation diagram. This independent check ensures that the correct number of equations was formed for each test model. And in order to ensure against any errors in card punching, the output from the data file was checked using the observation equation diagram for the particular test involved. And since all the angles of the test network were used in all the adjustments except the case for trilateration, it was easy to perform this checking operation after the first adjustment. The checking was confined to the few NTYPE cards (see § 6.1.7b) that were changed in order to accommodate the additional simulated observations (azimuth, distance and Doppler positioning). Any errors found were immediately rectified and the whole program re-run.

6.1.3 Program Structure

As has been stated in § 6.1.1, the program is composed of five constituent segments - the controlling program, three sub-routines and one function. The amount of storage required for each program segment will be found within the computer listing in the Annex. The main features of these segments are briefly described below:

(a) Program ANALYSIS

This is the controlling program. It reads all the data inputs except those used for computing the parameters of the error ellipses and the standard errors in azimuth and distance of the selected pairs of stations. It forms the normal equations from the coefficients of the observation equations computed in the subroutine - COEAMAT. It also computes the radii of curvature of the spheroid (ρ, ν) at every station. Some of the print instructions for the output are also performed here. The output sequence is as follows:

(i) The relevant heading for the particular operation.

(ii) Station numbers (serial numbers and local names) and their provisional co-ordinates. This in turn was used to check for any mistakes by the card reader in reading the data cards. This occurred sometimes when the machine (card reader) was developing a fault.

(iii) The number of equations formed.

(iv) Particulars of NAG inverse solution.

(b) Subroutine AZDIST

This sub-routine computes azimuths and distances between two points on the surface of a spheroid by using Robbins' formulae. The computation is performed radially from one central station at a time. The formulae used were taken from Robbins [1962]. By using these formulae, azimuths are computed exactly and distances are correct to (1/100) ppm at 1600 km. This sub-program serves the sub-routine COEAMAT.

(c) Subroutine COEAMAT

This sub-routine computes all coefficients for observation equations and loads the matrices A (the design matrix) and W (the weight matrix). Equations are developed radially from one central station at a time. Each line has associated with it a code number (see § 6.1.7) which specifies the combination of equations required for it; the maximum is obviously three - angles, distance and azimuth, and the number of permutations is seven.

The parameters EXTERNAL (Yes/Not), associated with a central station, controls whether or not an equation for an external angle is required. The Doppler observation equations at selected central stations are also incorporated in the sub-routine.

Also in order to make all the coefficients uniform (i.e. of the same order of magnitude) thereby overcoming the problem of truncation errors usually associated with large numbers, the coefficients of this subroutine were divided by 100. Thus the inverse elements should be divided by 10,000 to restore the correct units.

(d) Function CONRAD

This function converts the units of the provisional co-ordinates from sexagesimal system (degrees, minutes and seconds) to radian measure.

(e) Sub-routine ELLIPSE

This subroutine computes the standard errors of azimuth (in seconds of arc) and distance (in ppm) for a pair of stations within the test network. It also computes the absolute error ellipse elements for a number of stations. Except for the provisional co-ordinates and some constants which share the same common block with their equivalent in other segments, all the other data inputs required for computing the errors stated above, are read in here. The print instructions for these errors are also performed here.

6.1.4 The Inverse and the NAG Routines

The NAG (Numerical Algorithm Group) routine F01ACF was used for the inversion of the matrix of normal equations in this thesis. The routine is mainly used for the accurate inverse of a real symmetric positive definite matrix |NAG Mark 7, F01-F03, Vol. 3, 1978|. The minimum storage requirement for this library routine is space to hold the normal equations plus one extra row, and also a working space equivalent to the size of the normal equations and the extra row. Therefore in a work file limit of 1 Mb, this requirement represents the solution for a maximum size of normals of about 250 × 250. It is to be remarked that eight bytes of store are required to store one real number in a single precision mode. Other qualities of this routine are :

(a) there are no internally declared arrays.

(b) the computed inverse is correct to full machine accuracy.

The routine employs Cholesky's triangular decomposition method on calculating the inverse, and iterates if necessary to achieve full machine accuracy. The general principle for calculating the inverse is as follows:

The accurate inverse X say, of a real symmetric positive definite matrix, A is computed by solving the set of linear equations AX = I, where I is the identity matrix. The matrix A is decomposed into triangular form

$$A = LL^T$$

using Cholesky"s decomposition. The approximate inverse of X, is first calculated (i.e. the linear equations are solved just once) by forward and back substitution. Then the accurate inverse (or

rather the inverse computed to full machine accuracy) is obtained by successive corrections of this first approximation.

The residual matrix R = AX - I, where X is a computed inverse of A, conveys useful information. Firstly $||R||$ is a bound on the relative error in X and secondly $||R|| < \frac{1}{2}$ guarantees the convergence of the iterative process in the "corrected" inverse routines. Throughout the calculations with this routine, "additional precision" accumulation of inner products is used (i.e. Double precision arithmetic is used).

The sub-routine FO1ACF is usually stated in the following format/form

FO1ACF(N, EPS, A, IA, B, IB, Z, L, IFAIL)

The type of the listed parameters is as follows:

INTEGER (N, IA, IB, L, IFAIL)

REAL (EPS, A (IA, N), B(IB,N), Z(N))

In the call statement for using the routine the following parameters were used/specified in the order given below (to agree with the above parameters):

NUNKS This is an integer variable, of dimension, N, (in the program)

N = 184 for all the tests except the case for trilateration where it is 159). On entry it specifies the order of the matrix ANORM (the matrix to be inverted). It is unchanged on exit.

XO2AAF(X) : This is a real number. On entry it specifies the smallest positive number, such that 1.0 + XO2AAF(X) > 1.0 on the computer.

The value of this "machine constant" for the ICL 2980 computer of Oxford University is 2.0**(-37). Thus for the "correct inverse", this value of machine accuracy must be attained.

ANORM : This is a real array of dimension (IA, p) where p > NUNKS. Before entry, the array must contain the elements of the real symmetric positive definite matrix. On successful exit, the upper triangle remains unaltered and the lower triangle holds the lower triangular part of the inverse.

MUNKS : This is the same dimension as IA in the above section. It is an integer variable. On entry it specifies the first dimension of array ANORM as declared in the calling program. The routine requires that MUNK ≥ NUNKS + 1 (i.e. IA > N + 1).

BINV : This is a real array of dimension (IB, q) where q > N. It is used as a working space.

NUNKS : This is an integer variable of the same dimension as IB. On entry, it specifies the first dimension of array BINV as declared in the calling program. The routine requires that IB > N. It is unchanged on exit.

XVEC : This is a real array of dimension at least N. It is used as a working space.

L : This is an integer variable. On successful exit, L contains the number of corrections added to the approximate inverse to get the accurate inverse.

IFAIL : This is an integer and acts as an error detector. The routine uses it to carry out an internal test for positive definiteness. On calling the routine IFAIL should be assigned a coded value of the following form:

IFAIL = cba

when a = 0, the program is terminated immediately if the matrix is found not to be positive definite or ill-conditioned.

a = 1, the program continues to the next stage regardless of any defect in the matrix.

b = 0, the error message is supressed.

b = 1, the error message is printed out.

c = 0, the advisory message is supressed.

c = 1, the advisory message is printed out.

In the program IFAIL was assigned the value a = 1, b = c = 0.

On exit from the routine, IFAIL has the value 0, or 1 or 2, depending on whether the inversion is successful or not.

IFAIL = 0 means that the accurate inverse has been computed.

IFAIL = 1 indicates that the matrix is not positive definite, possibly due to rounding off errors.

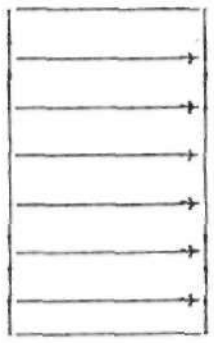
IFAIL = 2 represents that the refinement process fails to converge i.e. the matrix is ill-conditioned.

In this thesis, it was found that any failures related to IFAIL $\neq$ 0, were due to mistakes in the data.

6.1.5 Station numbering

Station numbering plays an important part as regards the storage and time-saving required in the processing of the normal equations. The matrix of the observation equations in a geodetic network is a sparse matrix and for the network under consideration it contains a maximum of six non-zero elements in each row. Thus for a particular case of this network where there are 396 observations and 184 unknowns, the non-zero elements are only about 3% of the observation equation matrix. However if the observation equations are suitably arranged, the non-zero coefficients of the normal equations will form a more or less narrow diagonal band. The solution of a large set of normal equations is much simplified if this band is kept as narrow as possible [Bomford, 1980]. This band width can be minimised by numbering the stations and unknowns in such order as will minimise the greatest difference between the serial numbers of unknowns occurring in any one equation. In other words, all the stations observed from any one station should be arranged such that they bear the least possible range of numbers. The band width of a matrix may be defined as its width at its widest point [Bomford, 1980, p. 138].

