COMPARATIVE STUDY OF THE PERFORMANCE OF MIMO EQUALIZERS FOR WIRELESS COMMUNICATION RECEIVERS

BY

Mohammed Dikko ALMUSTAPHA
MSC/ENG/06181/2010-2011
(md.almustapha@gmail.com)

A THESIS SUBMITTED TO THE DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING AHMADU BELLO UNIVERSITY ZARIA IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE AWARD OF MASTERS DEGREE IN ELECTRICAL ENGINEERING (COMMUNICATIONS ENGINEERING)

APRIL, 2014

Declaration

I hereby declare that the work in this thesis titled "Comparative Study of the Performance of Multiple-Input Multiple-Output (MIMO) Equalizers for Wireless communication Receivers" was performed by me in the Department of Electrical and Computer Engineering, Ahmadu Bello University, Zaria, under the supervision of Dr D.D Dajab and Prof. B.G. Bajoga. The information derived from the literature has been duly acknowledged in text and a list of references provided. No part of this work has been presented for another degree or diploma at any institution.

Mohammed Dikko ALMUSTAPHA **Date**

Certification

This project report/thesis/dissertation titled “Comparative Study of the Performance of Multiple-Input Multiple-Output (MIMO) Equalizers for Wireless Communication Receivers” meets the regulation governing the award of Master of Science (M.Sc) degree in Electrical Engineering of the Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

Dr D. D. Dajab		
Chairman Supervisory Committee	(Signature)	(Date)
Prof B. G. Bajoga		
Member, Supervisory Committee	(Signature)	(Date)
Dr M. B. Mua’zu		
Head of Department	(Signature)	(Date)
Prof A. A. Joshua		
Dean, School of Postgraduate Studies	(Signature)	(Date)

Acknowledgements

First of all I wish to thank Almighty God (Allah) for His blessings, guidance and protection during the period of this research. A Special thanks goes to my family for their love and support over the years.

I am very grateful to my supervisors, Dr. D. D. Dajab and Prof. B. G. Bajoga for their valuable comments, guidance, encouragement, and for giving me the opportunity to carry out this research or study.

I would also like to express my gratitude to Dr M. B. Muazu, Dr S. M. Sani, the Postgraduate Coordinator Dr O. Akinsanmi and the Seminar Coordinator Dr S. Garba whose enthusiasm and analytical capabilities supported me numerously during the course of my research.

M. D. Almustapha

April, 2014

Abstract

Multiple-Input Multiple-Output (MIMO) equalizers are of enormous importance in wireless communication systems due to their ability to combat the effect of Intersymbol Interference (ISI) in multipath environment. In this research, a MIMO 2X2, 2X3, 2X4, 2x5, 4X4, and 6X6 system transmission with Binary Phase shift keying (BPSK) modulation in Rayleigh fading channel was modeled and simulated using **MATLAB® V8.4.0.529 (R2009b)** Communication toolbox. The different equalization schemes namely Zero Forcing (ZF), Minimum Mean Square Error (MMSE) and Maximum Likelihood (ML) which mitigated the effect of ISI were compared to analyze the Bit Error Rate (BER) performance of the system. The results showed that the BER decreased as the antenna configuration is increased from 2X2, 4X4, to 6X6 for ML and MMSE case only. For a BER point 10^{-3} which is the benchmark for voice quality service, the ML equalizer outperformed the MMSE up to 11dB while the MMSE had a better performance over ZF with about 3 dB gain. Also results shows that a constant gain of 11 dB is maintained between ML and MMSE irrespective of the antenna configuration employed. This implied that, there is much reduction in transmitter power requirement when using ML equalizer as compared to MMSE and ZF. This is important with respect to energy savings and cost of radio equipment.

Table of Contents

2.2 Review of Prior Works

List of Tables

List of Figures

List of Appendices

Abbreviations and Symbols

AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
CP	Cyclic Prefix
CSI	Channel State Information
Eb	Energy per Bit
H	Channel Matrix
ICA	Independent Component Analysis
ICI	Inter Carrier Interference
iid	Independent Identically Distributed
ISI	Intersymbol Interference
ITU	International Telecommunication Union
MAI	Multiple Access Interference
MIMO	Multiple Input Multiple Output
MISO	Multiple Input Single Output
ML	Maximum Likelihood
MLSE	Maximum likelihood Sequence Estimation
MMSE	Minimum Mean Square Error

No	Noise Power Density
OFDM	Orthogonal Frequency Division Multiplexing
QPSK	Quadrature Phase Shift Keying
SIMO	Single Input Multiple Output
SISO	Single Input Single Output
SNR	Signal to Noise Ratio
STBC	Space Time Block Code
X	Transmitted Symbols
Y	Received Symbols
ZF	Zero Forcing

CHAPTER ONE

INTRODUCTION

1.1 Background

Wireless communication is a rapidly growing segment of the communications industry, with the potential to provide high-speed, high-quality information exchange between portable devices across the globe. The dramatic development of wireless communication over the last few decades has been tremendous. In the field of mobile communication, the demand for better technologies has surged, from voice communications requiring a data rate of a few Kbps to mobile ultra-broadband communication with a data rate of 100Mbps (as specified by the International Telecommunication Union (ITU)).

High data-rate wireless access is demanded by many broadband applications such as high speed computer networks, virtual navigation tele-medicine, and online education. Traditionally, more bandwidth is required for such higher data-rate transmission. Unfortunately, due to spectral limitations, it becomes impractical or at times very expensive to increase bandwidth (Li et.al., 2002). More so, increasing transmitter power for capacity to support high date-rate transmission is not a solution because mobile and other portable devices require the use of battery power, which is limited (Agrawal et al., 2012). In this case, using multiple transmit and receive antennas to form a Multiple-Input and Multiple-Output (MIMO) system for spectrally efficient transmission is an alternative solution.

MIMO systems have been one of the proposed solutions for enhancing spectrum utilization while fulfilling the data-rate required by the future wireless services. A considerable increase in data throughput and link range can be achieved with MIMO system without additional bandwidth or transmit power. MIMO system achieves this by higher spectral efficiency (more bits per second per Hertz of bandwidth) and link reliability or diversity (reduced fading). The multiple antennas can be used to provide high data rate transmission through multiplexing or to improve system performance through diversity (Goldsmith, 2005).

Digital modulation techniques especially the M-arry modulation schemes deliver higher data-rate effectively in multipath fading channels, as they achieve better bandwidth efficiency and give higher data-rate compared to other digital transmission systems (Suthar et al., 2010). When combined with MIMO system a blindfold of spectral efficiency is attained.

However, in mobile radio communications, the radio channel puts limits on the performance of communication systems due to various phenomena such as multipath propagation, time dispersion and fading. This introduces errors like Intersymbol interference (ISI), and other distortions into the signals transmitted over the wireless channel. To combat errors, equalization, convolution coding and other schemes are employed. Equalization compensates for channel-induced interferences whereas convolution coding enable for detection and correction of errors in a digital mobile radio communication system.

1.2 Statement of Problem

One of the major problems encountered in the wireless channel has been multipath propagation, and how to reduce its effect on the quality of information being transmitted. MIMO technology is employed due to its ability to convert multipath frequency selective fading channels to a non-multipath frequency fading channel. However, as the MIMO channel bandwidth becomes larger relative to the channel multipath delay spread, the channel suffers from ISI. One of the approaches employed in combating ISI in MIMO transmission is through the use of equalizers. However, the equalizer is much more complex in MIMO channels since the channel must be equalized over both space and time. This poses a major challenge to wireless communication receiver designers in selecting an optimal equalizer with better performance and low complexity to support future high data rate wireless applications. Hence, there is a need for the development of novel practical, low complexity equalization techniques. There is also need to understand the potentials and limitations of the equalizers when used in wireless communications systems characterized by very high data rates, high mobility and the presence of multiple antennas.

1.3 Research Aim and Objectives

The aim of the research is to investigate the performance of both Linear and non-Linear equalization algorithms for MIMO system.

The main objectives of the research are:

i. To develop mathematical model for the MIMO (2x2, 2x3, 2x4, 2x5, 4x4 and 6x6) based equalization schemes (ZF, MMSE and ML) with BPSK modulation in a Rayleigh fading channel

ii. To analyze the performance of the system by means of simulation models developed using the **MATLAB® V8.4.0.529 (R2009b)** Communication toolbox

iii. To compare the performance of the various schemes on the basis of the BER.

1.4 Scope and Significance of the Research

High data rate broadband wireless access is been demanded by many users application. In order to meet such user requirement, technologies such as MIMO are introduced for capacity and spectrum efficiency. However, due to the nature of the wireless environment especially the wideband where there exits the presence of large scatterers along the path, ISI degrades the quality of the received signal. To mitigate this effect Equalizer are employed compensate for the channel induced interference due to multipath ISI. Therefore, this research investigate the performance of linear and non-linear Equalization algorithms used in combating the effect of intersymbol interference (ISI) in a multipath fading channel. The Equalization algorithms under consideration are Zero Forcing, Minimum Mean Square Error and Maximum Likelihood Equalizers.

1.5 Limitations Of the Research

The equalization algorithms considered in this research largely depends on the condition where by the Channel Information is been known at the Receiver side. In order to meet such a requirement, the Channel must be estimated at the Receiver. Hence perfect Channel estimation is of utmost importance. Therefore a critical issue in this research is ensuring a perfect Channel estination especially at the Receiver side.

There is also a high computational requirement of the equalization algorithms which consequently implies high cost.

1.6 Thesis Outline

This thesis is organized into five chapters. The first chapter briefly introduced the research work and contains the research objectives, statement of problem and methodology. The review of related works and theoretical frame work is addressed in Chapter Two. Chapter Three discusses the methodology adopted in achieving the research objectives. Results analysis and discussion are presented in Chapter Four while Chapter Five discusses the limitations, conclusion and recommendations for further work.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter presents a review of Fundamental concepts and a review of prior works done similar to the study area. The principle of MIMO Transmission system and it gain has been presented. MIMO information theory, Digital modulation techniques and channel fading has also been discussed. Finally Equalization algorithms and mathematical modeling of the Equalizers was also presented.

2.2 Review of Prior Works

MIMO was originally conceived in the early 1970s by Bell Laboratory engineers trying to address the bandwidth limitations that signal interference caused in large, high-capacity cables. At the time, however, the processing power necessary to handle MIMO signals was too expensive to be practicable. Later on, in 1985, Jack Winters and Jack Salz published a paper on beamforming techniques describing a way of sending data from multiple users on the same frequency/time channel using multiple antennas at the transmitter and receiver (Salz, 1985). This ground breaking research opened ways for researches in the field of MIMO.

Winters (1987) studied the fundamental limits on the data rate of multiple antenna systems in a Rayleigh fading environment. With M-transmit and M-receive antennas, up to M-independent channels can be established in the same bandwidth. The results showed the potential for large capacity in systems with limited bandwidth. The research can only be viewed as an attempt to increase system capacity for systems with limited bandwidth using multiple transmit and multiple received antennas. There was no insight into the extent to which capacity growth is attained.

This led Foschini and Gans (1998), to look into the capacity of MIMO systems using multi-element array technology, which is processing the spatial dimension to improve wireless capacities in certain applications. The result showed that capacity increased linearly with the m= *min* (*M, N*) rather than logarithmically as in the case of MISO or SIMO. The drawback for this research was that, the linear relationship only holds true for independently identically distributed (i.i.d) flat Rayleigh fading channel and did not held true for all cases. Also for large number of antennas packed into small volumes, the linear relationship plateau out (collapsed) due to the effect of antenna correlation.

Apart from capacity gain, also using multiple antenna system can provide diversity by transmitting multiple copies of the same information across multiple fading channels to achieve higher reliability. Winters (1998) studied the ability of transmit diversity to provide diversity benefit to a receiver in a Rayleigh fading environment. The results showed that for 20-30 antennas, transmit diversity can achieve diversity gain of about 0.1dB for receive diversity. This implied that, the same diversity benefit can be obtained at remote and base station using multiple base-station antennas only. The main drawback of the research was the fact that an ideal MLSE with perfect channel estimation was considered. But in practice, when increasing the number of antennas, at some point the degradation due to channel estimation error may become greater than the increase in diversity gain.

Other studies have shown that MIMO systems provide a considerable increase in throughput and link range reliability without additional power or bandwidth augmentation (Li et al, 2002). This made it suitable for high data-rate multimedia application transmission. However, as the MIMO channel bandwidth became larger relative to the channel’s multipath delay spread, the channel suffered from ISI similar to the case of SISO (Goldsmith, 2005). Researchers proposed two approaches in dealing with ISI in MIMO channels. These included the use of a channel equalizer and/or a multicarrier modulation technique (Orthogonal frequency division multiplexing (OFDM)).

Choi and Cioffi (1999) proposed a combined Space- Time Block Code (ST-BC) with Multiple Input Multiple Output Equalizer (MIMO-EQ) for achieving transmit diversity in a frequency selective Rayleigh fading channel. In the proposed system, MIMO-EQ equalized the channel into a temporal Intersymbol Interference (IS1)-free channel, and then simple linear processing was used to perform the maximum likelihood (ML) decoding for ST-BC. The simulation results showed that the proposed system could achieve the full spatial diversity gain. This was due to the fact that the simple ML decoding of ST-BC required the orthogonality between signals at the receiver. Thus, MIMO-EQ was used to eliminate the temporal ISI, and thus maintain the orthogonality. However, the method employed in the research was based on Space-Time Block Codes (ST-BC) only. Space-Time Trellis Code which is another Space-Time Code type was not considered which had much more coding gain than the ST-BC.

Vicente and Asoke (2004) extended the application of independent component analysis (ICA) techniques based on higher-order statistics for scenarios composed of mutually independent non-Gaussian independently and identically distributed (i.i.d.) users signals in refining the performance of convectional linear detectors to a more generalized Multiple-Input Multiple-Output channel model with Minimum Mean Square Error (MMSE), a conventional equalization criterion. They proposed the use of ICA for the simultaneous extraction of the users' signals at their respective optimal MMSE equalization delays. The subsequent performance gains were achieved at only a modest increase in computational load relative to the conventional receiver. However, this work relied on mutual statistical independence between user's signals only; also the computational overhead involved in this method made it unsuitable for use.

Tarik and Samir (2005) addressed the problem of soft equalization and detection of coded MIMO symbols transmitted over a frequency selective fading channel. They developed a low complexity MMSE-based approach to iteratively equalizing MIMO-ISI channels where both interference cancellation and computation of the soft information were performed in a successive fashion. The approach decoupled ISI equalization and Multiple Access Interference (MAI) suppression, and allowed the ISI cancellation part to benefit from the soft output of the MAI resolution stage. Monte-Carlo simulations demonstrated attractive performance even under the presence of the proposed method of imprecise channel state information (CSI) at the receiver. The major drawback of the research was the fact that it was based on MMSE equalization only, other equalization types such as ZF, ML etc. were not considered.

Jiang et al (2005) analyzed the performances of the zero forcing (ZF) and minimal mean squared error (MMSE) equalizers applied to M × N wireless (MIMO) systems, in terms of output SNR, uncoded error and outage probabilities, diversity-multiplexing (D-M) gain tradeoff, and SNR gain. They showed that there was a gap between the output SNRs of ZF and MMSE equalizers, which converged with probability one to a random variable of scaled *F*-distribution as input SNR approaches to infinity. However, this work compared only two types of equalization algorithms ZF and MMSE. Other equalization schemes like ML DF and Blind equalizers were not considered.

Ernesto and Gerhard (2006) in their paper presented a novel approach to determining an approximation of the bias term, based on information available during the tree search in the main stack of the List Sequential (LISS) detector. LISS detection is an effective means of achieving near-capacity performance in iterative detection-decoding for MIMO systems. They proposed to approximate the bias term by considering the metric accumulated by the first path extended to a certain depth during the tree search. This required practically no implementation effort and achieved a complexity reduction comparable to that of the bias term based on an auxiliary stack. They extended further the LISS detector from ZF based to MMSE based MIMO detection and showed that the approach enabled the achievement of an excellent performance-complexity trade-off. The limitation of this work included, (i) LISS detection has shorter paths (ii) in the node extension process, tree grows very fast in width resulting in high detection complexity (iii) computational complexity and additional memory needed was very high.

Shaodan and Tung-Sang (2008) presented a semi-blind time-domain equalization technique for general (MIMO-OFDM) systems. In the proposed technique, the received OFDM symbols are shifted by more than or equal to the cyclic prefix (CP) length, and a blind equalizer is designed to completely suppress both intercarrier interference (ICI) and ISI using second-order statistics of the shifted received OFDM symbols. Simulations showed that, the proposed technique outperformed the existing techniques, and it was robust against the number of shifts in excess of the CP length. However, the authors assumed perfect time and frequency synchronization for the research which cannot be achieved in practical terms.

