

**ILLUMINATION DESIGN AND SIMULATION FOR OUTDOOR  
SPORTING ARENAS**

**BY**

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**A Thesis presented to the Department of Electrical Engineering, Ahmadu Bello  
University, Zaria, in partial fulfillment of the requirements for the award of  
Master of Science (M.Sc) Degree in Electrical Engineering**

## **DECLARATION**

I, Abubakar Buba Maude, hereby declare that this thesis titled “Illumination Design and Simulation for Outdoor Sporting Arenas” presented to the Department of Electrical Engineering, Ahmadu Bello University, Zaria is the effort of my own research and has never been submitted for any degree.

All sources consulted have been duly acknowledged.

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**Maude Abubakar Buba**

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**Date**

## CERTIFICATION

This thesis entitled “Illumination Design and Simulation for Outdoor Sporting Arenas” presented to the Department of Electrical Engineering Department meets the regulations governing the award of the degree of Master of Science in Electrical Engineering of Ahmadu Bello University, Zaria and is approved for its contribution to knowledge and literary presentation.

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## **DEDICATION**

This thesis is dedicated to my mother Hajiya Halimatu Suleiman and my late father Alhaji Mohammadu Abba Saleh. (Rabbi irham huma kama rabbayani sagira).

## **ACKNOWLEDGEMENTS**

I wish to thank my Supervisors Prof. B. G. Bajoga and Dr. J. Y Oricha for their commitment and support during the course of this work. I wish to express my appreciations to Engr. Sikiru Humble, Engr Yusuf Shaaban and Engr. Suleiman Garba for their advise, guidance and encouragement.

This work would not have been completed without the moral support of my friends like, Engrs Sani Musa, Dahiru Dalhat and Sufyan Nuhu.

The contribution of all my teachers in Ahmadu Bello University is well appreciated, for the knowledge they taught made life meaningful. I also wish to thank the two Post Graduate Coordinators, past and present Dr. M. B. Mu'azu and Boyi Jimoh, for their conscientious efforts in ensuring the smooth continuity of the postgraduate programmes of the Department.

I deeply thank members of my family, most especially my wife, Dr. Aisha M. Suleiman for patiently bearing with me, while I pursued my professional and academic ambitions. Finally, I wish to thank Mallam Saidu Shehu Adamu (Target) for corrections and typesetting and Tukur (Karami da Babba) of ABUDEE for their assistance generally.

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## **LIST OF ABBREVIATIONS**

ABUDEE	Ahmadu Bello University, Department of Electrical Engineering
CIBSE	Chartered Institute of Building Services Engineering
ESI	Equivalent Sphere Illumination
IEC	International Electro-technical Commission
IES	Illumination Engineering Society
IESNA	Illumination Engineering Society of North America
LUX	Lux (lumen/ m <sup>2</sup> )
MATLAB	Mathematical Laboratory
VCP	Visual Comfort Probability
IDA	International Dark sky Association

## ABSTRACT

Widespread demand for sporting activities in the night has led to the integration of lighting design in the development of sporting arenas due mainly to lower temperatures at night, increase in demand for sporting facilities and leisure considerations. Illumination of sporting arenas is essentially based on floodlighting. In this thesis the design procedures, calculations and factors for floodlighting were applied to a football field and the best aiming points to achieve the most uniform illumination using Monte Carlo Analysis programmed in MATLAB. A more uniformly illuminated field, would have less glare, but the more uniformly designed illumination was checked whether it would lead to less energy consumption and hence reduction in project cost. The illumination which was simulated by the Point-by-Point equation derived in algebraic form with adjustments for light intensity changes had the result depicted in Isolux contours. The contours showed visually that the Monte Carlo Analysis result was satisfactory and a further visual display of the uniformity gradient showed that the result at the best aiming point had the highest uniformity gradient of 0.027 Lux/m compared to other points that gave 0.021 Lux/m, while the field with random aiming points even gave a negative gradient of -0.001 Lux/m. As a field is a mirror image about its width and lengthwise centres, the result in the quadrant of study was applied to give the final illumination and isolux of the whole field. Further work could extend the study to other types of arenas and compare results with practical measurements using goniometer and photometer.

## CHAPTER ONE

### 1.1 PREAMBLE

Illumination design for outdoor sporting arenas is essentially floodlighting design to which most lighting fundamentals for interior lighting apply except for absence of reflection components and the fact that viewing of vertical and oblique surfaces become of primary importance. Floodlighting for arenas has to satisfy the requirements of players, referees, spectators and Television (TV) cameras, which often times are conflicting in providing uniform adequate illumination with minimal tolerable glare, hence the use of group of floodlights mounted on poles, masts, columns or towers for wide arenas or floodlights mounted on spectator stand fascias for smaller grounds (Joseph, 2003)

The two main methods of floodlighting calculations are point-to-point method and lumen method. The lumen method is used to determine the number of luminance and spread, while the point by point is used to determine the illuminance at each point. In this, Monte Carlo analysis is applied in determining the aiming point for each floodlight in each group of floodlights mounted on a column, mast, pole or tower to give the optimal uniform distribution of illuminance. These aiming points would otherwise have to be determined by trial and error to achieve the required uniform illuminance of the arena (Henderson and Marsden, 1972). It is pertinent to note that each aiming point consists of two angles, of tilt and azimuth. Choosing the aiming point of a large number of floodlights by trial and error will certainly be involving and take a long time.

An optimally designed illumination of sporting arena is expected to result from the above steps and consequently lead to less power consumption, which in the long run will lead to cheaper operational cost and faster project design delivery.

## **1.2 PROJECT MOTIVATION**

The increasing importance of sporting activities for healthy living, leisure and entertainment has made demand from the populace for sporting facilities in the form of Mega and Mini Stadia, Gymnasia, Playfields and various courts a regular request and therefore important to governments and investors. Integral to these sporting facilities is lighting for spectator stands, participant changing rooms, sporting fields and courts. It is a fact that the design of such lighting schemes usually involves costly and various avoidable errors and tight time schedules (Fink and Beatty, 1978). The design engineer is also usually faced with several client demands or even several clients with difficult inflexible schedules. It became critical to develop a more cost effective, better, faster way to undertake such designs for spectator stands, participant changing rooms, sporting fields and courts.

## **1.3 STATEMENT OF PROBLEM**

Floodlighting design for sporting arenas is time consuming, often fraught with error, even, involving trial and errors at some stages that lead to inefficiency in illumination, energy usage and cost.

This thesis seeks to develop a method using Monte Carlo Analysis programmed in MATLAB that will lead to an optimal illumination of any sporting arena. The result from the technique will be depicted in isolux contour visualizations for any chosen outdoor sporting arena.

#### **1.4 METHODOLOGY**

- i. Derivation of improved illumination formula
- ii. Application of Monte Carlo method programmed and run in MATLAB to obtain the best aiming points for a partitioned part of the sporting arena chosen (field).
- iii. Plotting the results for optimised and unoptimised aiming points as histograms, isolux contours and uniformity gradient comparing and analysing them.
- iv. Applying the results for a partition of the field to the entire field.
- v. Drawing conclusions from the analyses, looking at the limitations of the study and proffering suggestions for further work.

#### **1.5 THESIS OUTLINE**

Chapter one gives the general ideas behind this research work. In chapter two previous studies on related concepts used in these investigations are reviewed, some background information on the various issues in the work is also presented. Chapter three presents the derivations of the proposed design equations and Monte Carlo Analysis. While chapter four gives the results and analysis of the Monte Carlo Analysis programmed in MATLAB, along with considerations of energy and cost savings for the improved illumination design. Finally in chapter five the conclusions, limitations of the study and suggestions for further studies are given.



## **CHAPTER TWO**

### **THEORETICAL BACKGROUND AND LITERATURE REVIEW**

#### **2.1 INTRODUCTION**

This chapter seeks to introduce the subject of illumination and look at the theoretical issues involved in internal, external lighting and floodlighting. It also seeks to review some of the literature on studies in this area of illumination.

#### **2.2 Theoretical Background**

Illumination was one of the earliest applications of electrical engineering and is still in terms of utilisation, an area that affects the largest number of persons. This is because it is the main and most basic use of electricity in homes, offices and in all other facets of life (Starr, 1973). Illumination by electric lamps is cheaper and more convenient than by any other method. In fact it is likely that the electrical industry could not have developed at more than a fraction of its speed if it were not for the early profits it made from electric lighting (Starr, 1973).

There has always been a growing interest in getting abundant light at a very low cost. Lighting is very important in modern life and good lighting has several advantages, some of which include the following:

- i) clear visual perception of objects and surroundings,
- ii) increased efficiency, better appearance, reduced accidents,
- iii) better utilization of time by not depending on direct sunlight (which is usually available for limited hours),
- iv) high reliability, and
- v) reduction of eye strain and better health of the community

Illumination in exterior or interior lighting has extensive application in residential buildings, commercial complexes, shops, cinemas, street lighting, railway yards, aerodromes, factories, libraries, museums, amusement parks, operation theatres, and football fields. It is with a view thus to regulate and ensure provision of good illumination that the International Electro-technical Commission (IEC) and Chartered Institute of Building Service Engineers (CIBSE) published lighting codes. Good illumination must have the following qualities (Joseph, 2003).

- i) Clear perception without glare,
- ii) Adequate luminance,
- iii) Economy,
- iv) Aesthetics and beautiful appearance.

Illumination is broadly categorized into interior lighting and exterior lighting. Although the same lighting fundamentals apply to both, their applications are different for several reasons (Miheala, 2007)

- 1) Minimal reflected component is included in exterior lighting design
- 2) The night sky is dark often resulting in high contrast
- 3) Exterior lighting is often designed for a variety of persons and / or tasks,  
For example in sports fields, light must be provided for the players, the referees, the spectators and the TV cameras, each of these having different visual requirements.
- 4) The viewing of vertical, or perhaps oblique surfaces, is often of primary importance in exterior lighting.
- 5) Quite often, especially in sports lighting, the object being viewed is moving rapidly.