Different techniques are available in geodetic literature [Ashkenazi, 1967 and 1974; Schmitt, 1973, and Snay, 1976] on the optimum way of numbering the stations in a network in order to secure a minimum band width. Two numbering techniques have been considered in this work, and each gives a different pattern of the normal equations matrix. Parallel numbering (Fig. 6a and 6b) gives a normal equation matrix with elements clustered around the main diagonal and two secondary diagonals. Twisting numbering (Fig. 6c and 6d) gives a matrix with elements lying in branches perpendicular to the principal diagonal.

Fig. 6a 

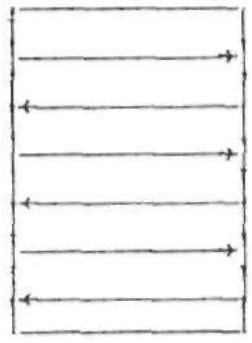
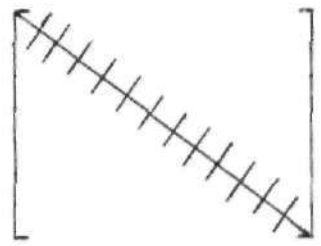
 Fig. 6c

Fig. 6b 

 Fig. 6d

Matrix of Normal equations; parallel numbering.

Matrix of Normal equations; twisting numbering.

The parallel numbering technique was mainly adopted in this work, though at places where it was not easy to apply the other technique and commonsense were used (see Fig. 6(ii)).

Also the use of serial numbers for the station numbering lends itself more easily to electronic computer processing than a system of local names and numbers which are mainly suitable for hand computations.

6.1.6 Angles or Directions

The problem of using either theodolite directions or angles as observed angular quantities in an adjustment has been a point of

argument by some geodesists [Rainsford, 1967; Vincenty, 1969; Allman and Bennet, 1966]. In general the use of either angles or directions as unknowns will lead to the same set of results (save for rounding off errors) provided that the exact geometrical correlation between the two sets of observation equations is accounted for [Ashkenazi et al, 1972]. It is therefore a matter of convenience to use either angles or directions. However it is to be remarked that in an adjustment by variation of co-ordinates, the use of angles results in one-third less unknowns and a narrower band than the use of directions. This is due to the inclusion of the orientation parameter z when directions are used. On the other hand, the incorporation of exterior angles in the method of angles is a serious addition if the band width is a critical matter. The method of angles has been used in this work because of its lesser storage requirement. Also as the exterior angles were not included in any of the adjustments considered, the main disadvantage of the method of angles was therefore not a problem.

6.1.7 Data Input

The structure of the input data is as follows :

(i) the provisional co-ordinates

(ii) the observation equation parameters (i.e. the parameters for computing the coefficients of the observation equations)

(iii) the stations and lines selected for the error analysis

(a) Provisional co-ordinates

These were punched on one card for each station, together with the computation number (serial number) and network number (local name) for the station.

(b) Observation Equation Parameters

There were two data cards per station, except in the case of the perimeter traverse, when they were four for only the stations considered. The first data card (the station card) contained information about the following:

(i) the serial number of the station (the central/observation station) being considered.

(ii) the total number of (radial) stations to which observations were made from the observation station.

(iii) whether external angles should be included (this was particularly useful for centre-point figures where observations have to close on the horizon). The three letter characters "YES" or "NOT" were used for this purpose. The "YES" indicated that observation equation for an external angles was required, and the "NOT" indicated the non-requirement of such an equation.

(iv) the serial numbers of the observed stations taken in a clockwise order. The total number of serial numbers should of course equal the number of observed stations, already indicated in (ii) above.

The second data card (the NTYPE card) indicated the type and number of observation equations required for a station/direction. For this work the following code (integer numbers were used for the purpose):

(i) 10 indicated that an observation equation for a Doppler fix was required; any other number (such as 12 used in this work) suggested the opposite. The integer variable NDO in the appropriate READ statement of the main program was used for reading in these numbers.

(ii) 1 indicated that only ANGLE observation equation is required.
2 only DISTANCE observation equation is required.
3 only AZIMUTH observation equation is required.
4 ANGLE and DISTANCE observation equations are required.
5 AZIMUTH and ANGLE observation equations are required.
6 AZIMUTH and DISTANCE observation equations are required.
7 ANGLE and DISTANCE and AZIMUTH observation equations are required.

One code number from the above seven codes was used for each of the radial stations listed in (iv) in a strict order in which these stations had been listed.

In the case of a perimeter traverse where four data cards were used for certain stations, the first two cards - the station card and the NTYPE card, represented the data information for the observation equations of the triangulation network alone. The other two cards - again a station and an NTYPE card represented the data information required for the observation equations of the traverse.

(c) Stations/Lines for the Error Analysis

For the computation of the point error ellipses, the serial numbers of the selected stations were punched on one card for each station.

For the relative standard errors in azimuth and distance, the serial numbers of the chosen pair of stations were punched on one card per pair.

SAMPLE OF OBSERVATION EQUATION DIAGRAM SHOWING STATIONS SELECTED FOR ADJUSTMENT (MODEL 1)

(Rules for selection are given in § 4.3.1)

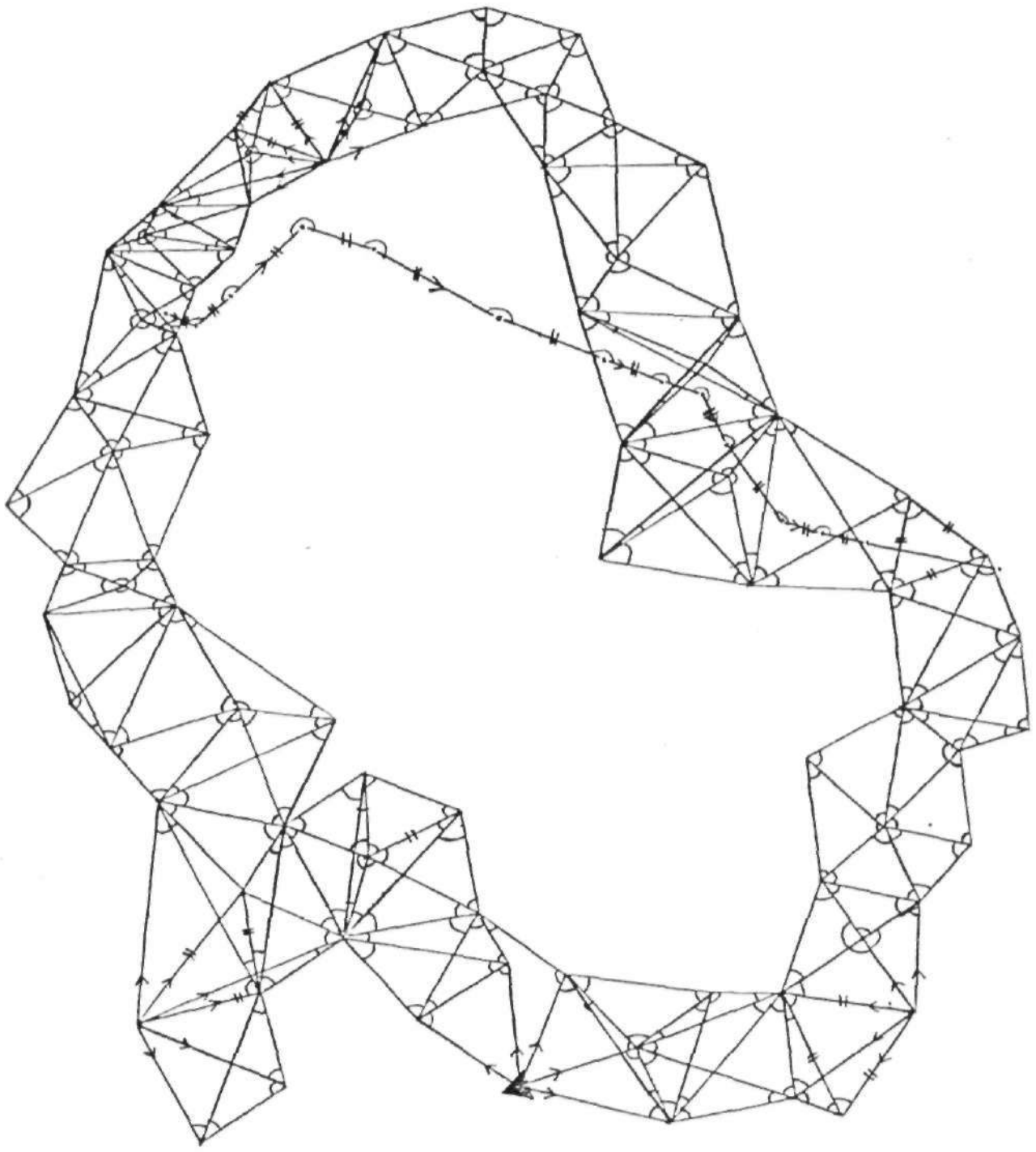
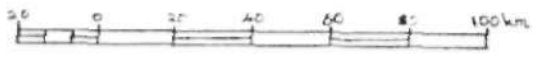


Figure 6(i)

Scale 1 : 2000 000



LEGEND

Observed Angle

Observed Azimuth (Bearing)

Observed Distance

Fixed Station

THE TEST NETWORK

(BLOCK OF NIGERIA'S HORIZONTAL GEODETIC NETWORK)