Bara'u and Xiaoheng (2011) studied the effect of Zero forcing and MMSE equalizers. They simulated a 2X2 MIMO system using MATLAB to determine the effect of these equalizers in combating ISI. They found out that MIMO transmission with MMSE equalization offers greater performance over ZF equalization in the region of about 3 dB which ultimately helped in reducing the effect of ISI in MIMO communication systems. The drawback for the research included ;(i) only two equalizers were compared which were both linear in nature not optimal (ii) The 2X2 MIMO system model was too basic as a better performance could be achieved by increasing the number of antennas.

In order to address these drawbacks faced in the work of Bara'u and Tan Xiaoheng (2011), **Sathish and Shankar (2011)** extended the work done by introducing an optimal equalizer which was none linear in nature. The non-linear equalizer introduced was the Maximum likelihood (ML) equalizer. The result showed that, the ML equalizer outperformed the two other linear equalizers (ZF and MMSE) although it had more complexity in design than the linear equalizers. However, the complex structure of this equalizer made it difficult to use especially with growing number of delay taps constituting a drawback.

Agrawal et al, (2012) investigated on improving the BER performance of MIMO receiver using ZF, MMSE and ML equalization. QPSK modulation was employed at the transmitter to modulate the symbols. Three types of antenna configurations were used for the simulation 2X2, 3X3 and 4X4 respectively. The simulation result showed that higher MIMO configuration (4X4) gave better BER performance. However, the drawbacks for the research included (i) QPSK modulation has low BER performance in multipath Rayleigh fading channel and hence can affect the overall system performance. (ii) The research considered only the case of equal number of transmits and receive antenna. A better performance could be achieved if the number of receiving antennas were more than the transmitting antennas because more receiving antennas translates to high diversity gain.

To overcome some of the limitations mentioned in the reviewed works of **Jiang et al (2005), Tarik and Samir (2005), Bara'u and Xiaoheng (2011), Agrawal et al, (2012)** ; (i) Less number of transmit and received antennas (ii) Non-optimal performance nature of ZF and MMSE .

This research will investigate the performance of both linear and non-linear equalization algorithms for MIMO systems. A MIMO 2X2, 4X4, 6X6, 2X3, 2X4, and 2X5 configuration will be employed so as to study the effects of antenna variations on the BER at both the transmitter and receiver side.

2.3 Principles of MIMO Transmission System

Consider the multiple-antenna system consisting of N-transmitters and M-receivers in Figure 2.1. A digital source in the form of binary data stream is fed to transmitting block for coding and mapping to complex modulation symbols (BPSK,M-QAM, etc.). This produces several separate symbol streams in which each is then mapped onto one the multiple transmit antennas. The signals are then launched into the wireless channel. At the receiver, the signals are captured by the multiple antennas and demodulation, decoding and demapping operations are performed to recover the information.

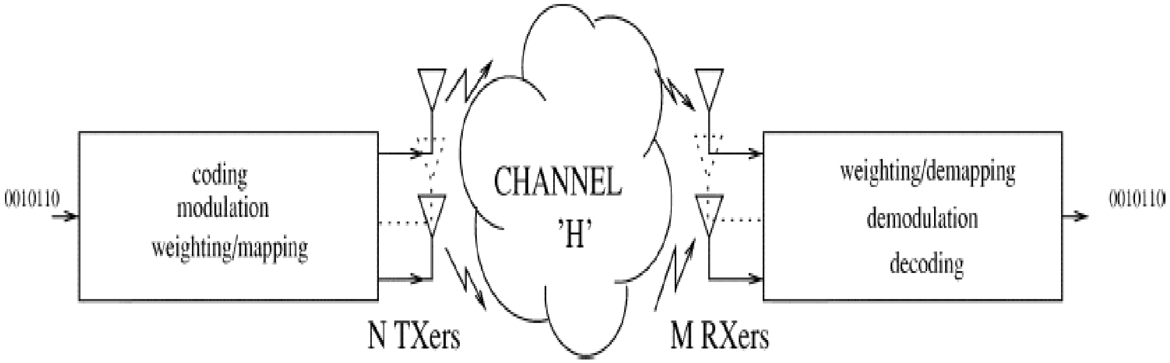


Figure 2.1: A MIMO wireless transmission system. (source: Gesbert et al (2003))

However, with MIMO systems two types of gain could be achieved, spatial multiplexing gain and diversity gain.

2.4 Multiplexing Gain

In a MIMO system configuration as in Figure 2.2, multiple independent spatial channels are created in order to provide multiplexing gain. Independent information sequences can be transmitted on each channel at the same time. The maximum number of channels is *min*(*M*,*N*), where M is the number to transmission antennas (Tx) and N is the number of receive antennas (Rx). As *min*(*M*,*N*) increases, the number of spatial channels increase linearly. The system capacity also increases linearly (Penchong and Hong, 2009).

Using singular value decomposition (SVD) from matrix theory, a matrix can be decomposed as follows;

$$H = USV^H \tag{2.1}$$

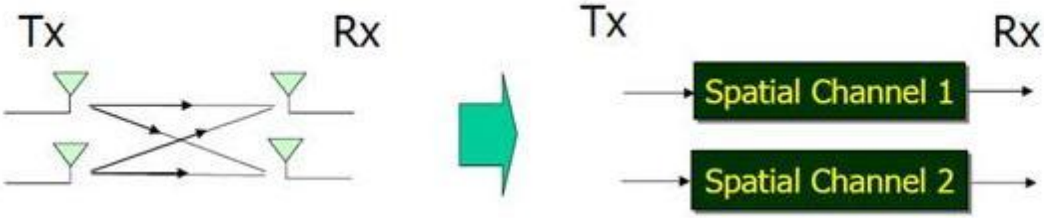


Figure 2.2: MIMO multiplexing feature (source: Murch ELEC 356)

Where U and V are unity matrix of size $N \times N$ and $M \times M$ respectively, and the superscript

H stands for Hermitian transform. Unity means $N \times N^{'} = I$. And *S* is a diagonal matrix.

The signal is processed by multiplying $U^{'}$ at the transmit side and *V* at the receive side, then the equivalent channel is given by

$$H^{'} = U^H HV = (U^H U)S(V^H V) = \begin{pmatrix} s_{11} & 0 \\ 0 & s_{22} \end{pmatrix} \tag{2.2}$$

H' is a diagonal matrix, with gains s_{11} and s_{22}. The channel equation will become

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} s_{11}x_1 \\ s_{22}x_2 \end{pmatrix} \tag{2.3}$$

This indicates that two independent channels are created when considering 2×2 antenna systems (Penchong and Hong, 2009).

2.5 Diversity Gain

Diversity is used in MIMO to combat channel fading. Since in MIMO systems, each pair of transmitting and receiving antennas provide a signal path from the transmitter to the receiver and each path carries the same information simultaneously, the signal achieved in the receive antenna is more reliable and the fading can be effectively decreased. For instance, instead of two independent information sources, we can send some processed representation of a single information source in order to achieve the diversity. The four channels can be seen as independent faded branches (Figure 3.3). So the MIMO channel now has a diversity order of 2 × 2 = 4. For the generalized MIMO channel, the diversity order is *M X N* (Bara'u and Tan Xiaoheng 2011).

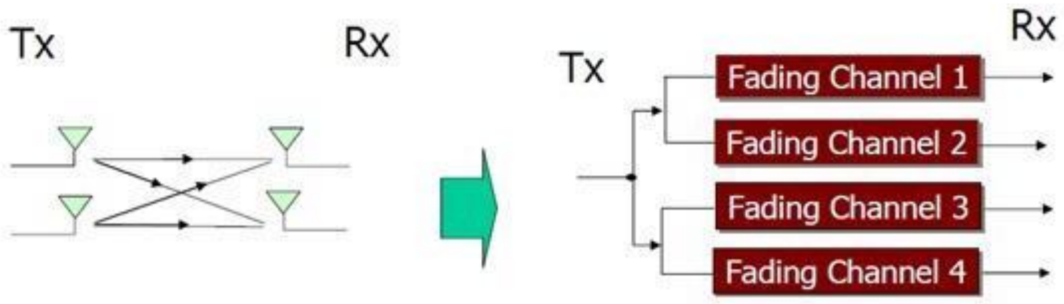


Figure 2.3 MIMO Diversity feature (source: Murch ELEC 356)

2.6 MIMO Information Theory

For a one-antenna system (Single-Input Single-Output (SISO) system), the maximum mutual information rate corresponding to the maximum data rate that can be transmitted over the channel with arbitrarily small error probability is given by the Shannon's formula (Goldsmith et al, 2002).

$$C = \log_2 \; 1 + \rho \, | h |^2 \quad \text{bps/Hz} \qquad (2.4)$$

Where h is the normalized complex gain of a fixed wireless channel or that of a particular realization of a random channel. ρ is the signal-to-noise ratio (SNR) at any RX antenna.

As the number of receiving antennas increased, the statistics of capacity improve and with N_R RX antennas, a Single-input Multiple-output (SIMO) system is formed. The expression for capacity is now given by

$$C = \log_2 \left(1 + \rho \sum_{m=1}^{N_R} | h_m |^2 \right) \quad \text{bps/Hz} \qquad (2.5)$$

Where h_m is the gain for RX antenna m^{th}. It is worth mentioning that in (2.5), increasing the value of N_R only results in a logarithmic increase in the average capacity (Gesbert et al, 2003).

Now, for the case of Multiple-input Single-output (MISO), where there are N_T transmit antennas for transmit diversity, in this common case the transmitter does not have the channel knowledge. So the capacity is given by (Foschini and Gans, 1998)

$$C = \log_2 \left(1 + \frac{\rho}{N_T} \sum_{n=1}^{N_T} | h_n |^2 \right) \quad \text{bps/Hz} \qquad (2.6)$$

Where h_n is the gain of the transmit antenna n^{th} transmit antennas. "The normalization by N_T ensures a fixed total transmitter power and shows the absence of any array gain as compared to (2.5) (Gesbert et al, 2003). Also as in the case of SIMO, the capacity for MISO grows logarithmically with N_T.

2.7 MIMO Systems Capacity

Now, let us consider the use of diversity at both transmitter and receiver to for a MIMO system. For N_T TX and N_R *RX* antennas, the equal power capacity equation is (Foschini and Gans, 1998).

$$C_{EP} = \log_2\left[\det\left(I_M + \frac{\rho}{N_T} HH^*\right)\right] \text{ bps/Hz} \qquad (2.7)$$

Where det(.) denotes the determinant of a matrix, I is an N_T X N_R identity matrix, ρ is the average received Signal to Noise Ratio (SNR), and H^* is the complex conjugate transpose of H. Foschini,(1998) and Telatar,(1995) both demonstrated that capacity in MIMO systems with equal power /uncollerated sources grew linearly with $m = \min(M, N)$ rather than logarithmically as in MISO and SIMO systems.

The characteristics of H can be investigated by performing singular value decomposition (SVD) of H to diagonalize H and find the eigen values. An SVD expansion of any matrix H$(N_R \times N_T)$ can be written as (Telatar, 1995)

$$H = UDV^* \qquad (2.8)$$

Where, U $(N_R \times N_R)$ and V $(N_T \times N_T)$ are unitary matrices, which means that $UU^* = VV^* = I$ D$(N_R \times N_T)$is non-negative and diagonal with entries specified by

$$D= diag(\sqrt{\lambda_1}, \sqrt{\lambda_2}, ..., \sqrt{\lambda_m}, 0, ..., 0) \qquad (2.9)$$

$\lambda_1, \lambda_2, ...\lambda_m$, are the nonzero eigenvalues of W, $m = \min(M, N)$, and

$$W = \begin{cases} HH^*, N_R \le N_T \\ H^*H, N_T < N_R \end{cases} \qquad (2.10)$$

The columns of U are the eigenvectors of HH^* and the columns of V are the eigenvectors of H^*H (Teletar, 1995). The SVD of equation (2.8) indicates that the channel matrix H can be diagonalized to a number of independent orthogonal sub channels, and the power gains of the i^{th} channel is λ_i (Gesbert et al 2003). Therefore equation (2.7) can be rewritten as

$$C_{EP} = \sum_{i=1}^{m} \log_2(1 + \frac{\rho}{N_T} \lambda_i) \text{ bps/Hz} \qquad (2.11)$$

Where $\lambda_1, \lambda_2, ...\lambda_m$ are the none zero eigenvalues of W in Equation (2.10).

2.8 Digital Modulation

Modulation is an integral part of any telecommunication system. There are 2 types of modulation: analog modulation and digital modulation. In analog modulation, an information-bearing analog waveform is impressed on the carrier signal for transmission whereas in digital modulation, an information-bearing discrete-time symbol sequence (digital signal) is converted or impressed onto a continuous-time carrier waveform for transmission. There exist at least three techniques for modulating digital information into an analog signal: amplitude shift keying (ASK) , frequency shift keying (FSK), and phase shift keying (PSK).In addition there is a fourth mechanism that combines changes in amplitude and phase called Quadrature amplitude modulation (QAM) which is used in many modern modems.

In this work PSK modulation scheme was employed because it is more immune to channel noise compared to the QAM scheme even though the later provide higher data rate transmission (Frenzel, 2005).

2.8.1 Phase Shift Keying

In phase shift keying (PSK), the phase of the carrier is varied to represent binary 0 or 1. Both peak amplitude and frequency remain constant as the phase changes. For example, if a phase of 0 degree represents binary 0, then the phase can be changed to 180 degrees to send binary 1. The phase of the signal during each bit duration is constant and its value depends on the bit (0 or 1). Figure 2.4 gives a conceptual view of a basic PSK scheme.

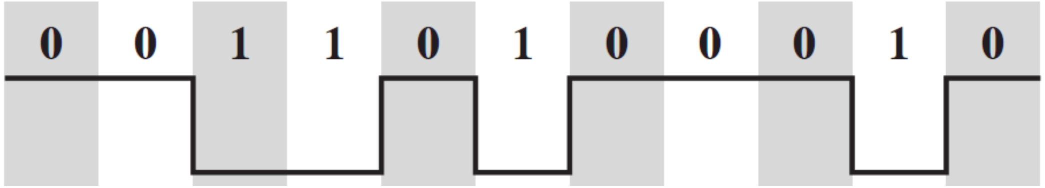


Figure 2.4: Basic PSK Scheme

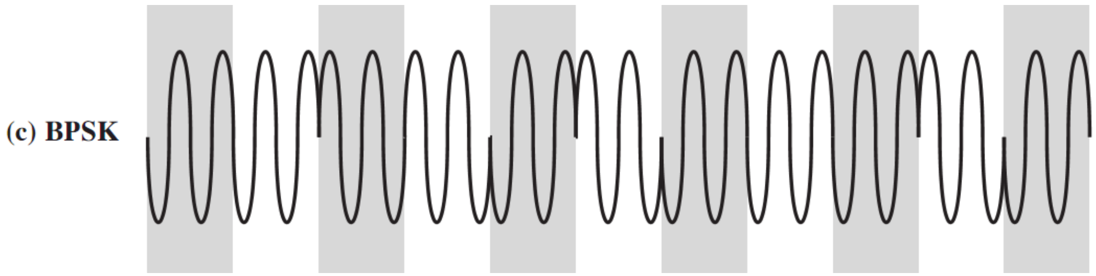
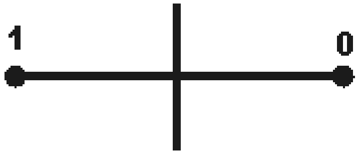


Figure 2.5: BPSK Modulation.

The scheme of Fig 2.5 is known as 2-PSK or binary PSK simply BPSK, because two different phases (0 and 180) are used. Fig 2.6 shows the relationship of phase to bit value. Fig. 2.6 (a) called a constellation or phase state diagram shows the same relationship by illustrating only the phases.