- 6) Very often exterior lighting conditions are such that vision is mesopic (<30 lx) involving both cones and rods.
- 7) The issues of light trespass and light pollution are of concern. Light trespass is unwanted light from luminaires on adjacent property. It manifests itself as increased ambient illumination or distracting glare. Light pollution is the sky glow produced by outdoor lighting, caused by reflected light from the ground and structures and from direct upward light from luminaires with improper cut off.

### **2.3 EXTERIOR LIGHTING**

Exterior lighting is achieved by the use of floodlights, hence is popularly referred to as floodlighting. Floodlighting means flooding of large surfaces with the help of light from powerful projectors (Rao, 2006). Several factors have to be considered in floodlighting design, some of which are:-

- i) The selection of aiming point
- ii) The location of floodlighting poles and configuration
- iii) The computation of coefficient of utilisation
- iv) The determination of pole height and spacing
- v) The choice of types of lamps to be used.

Choices of optimal value for each of the factors involve one or several computations which could be computerize. The use of computers in lighting design is a practical necessity if really useful results are to be obtained. Computer analysis will give useful point by point luminance figures plus the necessary data on Visual Comfort Probability (VCP) and Equivalent Sphere Illumination (ESI) for selected locations and viewing directions. The computer would also perform the calculations

accurately, rapidly and free the designer for other, less routine work. It also gives the designer the ability to change parameters repeatedly, without making the amount of calculation excessive. Several lighted sporting arenas are already in existence in Nigeria for example at Ranchers Stadium, Ahmadu Bello Stadium, Kaduna, and Township Stadia in Bauchi, Kano, Abuja, Lagos and so on. A look at them will show that their floodlighting schemes could be improved for now and also in future sporting arena developments through new ideas and techniques realisable by research.

#### **2.4 MATLAB CODE**

MATLAB has been adopted for simulation in this project due to its being well developed and tested, its user interface which provides an easy – to – use interactive environment that includes extensive numerical computation and visualization capabilities. It is also compact and extensible using toolboxes to add new commands and capabilities (Palm, 1998).

#### **2.5 ENERGY AND COST SAVINGS**

Cost savings could be realized if the cables, fittings and other materials will be downsized when there is substantial savings in the energy that could have resulted from a better designed illumination scheme (Jitka, 2007).

#### **2.6 LIGHT POLLUTION AND GLARE REDUCTION**

The International Dark-sky Association describes light pollution as;  
*“Any adverse effect of artificial light including sky glow, glare, light trespass, light clutter, decreased visibility at night, and energy waste”* (Homepage, IDA, 2011).

By far the most useful benefit, the properly designed and uniformly illuminated field will serve is to produce less light pollution and reduce wastage of energy thus, as the light will be focused to serve the purpose for which it was installed.

Non-uniformity of illumination in a play field will lead to glare in some places and dark patches in other areas. Glare could lead to discomfort or adverse effects (Thornlighting, 2008). Over-illumination and poorly designed lighting can lead to light trespass. The Illumination Engineering Society of North America (IESNA) has classified light emitted more than 80 degrees above nadir (the horizontal axis), to cut-off (0%), demi-cut-off (10%), and semi-cutoff (20%). While glare is classified into blinding glare, which leaves temporary or permanent vision deficiencies, disability glare, this leads to reduction in sight capabilities and discomfort glare which leads to annoying or irritating discomfort in vision and can cause fatigue. Any of these could result from a non-uniform field of vision.

Moreover, Mario Motta, President of the Massachusetts Medical Society said “glare from bad lighting is a public health hazard – especially the older you become. (Homepage, IDA, 2011).

## **2.7 ILLUMINATION DESIGN FOR SPORTING ARENAS**

As earlier mentioned, illumination design for sporting arenas is floodlighting design (Joseph, 2003). The step by step procedure employed is the same for any sporting arena and is as follows (Tyler, 1972).

1. Choose the type of lamp to use
  - High Intensity Discharge – Mercury (HID – Hg) Metal Halide Lamps
  - Fluorescent Lamps

- Filament Lamps
- Light Emitting Diode (LED) Lamps

Filament type lamps are gradually being phased out due to losses in heat generation, while mercury vapour floodlight lamps provide economical lighting with little maintenance.

2. Determine the recommended lighting levels from
  - Illumination Engineering Society Codes
  - Manufacturers' Data
3. Determine the recommended watts/square meters from IES charts or manufacturers' data.
4. Select the lighting fixture locations. There are several location arrangements. The choice of any one would depend on (i) availability of space, (ii) avoidance of obstruction to players during games, (iii) avoidance of obstruction to spectator view, (iv) degree of uniformity of distribution of illumination desirable, (v) type of sporting activity, (vi) size of the sporting arena. At this stage, the height, spacing and number of fixture positions would be chosen (Tyler, 1972).
5. Compute the number of lights required using
  - Lumen method also called Beam lumen method (especially in floodlighting) which generally gives an average result of the illumination over the arena; and involve more approximations and is used to determine the number of Lamps for a given illumination. (Fink, 1978).

- Point-by-point method, which gives a more detailed illuminance depiction and can even show whether and where we require supplementary lighting, and whether the layout we have used yields the desired uniformity (Stanley, 1980).
6. Determine the lamp arrangement based on the pole arrangement chosen. In floodlighting schemes this would involve training (adjustments) of the floodlighting with respect to vertical angle of elevation (aiming angles) and azimuth (tilting angle), which is done by trial and error to attain the best angles for which a more uniform illumination distribution would be attained (Henderson, 1972). This is basically an intensive part of the design especially where the floodlights are many. For most engineers the design ends here and may take hours and days depending on number and size of arenas involved and in each arena the number of floodlights.

## **2.8 Literature Review**

Illumination systems optimization is currently an active area of research. Various optimization algorithms including simplex, damped least squares, and global methods are being actively investigated (John, 2010).

John R. K. in his paper “Fractional Optimisation of Illumination Optics” describes the use of fractional optimization for illumination system design through using dynamic constraints in which he concludes that the method is useful for constraining dynamically complex illuminated surfaces (John, 2010).

Similarly, Allen et.al. in their paper “High Uniform Illumination of LEDs lighting with applying the multiple-curvature lens” mentioned that the promotion of illumination uniformity is an important key issue and in their work have proposed an

effective design to improve the uniformity of illumination of LEDs with lambertian radiation profiles by inserting into the lighting system, multiple-curvature lens (Allen et al., 2011).

Moreno, I. in his paper “Illumination uniformity assessment based on human vision” describes how he used sensitivity to spatial frequencies and Fourier transforms to propose a simple metric that assesses the uniformity of light distribution as the Human Visual System (HVS) does (Moreno, 2010).

Jonathan et. al. in their “Football lighting Optimisation” described how they have used sum of squares minimization, to optimise uniformity of illuminance distribution in the illumination design for a football field. Using this method they have arrived at a solution to tell the groundskeepers where each floodlight should be aimed to get the required illumination on the playing field (Jonathan et. al., 2010)

Deepa et.al. have reported on several papers on works using secondary optics such as diffusers, lenses, and reflectors for improvement in uniformity, but in their work have proposed the use of an automated, portable test-jig to measure illuminance in Lux at different points on the target surface and computes the uniformity of illumination which can be used as a metric to evaluate the performance of the luminaire (Deepa, et.al., 2012).

Considering the need for exploring optimisation of uniformity of illumination in outdoor sporting arenas and assessing its effects on energy and cost of illumination projects an attempt has been made in this work.

## **CHAPTER THREE**

### **PROPOSED ILLUMINATION DESIGN METHOD**

#### **3.1 INTRODUCTION**

Looking at the floodlighting design procedure stated in chapter two, a number of additional steps are hereby proposed to improve on the design process. These procedures have been made possible by the availability of computers and adoptable software packages that could be employed to speed up the computations and even depict the results in any chosen graphical form (Stanley, 1980). For these, MATLAB software would be employed due to its advantages (Palm, 1998).

The proposed procedures to improve the illumination design process for sporting arenas are as follows:

- i. Any of the several lighting fixture locations could be used and then compared for the best choice using graphical depictions
- ii. Various heights and spacing of fixture positions could be used and also compared for the best choice using graphical depictions.
- iii. The number of points on the point-by-point method computation could be varied to arrive at the optimal number of points to give the uniformity of distribution desired. To do this Monte Carlo Technique would be employed.
- iv. The arrangement of the floodlights in vertical and horizontal form on the tower or mast could be optimised too by looking at the choice of how many columns and rows would be best suited to give the most uniform illumination.
- v. The training (adjustments) of the floodlights for the best aiming angles and azimuth positions to give the best uniformity of illumination instead of being

by trial and error could be done by Monte-Carlo Technique (Richard and Travis, 2004).

### **3.2 MONTE CARLO ANALYSIS**

Monte Carlo method was invented by scientist working on the atomic bomb in the 1940s, who named it after the city of Monaco famed for its casinos and games of chance. Its core idea is to use random samples of parameters or input to explore the behavior of a complex system or process (Solver.com, 2010). The method tends to be used when it is infeasible, impossible or less efficient to compute an exact result with a deterministic algorithm. On the other hand, even when the data is exact, it is sometimes beneficial to deliberately introduce randomness into the search process as a means of speeding convergence and making the algorithm less sensitive to modeling errors (Woller, 2011).

Monte Carlo is a stochastic optimization method employed by engineers in probabilistic design for simulating and understanding the effects of variability (Woller, 2011). There is no single Monte Carlo method. Instead the term describes a large and widely used class of approaches. However, these approaches tend to follow a particular pattern:

1. Define a domain
2. Generate inputs randomly from the domain
3. Perform a deterministic computation using the inputs
4. Aggregate the results of the individual computations into the final result.

The method has two (2) common properties (Veah, 1997).