Station Numbering for Models 1 - 25

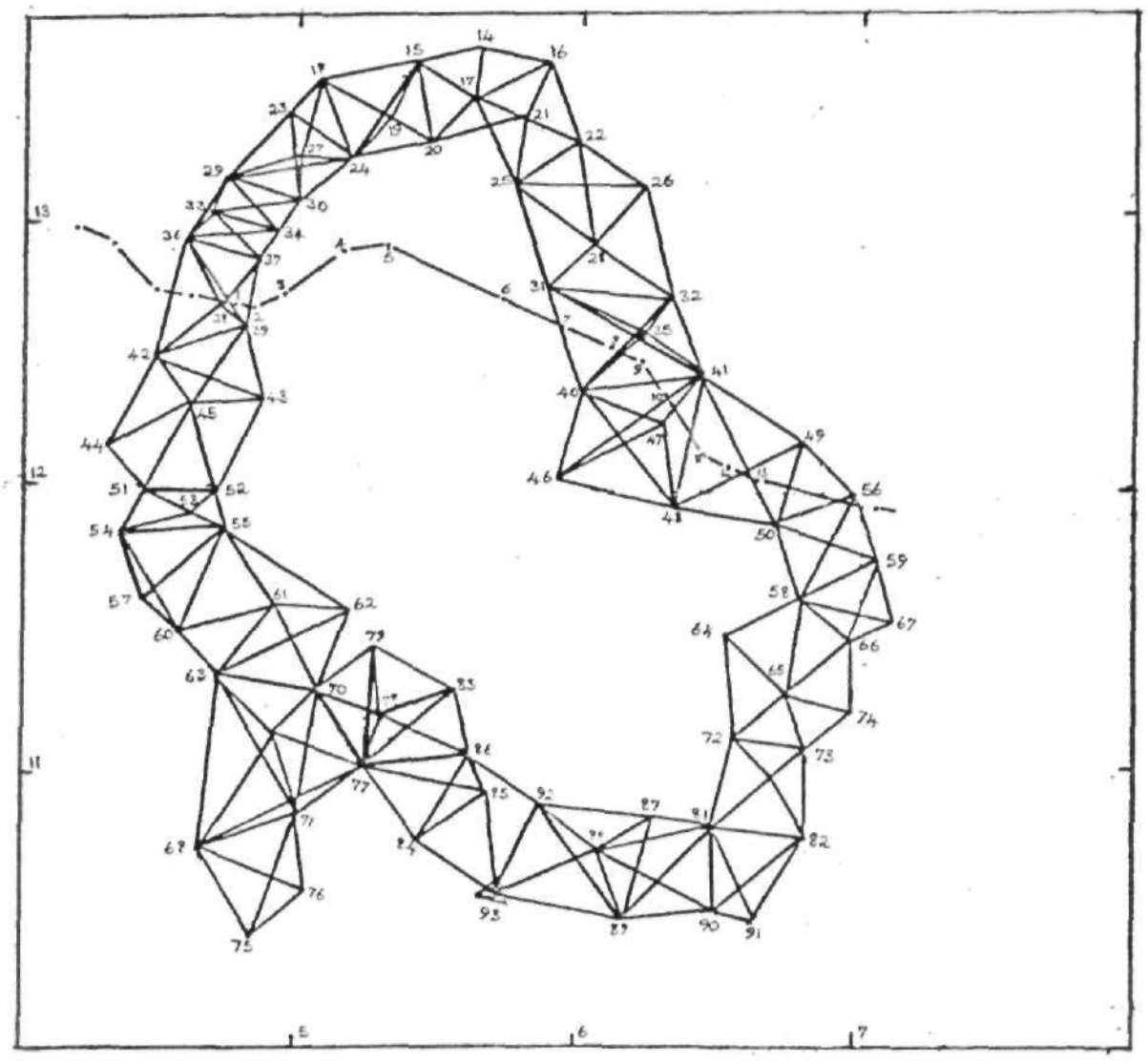
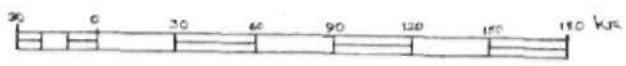


Fig 6(ii)a

Scale 1 : 3000 000



▲ Fixed station

THE TEST NETWORK

(BLOCK OF NIGERIA'S HORIZONTAL GEODETIC NETWORK)

Station Numbering for Models 26-31

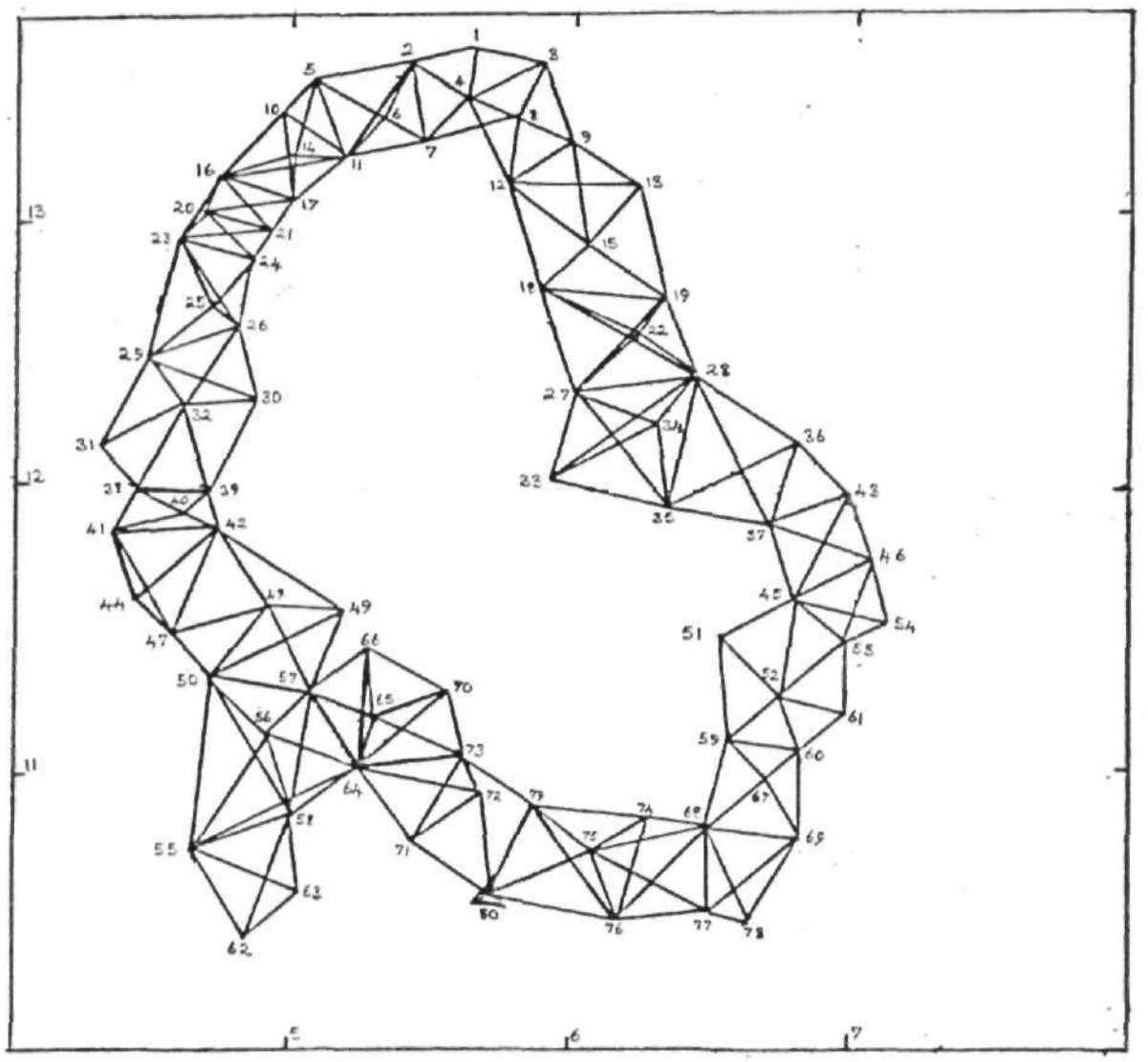
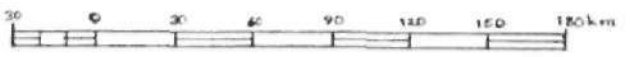


Fig 6(ii)b

Scale 1 : 3000 000



▲ Fixed Station

CHAPTER SEVEN

CONCLUSION

7.1 The present Nigerian geodetic horizontal network is obviously distorted by significant amounts both in scale and orientation. This is largely caused by the deficiency of astro-geodetic information which will pave the way for a satisfactory investigation of the geoid figure in the country.

7.2 The astrogravimetric method is probably the most feasible and economical method for determining the geoid figure in Nigeria. The method while making less demand of complicated mathematical theory and computation, provides a detailed geoid that has proper scale, shape and orientation. Moreover since the astrogravimetric technique is based on combination of two methods for determining geoid heights, the information content of the data in these methods is more completely exploited by the technique.