Bit	Phase
0	0
1	180

(a) Constellation Diagram

(b) Bits Mapping

Figure 2.6: BPSK Constellation (a) Constellation Diagram (b) Bits Mapping

PSK is not susceptible to the noise degradation of ASK, nor the bandwidth limitation of FSK. This means that smaller variations in the signal can be detected reliably by the receiver. The probability of error or BER for 1x1 systems with BPSK modulation in AWNG and Rayleigh fading channel is given by equations 2.12 and 2.13 respectively. (Harada and Prasad, 2002)

$$P_b = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_o}}\right) \tag{2.12}$$

$$P_b = \frac{1}{2}(1 - \frac{1}{\sqrt{1 + \frac{1}{E_b / N_o}}}) \tag{2.13}$$

2.9 Multipath Propagation

The multipath fading is a phenomenon caused by multiple copies of the transmitted signal arriving at the receiver via different routes. This occurs due to atmospheric ducting, ionospheric reflection, refraction, and reflection from water bodies and terrestrial objects such as mountains and buildings as the signal travels along the wireless channel. In digital radio communications, multipath can cause errors and affect the quality of information transmitted. The errors are due to intersymbol interference (ISI). Equalizers are used to combat the effect of the ISI. Alternatively, techniques such as orthogonal frequency division modulation and rake receivers may be used (Smalley 1994).

2.10 Rayleigh Fading Channel

The delays associated with different signal paths in a multipath fading channel changes in an unpredictable manner and can only be characterized statistically. When there are a large number of paths, the central limit theorem can be applied to model the time-variant impulse response of the channel as a complex-valued Gaussian random process. When the impulse response is modeled as a zeromean complex-valued Gaussian process, the channel is said to be a Rayleigh fading channel. Rayleigh fading is most applicable when there is no dominant propagation along a line of sight between the transmitter and receiver.

The Rayleigh probability density function (PDF) is given by (Tse and Viswanath, 2005).

$$P(r) = \begin{cases} \frac{r}{\sigma^2}\exp(-\frac{r^2}{2\sigma^2}) & for, r > 0 \\ 0 & otherwise \end{cases} \tag{2.13}$$

Where r is the envelop amplitude of the received signal, and 2 σ is the power of the received signal.

2.11 Wideband Fast Fading and its Effect

Multipath propagation is the major cause of fast fading in wireless mobile communication. Fast fading generally means the variation in signal amplitude that change rapidly with time. In a narrowband channel, usually the delay spread of the channel is less than the symbol duration of the transmitted signals as such the signals arrive at the receiver essentially at the same time. However, in a wideband channel the channel delay spread is larger than the symbol time duration and if there exist strong scatterers between the base and the mobile the signals arrive at the mobile in different time with various delays. The signal will then experience significant distortion, which varies across the channel bandwidth (Saunders and Zavala, 2007).

Now let us consider a wideband channel composed of energy which reaches the mobile via different path shown in Figure 2.7. If single scattering alone is considered, all scatterers lying on an ellipse with the base and mobile at its foci will contribute energy with the same delay given in terms of the distances between the terminals and the scatterer as

$$\tau = \frac{r_1 + r_2}{c} \tag{2.14}$$

Where c is the speed of light.

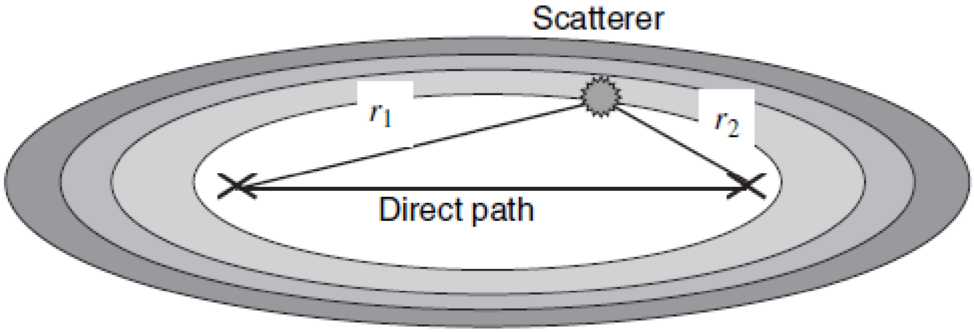


Figure 2.7: Equal-delay ellipses focused at the transmitter and receiver locations. (Source: Saunders and Zavala, 2007)

The received signal at the mobile y will be composed of a sum of waves from all scatterers, whose phase 'θ' and amplitude 'a' depend on the reflection and scattering characteristics of the scatterer, and whose time delay τ is given by (2.14) .Thus

$$y = a_1 e^{j(w\tau_1 + \theta_1)} + a_2 e^{j(w\tau_2 + \theta_2)} + \dots\dots\dots \tag{2.15}$$

Each of the components in this expression constitutes an "echo" of the transmitted signal. As the relative time delays for the arriving waves are significantly different, the channel varies with frequency and the spectrum of the received signal will be distorted.

However, in the time domain when a transmitted symbol is delayed in the channel, the energy becomes spread in time and the symbol arrives at the receiver with duration equal to the transmitted duration plus the delay range of the channel as depicted in Figure 2.8. The symbol is therefore still arriving at the receiver when the initial energy of the next symbol arrives, and this energy creates ambiguity in the demodulation of the new symbol. This process is known as intersymbol interference (ISI) (Saunders and Zavala, 2007).

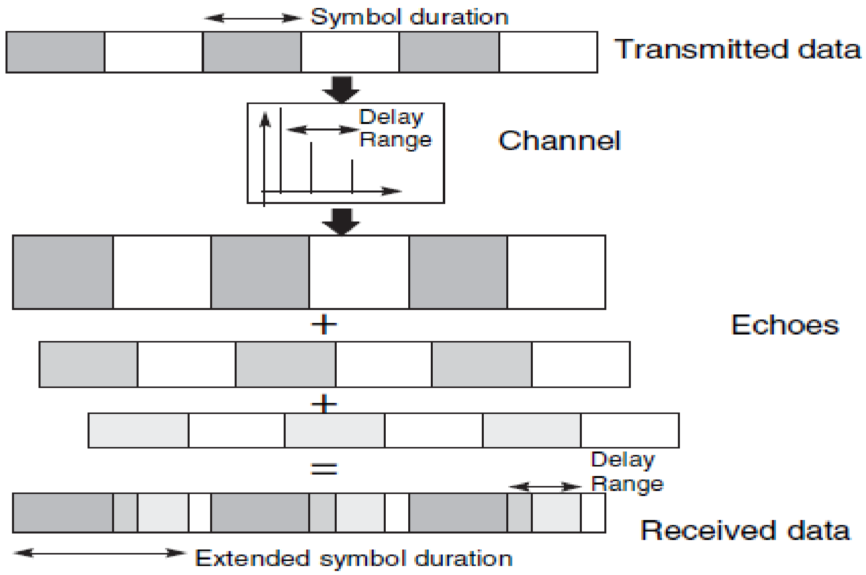


Figure 2.8: ISI due to echoes in wideband channel. (Source: Saunders and Zavala, 2007)

2.12 Overcoming Wideband Fast Fading

The wideband nature of the mobile channel imposes a significant limit on the quality of information transmitted through the channel. Due to multipath propagation, ISI degrades the quality of signal transmitted through the wideband channel as the received signal has a higher probability of been interpreted incorrectly. To overcome effect of ISI in wideband channel, the following techniques were proposed:

I. The use of Directional antennas
II. Small cells coverage
III. Diversity
IV. The use of Equalizer
V. Data rate

However, this research will focus on the equalizers used for overcoming the effect of ISI in MIMO communication systems.

2.13 Equalization

Equalization is defined as a signal processing technique used at the receiver to alleviate the problem of ISI caused by channel distortions (Goldsmith, 2005). This is achieved by the use of an adaptive digital filter (equalizer) at the receiver output. The channel frequency response of practical wireless channels is not flat due to multipath and other channel distortions. Hence, applying an equalizer flattens the channel frequency response and hence reduces the effect of ISI (Saunders and Zavala, 2007).

Equalizers fall into two broad categories: linear and nonlinear.

2.13.1 Linear Equalizers

Linear equalizers usually consist of a transversal filter with coefficients c_i chosen to best overcome the effects of the channel. Figure 2 .9 shows the structure of a linear equalizer with (2K + 1) coefficients. The coefficients are applied to versions of the received signals delayed by the symbol interval T, resulting in a symbol-spaced equalizer. The output of the filter is given by (Saunders and Zavala, 2007) as;

$$\hat{u}_k = \sum_{i=-M}^{M} c_i y_{k-i} \tag{2.16}$$

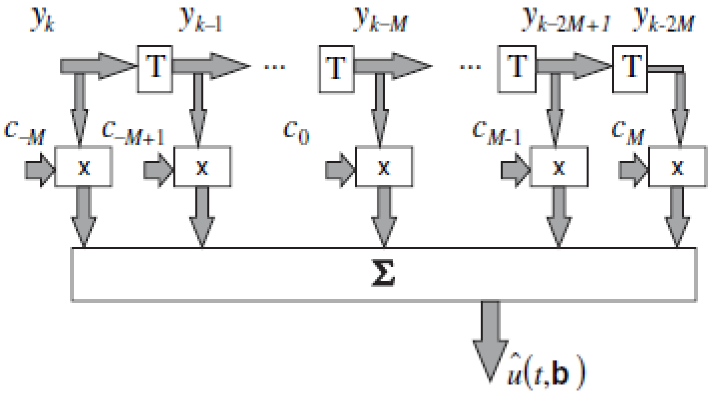


Figure 2.9: Structure of a linear equalizer (source: Saunders and Zavala, 2007)

There are two types of linear equalizers: Zero Forcing (ZF) and Minimum Mean Square Error (MMSE) which shall be discussed shortly. The former equalizer cancels all ISI, but can lead to considerable noise enhancement. The latter technique minimizes the expected mean squared error between the transmitted symbol and the symbol detected at the equalizer output, thereby providing a better balance between ISI mitigation and noise enhancement (Goldsmith, 2005).

2.13.2 Zero Forcing (ZF) Equalizer

The ZF Equalizer is a linear equalization algorithm used in communication systems proposed by Robert Lucky. It works by inverting the frequency response of the channel. In order to restore the signal before the channel, the equalizers apply the inverse of the channel to the received signal. The name Zero Forcing corresponds to bringing down the Inter Symbol Interference (ISI) to zero. But while doing so there may be an amplification of the noise term present (Sathish and Shankar, 2011).

Suppose F (*f*) is the frequency response of a channel, then the zero forcing equalizer *Z* (*f*) is constructed by (Yong et al, 2010)

$$Z(f) = \frac{1}{F(f)} \tag{2.17}$$

Therefore, the combination of channel and equalizer gives a flat frequency response and linear phase

$$F(f).Z(f) = 1 \tag{2.18}$$

The ZF equalizer is simple to implement but suffers from the problem of noise amplification hence not suitable for channels with high noise characteristics (Sathish and Shankar, 2011).

2.13.3 Mathematical Modeling of ZF equalization Matrix

Now consider a 2x 2 MIMO channel for simplicity, in the first time slot, the received signal on the first receive antenna is; (Sathish and Shankar, 2011)

$$y_1 = h_{1,1}x_1 + h_{2,2}x_2 + n_1 = [h_{1,1} \quad h_{1,2}]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \tag{2.19}$$

The received signal on the second receive antenna is given as, (Sathish and Shankar, 2011)

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1} \quad h_{2,2}]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \tag{2.20}$$

where;

y_1, y_2 are the received symbol on the first and second antenna respectively,

$h_{1,1}$ is the channel from 1st transmit antenna to 1st receive antenna,

$h_{1,2}$ is the channel from 2nd transmit antenna to 1st receive antenna,

$h_{2,1}$ is the channel from 1st transmit antenna to 2nd receive antenna,

$h_{2,2}$ is the channel from 2nd transmit antenna to 2nd receive antenna,

x_1, x_2 are the transmitted symbols and

n_1, n_2 are the noise on 1st and 2nd receive antennas

In matrix notation;

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \tag{2.21}$$

Equivalently,

$$y = Hx + n \tag{2.22}$$

Where;

Y = Received Symbol Matrix, H = Channel matrix, X = Transmitted symbol Matrix, N = Noise Matrix.

To solve for *x*, there is the need to find a matrix *W* which satisfies *WH* = *I*. The Zero Forcing (ZF) detector for meeting this constraint is given by,

$$W = (H^H H)^{-1} H^H \qquad (2.23)$$

Where: W - Equalization Matrix, and H - Channel Matrix. This matrix is known as the Pseudo inverse for a general m x n matrix. The term;

$$H^H H = \begin{bmatrix} h_{1,1}^{*} & h_{2,1}^{*} \\ h_{1,2}^{*} & h^{*}_{2,2} \end{bmatrix} \begin{bmatrix} h_{1,1} & h_{2,1} \\ h_{1,2} & h_{2,2} \end{bmatrix} = \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1}^{*} h_{2,1} + h_{2,1}^{*} h_{2,2} \\ h_{1,2}^{*} h_{1,1}^{*} + h_{2,2}^{*} h_{2,1} & |h_{1,2}|^2 + |h_{2,2}|^2 \end{bmatrix} \qquad (2.24)$$

This model can be extended to m x n antenna configuration.

2.13.4 Minimum Mean Square Error (MMSE) Equalizer.

The MMSE equalizer is based on the Minimum Mean Square Error algorithm which tries to obtain the Mean Square Error (MSE), which is a common measure of estimator quality. The main feature of MMSE equalizer is that it does not usually eliminate ISI completely but, tries to provide a tradeoff between ISI mitigation and noise enhancement by minimizing the total power of the noise and ISI components in the output.

Let x denote the original transmitted symbol, and x^ be the estimate of the transmitted symbol at the output of the equalizer. The MMSE equalizer tries to find a coefficient which minimizes the MSE between the original information symbol and the output of the equalizer as (Wong and Lok).

(2.25)

2.13.5 Mathematical Modeling of MMSE Equalization Matrix

To extract the two symbols which interfere with each other in the case 2x2 MIMO configuration, the received signal on the first received antenna is given by (kaur and singh, 2012);

$$y_1 = h_{1,1}x_1 + h_{2,2}x_2 + n_1 = [h_{1,1} \quad h_{1,2}]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \tag{2.26}$$

The received signal on the second receive antenna is, (kaur and singh, 2012).

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1} \quad h_{2,2}]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \tag{2.27}$$

Equivalently equation (2.27) is rewritten as

$$y = Hx + n \tag{2.28}$$

Where,

Y= received symbol is the channel matrix, x is the input symbol,

‘n’ is the noise getting added.

Equations (2.26) and (2.27) can be represented in matrix notation as follows

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \tag{2.29}$$

The Minimum Mean Square Error **(**MMSE**)** approach tries to find a coefficient W which minimizes the criterion,

$$E \ \left[W_{y-x}\right]\left[W_{y-x}\right]^H \tag{2.30}$$

To solve for χ, there is the need to find a matrix W which satisfies $WH = I$. The Minimum Mean Square Error (MMSE) detected for meeting this is constraint by

$$W = (H^{H}H + N_{o}I)^{-1}H^{H} \tag{2.31}$$

Where W -Equalization Matrix and H - Channel Matrix.

In fact, when the noise term is zero, the MMSE equalizer reduces to the Zero Forcing equalizer. This model can be extended to m x n antenna configuration.

2.14 Non-Linear Equalizers

Non-Linear equalizers are implemented using a lattice structure. The lattice filter employed in this type of equalizer has a more complex recursive structure than the transversal filter used by the linear equalizers. In exchange for this increased complexity relative to transversal structures, lattice structures often have better numerical stability and convergence properties and greater flexibility in changing their length (Ling and Proakis, 1985).

2.14.1 Maximum Likelihood (ML) Equalizer

The problem with most equalizers especially linear equalizers is the fact that they are not optimal in terms of minimizing the average symbol error probability. Because of the fact that the effect of a symbol is spread to other symbols, it is intuitive that the optimal receiver should observe not only the segment of received signal concerning the desired symbol, but the whole received signal instead

(Wong and Lok, 2005). Thus, the Maximum Likelihood equalizer provides the optimal solution to equalization problems by minimizing the probability of error over the entire sequence.