- i. Reliance on good random numbers

- ii. Slow convergence to a better approximation as more data points are sampled.

As a stochastic optimization method it can incorporate probabilistic (random) elements, either in the problem data (the objective function, the constraints, etc) or in the algorithm itself (through random parameter values, random choices, etc) or in both (Fu, 2011). The designer no longer thinks of each variable as a single value or number, instead each is viewed as a probability distribution. This could be one design option out of several that is found to be most robust.

Alternatively, it could be the only design option available, but with the optimum combination of input variable and parameters. This is sometimes referred to as robustification or parameter design. The typical goal is therefore to identify the design that will exhibit the smallest effects of random variability. This is robust Monte Carlo. Monte Carlo is often said to be sequential or non-sequential. In sequential Monte Carlo Simulation (MCS), the data are sequentially sampled, simulating the chronology of the stochastic process of system operation, while in non-sequential MCS, the states of the system are obtained by randomly sampling the components states space with no relation to the chronology of the events (Tiago, et.al., 2009).

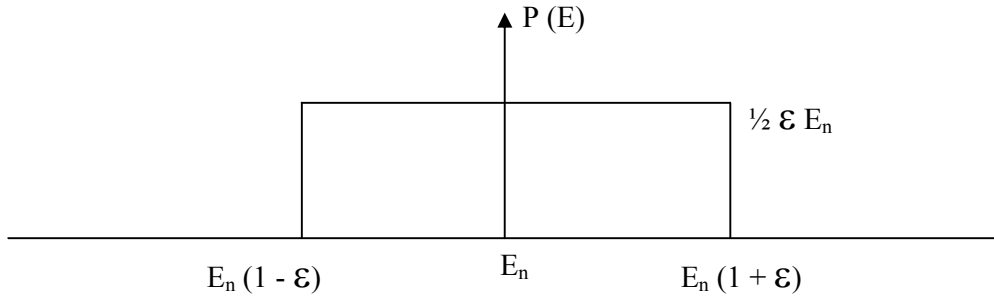
As mentioned earlier, the training (aiming) of flood lights during implementation of the usual design method involves trial and error. This trial and error is some kind of worst case design that may result in unnecessary over or under design. Thus the proposal for the use of Monte Carlo in this area of the design (Richard and Travis, 2004).

A mathematical model for symmetrical parameter variation is

$$P_n(1 - \varepsilon) < P < P_n(1 + \varepsilon) \quad (3.1)$$

$P_n$  is the nominal specification for the parameter,  $\varepsilon$  is the fractional variation (Richard and Travis, 2004). Where  $P$  is the probability of the parameter variation,  $P_n$  is the nominal specification for the parameter, while  $\varepsilon$  is the fractional variation of the parameter.

Equation (3.1) is visualized as a uniform distribution, applying this to illuminance distribution ( $E$ ), the probability density function  $P(E)$  for a uniformly distributed illuminance can be represented graphically in figure 1 below.



*Figure 1: Uniformly Distributed Illuminance*

Where  $E$  is the illuminance at any point and  $E_n$  is the nominal illuminance.

The probability that an illuminance value lies between  $E$  and  $(E + dE)$  is equal to  $P(E) dE$ . The total probability  $P$  must equal unity, so,

$$\int_{-\infty}^{+\infty} P(E) dE = 1 \quad (3.2)$$

Using this equation with the uniform probability density of figure 1 yields

$$P(E) = \frac{1}{2\varepsilon E_n} \quad (3.3)$$

To this Monte Carlo analysis can be readily implemented using MATLAB programming, using the uniform random number generator that is built into the software. Successive calls to this random number generator produce a sequence of pseudo-random numbers that are uniformly distributed between 0 and 1 with a mean of 0.5 as in figure 3.2

In MATLAB the function called rand can be used to generate random numbers with the mean centered at  $E_n$  and the width of the distribution set to  $(2\varepsilon) \times E_n$

$$\therefore E = E_n (1 + 2\varepsilon (\text{RAND}(m, n) - 0.5)) \quad (3.4)$$

Where m and n are the lower and upper limits of the random numbers range

For the uniform distribution described by equation (3.4)  $E_{\max}$ ,  $E_{\min}$ ,  $E_{ave}$  that is the maximum, minimum and average illuminances can be computed by Monte Carlo Analysis [Richard,2004].

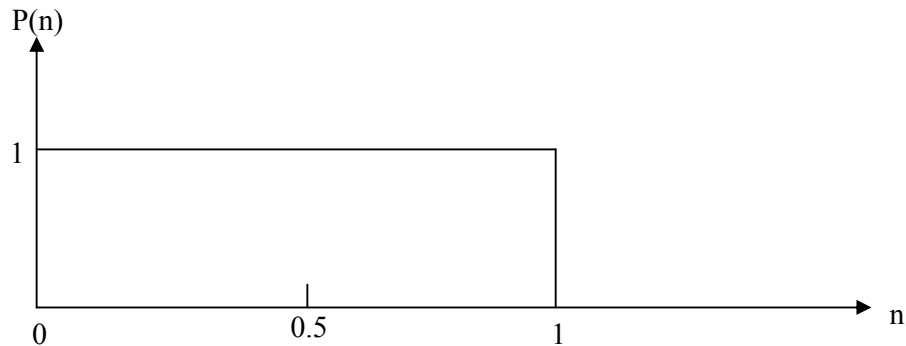


Figure 2: Distribution of Pseudo-random numbers from MATLAB

The uniformity of illumination can then be checked by taking the ratios:

$$A1 = E_{\max} / E_{\min} \quad 3.41$$

$$A2 = E_{\max} / E_{ave} \quad 3.42$$

$$A3 = E_{ave} / E_{\min} \quad 3.45$$

Where A1, A2, A3 are uniformity ratios

Computations of uniformity ratios from the above ratios could then be compared to assess what effect the Monte Carlo Analysis has yielded.

Consider the football field of allowed area, 120m x 80m. For this field as shown in figure 3.3 below, a four corner solution, for a 500lux using the four corner solution will give 12 x 2000w floodlights on each pole (Tyler, 1972). These floodlights would be placed on 4 x 30m poles each at a corner, arranged in 2 columns of 6 rows or 4 columns of 3 rows. For the 4 corner solution, the field is divided into 4 quadrants, with each quadrant floodlighted by a set of floodlights at the closest corner.

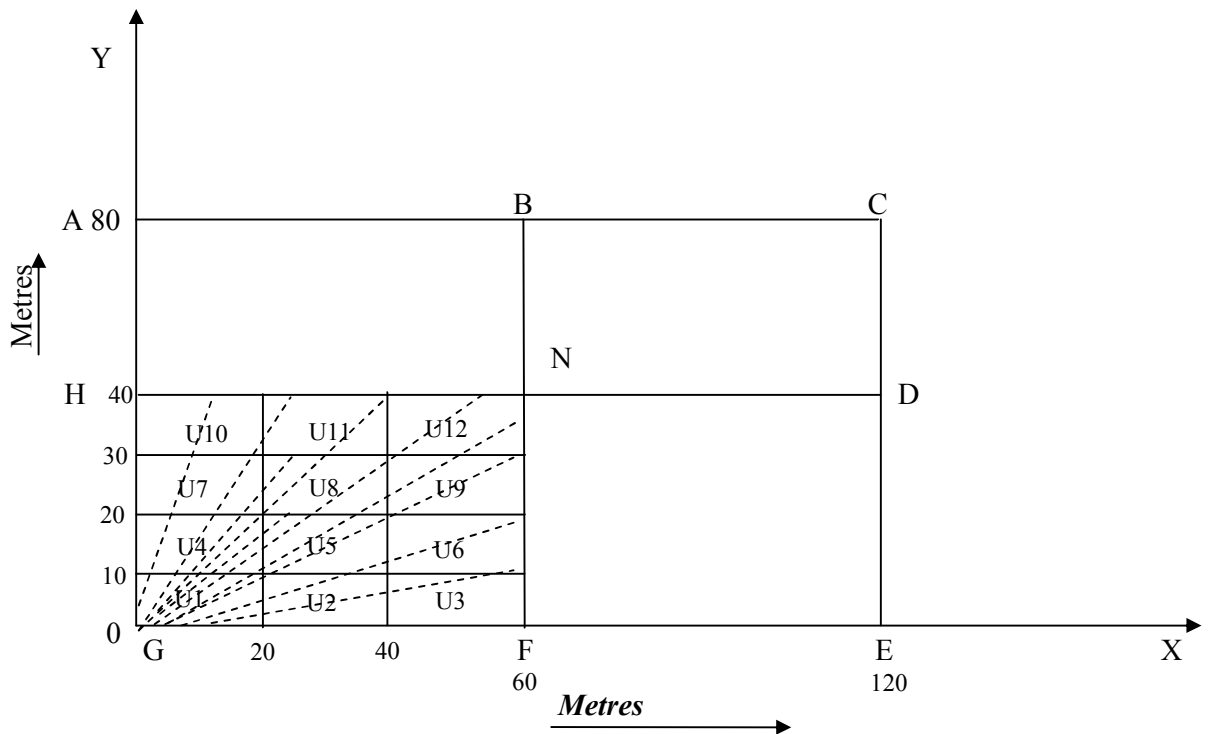


Figure 3.3 4 - Corner Solution floodlighting of a 120m x 80m field

Each quadrant would be divided into twelve cells as shown in quadrant GHNF in figure 3.3 The cells are labeled U1 to U12. Each of these cells is served by a single flood light aimed at it from the pole, so the floodlights can be labeled to the corresponding cell they serve.