Also the astrogravimetric method is flexible in the sense that it is adaptable easily for both a local regional spheroid and a mass centred geocentric system. In addition to the above qualities, the method provides information on the deflection of the vertical (i.e. the astrogravimetric deflection). This information may be used for the accurate reduction of angular measurements in the framework.

7.3 If the need for accurate and proper reductions of observations to the reference spheroid is accepted as the basis for all reduction procedure irrespective of the spheroid of reference, then the geo-centric spheroid with its wider application is probably the most suitable for Nigeria. Thus the adoption of this properly selected

world system may be used to great advantages to unify geodetic, gravity and satellite work, and also to provide a basis for other activities such as navigation.

7.4 The Development method used in the 1977 adjustment has the great disadvantage of providing co-ordinates that are greatly distorted. The distortions increase with the distances of these co-ordinates from the origin.

To reduce distortions, the network needs to be readjusted, but only after weak areas have been strengthened with additional observations and all observations have been properly reduced (i.e. to the reference spheroid and the geoid as required). The additional observations should be executed with the new techniques and instrumentation for improved accuracy and speed. And for the adjustments, the modified variation of co-ordinates formulae (which include scale and orientation bias parameters) should be used in order to overcome the problem of unknown systematic scale and orientation errors present in the network.

7.5 The uneven distribution of scale and orientation measurements within the framework have probably contributed to the distortions of the network. Strength analysis of individual blocks of the network should be performed to detect weak areas and the appropriate type of observations necessary to improve the strength. Only a network optimisation procedure, based on computer simulated observations, is likely to show the best economical techniques through which this can be done.

The work just carried out has demonstrated the power of computer simulation techniques. Obviously many qualitative conclusions on the methods of strengthening (or even designing) a network could be arrived at by commonsense and experience. But only computer simulation test can provide firm quantitative answers.

7.6 Lack of Laplace azimuths properly reduced to the mean polar axis (the CIO), will introduce systematic errors in orientation of unknown amounts in the network, particularly if the deflections of the vertical are appreciable. Thus adequate steps should be taken to determine the deflections of the vertical particularly at stations where azimuths are determined. The use of Black's method for azimuth observations on stars at reasonably high altitude, say 40^{o} - 50^{o} would probably solve the dual problem

7.7 It would be difficult to use the results obtained from the present analysis to make any quantitative conclusions on the amount of extra observations needed to improve the strength of the entire network. The reasons for this are that the strength of the network varies considerably from one section of the net to the other (generally the southern part of the network is weaker than the northern part), and also that the strength of a network depends on its geometrical configuration. However the results do indicate that :

(a) Surrounding the network with a form of super traverse (high order control traverse) provides a useful constraint to it; and thus improves its strength significantly.

(b) More scale and azimuth checks should be incorporated into the network, the exact number of which depends on the particular block being considered.

(c) For the block considered in this analysis, the optimum number of additional scale and azimuth controls required is six and three respectively. In other words, any other set of observations would have to be large and well-connected to appreciably reduce the network length and azimuth standard errors.

7.8 The incorporation of satellite-Doppler results into the Nigerian geodetic network can make a most useful contribution to the improvement in strength of the network and in the knowledge of the geoid-spheroid separation.

However the results of all the tests involving Doppler observations are remarkable in that the four Doppler positions had no significant effect on the scale and orientation errors of the network. In other words, the Doppler positions did not produce any improvement in the internal strength of the network as expressed by the average a posteriori standard errors of scale and azimuth. The reasons for this are :

(a) Firstly the overwhelming number of terrestrial observations were not being affected by the few Doppler positions and would not be unless extremely large weights i.e. near-infinite weights were inappropriately assigned to the latter. Thus, in order that Doppler observations can effectively contribute their proper share in improving the strength of a network, the scale and orientation bias parameters should be properly modelled to separate these errors from random errors.

(b) Secondly Doppler observations are absolute position measurements, and their incorporation into a network serves as a means of providing constraints on the network and not for defining the internal features of that network viz the shape, the scale and the (relative) orientation of the network.

Furthermore the existence of Doppler derived spheroidal co-ordinates in the network will help in positioning the reference system of this network with respect to the mass geocentre. Doppler derived positions have the additional advantage of being free from the type of systematic errors inherent in standard terrestrial observations.

7.9 In the event of the country still inclined to adopting a regional spheroid as a reference system, an investigation of the fit of the Clarke 1880 spheroid is highly desirable. Also the disturbing discrepancies between the different checks made for the deviation of the vertical at the origin together with the undefined height for this point, suggest that a redefinition of the origin of the National Datum is highly desirable.

7.10 The current proliferation of high precision and easy to operate light-wave EDM instruments, presents an ideal opportunity not only to increase the frequency of measured distances and thereby strengthening both the shape and scale of a triangulation network, but also to measure a sufficiently large number of distances for a pure trilateration solution of the whole network. Thus the need for a triangulation as a method of establishing controls becomes questionable.

On the other hand, the use of such a low-reliability scheme (i.e trilateration) should be dealt with caution because it is often characterised with an inability to expose blunders. In general the reliability of a network improves with the increase in the number of redundant observations.

Appendix

SAMPLE OF COMPUTER LISTING

(Model 1)

PROVISIONAL COORDINATES (B.N.TRIG.NET.)

	STATION	LATITUDE			LONGITUDE		
1	CFL5	12	41	43.3[illegible]	4	47	55.264
2	CFL6	12	41	59.313	4	[illegible]3	37.131
3	CFL7	12	44	4[illegible].094	5	[illegible]	41.574
4	CFL8	12	47	47.257	5	12	59.459
5	CFL9	12	46	5[illegible].416	5	[illegible]2	28.508
6	CFL10	12	37	4[illegible].166	5	4[illegible]	45.847
7	CFL11	12	35	49.4[illegible]3	5	53	54.[illegible]66
8	CFL12	12	32	27.41[illegible]	6	5	3.466
9	CFL13	12	28	1[illegible].85[illegible]	6	1[illegible]	51.107
10	CFL14	12	22	51.[illegible]23	6	16	21.[illegible]19
11	CFL15	12	16	5[illegible].[illegible]40	[illegible]	[illegible]9	45.552
12	CFL16	12	14	49.154	5	[illegible]4	34.145
13	CFL17	12	12	1[illegible].176	5	40	28.871
14	R31	13	38	11.3[illegible]	5	[illegible]7	33.467
15	R28	13	34	4[illegible].556	5	[illegible]3	50.[illegible]53
16	R32	13	34	1[illegible].6[illegible]3	5	[illegible]2	53.235
17	R30	13	26	1[illegible].53[illegible]	5	[illegible]5	39.561
18	R25	13	32	1[illegible].31[illegible]	5	2	22.250
19	R27	13	24	[illegible]5.[illegible]32	5	17	39.267
20	R29	13	16	5[illegible].9[illegible]	5	[illegible]6	15.219
21	R33	13	21	5[illegible].5[illegible]5	5	47	7.435
22	R34	13	17	17.14	5	[illegible]	50.[illegible]69
23	R24	13	25	51.[illegible]97	4	[illegible]8	27.322
24	R26	13	14	22.1[illegible]8	5	[illegible]	10.[illegible]01
25	R35	13	7	4[illegible].11[illegible]	[illegible]	.4	22.550
26	R36	13	[illegible]	1.7[illegible]	[illegible]	13	31.[illegible]89
27	R23	13	14	[illegible]4.[illegible]61	4	[illegible]7	11.141
28	R37	13	54	[illegible].[illegible]93	5	1	41.[illegible]19

29	R21	13	1[illegible]	15.11[illegible]	4	41	2.274
30	R22	13	4	31.3[illegible]	4	37	24.023
31	R38	12	44	4[illegible].714	5	51	47.792
32	R34	12	41	39.844	6	19	14.547
33	R20	13	2	33.93[illegible]	4	37	21.714
34	R19	12	58	0.29[illegible]	4	33	1.970
35	R40	12	34	49.[illegible]24	5	13	2.374
36	R17	12	57	19.51[illegible]	4	21	37.462
37	R18	12	52	[illegible].023	4	[illegible]9	20.944
38	R16	12	41	35.333	4	39	24.681
39	R15	12	37	21.677	4	45	16.994
40	R42	12	2[illegible]	5[illegible].173	5	59	40.060
41	R41	12	25	[illegible].07[illegible]	5	35	59.004
42	R14	12	3[illegible]	4[illegible].414	4	34	54.911
43	R13	12	2[illegible]	4[illegible].4[illegible]2	4	4[illegible]	9.783
44	R11	12	11	57.55[illegible]	4	19	20.334
45	R12	12	20	[illegible].579	4	31	57.351
46	R43	12	1	16.54[illegible]	5	[illegible]	43.243
47	R45	12	14	4.31[illegible]	6	17	5.894
48	R44	11	54	44.936	5	33	15.361
49	N127	12	6	54.735	5	49	0.502
50	N126	11	51	1[illegible].757	5	[illegible]7	10.475
51	R10	12	1	25.47[illegible]	4	[illegible]2	10.495
52	R9	12	[illegible]	2[illegible].[illegible]6	4	[illegible]7	53.293
53	R[illegible]	11	54	5[illegible].43[illegible]	4	[illegible]2	48.635
54	R7	11	5[illegible]	45.1[illegible]	4	17	27.208
55	R5	11	52	35.[illegible]57	4	[illegible]8	52.477
56	N123	11	[illegible]7	1[illegible].27[illegible]	7	[illegible]3	31.1[illegible]6
58	N125	11	34	25.9[illegible]5	6	47	14.264
59	N124	11	44	[illegible].9[illegible]8	7	4	48.516
60	R3	11	25	5[illegible].[illegible]8[illegible]	4	[illegible]6	6.728