The ML equalizer chooses the transmitted input sequence that maximizes the log-likelihood of the received signal sequence at the receiver when the channel information is been known at receiver side. The algorithm was first proposed by Forney and later implemented with Viterbi algorithm to reduce computational complexity

2.14.2 Mathematical Modeling of (ML) Equalizer

The ML equalization approach determines the estimate of the transmitted signal vector x as (kaur and singh, 2012)

$$K = \left| y - Hx \right|^2 \tag{2.32}$$

Expanding,

$$K = \left| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{1,2} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right|^2 \tag{2.33}$$

Using BPSK modulation technique, the possible values of x_1 is +1 or -1 . Similarly x_2 also take values +1 or -1. In finding the Maximum Likelihood solution, there is the need to find the minimum from the all four possible combinations of x_1 and x_2

$$K_{+1,+1} = \left| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1.1} & h_{1,2} \\ h_{1,2} & h_{2,2} \end{bmatrix} \begin{bmatrix} +1 \\ +1 \end{bmatrix} \right|^2 \tag{2.34}$$

$$K_{+1,-1}=\left\|\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}-\begin{bmatrix} h_{1.1} & h_{1,2} \\ h_{1,2} & h_{2,2} \end{bmatrix}\begin{bmatrix} +1 \\ -1 \end{bmatrix}\right\|^2 \qquad (2.35)$$

$$K_{-1,+1}=\left\|\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}-\begin{bmatrix} h_{1.1} & h_{1,2} \\ h_{1,2} & h_{2,2} \end{bmatrix}\begin{bmatrix} -1 \\ +1 \end{bmatrix}\right\|^2 . \qquad (2.36)$$

$$K_{-1,-1}=\left\|\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}-\begin{bmatrix} h_{1.1} & h_{1,2} \\ h_{1,2} & h_{2,2} \end{bmatrix}\begin{bmatrix} -1 \\ -1 \end{bmatrix}\right\|^2 \qquad (2.37)$$

The estimate of the transmit symbol is chosen based on the minimum value from the above four values i.e

if the minimum is $K+1,+1 \Rightarrow 1 \quad 1$

if the minimum is $K+1,-1 \Rightarrow 1 \quad 0$

if the minimum is $K-1,+1 \Rightarrow 0 \quad 1$

if the minimum is $K-1,-1 \Rightarrow 0 \quad 0$

This analysis can be applied to a MIMO N x M case.

2.14.3 Decision Feedback Equalizer

The Decision Feedback Equalizer (DF) consists of a Feedfoward filter and a Feedback filter. The DFE determines the ISI contribution from the detected symbols by passing them through the feedback filter that approximates the combined discrete equivalent baseband channel. The resulting ISI is then subtracted from the incoming symbols. However, the feedback filter of the DFE does not suffer from noise enhancement because it estimates the channel frequency response rather than its inverse. For channels with deep spectral nulls, DFEs generally perform much better than linear equalizers (Goldsmith, 2005).

The detection process in the DF relies on decisions being correct, however; when detection error is made, the error keeps on propagating which give catastrophically wrong results. Figure 2.10 shows the structure of the DF equalizer.

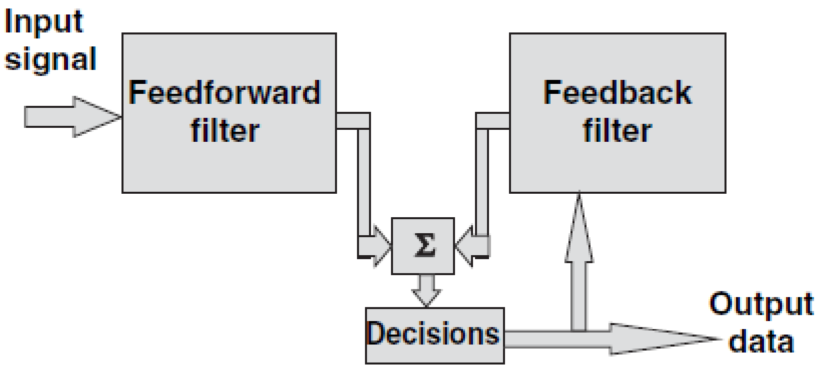


Figure 2.10: Decision-Feedback Equalizer Structure (Source: Saunders and Zavala, 2007)

2.15 Other Equalization Methods

Apart from the linear and non-linear equalizers discussed so far, other equalization method does exist such as the Turbo Equalizers, Adaptive Equalizers and Blind Equalizers. The Turbo equalizer has turbo decoding principles applied to the equalizer design. It iterates between a MAP equalizer and a decoder to determine the transmitted symbol. On the other hand, the Adaptive Equalizer uses training and tracking mechanism incorporated into the Equalizer design. During training the coefficients of the equalizer are updated at time t based on a on a known training sequence that has been sent over the channel. The tracking mechanism is used to adjust the channel estimate inherent in the equalizer. Blind equalizers do not use training: they learn the channel response via the detected data only (Goldsmith, 2005).

However, this research will focus on Zero Forcing (ZF), Minimum Mean Square Error (MMSE) and Maximum Likelihood (ML) equalizers.

2.16 Conclusion

In this chapter, the fundamental concepts of MIMO systems and Equalization techniques has been discussed and presented. A mathematical modelling of the Equalizer has also been presented. Finally a number of similar prior works were also reviewed and presented.

CHAPTER THREE

METHODOLOGY

3.1 Introduction

This chapter provides the detailed description of the materials and methodology adopted for the research. Firstly, a MIMO system with different configuration was modelled and simulated with the three different Equalization schemes employed in MATLAB. Lastly, the result obtained was compared to analyse the effect of the different Equalization schemes employed.

3.2 Choice of Modulation Scheme

In this research, M-ary PSK modulation scheme was chosen because it provides better bandwidth efficiency with less bit energy to noise power requirement than other M-ary modulation schemes like QAM, ASK etc. That is to say they are more immune to channel noise and are easy to demodulate in the presence of noise. Figure 3.1 shows the BER performances for the various digital modulations schemes. As depicted in Figure 3.1, BPSK has the best BER performance at lower carrier noise ratio (dB). Hence, the choice of BPSK modulation as modulation scheme to be employed in this research.

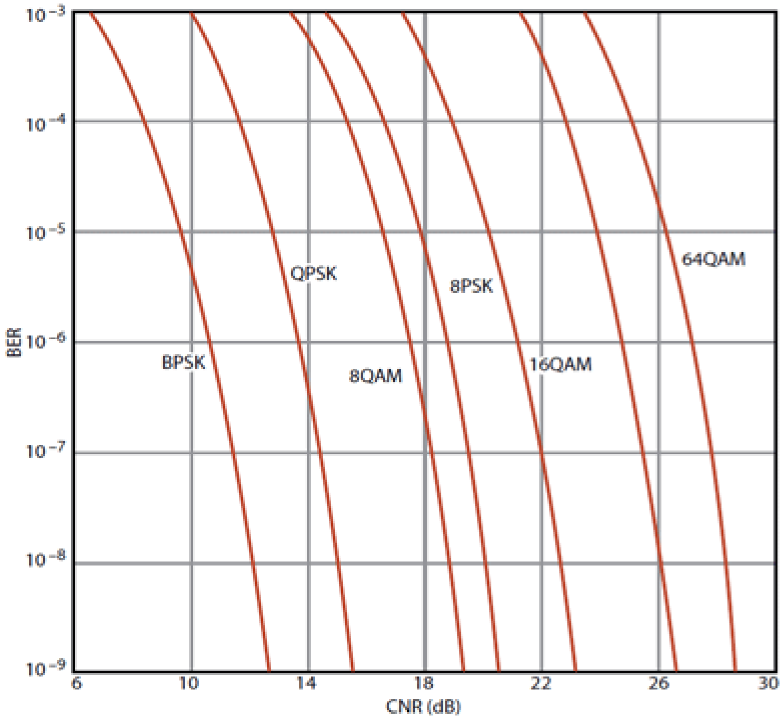


Figure 3.1: *Comparison of several popular modulation methods and their spectral efficiency expressed in terms of BER versus CNR. (Source; Frenzel, 2012).*

3.3 MIMO System Model

In this research, a MIMO M x N transmission system consisting of M-transmit and N-receive antennas was chosen. A single-Tap multipath fading channel with Rayleigh distribution was employed. Rayleigh model was chosen because it the most widely used model for indoor and urban channels signal propagation in wireless communication (Burel, 2003).

Assuming that the channel experienced by each transmit antenna is independent from the channel experienced by other transmit antennas. For the i^{th} transmit antenna to the j^{th} each transmit symbol gets multiplied by a random varying complex number $h_{i,j}$. As the channel is a Rayleigh channel, the real and imaginary parts of $h_{i,j}$ are Gaussian distributed having a mean and variance $\mu h_{i,j} = 0$ and $\sigma^2 h_{i,j} =$ — . Where by the channel experience between each transmit to receive antenna is independent and varying in time. On the receive antenna, the noise n has the

Gaussian probability density function with $p(n) =$ ═══ ——— (3.1)

with $\mu = 0$ and $\sigma^2 =$ — (3.2)

It's assumed that the channel is known at the receiver.

3.4 Simulation to Determine the Performance of MIMO Equalizers

In order to determine the performance of the equalizers in reducing the effect of ISI experienced by the signal due to multipath propagation, a MIMO N X M equalization system was modeled and simulated using MATLAB version 8.4(R2009b). Matlab provides graphic user interface for simulating dynamic systems. Matlab was chosen as simulation tool since it has a communication toolbox optimized for handling MIMO system simulation by just using simple mathematic expression, matrix and vector manipulation. It provides a platform for simulating contemporary digital communication systems. The results obtained can easily be displayed by the help of spectrum scopes and time scopes during the runtime thus giving a feel of the actual communication system. The system is modeled as shown in Figure 3.2. The modeled system consists of transmitters, a channel and receivers.

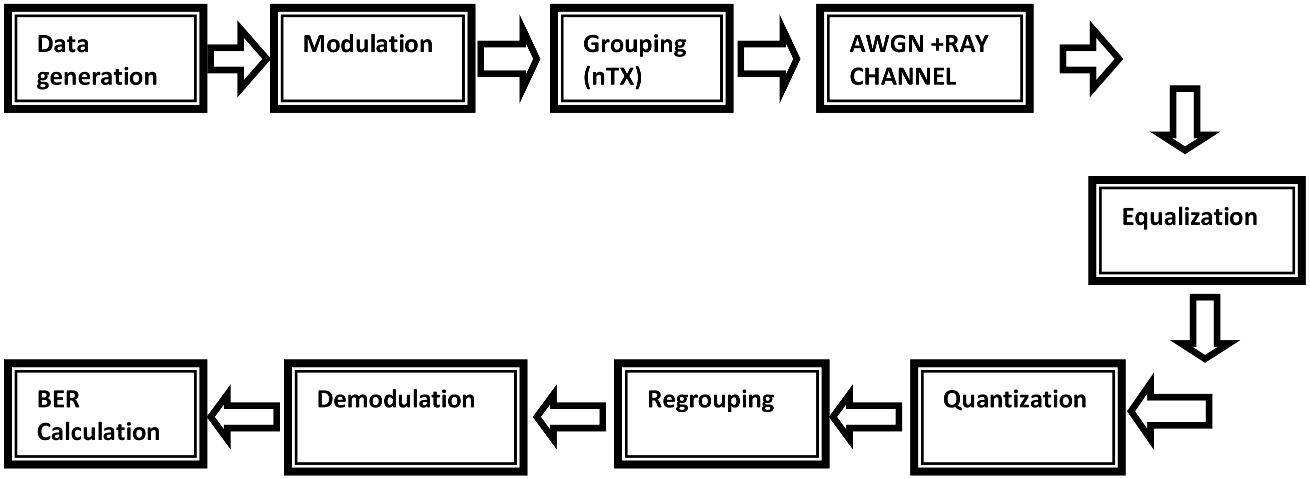


Figure 3.2: MIMO Equalization Simulation Block Model

3.4.1 Data Generation

Data generation block is used to generate a serial random binary data in form of 1’s and 0’s. This binary data stream models the raw information that is going to be transmitted. The data generated is then fed into the modulation block.

3.4.2 Modulation

In this block, binary data streams are mapped using BPSK modulation format. Each symbol is mapped to a phase angle of 0 and 180 degrees. The use of phase shift keying produces a constant amplitude signal and was chosen for its simplicity and to reduce problems with amplitude fluctuations due to fading. Coding and other forward error control mechanisms could also be implemented at this stage.

3.4.3 Grouping

The modulated data streams are now grouped into pairs of symbols depending on the number of available transmit antennas and then sent over the transmission channel.

3.4.4 Transmission Channel

The transmission channel consists of the additive white Gaussian noise (AWGN) channel and Rayleigh fading channel. In AWGN channel model, the only impairment is the linear addition of wideband or white noise with a constant spectral density (expressed as watts per hertz of bandwidth) and a Gaussian distribution of amplitude. The model does not account for the phenomena of fading, frequency selectivity, interference, nonlinearity or dispersion unlike the Rayleigh fading channel model.

3.4.5 Equalization

In this block an equalizer is applied to the received transmitted symbols in order to mitigate the effect of ISI present due to multipath propagation on the frequency selective fading channel (Rayleigh fading noise). The equalizers were modeled using equations (2.23), (2.31) and (2.32) respectively

3.4.6 Quantization

The received symbols from the equalizer are quantized and passed to the receiving antennas for reshaping.

3.4.7 Regrouping and Reshaping

The quantized pairs of symbols are regrouped and reshaped into serial data streams by the receiving antennas to resemble the original bit of data streams that was sent.

3.4.8 Demodulation

In this block, demodulation of received data streams is done.

3.4.9 Bit-error Rate (BER) Calculation.

After decoding and demodulation, the received signal is then compared to the original transmitted signal to calculate the bit error rate (BER).

3.5 MIMO System with Zero Forcing Equalizer

A MIMO MxN system was simulated using BPSK modulation and passed through a multipath fading channel with Additive White Gaussian Noise. The Zero forcing Equalizer was then applied to reduce the effect of ISI due to multipath propagation. The bit-error rate of the system is then determined. The simulation was carried out using MATLAB M-File script (see Appendix A2).

The procedure used to actualize this simulation is given as follows;

(i) A random binary sequence of +1's and 0's was generated as the data input

(ii) The generated data is then modulated using BPSK modulation

(iii) The modulated data was then grouped into pairs of symbols and sent in on time slot

(iv) The signals are then Passed through a multipath fading channel with Additive White Gaussian Noise

(v) After passing through a multipath channel, equalization (Zero Forcing) is performed on the received signal

(vi) The received symbols were the Demodulate using BPSK demodulation

(vii) The Bit Error Rate was then Calculated by comparing the transmitted signal to the received signal

The parameters used in this simulation are listed in Table 3.1

Table 3.1: Parameters for MxN MIMO with BPSK modulation using ZF/MMSE/ML over Rayleigh channel

PARAMETERS	**VALUE**
Number of bits or symbols	10^6 bits or symbols

Number of transmitters	[2-6]
Number of receivers	[2-6]
Eb/No	0:25
Modulation	BPSK
Equalization	ZF/MMSE/ML
Channel	Rayleigh channel + AWGN channel
Bit rate	1Mbps

3.6 MIMO MxN System using BPSK with MMSE Equalization

In this simulation, the effect of MMSE equalizer on MIMO signal transmitted using BPSK modulation was investigated. The MMSE equalizer though does not remove the effect of ISI but in essence provides a trade-off between ISI cancellation and noise amplification. The simulation was carried out using MATLAB M-File script (see Appendix A3).

The following algorithm was used for this simulation

(i) A random binary sequence of +1's and 0's was generated as the data input

(ii) The generated data was encoded using BPSK encoding scheme

(iii) The modulated data is then Grouped into pair of symbols and sent in time slot

(iv) Multiply the symbols with the channel and then add white Gaussian noise.

(v) The received symbols are then equalized using MMSE equalization

(vi) BPSK demodulation is then performed after equalizing the received signal

(vii) The bit error rate of the system is then calculated

The parameters used in this simulation are similar to that used for Zero forcing equalization simulation except for the equalization scheme used which is MMSE in this case (see Table 3.1)

3.7 MIMO MxN System using BPSK with ML Equalization

A MIMO MxN system was simulated using BPSK modulation and passed through a multipath fading channel with Additive White Gaussian Noise. The Maximum Likelihood Equalization was applied to reduce the effect of ISI due multipath propagation. The bit-error rate of the system is then determined. The simulation was carried out using MATLAB M-File script (see Appendix A4).

The simulation process consist the following steps;

(i) A random binary sequence of $+1'$s and $0'$s was generated as the data input

(ii) The generated data is then Modulated using BPSK modulation

(iii) After modulation, the data is grouped into pair of symbols and send time slot

(iv) The symbols are then multiplied with the channel and then white Gaussian noise is added.

(v) The minimum among the possible transmit symbol combinations is then determined

(vi) Based on the minimum the estimate of the transmit symbol is chosen

(vi) Hard decision BPSK demodulation is applied on the received symbols

(vii) The bit error rate of the system is then determined.

The flow chart for the simulation of the three different equalization schemes employed is depicted in Figure 3.3. Also the simulation results obtained for the three various Equalization algorithms using the different MIMO systems configuration is presented in Appendix A4.