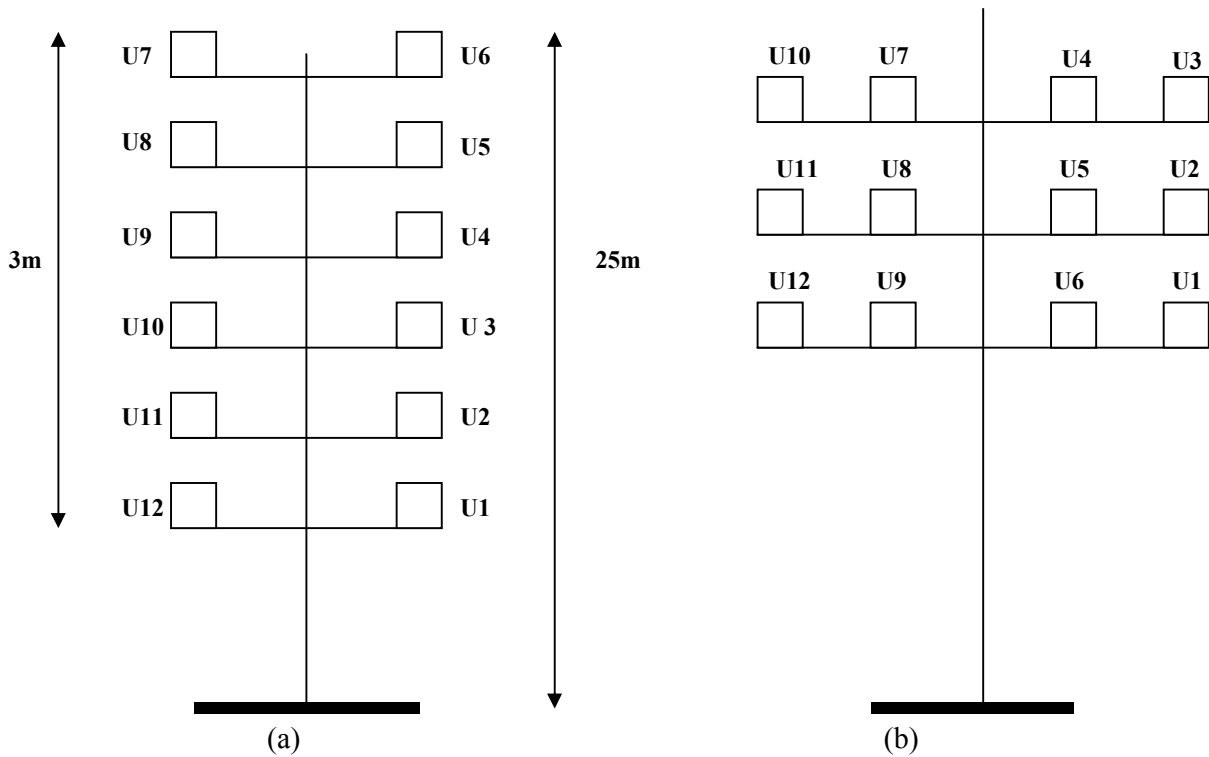


Figure 3.4: Floodlights on poles in (a) 2 columns of 6 rows and (b) 4 columns of 3 rows

Referring to figures 3.5 and 3.6, from inverse square and cosine laws of illumination.

The illumination at any point within cell U1 is

$$E = \frac{I \cos \phi}{d^2} \quad (3.5)$$

Considering the right angle triangle  $\triangle ABC$  in figure 3.5,

$$d \cos \phi = h \quad (3.6)$$

$$\text{and } d^2 \cos^2 \phi = h^2 \quad (3.7)$$

Substituting (3.7) in (3.5)

$$E = \frac{I \cos^3 \phi}{h^2} \quad (3.8)$$

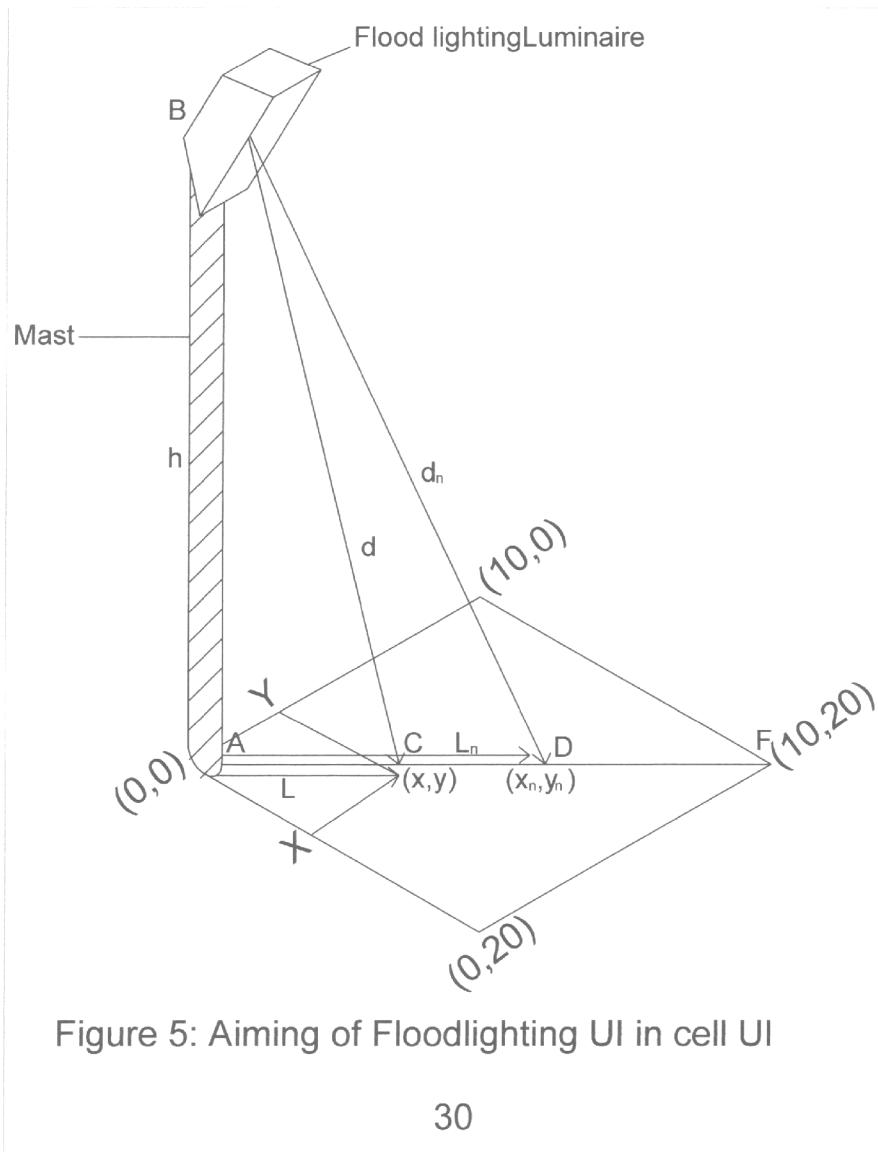
E = Illumination in Lux

I = Light Intensity in lumens or candela

h = floodlight mounting height in meters

$\phi$  = Angle subtended by the floodlight peak intensity (PI) at point C

The aiming of the peak intensity (PI) point of the floodlight is to obtain equal illuminance at C and D and any other point that is at the rear and far sides of the rectangular area, U1



See correct page attached

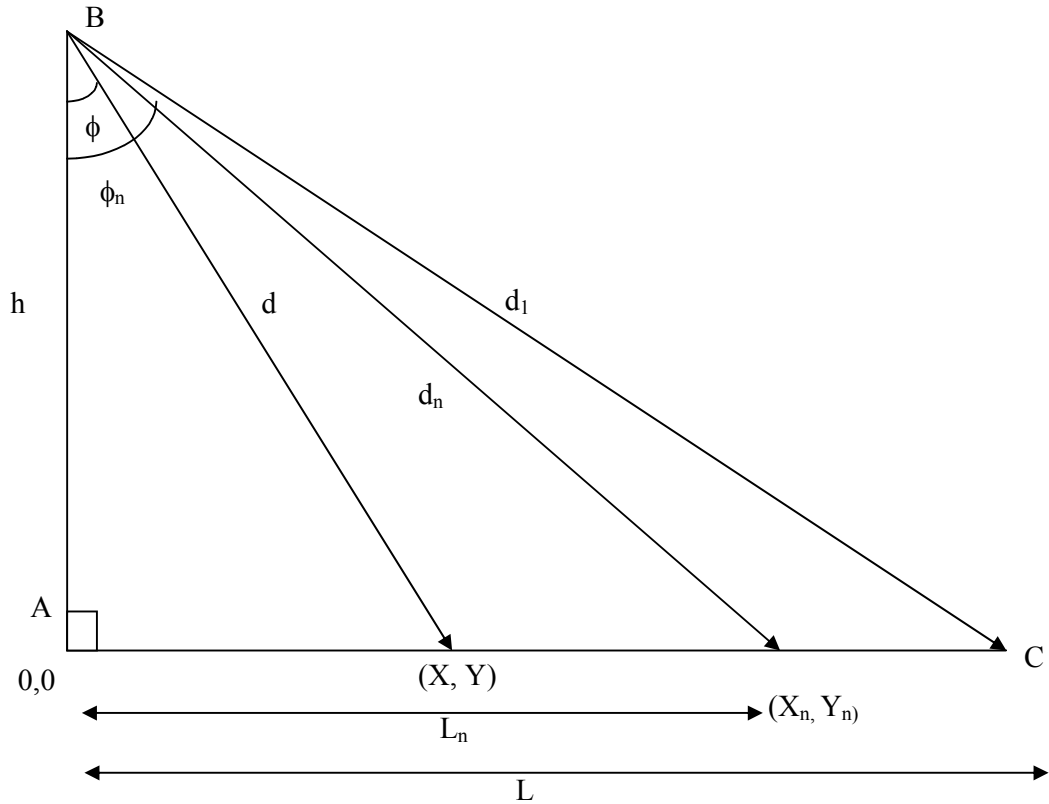


Figure 3.6: Floodlight aiming profile

For uniformity in illumination it means

$$E_{(XY)} = E_{(X_n Y_n)} \quad (3.9)$$

That is,

$$\frac{I \cos^3 \phi}{h^2} = \frac{I_n \cos^3 \phi_n}{h^2} \quad (3.10)$$

$$\Rightarrow \frac{I_n}{I} = \frac{\cos^3 \phi}{\cos^3 \phi_n} = K \quad (3.11)$$

$$\Rightarrow I_n = KI \quad (3.12)$$

Where K is the uniformity ratio and is constant

That is, the intensity toward the far side needs to be K times that of the closer side.