61	R2	11	32	24.935	4	50	34.435
62	R1	11	31	6.497	5	7	41.262
63	D21	11	16	9.491	4	37	40.951
64	N122	11	27	13.217	6	31	33.187
65	N120	11	15	20.917	6	45	2.892
66	N123A	11	20	27.065	6	59	33.824
67	N123	11	29	50.192	7	5	45.307
68	D17	10	45	38.310	4	33	39.405
69	D22	11	5	27.204	4	49	40.958
70	D20	11	13	8.315	5	1	44.902
71	D18	10	46	24.739	4	54	35.282
72	N116	11	5	10.650	5	33	53.758
73	N115	11	1	44.901	6	49	14.397
74	N119	11	16	29.724	5	59	19.256
75	D15	10	25	52.933	4	44	51.722
76	D16	10	35	47.417	4	56	38.918
77	D19	10	55	53.662	5	12	23.079
78	D28	11	7	57.059	5	15	2.734
79	D29	11	23	15.417	5	13	5.051
80	N118	10	57	42.511	5	2	4.474
81	N117	10	45	55.447	6	29	1.194
82	N114	10	43	43.204	5	49	25.772
83	D30	11	14	14.733	5	31	53.959
84	D32	10	41	42.915	5	24	29.947
85	D33	10	51	31.715	5	19	14.704
86	D31	10	59	55.765	5	35	0.360
87	D27	10	46	23.561	6	14	44.679
88	D26	10	41	15.417	6	3	37.125
89	D24	10	24	10.832	6	9	2.616
90	N111	10	37	37.913	6	28	50.295
91	N110	10	25	.915	6	27	29.503
92	D34	10	49	33.95	5	50	54.539
93	D35	10	29	54.655	5	41	46.524 FIN STN.

NO. OF ITERATIONS = 4

NO. OF EQUATIONS = 396

IFAIL = 0

ABSOLUTE ERROR ELLIPSE AXIS

STN.NO.	SIGMAX (METRES)	SIGMIN (METRES)	MAJOR-AXIS (AZIMUTH IN DEG.)
14	0.8	0.7	39.
22	0.8	0.6	49.
29	0.8	0.7	26.
41	0.6	0.4	57.
55	0.4	0.4	176.
59	0.4	0.4	168.
70	0.3	0.2	146.
75	0.3	0.3	159.
82	0.3	0.2	71.
92	0.2	0.1	16.

STANDARD ERRORS IN AZIMUTH AND DISTANCE

FROM	TO	LENGTH (KM)	AZIMUTH (ARC SEC.)	DISTANCE (PPM)
14	17	22.	0.87	4.7
22	25	31.	0.87	4.7
24	19	22.	0.51	3.7
32	35	17.	0.94	6.3
33	29	16.	0.94	5.7
39	38	13.	0.96	6.3
49	56	36.	1.03	2.5
55	54	39.	0.95	4.3
59	50	43.	0.94	2.3
62	63	60.	0.64	3.9
75	68	42.	0.70	3.2
87	89	46.	0.59	3.7
14	89	362.	0.45	2.2
57	67	316.	0.41	1.9

MEAN S.E. IN AZIMUTH = 0.72 ARC SEC.