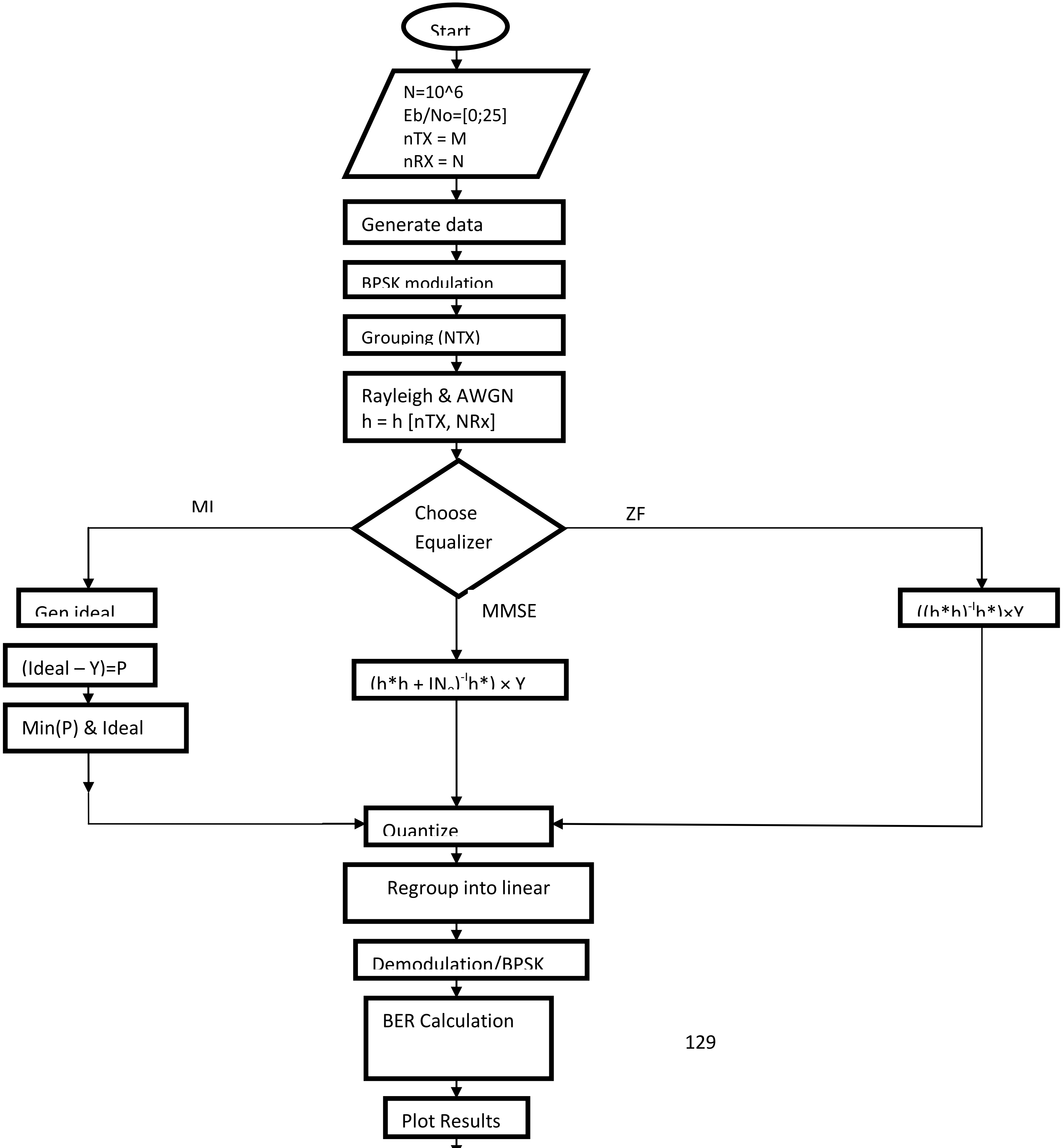

Start
N=10^6
Eb/No=[0;25]
nTX = M
nRX = N
Generate data
BPSK modulation
Grouping (NTX)
Rayleigh & AWGN
h = h [nTX, NRx]
Choose Equalizer
Ml
ZF
MMSE
Gen ideal
(Ideal – Y)=P
Min(P) & Ideal
$(h^*h + IN_0)^{-1}h^*) \times Y$
$((h^*h)^{-1}h^*)xY$
Quantize
Regroup into linear
Demodulation/BPSK
BER Calculation
Plot Results

Figure3.3: Flow chart for the Simulation process for the three different Equalization Schemes

3.8 Conclusion

The material and methodology adopted for the research has been presented in this chapter. A MIMO system with different configuration was modeled and simulated in MATLAB and the three different Equalization algorithms were applied to reduce the effect of ISI.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Introduction

In this chapter, the simulation results are analyzed and discussed. The simulation results are plotted in term of the performance of system that is Bit Error Rate (BER). Also the performance of the three different Equalization algorithms were compared for different MIMO system configuration.

4.2 Performance Measure

The performance of most digital communication system is evaluated in terms of Bit Error Rate (BER) versus Energy per bit- to- Noise density (Eb/No).

In digital communication BER is the percentage of bits in error relative to the total number of bits received in a transmission, usually expressed as ten to a negative power. Mathematically, BER is given as: (TimbercCon, 2012)

$$\text{BER} = \frac{\qquad\qquad}{\qquad\qquad} \qquad (4.1)$$

The Eb/No is defined as the ratio of Energy per bit divided by the noise power density. It is in determining the strength of the signal of the energy of each bit in a transmission to the strength of the background noise. The Eb/No allows for comparing the BER performance of digital modulation schemes without taking bandwidth into account (Hills, 2008).

Table 4.1 shows the Expert's benchmark for BER for voice and data quality of service. This is the minimum acceptable BER value for good quality of service to be attained.

Table 4.1: ITU Recommended BER Benchmark.

S/N	Bit Error Rate Value	Application
1	10^{-3}	Voice
2	10^{-6}	Data

4.3 MIMO with Zero Forcing (ZF) Equalization

Figure 4.1 shows the simulation result of MIMO 2x2, 4x4, and 6x6 with BPSK modulation using ZF equalization. The simulation results of the 2x2, 4x4, and 6x6 all shows a matching result. As depicted by the figure 4.1, at a BER point 10^{-3}; the Eb/No value is 24dB for all the MIMO configurations employed (2x2, 4x4, and 6x6). This implies that, the zero forcing equalizer helps us to achieve the data rate gain, but NOT take advantage of diversity gain as for the number of antennas present (Tse and Viswanath, 2007). Therefore, increasing the antenna configuration does not give any improvement in terms of BER performance. This is because the ZF equalizer tries to cancel out the interfering terms when performing the equalization and while doing so, there can be an amplification of noise. Hence the ZF is not suitable for application in noisy channels. However, it is simple and easy to implement.

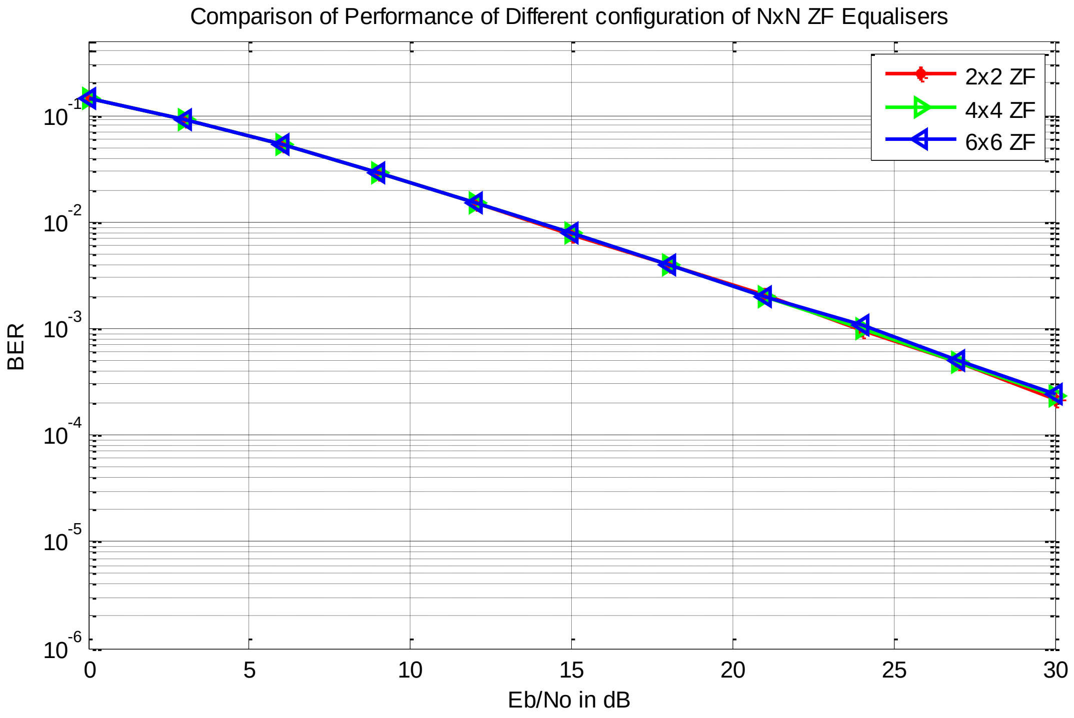


Figure 4.1: Comparison of BER for MIMO 2x2, 4x4 and 6x6 with ZF equalization

The simulation results of MIMO 2x2, 2x3, 2x4, and 2x5 with ZF equalizer is presented is in Figure 4.2. By keeping the transmitting antennas constant say two in this case and varying the number of receiving antennas, the system BER performance has improved. As can be seen from Figure 4.2, at a **BER point 10^{-3}**; with MIMO 2x2, the Eb/No value is 24 dB, with 2x3 the Eb/No value is 11 dB, 2x4 we have 7 dB and finally with 2x5 the Eb/No value is 4 dB. However, at **BER point 10^{-6}**; Eb/No values are 12dB, 17dB, and 27dB for 2x5, 2x4, and 2x3 respectively. But for the 2X2 no point within the values set for the simulation. This implies that, the 2X2 configuration does not meet the **10^{-6} BER** requirement for data within 0-30dB Eb/No values the Simulation can support. Hence, a considerable gain of about 17dB was achieved at **BER point 10^{-3}** with the MIMO 2x5 Configuration as against the MIMO 6x6, 4x4 and 2x2 for ZF equalization.

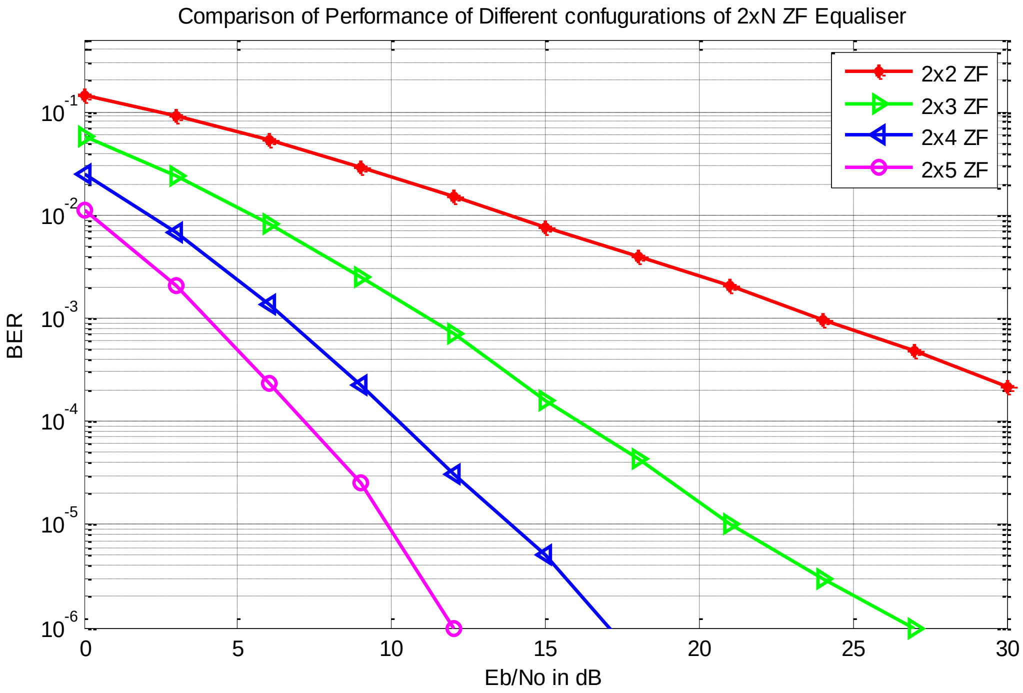


Figure 4.2 Comparison of BER for MIMO 2x2, 2x3, 2x4, 2x5 with ZF equalization

4.4 MIMO with Minimum Mean Square Error (MMSE) Equalization.

This section presents a plot showing the BER performance of MIMO NxM with MMSE equalization in Rayleigh fading channel. As depicted in Figure 4.3 there is an improvement in the BER performance of the system when the antenna configuration is increase from

2x2 to 6x6. This is true due to the fact that, the MMSE equalizer provides a trade-off between ISI cancellation and noise amplification. Unlike the ZF equalizer which suffers from noise amplification while nulling out the ISI component present.

As shown in Figure 4.3, there is a constant gain of 4 dB as the antenna configuration is increased from 2x2, 4x4, and 6x6.

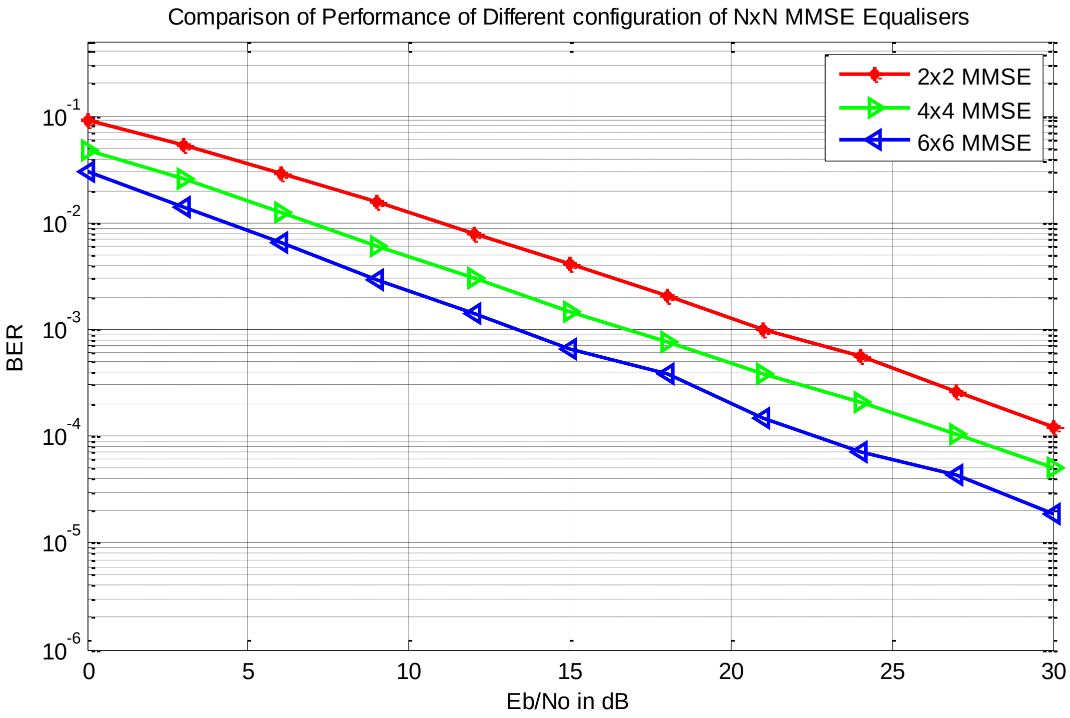


Figure 4.3: Comparison of BER for MIMO 2x2, 2x2, 6x6 with MMSE equalization.

Figure 4.4 shows the BER performance of MIMO 2xN with MMSE equalization. By putting two antennas at the transmitter and varying the number of antennas at the receiving side, a considerable gain in dB is achieved. As can be observed from Figure 4.4, at

BER point 10^{-3}; with 2x2 the Eb/No is 21 dB, 2x3 it is 10 dB, with 2x4 and 2x5 the Eb/No value is 6 dB and 4 dB respectively. Also at **BER point 10^{-6}**; the values are 12dB, 17dB and 24dB, for 2X5, 2X4, and 2X3 respectively. However the 2X2 have no value within the specified range. However, increasing the number of receiving antennas improves the overall system performance (i.e. a gain of 17dB is obtained for 2x5 over 2x2 configuration).