This means the intensity of the floodlight has to vary according to the aiming point. This is difficult to achieve for practical purposes. In general a 10% variation in intensity represents the limit of coverage of a floodlight [Joseph, 2003]. Thus to determine the actual aiming point of  $X_n$   $Y_n$  in each situation, the intensity distribution of the floodlight must be considered and a trial and error must be made to arrive at the best point to give the most uniform illumination distribution possible in the rest of the field. It is usually advised that the floodlights be aimed at  $\frac{2}{3}$  or  $\frac{3}{4}$  of the distance along the diagonal line L as shown in figure 3.5 (Joseph, 2003).

### 3.3 IMPROVED UNIFORM ILLUMINATION

To improve the computations of uniform illumination within cell U1, considering figure 3.5 and from equation (3.8)

$$E = \frac{I \text{Cos}^3 \phi}{h^2} \quad (3.13)$$

$$\text{Cos}^3 \phi = \frac{h^3}{d^3} \quad (3.14)$$

$$d = (h^2 + L^2)^{\frac{1}{2}} \quad (3.15)$$

$$\Rightarrow E = \frac{I}{h^2} \bullet \frac{h^3}{(h^2 + L^2)^{\frac{3}{2}}} \quad (3.16)$$

therefore,`

$$E = \frac{I h}{(h^2 + L^2)^{\frac{3}{2}}} \quad (3.17)$$

As the floodlight peak intensity is aimed at  $(X_n, Y_n)$  the intensity variation along the diagonal will be reflected by a continuous change with respect to position, within the 10% limit if a factor  $f$ , where:

$$f = (0.1 I_n) \frac{L_n - L}{L_n} \quad (3.18)$$

is introduced for  $0 < L_n < L$  which along X-axis is  $0 < X < X_n$

and

$$f = (0.1 I_n) \frac{L - L_n}{L} \quad (3.19)$$

for  $L_n < L < 20$  which along x - axes is  $X_n < X < 20$

hence, the improved illumination to reflect changes in intensity from  $I_n$  (peak intensity) at other positions at  $0 < X < X_n$

$$E = \left[ I_n - \left[ 0.1 I_n \frac{L_n - L}{L_n} \right] \right] \left[ \frac{h}{(h^2 + L^2)^{\frac{3}{2}}} \right] \quad (3.20)$$

and

$$E = \left[ I_n - \left[ 0.1 I_n \frac{L - L_n}{L} \right] \right] \left[ \frac{h}{(h^2 + L^2)^{\frac{3}{2}}} \right] \quad (3.21)$$

at  $X_n < X < 20$

Hence, it is clear that for any other position,  $f$  and  $E$  could be computed. This will clearly be cumbersome for several points; hence it is converted to a MATLAB code in Appendix I. For several points  $(X, Y)$  within cell U1 the illumination  $E$  can now be computed and a uniformity check applied to see how uniform the illumination is with this factor.

### 3.4 APPLYING MONTE CARLO METHOD

Another step proposed towards achieving more uniformity of illumination distribution for any field in arena floodlighting design is to use Monte Carlo Method earlier discussed, to determine the point  $(X_n, Y_n)$  at which the Peak Intensity of the floodlight (PI) will be aimed to attain the most uniform illumination distribution of adequate level.

Using Monte Carlo Method hundreds or thousands of  $(X, Y)$  could be generated by the random number function and hundreds or thousands of  $E$  computed and through the analysis several  $E_{\text{maximum}}$ ,  $E_{\text{minimum}}$ ,  $E_{\text{average}}$  computed, uniformity as the ratios  $E_{\text{maximum}}$  to  $E_{\text{minimum}}$ ,  $E_{\text{maximum}}$  to  $E_{\text{average}}$  and  $E_{\text{average}}$  to  $E_{\text{minimum}}$  could then be checked till the best  $(X_n, Y_n)$  point is found for every floodlight and every cell.

To generate random numbers uniformly distributed in any interval other than  $(0, 1)$ , we use

$$y = (b - a)x + a \quad (3.22)$$

where  $x$  is a random number uniformly distributed in the interval  $(0, 1)$  and  $(a, b)$  represent the interval other than  $(0,1)$ .

Thus, in this case our random number range will be given by  $(b - a)$ , hence

$$X_n = (20 - 0) \times \text{rand}(1, 1000) \quad (3.23)$$

A thousand random values of  $X$  in the interval  $(0 - 20)$  cell U1 will be generated

Similarly, from

$$y = mx + C \quad (3.24)$$

where  $m$  is the gradient and  $c$  is a constant in the equation

$$\text{looking at the UI cell area, we can see that } C = 0, \text{ (figure 3.5)} \quad (3.25)$$

$$m = \frac{10 - 0}{20 - 0} = \frac{1}{2} \quad (3.26)$$

$$\text{hence, } Y_n = \frac{X_n}{2} \quad (3.27)$$

It can be seen from figure 3.5 that,

$$L = (X^2 + Y^2)^{\frac{1}{2}} \quad (3.28)$$

$$L_n = (X_n^2 + Y_n^2)^{\frac{1}{2}} \quad (3.29)$$

$I_n = 2000$  candelas from the floodlight manufacturers data in Appendix VIII

$0.1I_n = 200$  candelas

and as shown earlier on,

for  $0 < L < L_n$  which along X – axis is  $0 < X < X_n$

$$E = \left[ I_n - \left[ 0.1 I_n \frac{L_n - L}{L_n} \right] \right] \left[ \frac{h}{(h^2 + L^2)^{\frac{1}{2}}} \right] \quad (3.30)$$

and

for  $L < L_n < 20$  which along x-axis is  $X_n < X < 20$

$$E = \left[ I_n - \left[ 0.1 I_n \frac{L - L_n}{L} \right] \right] \left[ \frac{h}{(h^2 + L^2)^{\frac{1}{2}}} \right] \quad (3.31)$$

For the several values of  $(X_n, Y_n)$ , and the thousands of values of X and Y,  $E_{\max}$ ,  $E_{\min}$ ,  $E_{\text{ave}}$  can be obtained from the MATLAB code for Monte Carlo Analysis in Appendix I, at each and several values of  $(X_n, Y_n)$ , the computation can be repeated to find several value of E,  $E_{\max}$ ,  $E_{\min}$ ,  $E_{\text{ave}}$  and the uniformity ratios also computed. Hence uniformity ratios at the different  $(X_n, Y_n)$  will be compared and the best chosen. This is run as a MATLAB code I shown in Appendix I. The results obtained

from the code is analysed and discussed in chapter four (4), using isolux contours obtained from MATLAB Codes II and III in Appendices II and III respectively. So also checks are made to assess if energy and cost savings are realisable as a consequence of better design of the illumination of the field.

## CHAPTER FOUR

### RESULTS AND ANALYSES

#### 4.1 BEST AIMING POINT

The MATLAB code I, when ran, each time went through three iteration loops for,

K=1 to 10000;

Xn=1 to 20;

n = 1 to 1000;

These iteration loops will produce 20000 random values of X in the range Xn =1 to 20 and compute the values of the illumination and find its highest and the lowest values, then compute the uniformity as the ratio of the maximum to the minimum. These ratios will then be checked for the very best that will give the most uniform illumination distribution called Xbest.

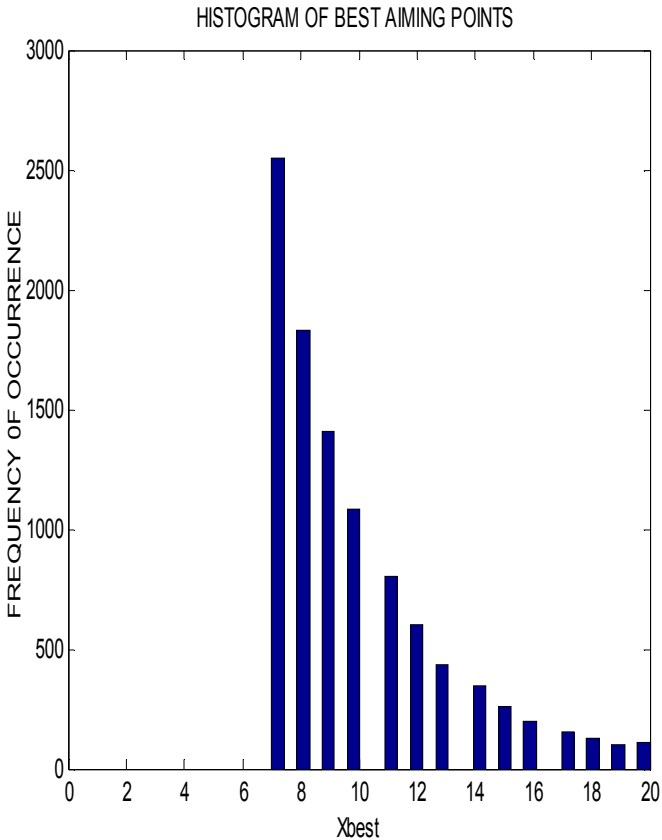
The histogram in Figure 4.1 shows the various points at which aiming points that give the most uniform illumination within cell U1 of the football field, referred to as Xbest occur. By Monte Carlo Analysis the best of all these points is the mode which occurs at Xbest is 7 from which the aiming point was computed as

$$\begin{aligned} X_{best} &= X_n = 7; \\ Y_{best} &= X_n / 2 = 3.5 \end{aligned} \quad (4.1)$$

$$\begin{aligned} L_n &= (X_n^2 + Y_n^2)^{1/2} \\ &= (7^2 + 3.5^2)^{1/2} \\ &= 7.82 \text{ meters} \end{aligned} \quad (4.2)$$