MEAN S.E. IN DISTANCE = 4.3 PPM

END OF LISTING OF FILE :SURVEY.ICL81012104164710(1,*,1) FOR USER SURVEY AT 19

```
RATING SYSTEM     JOB: CTMA4     USER: SURVEY 19?1/?1/2?__04:1?:31
                  RUN UNDER PROFILE OXB3 FROM CR:5

OLC1 STDFSD(,CENT)
OLC2 OXFORTRAN(OUT=*OXLFF,LIB=*NAGF)
OXTIME: limit 716 seconds
LIST REQUEST QUEUED: LENGTH OF QUEUE IS NOW 62
OUTPUT TO :SURVEY.ICL510128041848471?(1,*,1)
FORTRAN F1.F.70(27)  CF.50.C1,MAL:FFFF1
LISTING TO
(WORK FILE)
SOURCE READ FROM
:STD.STDAT(6,*,1)
0003
0004          PROGRAM ANALYSIS
0005 C
0006 C        THIS PROGRAM CARRIES OUT STRENGTH ANALYSIS OF A BLOCK OF NIGERIA'S
0007 C        PRIMARY TRIANGULATION NETWORK,(E.N.TRIG.NET.).
0008 C        ALL COMPUTATIONS ARE BASED ON CLARK. 1880 SPHEROID;AND THE METHOD
0009 C        OF VARIATION OF COORDINATES IS USED.
0010 C
0011 C
0012          DIMENSION IFST(12),ATYPE(12),ALATR(93),ALONR(93),ROE(93),RNU(93),
0013         +DIST(12),AZ12(12),AZ21(12),A(500,184),AINORM(185,184),BINV(184,184)
0014         +,XVEC(184),WT(500),AT(500),ATW(500),TN(184)
0015 C
0016 C        NTP=TOTAL NUMBER OF STATIONS IN THE NETWORK
0017 C        NOSTAT=TOTAL NUMBER OF OBSERVED STATIONS IN THE NETWORK
0018 C        NF=TOTAL NUMBER OF FIXED STATIONS
0019 C        NFSTS=TOTAL NUMBER OF FORE STNS. OBSERVED FROM A STATION
0020 C        NST=SERIAL NUMBER OF STATIONS
0021 C        NSFF=SERIAL NUMBER OF THE FIXED STATION
0022 C        ICST=SERIAL NUMBER OF AN OCCUPIED STATION (OBSERVATION STATION)
0023 C        IFST=SERIAL NUMBERS OF OUTSIDE STATIONS
0024 C        AK=STATION NUMBER
0025 C        CONRAD=CONVERSION FROM DEGREES TO RADIANS
0026 C
0027          CHARACTER*5 EXTERNAL
0028          COMMON/ELKT/ALATR,ALONR,ROE,RNU
0029          COMMON FIF/ECCONS/EE,E1,EF
0030          COMMON/ELKS/SEMAS,SEES
0031          EQUIVALENCE (A(1),AINORM(1)),(A(34041),BINV(1))
0032          EQUIVALENCE (AT(1),ATW(1))
0033 C
0034          DATA NF,NTP,NOSTNT,NSFF,*,NUNKS,NUNKS,IFAIL/1,93,92,93,500,184,
0035         +185,1/
0036          SMA=6378249.145
0037          SME=6356514.870
0038          EE=SQRT(SMA**2-SME**2)/SMA
0039          E1=SME/SMA
0040          EF=EE/E1
0041          PI=4.*ATAN(1.)
0042          SEMAS=0.
0043          SEES=0.
0044          WRITE(2,9)
0045        9 FORMAT(1X,4(/),1X,25X,'PROVISIONAL COORDINATES (E.N.TRIG.NET.)',
0046         +//,1),23X,'STATION',9X,'LATITUDE',11X,'LONGITUDE')
0047          DO 60 J=1,NTP
0048          READ(1,10)ICST,AK,LATD,LATM,SLATS,LOND,LONM,ALONS
0049       10 FORMAT(I4,2X,A5,4X,I2,2X,I2,1X,F7.3,3X,I2,2X,I2,1X,F7.3)
```

```
      ,ALATS)
      ,ALONS)
      )**2)
      TS,ALATS,LONS,LONA,ALONS
     +5X,I2,2X,I2,F8.2,5X,I2,2X,I2,F8.3)
      TS,ALATS,LONS,LONA,ALONS
     +5X,I2,2X,I2,F8.3,5X,I2,2X,I2,F8.3,
      SUBROUTINE SPHERE(ALATS,ALONS,AZ12,AZ21,DIST,IFST,IOST,NFST
     +,ALATR,ALONR,AZ12,AZ21,DIST,
     +NFSTS,NUNKS)
      DIMENSION IFST(KD),I=1,NFSTS)
      DIMENSION ALATS(NFSTS),
     +ALONS(NFSTS)
      DIMENSION ALATR,ALONR,AZ12,AZ21,DIST,IFST,IOST,NFST
      DIMENSION ALATS,ALONS,AZ12,AZ21,DIST,
     +NFSTS,NUNKS)
      COMMON/ANGER/NUNKS,BINV,NUNKS,XVEC,L,IFAIL)
      WRITE(6,11)IOST,AK,LATS,LA
      FORMAT(1X,//,1X,20X,13,2X,
     +'TH FIXED STA.')
      GOTO 60
    5 WRITE(2,13)IOST,AK,LATS,LA
   13 FORMAT(1X,//,1X,20X,I3,2X,
     +'TH FIXED STA.')
   60 CONTINUE
      DO 65 J=1,NUNKS
      DO 65 I=1,
      A(I,J)=0.
   65 CONTINUE
C
      NFG=1
      DO 70 J=1,NOSTNT
      READ(1,12)IOST,NFSTS,EXTE
      READ(1,14)RDD,(NTYPE(K),K
   12 FORMAT(I5,2X,I2,1X,A2,1X,
   14 FORMAT(I2,2X,14(I1,1X))
      CALL AZDIST(ERF,SMA,RHO,A
     +,ATF)
      CALL COMPAT(IOST,IFST,NF
     +NTYPE,NTH,NFSTS,NTS,EXTER
   70 CONTINUE
      NFG=NFG-1
      DO 75 J=1,NTG
   75 CONTINUE
      DO 85 J=1,NUNKS
      DO 85 I=1,NTG
      AT(I)=A(I,J)
      ATW(I)=WT(I)*AT(I)
   85 CONTINUE
      DO 90 K=J,NUNKS
      TN(K)=0.
      DO 95 I=1,NTG
      TN(K)=ATW(I)*A(I,K)+TN(K)
   95 CONTINUE
      ANORM(K,J)=TN(K)
   90 CONTINUE
      DO 100 J=1,NUNKS
      DO 100 I=J,NUNKS
      ANORM(J,I)=ANORM(I,J)
  100 CONTINUE
      CALL FORMCF(NUNKS,XCZARF(
      DO 105 J=1,NUNKS
      DO 105 I=J,NUNKS
      BINV(I,J)=ANORM(I+1,J)
      BINV(J,I)=BINV(I,J)
  105 CONTINUE
      WRITE(6,21)DL,AKG,IFAIL
      CALL SLLIPS(BINV)
      STAT=SECS/12.
      SLP=SECS/12.
   21 FORMAT(1H1,(/),20X,'RC
     +IGHT = ',I2,//,21X,'IFAI
```

```
      PRINT(6,111)SMA,SMB
  111 FORMAT(1X,///,24X,'EQUATORIAL RADIUS = ',F14.2,1X,'ARC SEC.',//,
     +24X,'EARTH ECC. IN DISTANCE = ',F5.1,1X,'PPM')
      STOP
      END
C
C
C
C     SUBROUTINE AZDIST(SMA,SMB,FNTP,ALATR,ALONR,AZ12,AZ21,DIST,IFST,IOST
C    +,NFSTS)
C     THIS SUBROUTINE COMPUTES THE AZIMUTH AND THE DISTANCE
C     BETWEEN TWO POINTS ON THE SURFACE OF A SPHEROID BY USING ROBBINS'
C     FORMULA
      DIMENSION IFST(NFSTS),AZ12(NFSTS),AZ21(NFSTS),DIST(
     +NFSTS)
      COMMON FILE/ECCONS/PI,RP
      PI=4.*ATAN(1.)
      RP=SQRT(SMA**2-SMB**2)/SMA
      F1=SMB/SMA
      RP=F1/F1
      DO 150 J=1,NFSTS
      RNLSIN1=RP*(1.-F1)*SIN(ALATR(IOST))
      RNCOSN1=RP*COS(ALATR(IOST))
      RNLSIN2=RP*(1.-F1)*SIN(ALATR(IFST(J)))
      RNCOSN2=RP*COS(ALATR(IFST(J)))
      CK1=ATAN2(RNLSIN1,RNCOSN1)
      CK2=ATAN2(RNLSIN2,RNCOSN2)
      DLON12=ALONR(IFST(J))-ALONR(IOST)
      DLON21=-DLON12
      AZ12(J)=ATAN2(SIN(DLON12)*COS(ALATR(IFST(J)))
     +,COS(ALATR(IOST))*SIN(ALATR(IFST(J)))-SIN(ALATR(IOST))
     +*COS(DLON12))
      AZ21(J)=ATAN2(SIN(DLON21)*COS(ALATR(IOST))
     +,COS(ALATR(IFST(J)))*SIN(ALATR(IOST))-SIN(ALATR(IF
     +ST(J)))*COS(DLON21))
      IF(AZ12(J).LT.0.)GOTO 1
      IF((ALATR(IOST)-ALATR(IFST(J)).GT.0.)GOTO 2
      AZ21(J)=PI+AZ21(J)
      IF(SIN(AZ12(J)).GT.0.)GOTO 4
      GOTO 5
    2 AZ12(J)=PI+AZ12(J)
      IF(-SIN(AZ12(J)).GT.0.)GOTO 3
      GOTO 5
    1 IF((ALATR(IOST)-ALATR(IFST(J)).GT.0.)GOTO 4
      AZ12(J)=2*PI+AZ12(J)
      AZ21(J)=PI+AZ21(J)
      IF(-SIN(AZ12(J)).GT.0.)GOTO 4
      GOTO 5
    3 AZ12(J)=PI+AZ12(J)
      AZ21(J)=2*PI+AZ21(J)
    4 IF(SIN(AZ12(J)).GT.0.)GOTO 6
    5 SS=ASIN(SIN(DLON21)*COS(CK2)/
     +SIN(AZ12(J)))
    6 AM=.5*(SIN(ALATR(IOST))*SIN(ALATR(IOST))*COS(AZ12(J)))
      C=SQRT(1.-AM)
      DIST(J)=RNCOSN1*SS*(1.-F1*AM*(1.+C)/4.+F1*F1*AM*AM*(1.-
     +2.*AM)/8.-F1*F1*F1*AM*AM*(1.-2.*AM)/16.)
     +-SS*F1*F1*AM*AM*(1.-2.*AM)/32.
      CONTINUE
      RETURN
      END
```

```
      SUBROUTINE COEFMAT(IOST,IFS
     - (PROJECT FILE CREATED -  (VIRTUAL STOR
     4     +DIST,NTYP=,NTP,NFSTS,NSG,X
     4
     - THIS SUBROUTINE COMPUTES THE COEFFICIENTS OF THE ELEMENTS
     - OF THE A-MATRIX IN THE OBSERVATION EQUATIONS FOR DOPPLER FIX,ANGLE(INTERIOR),AZIMUTH,
     - DISTANCE,AND ANGLE(EXTERNAL
     -
     -  INTEGER IFST(NFSTS),NTYP(N
     -  REAL RGT(NTP),RNU(NTP),ALAT
     +S),DIST(NFSTS)
     -  REAL A(M,NUNKS),WT(M)
     -  CHARACTER*7 EXTERNAL
     -  COMMON PI
     -  PI=4.*ATAN(1.)
     -  FORMATION OF THE EQUATIONS
     -  IF(NTYP.EQ.1)GOTO 1
     -  GOTO 110
     -1 IF(IOST.EQ.93)GOTO 110
     -  A(NSG,2*IOST-1)=1./100.
     -  WT(NSG)=1./(.0227**2)
     -  NSG=NSG+1
     -  A(NSG,2*IOST)=1./100.
     -  WT(NSG)=1./(.0227**2)
     -  NSG=NSG+1
     - 110 IF(NTYP(J).EQ.2)GOTO 127
     -  COMPTN. OF COEFFICIENTS FOR  ANGLE & AZIMUTH EQUATIONS
     -  AZK=ROE(IOST)*SIN(AZ12(J))/
     -  AZL=RNU(IFST(J))*COS(ALAT(
     -  AZM=RGT(IFST(J))*SIN(AZ21(J
     -  AZI=-AZL
     -  IF(J.GT.1)GOTO 120
     -  AZO=AZ12(J)
     -  AZK1=AZK
     -  AZL1=AZL
     -  AZM1=AZM
     -  AZN1=AZN
     -  IFST1=IFST(J)
     - 120 IF(J.EQ.1)GOTO 121
     -  IF(NTYP(J).EQ.6.OR.NTYP(J
     -  FORMATION OF THE ANGLE
     -  IF(IOST.EQ.93)GOTO 130
     -  A(NSG,2*IOST-1)=(AZK-AZKS)/
     -  A(NSG,2*IOST)=(AZL-AZLS)/1
     -  IF(IFST(J).EQ.93)GOTO 140
     - 130 IF(IFST(J).EQ.93)GOTO 131
     -  A(NSG,2*IFST(J)-1)=AZM/1
     -  A(NSG,2*IFST(J))=AZN/1
     - 140 IF(IFSTS.EQ.93)GOTO 132
     - 131 IF(IFST(J).EQ.93)GOTO 121
     -  A(NSG,2*IFSTS-1)=-AZM/1
     -  A(NSG,2*IFSTS)=-AZN/1
     - 132 WT(NSG)=1./(.7**2)
     -  NSG=NSG+1
     - 121 AZKS=AZK
```

```
      AZNS=AZN
      AZNS=AZN
      AZOS=AZ12(J)
      IFSTS=IFST(J)
      IF(NTYP.EQ.7.OR.NTYPE(J).EQ.6.OR.NTYPE(J).EQ.5.OR.NTYPE(J).EQ.
     +3)GOTO 124
      GOTO 125