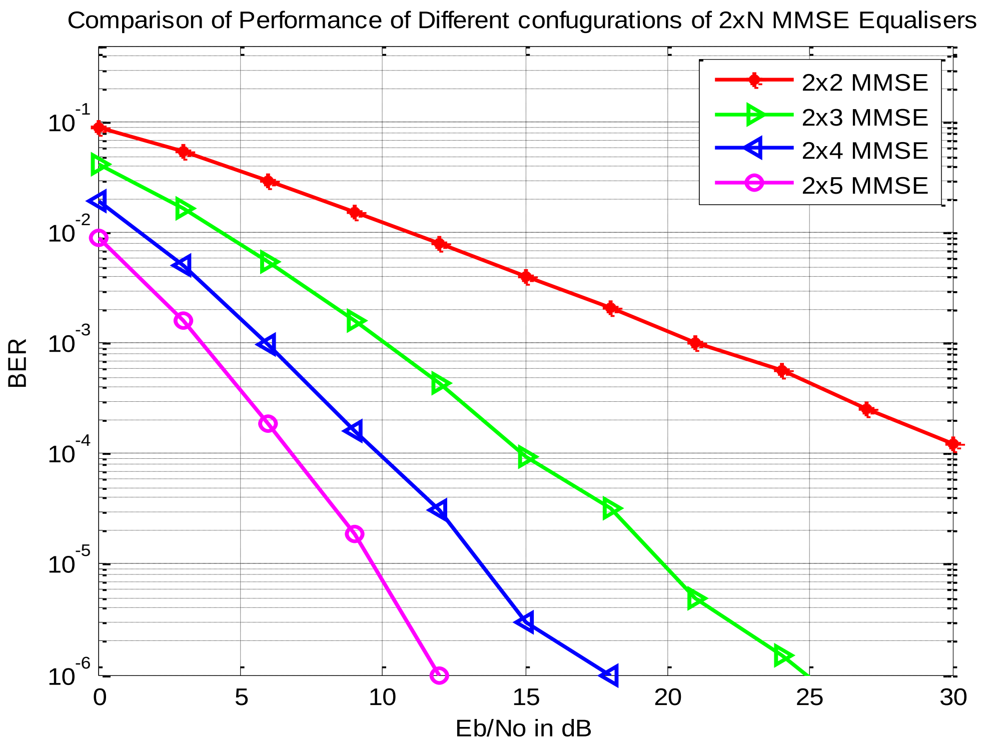


Figure 4.4 Comparison of BER for MIMO 2x2, 2x3, 2x4, 2x5 with MMSE equalization.

4.5 MIMO with Maximum Likelihood (ML) Equalization

The result of MIMO 2x2, 4x4, and 6x6 with ML equalization is shown in Figure 4.5. As depicted in Figure 4.5 a high gain and good error performance is achieve when the number of both transmit and receive antenna is increased. Although the gain in dB is not constant for multiple configurations as in MMSE but there is a remarkable gain as the configuration of antennas is increased. At a **BER point 10^{-3}**; with 2x2 the Eb/No value is 11 dB, 4x4 4 dB and lastly 6x6 it is 1 dB. Also at **BER point 10^{-6};** the values obtained are, 26dB, 13dB, and 7dB respectively for 2x2, 4x4, and 6x6.

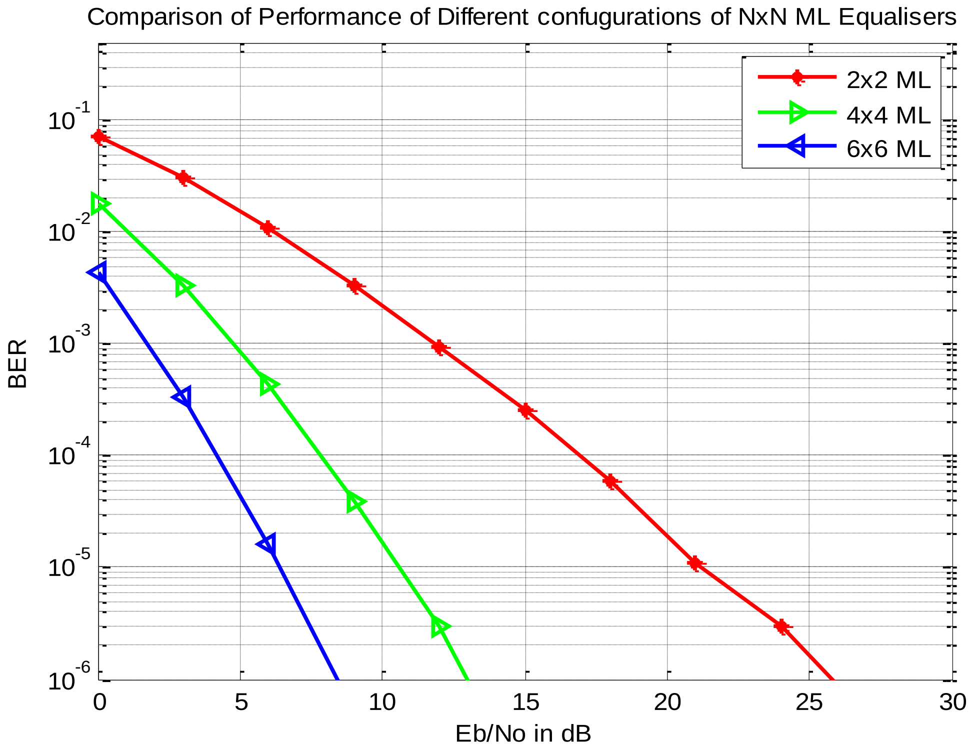


Figure 4.5 Comparison of BER for MIMO 2x2, 4x4, 6x6 with ML equalization

Figure 4.6 shows the performance of MIMO 2x2, 2x3, 2x4 and 2x5 with ML equalization. It is observed that by increasing the number of antennas at the receiver side the BER performance improves drastically. Although the performance is not as good as obtained when higher antenna configuration is employed.

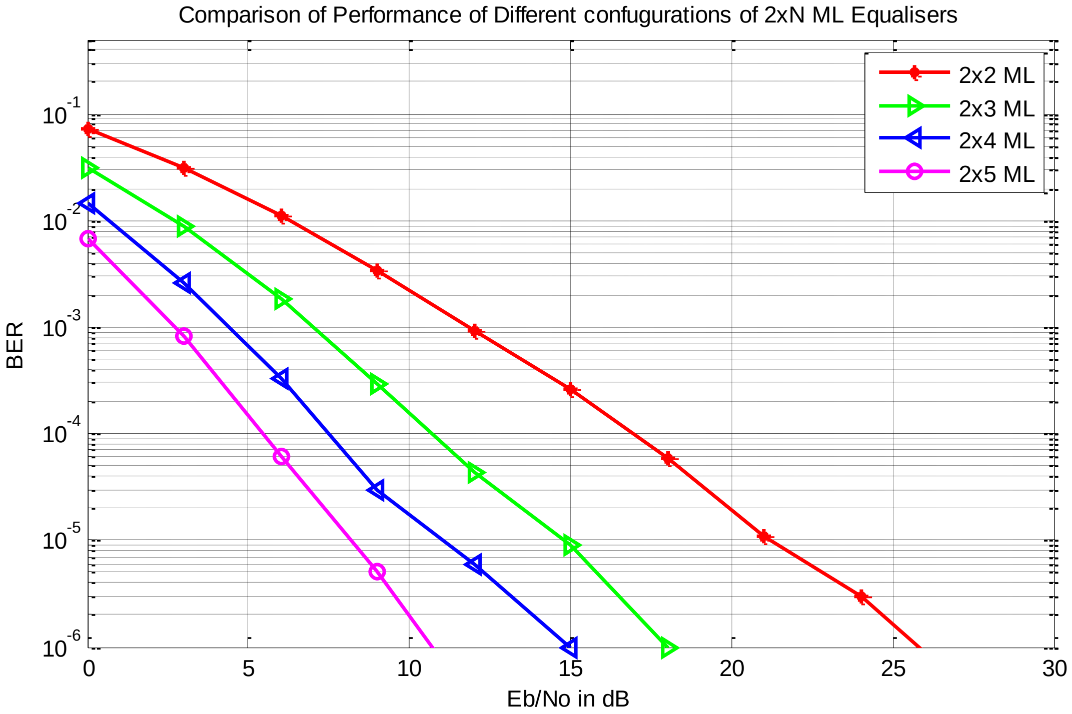


Figure 4.6: Comparison of BER for MIMO 2x2, 2x3, 2x4, 2x5 with ML equalization.

4.6 Comparing the Performance of ZF, MMSE and ML Equalization.

This section presents the comparative analysis of the performance of the three different equalization schemes employed so far in this study.

Figure 4.7 shows the performance of the ZF, MMSE and ML equalization for MIMO 2x2 with BPSK in rayleigh fading channel. As depicted in figure 4.7, at **BER point 10^{-3}**; with ZF equalizer the Eb/No value is 24 dB which corresponds to the result obtained without MIMO Equalization employed (i.e using Equation (2.13), with MMSE there is a gain of around 3 dB (i.e Eb/No value 21) and lastly with ML equalizer there is a gain of 11 dB as compared to the MMSE and 14 dB when compared with ZF. Also at **BER point 10^{-6}**; only the ML equalizer has a value within the specified range for the simulation (24dB).

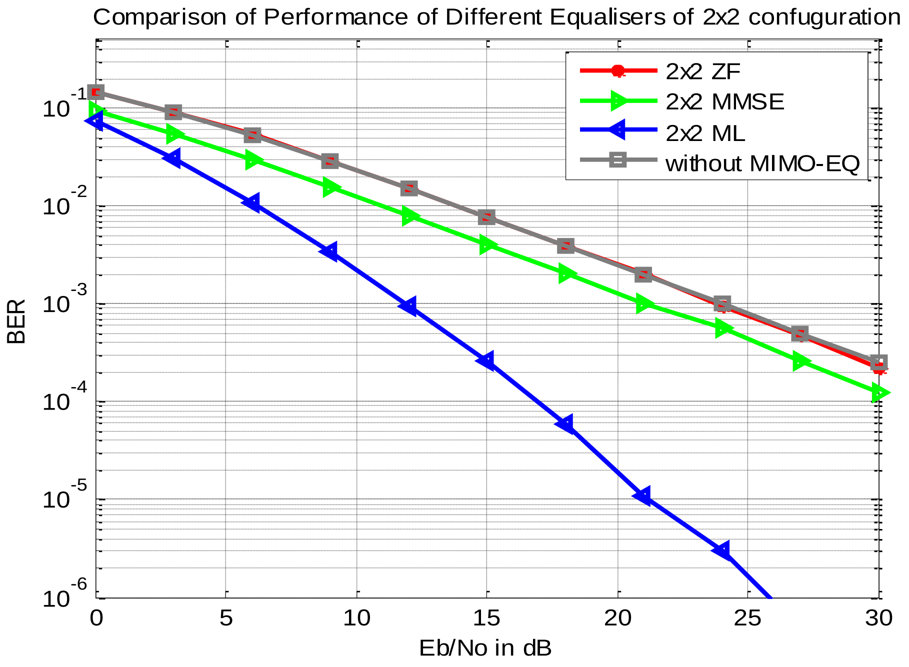


Figure 4.7: Comparison of performance of ZF, MMSE and ML equalizers 2×2 configuration

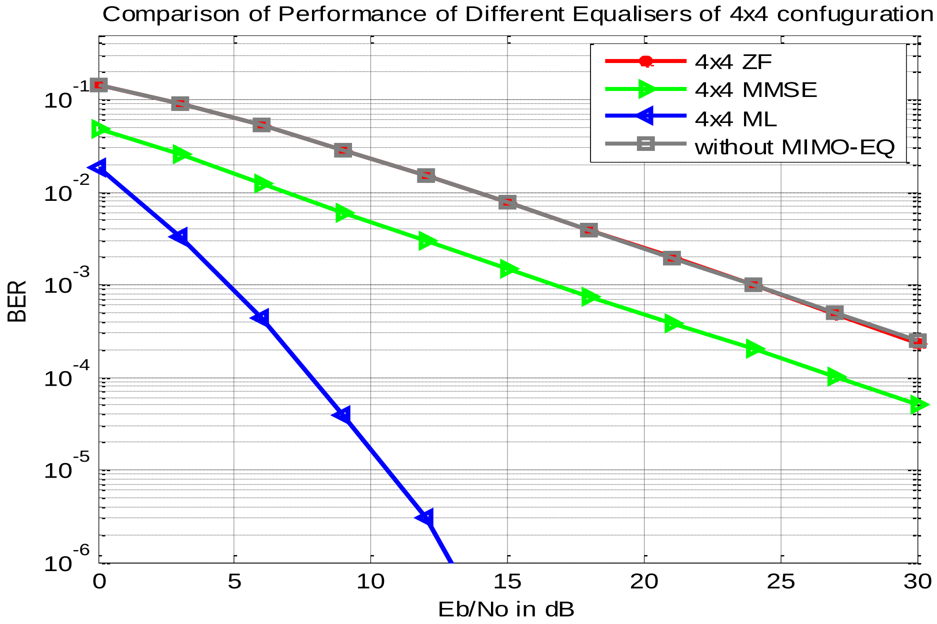


Figure 4.8: Comparison of performance of ZF, MMSE and ML equalizers 4×4 configuration

The performance of ZF, MMSE and ML equalizers with MIMO 4×4 configuration is depicted in Figure 4.8. at a **BER point 10^{-3}**; Eb/No of ZF is 24dB corresponding with result obtained using Eqn. (2.13), MMSE is 16dB and that of ML is 5dB. A gain of 11dB was maintained between ML and MMSE whereas ZF showed no improvement with antenna configuration increased from 2×2 to 4×4. Similarly at a **BER point 10^{-6}**; the ML equalizer had Eb/No value of 14dB while ZF and MMSE equalizers have no value within the specified range of values.

Figure 4.9 shows the performance of ZF, MMSE and ML equalizers with MIMO 6×6 configuration. This figure depicts, at a **BER point 10^{-3}**; Eb/No of ZF is still 24dB (the same as obtained using Eqn. (2.13)), MMSE is 13dB and ML 2dB. Similarly, at a **BER point 10^{-6}**; the Eb/No value for ML is 8dB while for ZF and MMSE the value fall outside the specified range.

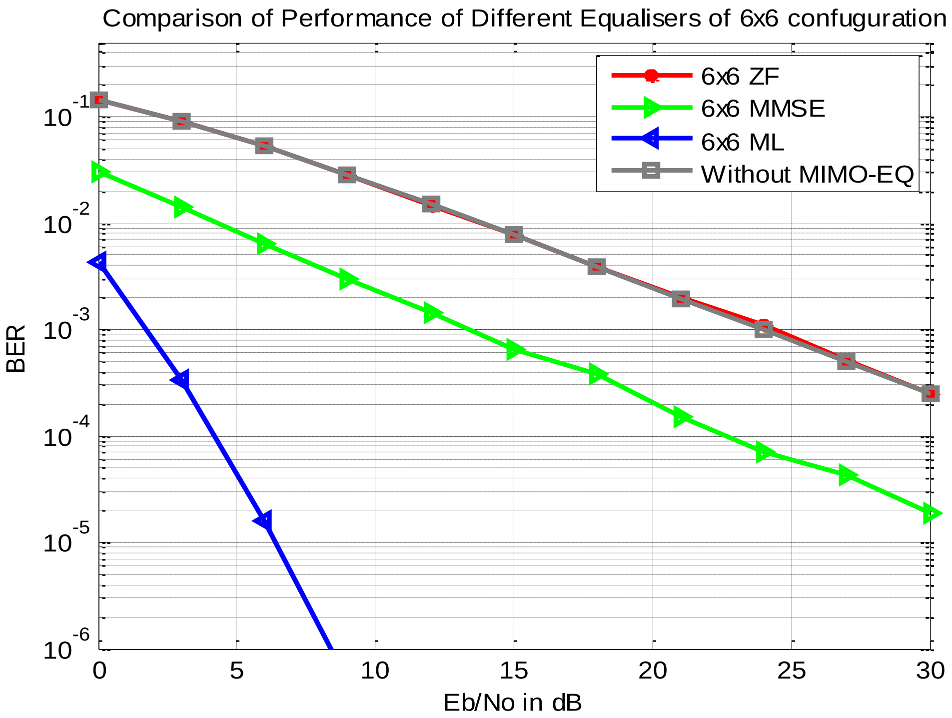


Figure 4.9: Comparison of performance of ZF, MMSE and ML Equalizer 6×6 configuration

The summary of the performance comparison of the three equalizers is shown in Table 4.1

Table 4.2: Comparison of ZF, MMSE and ML Equalization schemes.