A comparison of the isolux contours plotted from the Matlab Code II in Appendix II for eight (8) of the Fourteen (14) points of the histogram in Figures 4.1

show that of all the isolux contours, the contours for  $X_n = 7$  has the most uniformly spread contours, with contours varying in steps of 2, and also 11 contour lines. All other points as seen in Figure 4.2 either have less number of contours or varying in steps of 1 or having more clustering of contours. This shows that the  $X_n=7$  gives the most uniform illumination of the U1 field. A plot of the uniformity gradient for  $X = 7, 8, 10$  and random in the 12 cells of Quadrant A shows that  $X = X_{best} = 7$  gives the best uniformity gradient of 0.027 lux/m, with the case for random  $X$  points in the cell even yielding a negative gradient of 0.001 lux/m as shown in figure 4.3



**Figure 4.1: Histogram of Best Aiming Points of Floodlight U1**

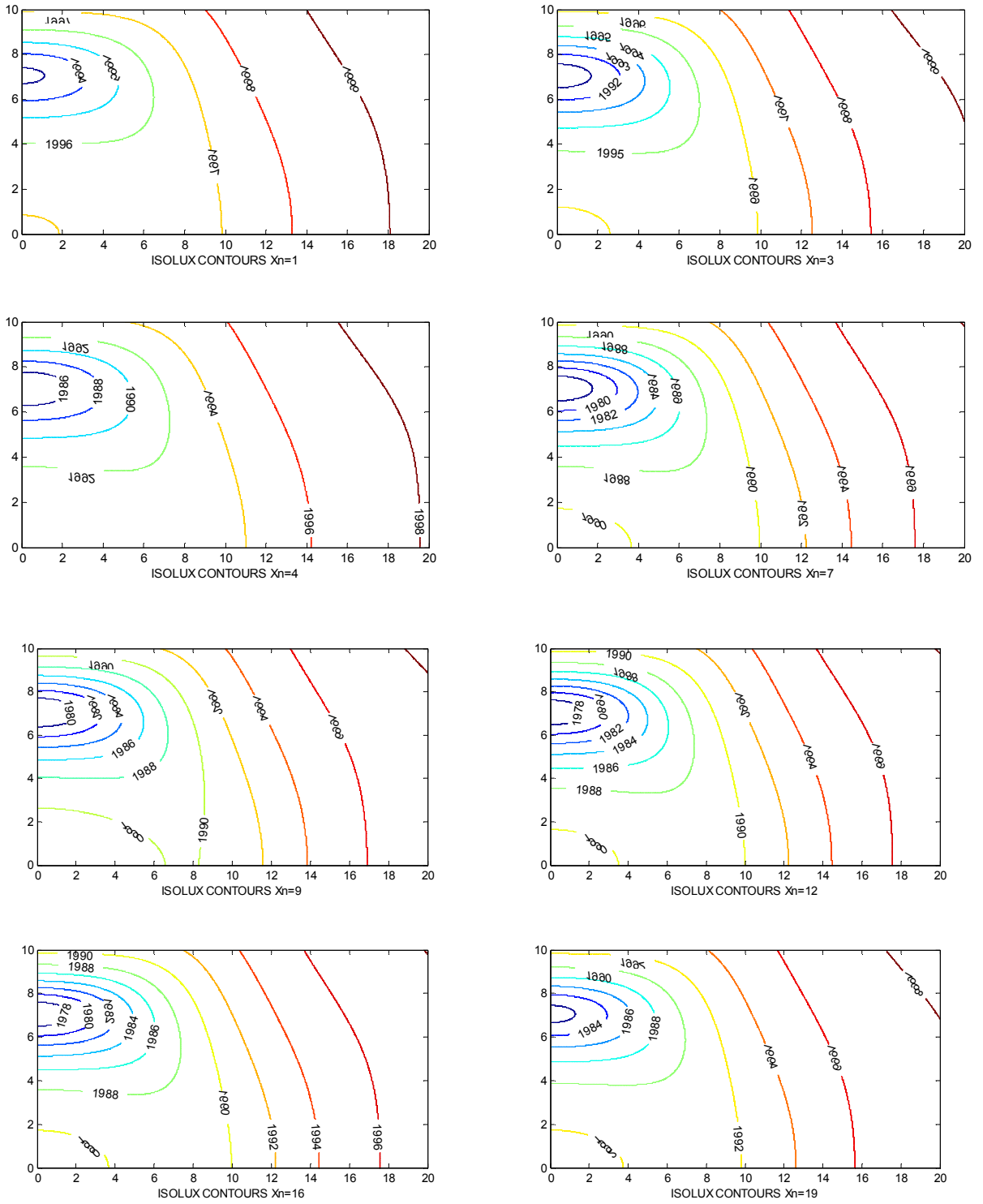
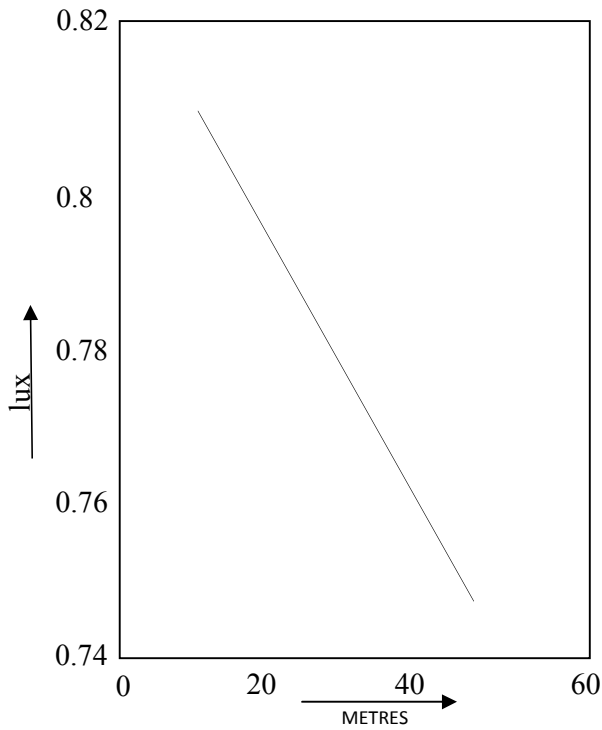
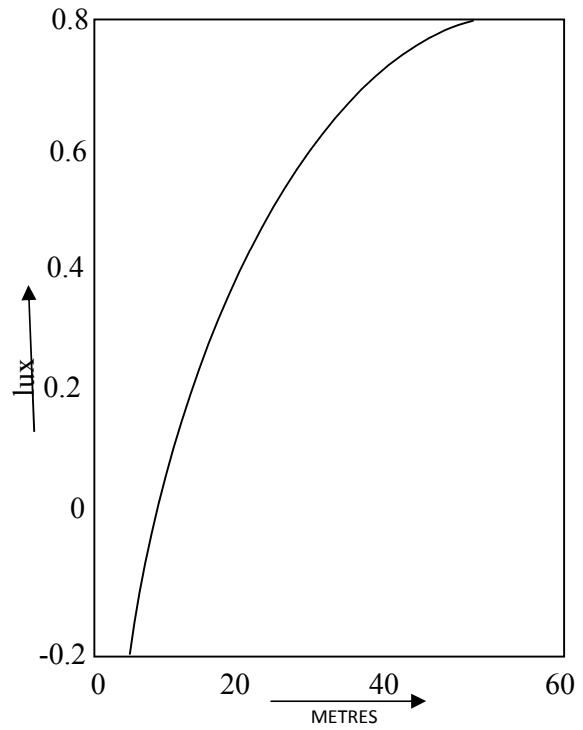


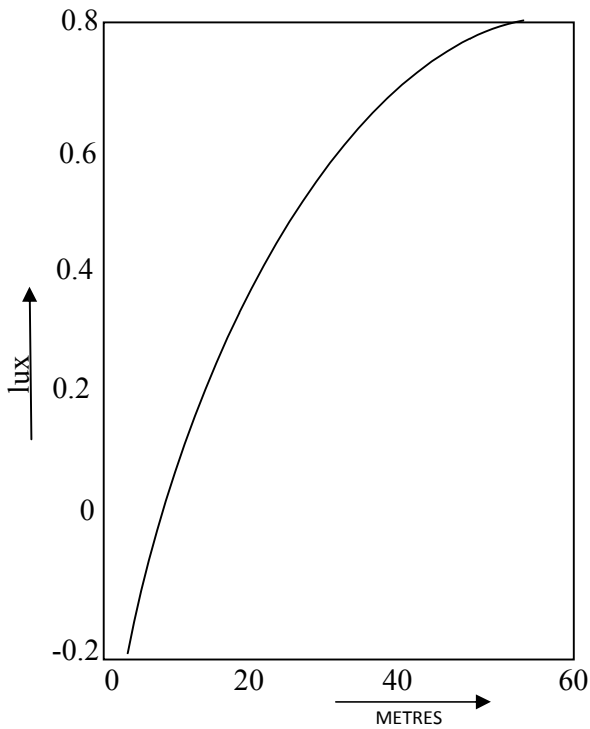
Figure 4.2: Isolux Contours for  $X_n$  is 1, 3, 4, 7, 9, 12, 16 and 19