C     FORMATION OF THE AZIMUTH EQUATIONS
  124 IF(ICST.EQ.93)GOTO 133
      ACOS=(AZ12(J)-AZK)*100.
      ACOS=(AZL-SIN(ALATK(IFST(J)))*SIN(ALATI(IFST(J)))*COS(AZ12(J)))/
     +(COS(ALATK(IFST(J)))*COS(ALATI(IFST(J))))
      IF(IFST(J).EQ.93)GOTO 134
  133 IF(IFST(J).EQ.93)GOTO 122
      ACOS=(AZ12(J)-1)*SIN(1SEC)/100.
      ACOS=(AZ12(J)-AZK)*SIN(1SEC)/100.
  134 IF(ICST.EQ.24.OR.ICST.EQ.25.OR.NTYPE(J).EQ.7)GOTO 114
      WT(NIG)=1./(.7**2)
      GOTO 123
  114 WT(NEG)=1.
  116 NTG=NIG+1
  125 IF(NTYPE(J).EQ.4.OR.NTYPE(J).EQ.6)GOTO 127
      GOTO 122
C     FORMATION OF DISTANCE EQUATIONS
  127 IF(ICST.EQ.93)GOTO 135
      ACOS=(AZ12(J)-1)*SIN(1SEC)/100.
      ACOS=(AZ12(J)-AZK)*SIN(1SEC)/100.
  135 IF(IFST(J).EQ.93)GOTO 125
      ACOS=(AZ12(J)-1)*SIN(1SEC)/100.
      ACOS=(AZ12(J)-AZK)*SIN(1SEC)/100.
      +SIN1SEC)/100.
  136 IF(ICST.EQ.15.OR.ICST.EQ.22.OR.ICST.EQ.55.OR.ICST.EQ.
     +.55.OR.ICST.EQ.56.OR.ICST.EQ.
     +7.OR.21.OR.ICST.EQ.59.OR.ICST.EQ.
      WT(NIG)=((110.**6)/(2.*DIST(J)
      GOTO 117
  117 WT(NIG)=((110.**6)/(2.5*DIST(J
  118 NTG=NTG+1
C     FORMATION OF ANGLE EQUATIONS
  123 IF(J.LT.NFSTS)GOTO 122
      IF(NTG.EQ.NTG)GOTO 74
      GOTO 12
   34 IF(ICST.EQ.93)GOTO 137
      ACOS=(AZK1-AZK)/1
      ACOS=(AZL1-AZLS)/1
      IF(IFST1.EQ.93)GOTO 141
  137 IF(IFST1.EQ.93)GOTO 13
      ACOS=(AZ1-AZ1)/100.
      ACOS=(AZ1-AZ1)/100.
  141 IF(IFSTS.EQ.93)GOTO 12
  121 IF(IFSTS.EQ.93)GOTO 123
      ACOS=(AZK-AZKS)/1
      ACOS=(AZLS-AZL)/1
  136 WT(NIG)=1./(.7**2)
      NTG=NTG+1
  122 CONTINUE
      RETURN
      END
C
C
C
```

```
      FUNCTION CONRAD(ID,P,S)
 OBJECT FILE CREATED -  (VIRTUAL STORE)
      CONRAD=(ID+P/60.+S/3600.)*4.*ATAN(1.)/18
      RETURN
      END
C
C
C
      SUBROUTINE ELLIPSE(A)
 OBJECT FILE CREATED -  (VIRTUAL STORE)
C
C     THIS SUBROUTINE ELLIPSE COMPUTES LENGTHS OF THE AXES AND
C     ORIENTATIONS OF THE MAJOR AXIS OF THE ERROR ELLIPSES;AND
C     ALSO COMPUTES THE STANDARD ERRORS(S.E.) IN AZIMUTH AND DISTANCE
C     OF SOME SELECTED PAIRS OF STATIONS IN THE NETWORK.
C
      DIMENSION B(2,2),C(4,4),A(184,184),ALATR(93),ALONR(93),ROE(93),
     +RNU(93)
      DIMENSION AA(4),AD(4)
      COMMON/ELNE/ALATR,ALONR,ROE,RNU
      COMMON/BLKS/SEMAS,SEMS
      SMA=6378249.145
      SMB=6356514.870
      PIE=4.*ATAN(1.)
      SIN1SEC=SIN(PIE/(180*3600))
      WRITE(2,200)
  200 FORMAT(1X,71(/),27X,'ABSOLUTE ERROR ELLIPSE AXES',//,2X,15X,
     +'STN.NO.',3X,'SIGMAX',4X,'SIGMIN',7X,'MAJOR-AXIS',/,2X,24X,
     +'(METERS)',2X,'(METRES)',3X,'(AZIMUTH IN DEG.)',/)
   50 READ(1,*)I,J
      IF(I.NE.0)GOTO 20
      LOOP=LOOP+1
      IF(J.NE.0)GOTO 35
      C(1,1)=A(2*I-1,2*I-1)
      C(1,2)=A(2*I-1,2*I)
      C(1,3)=A(2*I-1,2*J-1)
      C(1,4)=A(2*I-1,2*J)
      C(2,2)=A(2*I,2*I)
      C(2,3)=A(2*I,2*J-1)
      C(2,4)=A(2*I,2*J)
      C(3,3)=A(2*J-1,2*J-1)
      C(3,4)=A(2*J-1,2*J)
      C(4,4)=A(2*J,2*J)
      DO 40 K=1,4
      DO 40 L=K,4
      IF(L.EQ.K)GOTO 40
      C(L,K)=C(K,L)
   40 CONTINUE
      GOTO 60
   35 DO 37 LI=1,2
      DO 37 LJ=1,2
   37 B(LI,LJ)=0.
      B(1,1)=A(2*I-1,2*I-1)
      B(1,2)=A(2*I-1,2*I)
      B(2,2)=A(2*I,2*I)
      B(2,1)=B(1,2)
      SIGXY=B(2,1)
      SIGY2=B(1,1)
      SIGX2=B(2,2)
      AZAMAX=ATAN(2*SIGXY/(SIGY2-SIGX2))/2.
      IF(SIGXY.LT.0..AND.SIGY2-SIGX2.LT.0)AZAMAX=AZAMAX+PIE/2.
      IF(SIGXY.GT.0..AND.SIGY2-SIGX2.LT.0)AZAMAX=AZAMAX+PIE/2.
```

```
      IF(SIGX.LT.  .AND.SIGX>SIGX
      SIGMAX=SQRT(SIGX2*COS(ALAT(
     +))
      SIGMIN=(SIGSUM)*SQRT(RO (I)*R
     +FIE/2.))
      SIGMAX=(SIGSUM)*SQRT(RO (I)*R
      SIGMIN=(SIGSUM)*SQRT(RO (I)*R
      AE=AZAMAX*180./PI
      AZ=AE
      IF(J.EQ.0)GO TO 61
   60 SZ=(SMA**2-SME**2)/SMA**2
      DLAT=ALAT(J)-ALAT(I)
      DLON=ALON(J)-ALON(I)
      TPS=SQRT((SIN(DLAT/2.)**2+SIN(ALAT(I))*
      PSI=ATAN((1.-E2)*TAN(ALAT(
     +COS(ALAT(J)))
      PSI=ATAN((1.-E2)*TAN(ALAT(
     +COS(ALAT(I))))
      IF(AES(DLAT).GT.ABS(DLON))GO
      AJI=PI/2.-ATAN(X1/Y)
      AJI=PI/2.-ATAN(X2/(-Y))
      IF(DLON.LT.0.)AJI=AJI+PI
      IF(DLON.GT.0.)AJI=AJI-PI
      SISG=SIN(DLON)*COS(PSI2)/SIN
      COSG=SQRT(1.-SISG*SISG)
      IF(SISG.LT.0.)AJI=ATAN(SISG
      IF(SISG.GT.0.)AJI=PI/2.-ATAN
      GOTO 9
   61 AJI=ATAN(Y/X1)
      AJI=ATAN((-Y)/X2)
      IF(DLAT.LT.0.)AJI=AJI+PI
      IF(DLAT.GT.0.)AJI=AJI+PI
      SISG=COS(PSI2)*X1/COS(AJI)
      COSG=SQRT(1.-SISG*SISG)
      IF(SISG.GT.0.)AJI=ATAN(SISG
      IF(SISG.LT.0.)AJI=AJI+TWOPI
    9 G=1.E5*SIN(ALATR(I))
      H=1.E5*COS(ALATR(I))*COS(AJI)
      SIJ=RNU(I)*SQ*(1.-SQ*SG*H*H/2.
     +)+(SQ**2)*(1./12.)*(3.*H*(4.
     +(1./72.)*(H
      IF(AJI.LT.0.)AJI=AJI+TWOPI
      IF(AJI.LT.0.)AJI=AJI+TWOPI
      IF(AJI.GT.TWOPI)AJI=AJI-TWO
      IF(AJI.GT.TWOPI)AJI=AJI-TWO
      AA(1)=EQR(I)*SIN(AJI)/SIJ
      AA(2)=RNU(J)*COS(ALATR(J))*C
      AA(3)=EQR(J)*SIN(AJI)/SIJ
      AA(4)=-AA(2)
      AB(1)=-EQ (I)*COS(AJI)*SIN
      AB(2)=EQR(J)*COS(ALATR(J))*S
      AB(3)=-EQ (J)*COS(AJI)*SIN
      AB(4)=-AB(2)
      SUM=0.
      DO 55 K1=1,4
      DO 55 K2=1,4
      SUM=SUM+AA(K1)*AA(K2)*COEL,K
   55 SUM=SUM+AB(K1)*AB(K2)*COEL,K
```

```
                SLA=(SQRT(SIA))/100.
                SID=(SQRT(SID)*1.0E+6/ABS(SIJ))/100.
                IF(LCOP.GT.22)GOTO 222
                SUMAS=SUMAS+SIA
                SUMDS=SUMDS+SID
            222 SIJ=SIJ/1000.
                DST=ABS(SIJ)
                GOTO 71
             61 WRITE(2,48)I,SIGMAX,SIGMIN,AZ
             48 FORMAT(1X,/,20X,I2,6X,F4.1,5X,F4.1,10X,F5.0)
             71 IF(I.EQ.14.AND.J.EQ.17)WRITE(2,204)
            204 FORMAT(1H1,9(/),1X,20X,'STANDARD ERRORS IN AZIMUTH AND DISTANCE
               +',//,1X,18Y,'FROM',3X,'TO',4X,'LENGTH',4X,'AZIMUTH',5X,'DISTANCE',
               +/,1X,32Y,'(KM)',4X,'(ARC SEC.)',4X,'(PPM)')
                IF(J.NE.0)WRITE(2,49)I,J,DST,SLA,SID
             49 FORMAT(1Y,/,20X,I2,4X,I2,4),F5.0,7Y,F5.2,6X,F4.1)
                GOTO 50
             20 RETURN
                END
 0437 ++++
 OBJECT FILE CREATED -  (VIRTUAL STORE)
 COMPILATION COMPLETE - NO DIAGNOSTICS
 UNIT 2 OPENED
 UNIT 1 OPENED
 0439
 0439      1     CEL5     12  41  43.378     4  47  55.174
 0440      2     CEL6     12  41  59.318     4  53  37.191
 0441      3     CEL7     12  44  48.094     5   0  41.574
 0442      4     CEL8     12  47  47.257     5  12  59.489
 0443      5     CEL9     12  46  56.418     5  22  28.518
 0444      6     CEL10    12  37  43.166     5  46  47.947
 0445      7     CEL11    12  35  46.403     5  53  54.966
 0446      8     CEL12    12  32  27.416     6   5   2.466
 0447      9     CEL13    12  28  19.456     6  10  51.107
 0448     10     CEL14    12  22  51.728     6  16  21.919
 0449     11     CEL15    12  16  59.048     6  29  45.952
 0450     12     CEL16    12  14  49.154     6  34  34.195
 0451     13     CEL17    12  12  10.176     6  40  21.071
 0452     14        R31   13  38  11.369     5  37  33.467
 0453     15        R28   13  34  43.556     5  23  51.153
 0454     16        R32   13  34  13.653     5  52  52.235
 0455     17        R30   13  26  13.530     5  35  31.571
 0456     18        R25   13  32  13.816     5   2  27.960
 0457     19        R27   13  22  35.062     5  17  39.367
 0458     20        R29   13  16  52.913     5  26  15.219
 0459     21        R33   13  21  59.595     5  47   7.435
 0460     22        R34   13  17  17.149     5  58   5.449
 0461     23        R24   13  23  51.497     4  53  27.382
 0462     24        R26   13  14  22.193     5   9   1.661
 0463     25        R35   13   7  46.112     5  44  23.538
 0464     26        R36   13   8   1.770     5  13   1.809
 0465     27        R23   13  14  34.361     4  57  11.141
 0466     28        R37   12  54   3.243     5   1  41.419
 0467     29        R21   13  10  15.116     4  41   2.374
 0468     30        R22   13   4  31.368     4  57  24.033
 0469     31        R38   12  44  41.714     5  51  49.732
 0470     32        R39   12  41  39.844     5  19  11.487
 0471     33        R20   13   2  33.018     4  37  21.714
 0472     34        R19   12  58   9.299     4  52   1.970
 0473     35        R40   12  34  45.634     5  13   7.574
 0474     36        R17   12  57  12.512     4  31  37.442
 0475     37        R18   12  52   5.778     4  43  27.644
```