	Eb/No values in (dB) at BER point 10^{-3}			Eb/No values in (dB) at BER point 10^{-6}		
Config.	ZF	MMSE	ML	ZF	MMSE	ML
2x2	24	21	10	-	-	26
4×4	24	15	5	-	-	14
6×6	24	13	2	-	-	8

Therefore, based on the results obatined from the simulation it is obvious that the ML equalizers outperformed the MMSE and ZF interms of the performance metric. This implies that with ML equalization, the probability of error is minimal at lowest possible bit energy to noise density as compared to ZF and MMSE which is very important to service operators in link budgeting.

4.7 Proposed Validation

The validation process is an important component in research which determines whether the simulated response of the developed model compares reasonably with the measured results thus establishing its viability.Will perform in real life according to the design specification quality attributes.

Two types of experimental approaches are proposed for the validation of the simulated results.

(i) Laboratory Based Experiment: In order validate the equalizer functionality through laboratory based experimentation the following equipments are needed.

(a) Signal generator

(b) Signal Analyzer

(c) Noise generator

(d) DSP processor board/chip

(e) A PC with MATLAB software

The block diagram for the proposed laboratory based experimental setup is illustrated in Figure 4.10

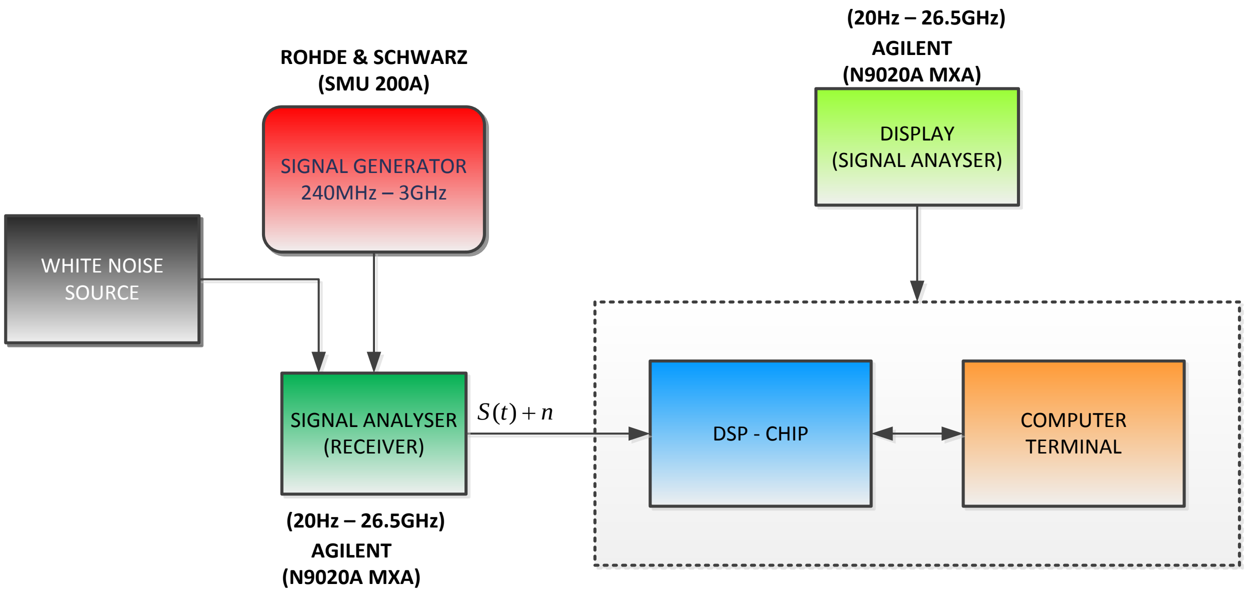


Figure 4.10 Proposed Laboratory based experimental setup.

The signal to be analyzed shall be generated using the SMU200A signal generator which has support for RF range between 240MHz to 3GHz. It has support for I/Q Modulators so the BPSK option can be easily selected. The SMU200A is equiped with simulator for different fading scenarios including Rayleigh fading. The generated signal shall then be passed to the receiver which shall be an N9020A MXA signal analyzer. The white noise shall be added to the received signal by using a White noise source Generator connected to it.

The equalization algorithms shall be implemented on a programable DSP processor via a connection to computer terminal. The equalized signal shall then be observed at the signal analyzer for analysis.

(ii) Outdoor Experiment: The equipment setup for the proposed outdoor-based experiment is shown in Figure 4.11.The system consists of two parts; the transmitter side and the receiver side. At the transmitter the data shall be generated using a MATLAB program which will be fed to a custom designed FPGA board with Digital to Analog converters via PCI board. The hardware shall be able to support the maximum number of transmit/receive antenna. A low-bass filter is needed to filter high harmornics and carrier signal for up conversion.

At the receiver, similar topology shall be adapted with a low-pass filter to reduce distorsions that may result due to interference neigbouring transmitters.

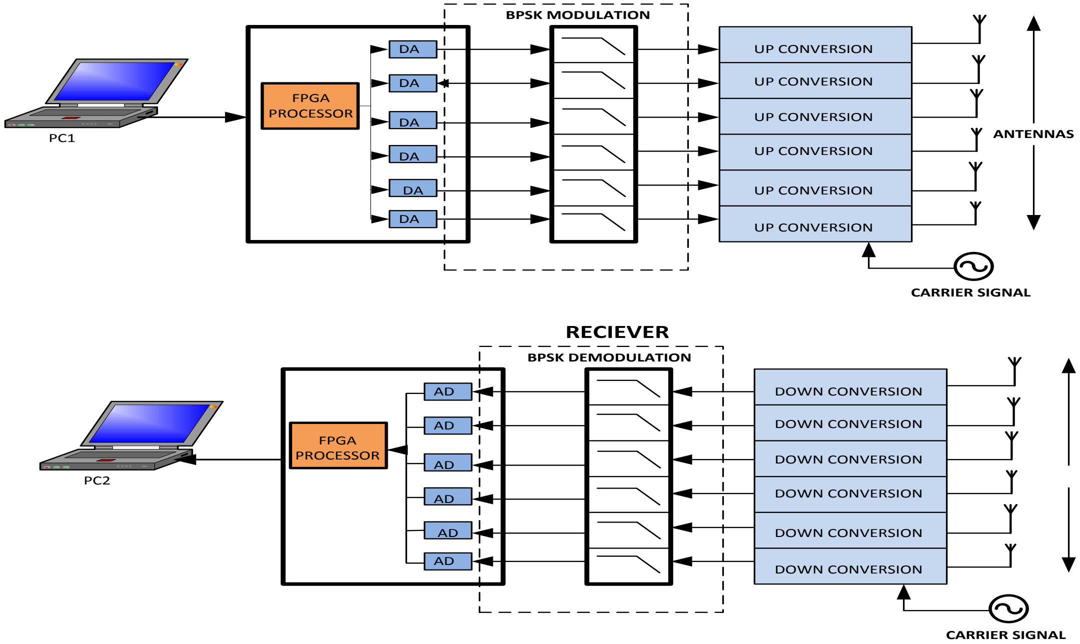


Figure 4.11 Proposed Outdoor Based Experimental setup for Validation. (source:Warnas et al, 2008).

However, due to none availability of the equipments needed for the experiment at present, validation through hardware implementation was not possible.

4.8 Conclusion

The results obtained from the simulation carried out in chapter Three were analysed and discussed. A comparative analysis of the three different Equalizers with different MIMO configuration was presented. Lastly two proposed validation approaches were presented in this chapter for testing the viability of the results obtained.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION FOR FURTHER WORK

5.1 Introduction

This research is aimed at analysing the performance of MIMO equalizers used for wireless communication receiver design. MIMO is a spectral efficient technique which provide increased data throughput and link reliability without additional bandwidth or transmit power. It acheives this by transmission of more bits per second per hertz of bandwith and reduce fading. It is currently being employed in conjuction with other techniques such as OFDM for the next generation mobile systems.

Equalizers are of enormous importance in wireless communication receiver design due to their ability to combat the effect of ISI in multipath fading enviroment. In MIMO systems, equalization is much more complex due to fact that the channel must be equalized over both space and time. This poses a challenge to wireless communication receiver designers. One critical issues with linear equalizers is the fact that they are not optimal in terms of minimizing the average symbol error probability. This is because the effect of a symbol is spread to other symbols. This has been addressed in optimal receiver like the ML equalizer were the entire segment of the received signal is observed not only the desired symbol.

5.3 Conclusion

In this research, the performance of MIMO equalizers used for mitigating the effect of ISI was studied and presented. For MIMO system, the effect of varying antenna configuration was investigated and results were compared for all the three equalizers under study. In all the cases, a better performance was achieved when the number of receiving antennas exceeded that of transmitting antennas. The MIMO system simulation for the equalizers was implemented in MATLAB version 7.4(R2009b) employing BPSK as modulation and Rayleigh fading channel as Transmission channel.

The result showed that, ML equalization has the best performances as compared to MMSE and ZF equalization with about 11dB at BER point 10^{-3}. This implied that, there is a much reduction in transmitter power requirement when using ML equalizer as compared to MMSE and ZF. This is important toward energy savings and cost of radio equipment. However comparison at BER point 10^{-6} was not possible for 2X2, 4X4, and 6X6 configuration because some of the equalizers (ZF and MMSE) have no values at that point. In order to achieve that BER point there is the need to increase the sample size to 10^8 bits and the Eb/No range to exceed 30dB. Furthermore, two possible methods of validation have been proposed. However due to none-availability of the equipment necessary for the experimental validation and cost in acquiring them, and also the period required for the completion of this work, the research was only limited to providing detailed procedure and scenarios for the experimental validation.

5.4 Recommendations and Further Work

Even though the aim and objectives of this work have been achieved to a certain level, a lot of improvements can be done to make it better. The following are suggested ways of improving this work.

i. The validation method proposed in this research should be implemented so as to ascertain the viability of the simulated results obtained
ii. Since this work does not include any form of Forward Error Correction, Channel Estimation and lnterleaving, the effect of these techniques can be investigated and incorporated into a single model to ascertain and analyze the overall system
iii. Application of Successive Interference Cancellation (SIC) and optimal Ordering to ZF and MMSE equalizers
iv. An investigation into other equalization technique like Adaptive Blind and Decision Feedback can also be carried out for better understanding of the concept
v.
vi.

REFERENCES

Agrawal S, Basha S.J and Kumar S. (2012), “Data Recovery using ML Equalizer in NLOS Communication Enviroment”. International Journal of Engineering and Research Applications (IJERA) ISSN:2248-9622 Vol.2, Issue3, May-June, pp835-839.

Bara’u G. N. and Tan Xiaoheng (2011) “A Comparative Performance Analysis of Multiple Input Multiple-Output using MATLAB with Zero Forcing and Minimum Mean SquareError Equalizers”. American J. of Engineering and Applied Sciences 4 (3): 425-428, 2011 ISSN 1941-7020.

Burel G., (2003), “About the Reliability of Simulations of MIMO Transmission Systems”, WSEAS Transaction on Communications Vol. 2, Issue 1,pp. 33-38

Choi W. J. and Cioffi J. M. (1999), “Multiple Input/Multiple Output (MIMO) Equalization for Space-Time Block Coding”, IEEE 0-7803-5582-2/99

Ernesto Z. and Gerhard F. (2007) “Improved Length Term Calculation and MMSE Extension for LISS MIMO Detection”. In Information Theory Workshop, 2006. ITW'06 Punta del Este. IEEE (IEEE), pp. 650-654.

Foschini G.J and Gans M. J (1998) “On Limits of Wireless Communications in a Fading Environment when Using Multiple Antennas”. Wireless Personal Communications **6:** 311–335, 1998.

Frenzel L. E (2012) “Understanding Modern Digital Modulation Techniques”, A tutorial Retrieved from http://electronicdesign.com/communications/understanding-modern digital-modulation-techniques. Jan 23, 2012

Gesber D. Shafi M., Shiu D. Smith P.J, and Naguib A (2002) “From Theory to Practice: An Overview of MIMO Space–Time Coded Wireless Systems”. IEEE journal on selected areas in communications, vol. 21, no. 3, april 2003 pp: 281

Goldsmith, A., (2005). “Wireless Communications.” First Edn., Cambridge University Press, Cambridge ISBN: 0521837162, pp: 644.

Goldsmith, A., Jafar, S.A., Jindal, N., and Vishwanath, S. (2002). Fundamental capacity of MIMO channels. IEEE J. Select. Areas Commun. Retrieve http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.1.4446&rep=rep1&type=pdf

Harada, H., and Prasad, R. (2002). Simulation and software radio for mobile communications, Vol 1 (Artech House). ISBN: 1580530443, pp: 79

http://www.timbercon.com/BER-Bit-Error-Rate.html (2012)

Hill G., (2008). “The cable and Telecommunications professionals” Third Edition . Jordan Hill, Oxford UK.

Jiang Y. Varanasi M. K. Li J. (2005), “Performance Analysis of ZF and MMSE Equalizers for MIMO Systems: An In-Depth Study of the High SNR Regime”. Department of Electrical and Computer Engineering, University of Colorado. Retrieved form http://www.sal.ufl.edu/yjiang/papers/VbitwcR6.pdf

Kaur, J. and Singh M. (2012)."Performance analysis of various channel estimation techniques for higher order modulation in a MIMO system." International Journal of Engineering and Innovative Technology (IJEIT) Volume 2, Issue 2, August 2012. ISSN: 2277-3754**.** Pp 114-117.

Li, Y. Winters J.H and sollenbeger R.N (2002), “MIMO-OFDM for wireless communication: Signal detection with Enhanced Channel Estimation”. IEEE transactions on communications, vol. 50, no. 9, September 2002, pp 1427.

Ling, F and Proakis J. (1985). “ Adaptive lattice decision Feedback Equalization; Their performance and Application to Time-Variant multipath channel”. IEEE Trans. Commun. Vol.33, No.4 pp.348-356.

Murch R. D ELEC 536 “Digital Communications”, Lecture Notes. Download information:http://course.ee.ust.hk/elec536

Penchong, J. and Z. Hong, 2009 “Development and simulation of a MIMO communication system based on IEEE 802.11n”. University of Science and Technology. http://people.ee.duke.edu.

Salz, J.(1985). “Digital transmission over cross-coupled linear channels,” AT&T Technical Journal, vol. 64, no. 6, pp. 1147-1159.

Sathish K. N. and Shankar K. R (2011) “Bit Error Rate Performance Analysis of ZF, ML and MMSE Equalizers for MIMO Wireless Communication Receiver”, European Journal of Scientific Research ISSN 1450-216X Vol.59 No.4 pp. 522-532

Saunders, S. and Zavala, S. (2007). “Antennas and propagation for wireless communication systems”, Second Edition, John Wiley & Sons. ISBN: 0470848790 pp: 242, 244, 417.

Shaodan M. and Tung-sang N., (2008) “Semi-Blind Domain Equalization for MIMO-OFDM systems”, IEEE Transaction on Vehicular Technology. Vol.57 No.4 pp 2219-2227.

Smalley D. (1994) “Equalization concept: A tutorial Application Report,” Atlanta Regional Technology Center. SPRA140. Retrieved from http://dsp-book.narod.ru/spra140.pdf

Suthar R. Joshi S. and Agrawal N. (2010) “ Performance Analysis of Different M-AR Modulation Techniques in Cellular Mobile Communication”. IP Multimedia Communications A Special Issue from IJCA - www.ijcaonline.org.

Tarik A. and Samir S. (2005), “Successive Interference Cancellation Turbo Receivers for Frequency Selective Fading MIMO Systems”, Signals and Telecommunications Group , INPT, Madinat Al Irfane, Rabat, Morocco. www.eurasip.org/Proceedings/Ext/ISCCSP2006/.../cr1303.pdf

Teletar E. I. (1995) “ Capacity of Multi-antenna Guassian Channels,” AT&T Bell laboratories, Tech. Memo.

Tse D. and Viswanath P. (2005) “Fundamentals of wireless Communications,” Cambridge, www.cambridg e.org /9780521845274

Vicente, Z and Asoke, K. (2004) “Blind MIMO equalization with optimum delay using Independent component analysis”. Int. J. Adapt. Control and Signal Precessing. 18:245-263(DOI:10.1002/acs.792).

Warnas, J., Shao, X., Schiphorst, R., and Slump, C.H. (2008). “MIMO System Setup and Parameter Estimation”. Retrieved from http://doc.utwente.nl/62690/1/document.pdf.