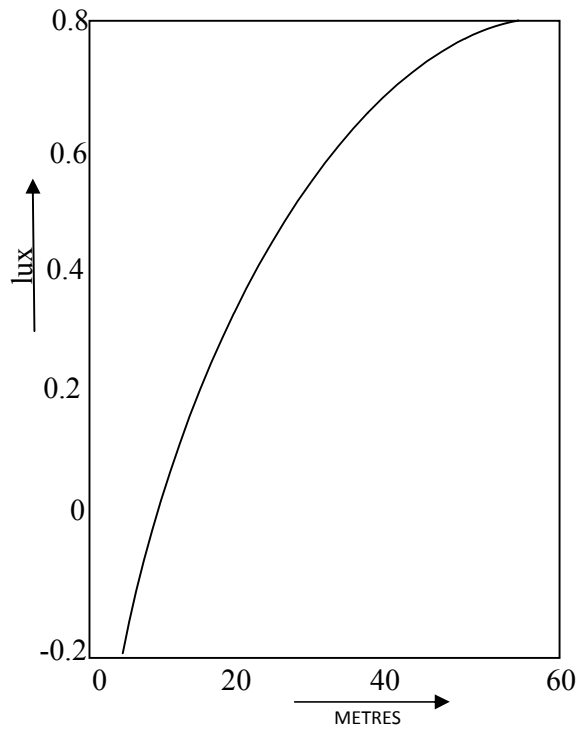
UNIFORMITY GRADIENT WITH RANDOMLY CHOSEN X IN THE 12 CELLS OF QUADRANT A: GRAD = -0.001



UNIFORMITY GRADIENT WITH  $X=X_{best}$  IN THE 12 CELLS OF QUADRANT A: GRAD = 0.027

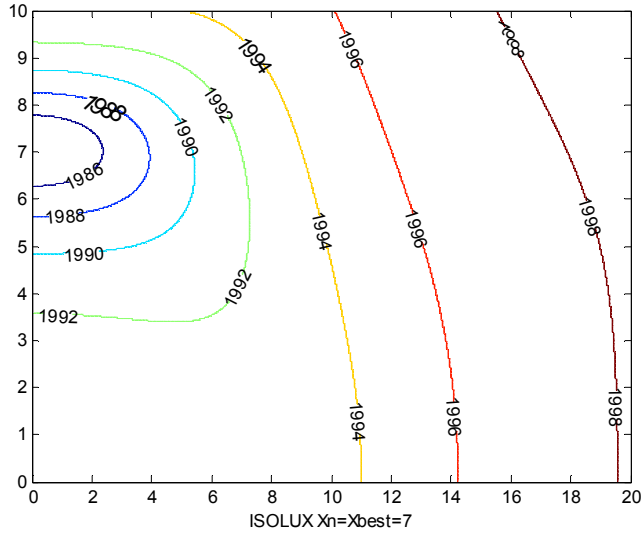


UNIFORMITY GRADIENT WITH  $X=8$  IN THE 12 CELLS OF QUADRANT A: AGRAD=0.021

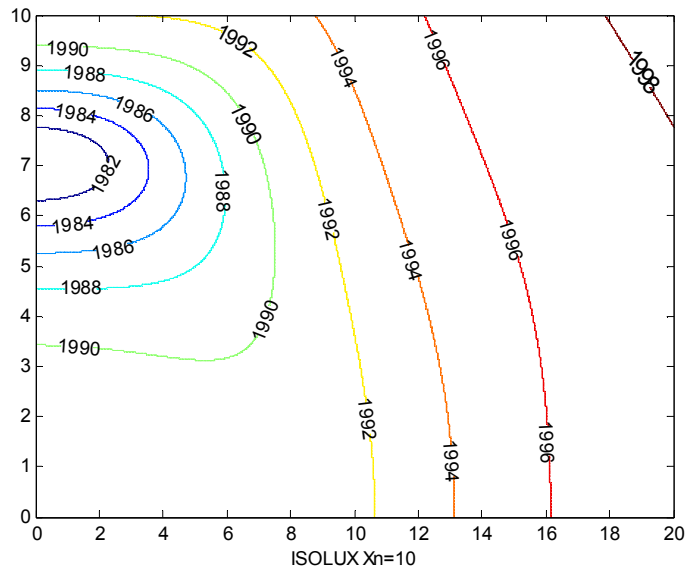


UNIFORMITY GRADIENT WITH  $X=10$  IN THE 12 CELLS OF QUADRANT A: AGRAD=0.020

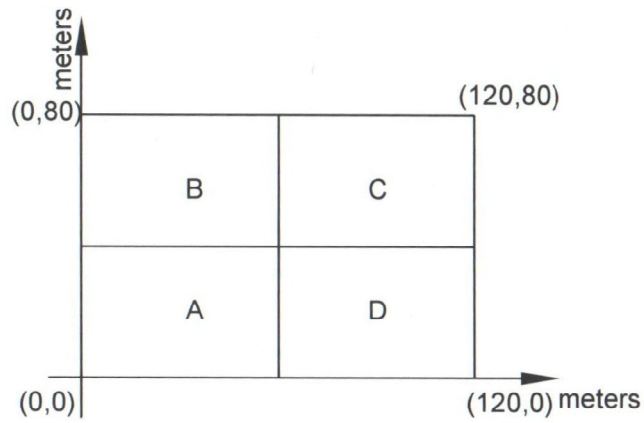
**Figure 4.3: Uniformity Gradients for Different Values of X**



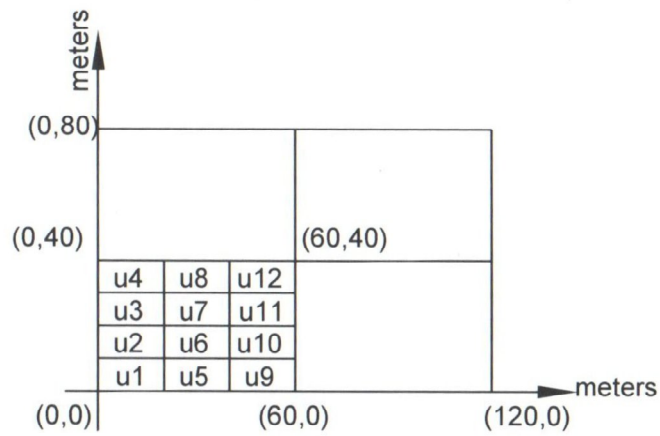
**Figure 4.4: Isolux Contours  $X_n = X_{best} = 7$**



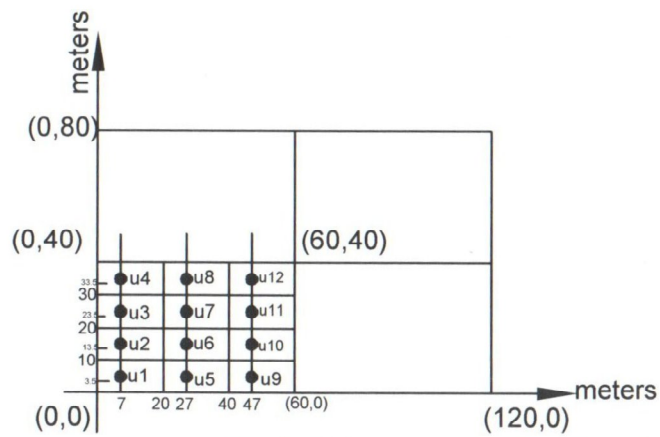
**Figure 4.5: Isolux Contours for  $X_n = 10$**



**Figure 4.6:** Field divided into 4 Quadrants A,B,C,D. mirror images, Length and Breath.



**Figure 4.7:** The 12 Cells of Quadrant A



**Figure 4.8:** The best aiming points for each cell

## 4.2 ENERGY AND COST SAVINGS

The aiming points  $X_n = X_{best} = 7$  and  $X_n = 10$  having their isolux points plotted as shown in figures 4.4 and 4.5 from Matlab Code II respectively are compared to check for energy and cost savings, thus;

For  $X_n = 7$  then,  $Y_n = 3.5$

$$L_n = (X_n^2 + Y_n^2)^{1/2} \quad (4.3)$$

$$= 7.82$$

Using  $X = 2$  then  $Y = 1$

$$E_7 = I_{n7} - (0.1 I_{n7} (L_n - L) / L) (h / (h^2 + L^2)^{3/2}) \quad (4.4)$$

$I_n = 2000 \text{cd}$ ,  $h = 27 \text{m}$

Substituting values we have,

$$E_7 = 1999.8072$$

Similarly,

At  $X_n = 10$ ,  $Y_n = 5$

$$L_n = (10^2 + 5^2)^{1/2} = 11.1803$$

Using  $X = 2$   $Y = 1$  and  $L = 2.25$

$$E_{10} = I_{n10} - (0.1 I_{n10} (L_n - L) / L) (h / (h^2 + L^2)^{3/2}) \quad (4.5)$$

$$= I_{n10} - 0.0001 I_{n10} \quad (4.6)$$

$$= 0.9999 I_{n10}$$

For  $X_7$  and  $X_{10}$  to give the same uniform illumination, then

$$E_{10} = E_7 \quad (4.7)$$

$$\text{i.e., } 0.9999 I_{n10} = 1999.8072 \quad (4.8)$$

$$\text{then } I_{n10} = \frac{1999.8072}{0.9999}$$

By proportion the additionally required intensity of  $0.0072 \text{cd}$  will be provided by

$$\frac{2000 \text{W} \times 2000.0072 \text{cd}}{2000 \text{cd}}$$

$$\text{i.e., } P_{FL10} = 2000.0072 \text{ watts}$$

$$P_{FL10} = 2000 \text{ watts}$$

$$\begin{aligned}
P''_{FL10} &= P'_{FL10} - P_{FL10} && (4.9) \\
&= 2000.0072 - 2000 \\
&= 0.0072 \text{ watts}
\end{aligned}$$

Where PFL10 is the power rating of the U10 floodlight (2000W)

$P'_{FL10}$  is the newly required U10 floodlight power (2000.0072W) and  $P''_{FL10}$  is the additional power required (0.0072W) to produce the additional intensity required to attain the best uniformity as for  $X_n=7$

### 4.3 APPLYING THE RESULTS TO THE ENTIRE FIELD

The field which is 120m x 80m was initially partitioned into 4 equal parts as shown in figure 4.6 as quadrants A,B,C and D. each of these quadrants is a mirror image of the other lengthwise or breadth wise, hence whatever is attained for any quadrant say A is mirrored for the rest.