0476	39	R16	12	41	33.533	4	39	24.561
0477	40	R15	12	37	21.677	4	45	12.954
0478	41	R42	12	20	50.173	5	59	47.069
0479	41	R41	12	25	8.078	6	25	59.[illegible]4
0480	42	R14	12	30	42.412	4	24	57.911
0481	43	R13	12	20	46.482	4	46	9.[illegible]3
0482	44	R11	12	11	57.550	4	13	21.394
0483	45	R12	12	20	8.589	4	31	57.351
0484	46	R43	12	1	16.546	5	53	43.243
0485	47	R45	12	14	4.318	5	17	5.[illegible]4
0486	48	R44	11	54	44.926	6	20	15.561
0487	49	N127	12	8	54.735	6	49	[illegible].562
0488	50	N126	11	51	10.757	5	42	1[illegible].475
0489	51	R10	12	1	26.446	4	22	1[illegible].455
0490	52	R9	12	0	35.236	4	37	57.223
0491	53	R8	11	54	58.422	4	32	4.[illegible]35
0492	54	R7	11	50	45.138	4	17	27.218
0493	55	R5	11	52	25.067	4	38	52.477
0494	56	N128	11	57	10.272	7	0	31.198
0495	57	R6	11	37	17.691	4	20	39.911
0496	58	N125	11	34	25.966	6	47	14.214
0497	59	N124	11	44	3.939	7	4	4[illegible].516
0498	60	R3	11	28	58.030	4	22	[illegible].726
0499	61	R2	11	32	24.925	4	50	34.435
0500	62	R1	11	31	6.497	5	7	41.262
0501	63	D21	11	18	9.433	4	37	47.951
0502	64	N122	11	27	13.217	6	31	33.197
0503	65	N120	11	15	23.937	6	45	[illegible].542
0504	66	N123A	11	26	28.065	6	59	38.[illegible]4
0505	67	N123	11	29	58.192	7	8	45.857
0506	68	D17	10	45	35.370	4	35	35.415
0507	69	D22	11	5	27.236	4	49	41.558
0508	70	D25	11	13	8.315	5	10	44.[illegible]2
0509	71	D18	10	46	24.758	4	54	35.212
0510	72	N116	11	5	16.650	6	33	57.758
0511	73	N115	11	1	44.981	6	49	14.[illegible]97
0512	74	N119	11	10	29.724	6	59	1[illegible].206
0513	75	D15	10	25	52.823	4	44	51.722
0514	76	D16	10	35	43.410	4	56	2[illegible].518
0515	77	D19	10	55	53.652	5	12	22.[illegible]79
0516	78	D28	11	7	53.052	5	15	2.714
0517	79	D29	11	23	16.437	5	13	5.[illegible]51
0518	80	N118	10	57	42.552	5	42	[illegible].474
0519	81	N117	10	45	55.492	5	29	1.194
0520	82	N114	10	43	43.294	5	49	23.712
0521	83	D30	11	14	14.758	5	31	53.959
0522	84	D32	10	40	42.916	5	24	29.947
0523	85	D33	10	51	31.815	5	39	17.[illegible]4
0524	86	D31	10	59	55.765	5	35	[illegible].560
0525	87	D37	10	48	23.561	5	14	44.[illegible]79
0526	88	D36	10	40	10.4[illegible]8	5	3	7[illegible].15
0527	89	D38	10	24	10.[illegible]95	5	9	[illegible].516
0528	90	N111	10	27	37.9[illegible]2	6	20	5 .[illegible]55
0529	91	N110	10	25	5.915	6	37	2[illegible] 3
0530	92	D34	10	49	33.950	5	56	54.5[illegible]9
0531	93	D35	10	29	54.655	5	41	4 . [illegible]4
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0533	12	0						
0534	2	2 NOT	1	3				
0535	12	1 4						
0536	3	2 NOT	2	4				
0537	12	1 7						

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0733 55     54
0734 59     50
0735 62     63
0736 75     68
0737 87     89
0738 14     99
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0740 0       0
STOP
0741 ++++
UNIT 1 CLOSED
UNIT 2 CLOSED
Used 434.568 seconds
0742 0Y3J
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RESOURCES CHARGED TO ACCOUNT :ACCOUNTS.SURVEY FOR JOB :SURVEY.CTMA4

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TRUS = 3710 VSIS = 1299 VS DISC XFERS = 303 VS OCCUPANC

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JOB  SURVEY      .CTPAA                FORTRAN F1.B.7L(27)          COMPILATION

                                 END OF COMPILATION SUMMARY

LINE  SECTION NAME                 ERRORS  WARNINGS      SIZE      ATTRIBUTE
                                                      (*) C COMMON)

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  117  AZDIST                          0         0       176       SUBROUTINE
  173  COMPAR                          0         0       239       SUBROUTINE
  293  CONRAD                          0         0       212       FUNCTION
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