Winter, J. (1987) “On the Capacity of Radio Communication Systems with Diversity in a Rayleigh Fading Environment” **10.1109/JSAC.1987.1146600** ISSN: 0733-8716 IEEE.

Winters J. H., (1998) “The Diversity Gain of Transmit Diversity in Wireless Systems with Rayleigh Fading”, IEEE transactions on vehicular technology, vol. 47, no. 1, February 1998. PP 119.

Wong, T. F. and T. M. Lok (2004). "Theory of Digital Communication. Chapter 4 ISI and Equalization." University of Florida. Retreived from www.scribd.com/doc/125250332/817314480-ch4 ISI and Equalization. Feb 13, 2013.

Zhang X. and Kung S., (2003), “Capacity analysis for parallel and sequential MIMO equalizers” IEEE Transactions on Signal Processing, vol. 51. PP 2989-300

APPENDICES

Appendix A1: MATLAB script for computing the BER of MIMO system with BPSK modulation in Rayleigh channel using ZF equalizers.

```
clear;

n= 10^6; %Number of bits
Tx = N; %Number of transmitting antennas
Rx = N; %Number of recieving antennas
Eb_No_dB = 0:3:30;  %Signal to noise ratio in dB

 BERR = zeros(1,length(Eb_No_dB));
for k= 1:length(Eb_No_dB)

    %Data generation and modulation
    DataDig = randi([0,1],[1,n]);%Generating random digital numbers (0s and 1s)
    DataMod = 2*DataDig-1;       %Converting to 1s(for 1s) and -1s(for 0s) for BPSK

    DataMod = reshape(DataMod,[Tx,length(DataMod)/Tx]); %Grouping for spatial antenna 
transmit diversity

    %Transmission
    [DataNoi, h] = raynoise(DataMod, Eb_No_dB(k), Rx);  %Passing the data through 
Rayleigh noise channel

%Reception
%       DataEqu = zefo(DataNoi, h);               %Applying Zero Forcing nTx by nRx 
Equaliser
    DataEqu = real(reshape(DataEqu, [1,numel(DataEqu)])); %Demodulating the signal(Hard 
Decision)
    DataDem = 0.5*(1+abs(DataEqu)./DataEqu);    %Quantization
```

```
for
    [~, BER] = biterr(DataDig, DataDem); %Calculating BER
    BERR(k) = BER;
end

% 'Zero Forcing' equalising receiver modeling

function [DataEqu] = zefo(DataNoi, h)
Rx = size(DataNoi,1);%numel(DataNoi)/length(DataNoi);
Tx = size(h,2);
n_Tx = size(DataNoi,2);
hinv = zeros(Tx, Rx, n_Tx);
DataEqu = zeros(Tx,n_Tx);

for k=1:n_Tx
    hinv(:,:,k) = ((h(:,:,k))'*h(:,:,k))\(h(:,:,k))';
    DataEqu(:,k)= hinv(:,:,k)*DataNoi(:,k);
end

%Rayleigh fading channel modelimg
function [DataNoi, h] = raynoise(DataMod, Eb_No_dB, Rx)
Tx = size(DataMod,1);%numel(DataMod)/length(DataMod);
n_Tx = size(DataMod,2);

DataNoi = zeros(Rx,n_Tx);
awgn = sqrt(0.5)*(randn(Rx,n_Tx) + 1i*randn(Rx,n_Tx));
h = sqrt(0.5)*(randn(Rx,Tx,n_Tx) + 1i*randn(Rx,Tx,n_Tx));

if Tx == 1
    h = reshape(h,[Rx,n_Tx]);
    DataNoi = h.*kron(DataMod,ones(Rx,1))+(10^(-Eb_No_dB/20))*awgn;
    h = reshape(h,[Rx,Tx,n_Tx]);
else
    for k = 1:Rx
        DataNoi(k,:) = sum(squeeze(h(k,:,:)).*DataMod) +(10^(-Eb_No_dB/20))*awgn(k,:);
    end
end
```

```
clear

Eb_No_Li = 10.^(Eb_No_dB/10);
TheoBer1x1 = 0.5.*(1-1*(1+1./Eb_No_Li).^(-0.5));
p = 1/2-1/2*(1+1./Eb_No_Li).^(-1/2);
TheoBer1x2 = p.^2.*(1+2*(1-p));
clear Eb_No_Li k n p;
figure;
semilogy(Eb_No_dB, BERR, '-r*',Eb_No_dB,TheoBer1x2,'-ko',Eb_No_dB,TheoBer1x1,'-ms');
axis([0 22 10^-6 0.5])
grid on;
legend('Simulated  ZF ', '', '');
xlabel('Eb/No in dB');
ylabel('BER');
title('BER against Eb/No for Tx by Rx ZF on Rayleigh Channel');
```

Appendix A2

MATLAB script for computing the BER of MIMO system with BPSK modulation in Rayleigh channel using MMSE equalizers.

```
clear;

n= 10^6; %Number of bits
Tx = N; %Number of transmitting antennas
Rx = N; %Number of recieving antennas
Eb_No_dB = 0:3:30;  %Signal to noise ratio in dB

 BERR = zeros(1,length(Eb_No_dB));
for k= 1:length(Eb_No_dB)

    %Data generation and modulation
    DataDig = randi([0,1],[1,n]);%Generating random digital numbers (0s and 1s)
    DataMod = 2*DataDig-1;       %Converting to 1s(for 1s) and -1s(for 0s) for BPSK

    DataMod = reshape(DataMod,[Tx,length(DataMod)/Tx]); %Grouping for spatial antenna
transmit diversity

    %Transmission
    [DataNoi, h] = raynoise(DataMod, Eb_No_dB(k), Rx);  %Passing the data through
Rayleigh noise channel

%       DataEqu = mmse(DataNoi, h, Eb_No_dB(k));  %Applying MMSE nTx by nRx Equaliser


    DataEqu = real(reshape(DataEqu, [1,numel(DataEqu)])); %Demodulating the signal(Hard
Decision)
    DataDem = 0.5*(1+abs(DataEqu)./DataEqu);    %Quantization
for
    [~, BER] = biterr(DataDig, DataDem); %Calculating BER
    BERR(k) = BER;
```

```
end
clear

Eb_No_Li = 10.^(Eb_No_dB/10);
TheoBer1x1 = 0.5.*(1-1*(1+1./Eb_No_Li).^(-0.5));
p = 1/2-1/2*(1+1./Eb_No_Li).^(-1/2);
TheoBer1x2 = p.^2.*(1+2*(1-p));
% 'Minimum Mean Square Error' equalising receiver modeling

function [DataEqu] = mmse(DataNoi, h, Eb_No_dB)
Rx = size(DataNoi,1);
Tx = size(h,2);
n_Tx = size(DataNoi,2);
hinv = zeros(Tx, Rx, n_Tx);
DataEqu = zeros(Tx,n_Tx);

for k=1:n_Tx
    hinv(:,:,k) = ((h(:,:,k))'*h(:,:,k)+10^(-0.1*Eb_No_dB)*eye(Tx))\(h(:,:,k))';
    DataEqu(:,k)= hinv(:,:,k)*DataNoi(:,k);
end

%Rayleigh fading channel modelimg
function [DataNoi, h] = raynoise(DataMod, Eb_No_dB, Rx)
Tx = size(DataMod,1);%numel(DataMod)/length(DataMod);
n_Tx = size(DataMod,2);

DataNoi = zeros(Rx,n_Tx);
awgn = sqrt(0.5)*(randn(Rx,n_Tx) + 1i*randn(Rx,n_Tx));
h = sqrt(0.5)*(randn(Rx,Tx,n_Tx) + 1i*randn(Rx,Tx,n_Tx));

if Tx == 1
    h = reshape(h,[Rx,n_Tx]);
    DataNoi = h.*kron(DataMod,ones(Rx,1))+(10^(-Eb_No_dB/20))*awgn;
    h = reshape(h,[Rx,Tx,n_Tx]);
else
    for k = 1:Rx
        DataNoi(k,:) = sum(squeeze(h(k,:,:)).*DataMod) +(10^(-Eb_No_dB/20))*awgn(k,:);
```

```
    end
end

clear Eb_No_Li k n p;
figure;
semilogy(Eb_No_dB, BERR, '-r*',Eb_No_dB,TheoBer1x2,'-ko',Eb_No_dB,TheoBer1x1,'-ms');
axis([0 22 10^-6 0.5])
grid on;
legend('Simulated   MMSE', '', '');
xlabel('Eb/No in dB');
ylabel('BER');title('BER against Eb/No for Tx by Rx MMSE on Rayleigh
Channel');
```

Appendix A3

MATLAB script for computing the BER of MIMO system with BPSK modulation in Rayleigh channel using ML equalizers.

```
clear;

n= 10^6; %Number of bits

Tx = N; %Number of transmitting antennas
Rx = N; %Number of recieving antennas
Eb_No_dB = 0:3:30;  %Signal to noise ratio in dB

 BERR = zeros(1,length(Eb_No_dB));
for k= 1:length(Eb_No_dB)

    %Data generation and modulation
    DataDig = randi([0,1],[1,n]);%Generating random digital numbers (0s and 1s)
    DataMod = 2*DataDig-1;       %Converting to 1s(for 1s) and -1s(for 0s) for BPSK

    DataMod = reshape(DataMod,[Tx,length(DataMod)/Tx]); %Grouping for spatial antenna
transmit diversity

    %Transmission
    [DataNoi, h] = raynoise(DataMod, Eb_No_dB(k), Rx);  %Passing the data through
Rayleigh noise channel

%Reception
DataEqu = maxl(DataNoi, h);              %Applying Maximum Likelyhood nTx by nRx
Equaliser

    DataEqu = real(reshape(DataEqu, [1,numel(DataEqu)])); %Demodulating the signal(Hard
Decision)
    DataDem = 0.5*(1+abs(DataEqu)./DataEqu);    %Quantization
```

```
for
    [~, BER] = biterr(DataDig, DataDem); %Calculating BER
    BERR(k) = BER;
end
% 'Maximum Likelihood' equalisisng receiver Modeling
function [DataEqu] = maxl(DataNoi, h)
Tx = size(h,2);
n_Tx = size(DataNoi,2);
DataEqu = zeros(Tx,n_Tx);

PossData = zeros(Tx, (2^Tx)); %Possible data
for k = 1:Tx
    for m = 1:(2^Tx)
        PossData(k,m) = 2*floor(mod((m-1),(2^k))/(2^(k-1)))-1;
    end
end
 for k = 1:n_Tx
    Inter = kron(DataNoi(:,k),ones(1,2^Tx))-h(:,:,k)*PossData;
    [~,MinIndex] = min(sum(abs(Inter)),[],2);
    DataEqu(:,k) = PossData(:,MinIndex);
End

%Rayleigh fading channel modelimg
function [DataNoi, h] = raynoise(DataMod, Eb_No_dB, Rx)
Tx = size(DataMod,1);%numel(DataMod)/length(DataMod);
n_Tx = size(DataMod,2);

DataNoi = zeros(Rx,n_Tx);
awgn = sqrt(0.5)*(randn(Rx,n_Tx) + 1i*randn(Rx,n_Tx));
h = sqrt(0.5)*(randn(Rx,Tx,n_Tx) + 1i*randn(Rx,Tx,n_Tx));

if Tx == 1
    h = reshape(h,[Rx,n_Tx]);
    DataNoi = h.*kron(DataMod,ones(Rx,1))+(10^(-Eb_No_dB/20))*awgn;
    h = reshape(h,[Rx,Tx,n_Tx]);
else
    for k = 1:Rx
```

```
        DataNoi(k,:) = sum(squeeze(h(k,:,:)).*DataMod) +(10^(-Eb_No_dB/20))*awgn(k,:);
    end
end
clear

Eb_No_Li = 10.^(Eb_No_dB/10);
TheoBer1x1 = 0.5.*(1-1*(1+1./Eb_No_Li).^(-0.5));
p = 1/2-1/2*(1+1./Eb_No_Li).^(-1/2);
TheoBer1x2 = p.^2.*(1+2*(1-p));
clear Eb_No_Li k n p;
figure;
semilogy(Eb_No_dB, BERR, '-r*',Eb_No_dB,TheoBer1x2,'-ko',Eb_No_dB,TheoBer1x1,'-ms');
axis([0 22 10^-6 0.5])
grid on;
legend('Simulated   ML', '', '');
xlabel('Eb/No in dB');
ylabel('BER');title('BER against Eb/No for Tx by Rx ML on Rayleigh Channel');
```

Appendix A4

Simulation Results for the different MIMO system configuration using ZF, MMSE, and ML Equalization with BPSK modulation in Rayleigh fading Channel.

Config.	Equalizer												
2x2	ZF	Eb/No Value	0	3	6	9	12	15	18	21	24	27	30
		BER value	0.14631	0.091706	0.053729	0.028642	0.01525	0.007629	0.003929	0.002027	0.000943	0.000486	0.00022
4x4	ZF	Eb/No Value	0	3	6	9	12	15	18	21	24	27	30
		BER value	0.1463	0.092157	0.052877	0.02883	0.015068	0.00775	0.003904	0.002023	0.000994	0.000491	0.000236
6x6	ZF	Eb/No Value	0	3	6	9	12	15	18	21	24	27	30
		BER value	0.146104	0.091346	0.053576	0.028732	0.014868	0.007759	0.003946	0.00199	0.001093	0.00051	0.000246
2x3	ZF	Eb/No Value	0	3	6	9	12	15	18	21	24	27	30
		BER value	0.058274	0.02392	0.008183	0.002513	0.000697	0.000162	0.000043	0.00001	0.000003	0.000001	0
2x4	ZF	Eb/No Value	0	3	6	9	12	15	18	21	24	27	30
		BER value	0.024972	0.006717	0.001367	0.000223	0.000031	0.000005	5E-07	0	0	0	0
2x5	ZF	Eb/No Value	0	3	6	9	12	15	18	21	24	27	30
		BER value	0.011063	0.002048	0.000235	0.000025	0.000001	0	0	0	0	0	0
2x2	MMSE	Eb/No Value	0	3	6	9	12	15	18	21	24	27	30
		BER value	0.09236	0.054171	0.015597	0.007991	0.004032	0.002077	0.001012	0.000562	0.000259	0.000259	0.000123
4x4	MMSE	Eb/No Value	0	3	6	9	12	15	18	21	24	27	30
		BER value	0.048171	0.025589	0.012449	0.006052	0.003022	0.001487	0.000753	0.000385	0.000206	0.000103	0.00005
6x6	MMSE	Eb/No Value	0	3	6	9	12	15	18	21	24	27	30
		BER value	0.030227	0.014136	0.006509	0.002943	0.001421	0.000643	0.000377	0.000149	0.00007	0.000043	0.000019
2x3	MMSE	Eb/No Value	0	3	6	9	12	15	18	21	24	27	30
		BER value	0.042004	0.01645	0.005469	0.00163	0.000437	0.000096	0.000032	0.000005	1.5E-06	4E-07	0
2x4	MMSE	Eb/No Value	0	3	6	9	12	15	18	21	24	27	30
		BER value	0.019405	0.005036	0.000965	0.000163	0.000031	0.000003	0.000001	0	0	0	0
2x5	MMSE	Eb/No Value	0	3	6	9	12	15	18	21	24	27	30
		BER value	0.008983	0.001618	0.000186	0.000019	0.000001	0	0	0	0	0	0
2x2	ML	Eb/No Value	0	3	6	9	12	15	18	21	24	27	30
		BER value	0.072772	0.031036	0.01093	0.003351	0.000936	0.000258	5.90E-05	1.10E-05	3.00E-06	3.00E-06	0.00E+00
4x4	ML	Eb/No Value	0	3	6	9	12	15	18	21	24	27	30
		BER value	0.018346	0.003347	0.000435	0.000039	0.000003	1E-07	0		0	0	0

6x6	ML	Eb/No Value	0	3	6	9	12	15	18	21	24	27	30
		BER value	0.004294	0.000334	0.000016	5.00E-07	0	0	0	0	0	0	0
2x3	ML	Eb/No Value	0	3	6	9	12	15	18	21	24	27	30
		BER value	0.031818	0.008803	0.001849	0.000291	0.000043	0.000009	0.000001	0	0	0	0
2x4	ML	Eb/No Value	0	3	6	9	12	15	18	21	24	27	30
		BER value	0.014419	0.002604	0.000332	0.00003	0.000006	0.000001	0	0	0	0	0
2x5	ML	Eb/No Value		3	6	9	12	15	18	21	24	27	30
		BER value	0.006835	0.000834	0.000061	0.000005	3E-07	0	0	0	0	0	