For the ease of derivation of equations, application of Monte Carlo Analysis and initial analysis of results quadrant A which was chosen was divided into 12 cells as shown in Figure 4.7. Each cell is served by a floodlight with corresponding label that is U1 to U12. The results obtained for U1 is then applied to the other cells as all the cells are the same in dimension. In Figure 4.8 the best aiming point for each floodlight in each cell is shown as dot points corresponding to the results of  $X_n=X_{best}=7$  from cell U1.

Using MATLAB Code III the Isolux contours for quadrant A taking random points of  $X_n$  was produced as shown in Figure 4.9, while the contours for the quadrant using the best aiming points of each cell is as shown in Figure 4.10. Finally the Isolux contours for the whole field using the best aiming points in all cells is shown in Figure 4.11 and it shows clearly the mirror image nature of the quadrants.

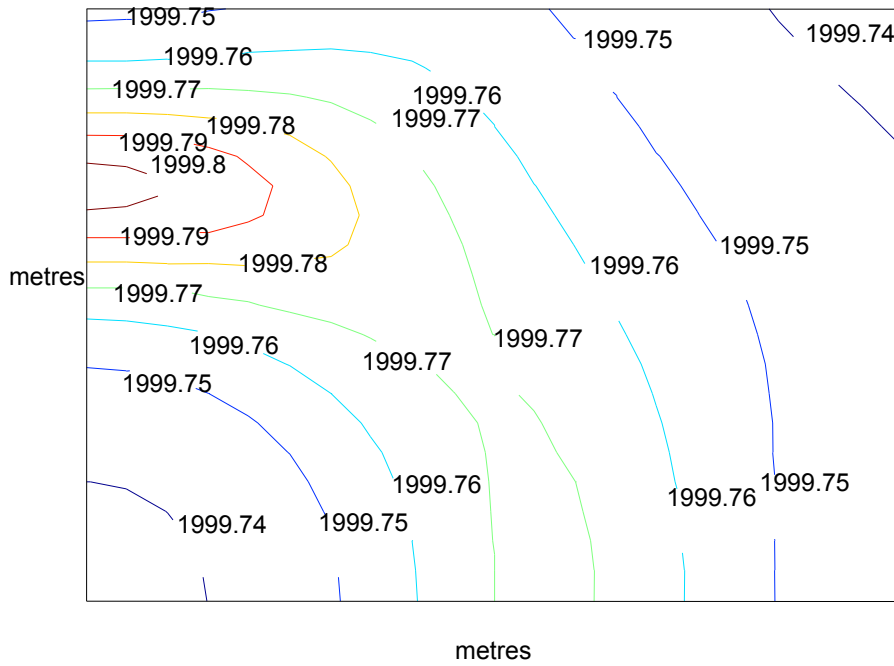


Figure 4.9: Isolux contours for random  $X_n$  for Quadrant A

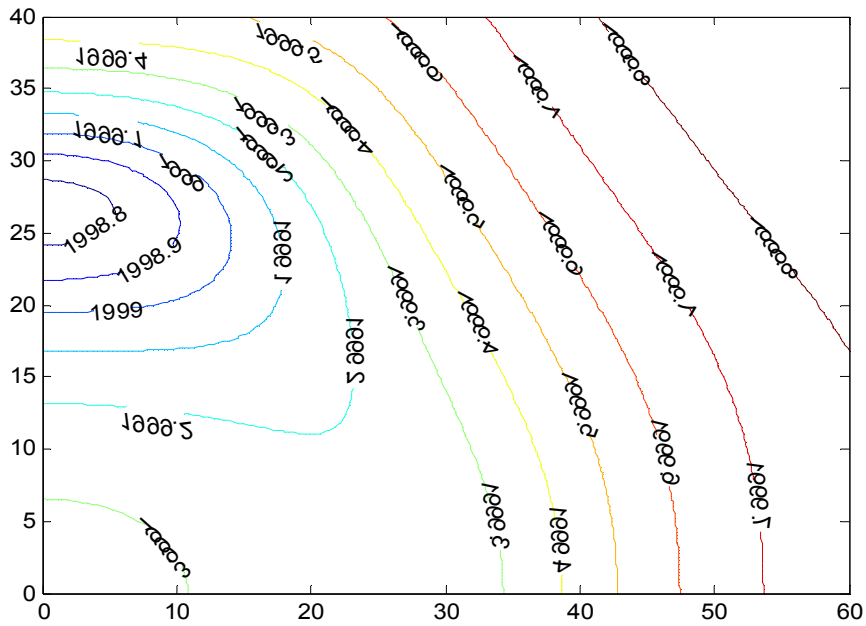
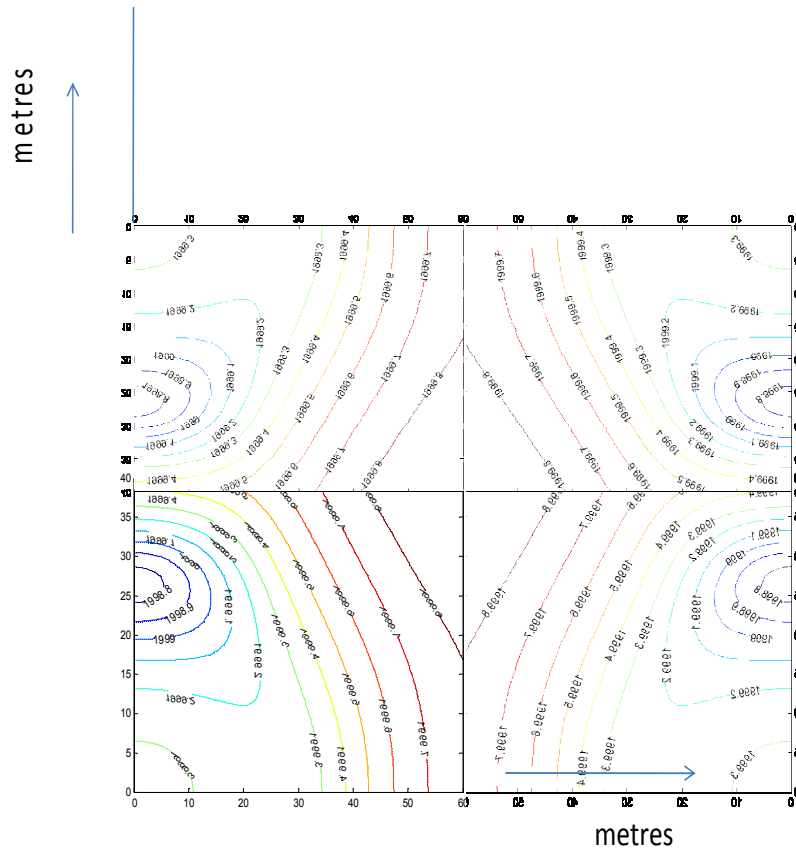


Figure 4.10: Isolux contours for  $X_n = X_{best}$  for Quadrant A



**Figure 4.11: Isolux Contours for best aiming points of Xn for the whole field (four quadrants mirror imaged)**

## **CHAPTER FIVE**

### **CONCLUSION**

#### **5.1 INTRODUCTION**

This chapter seeks to highlight the major contributions of this research work, to enumerate the limitations of the study and to give suggestions for further work in illumination research.

This study has focused mainly on using Monte Carlo Analysis programmed using MATLAB to optimize illumination design, to simulate the illumination equations, display the results in graphical form using isolux contours and assess energy and cost savings potential in a more uniformly illuminated sporting arena.

#### **5.2 SIGNIFICANCE OF THE STUDY**

A factor of light beam intensity variation within 10% of peak intensity over the arena, illuminated for each floodlight was introduced into the illumination equation, to take care of the facts that the beam intensity varies with aiming point over the area illuminated (Joseph, 2003).

The use of Monte Carlo Method to determine the best aiming points of the floodlights as an alternative to having to do it by trial and error on the field at installation stage was investigated and found to be satisfactory in achieving uniform illumination of the field. Moreover, finding the best aiming points by trial and error would require the use of equipment like the goniometer and photometer, which may not be readily available. Using Monte Carlo Analysis in this way in illumination projects will always lead to savings in project design and delivery time and hence in cost.

### **5.3 LIMITATION OF THE STUDY**

Detailed experimental study to practically measure the various illuminances at various stages of the work could have been conducted but for non-availability of equipment such as photometer and goniometer.

### **5.4 CONCLUSION**

In conclusion, it can be said that this research work achieved the following:

- i. Proposed a factor of light beam intensity variation to take care of the fact that the beam intensity varies with aiming point over the area illuminated.
- ii. Proposed a method of determining the best aiming points for floodlights on a sporting arena using Monte Carlo Analysis rather than the trial and error along with the use of special equipment such as the goniometer and photometer.
- iii. Developed the illumination equations from trigonometric form to Cartesian form to enable the application of Monte Carlo Analysis.
- iv. Analysed that a more uniformly illuminated field will not only lead to less glare, less lighting pollution, but will also lead to some energy savings, though seemingly meagre instantaneously, but could become considerable if viewed from the point of view of the life-span of a floodlight, the hours of operation daily ,and will thus run into several Gigawatt-hours of electricity. These Gigawatt-hours of electricity is responsible for millions of tons of CO<sub>2</sub> emissions (IDA, 2011) and is a major source of concern to the environment.
- v. Developed MATLAB Codes which could be used to speed up floodlighting design with more accuracy. They could be used by any designer easily and be adopted for use for other arenas apart from a football field.

## **5.5 SUGGESTIONS FOR FURTHER WORK**

With the availability of equipment like photometer, goniometer, reflectors, beam optical concentrators, diffusers, lasers, masers, several measurements and studies could be conducted to see other factors that affect variability of uniformity of illumination, glare, light pollution, luminous efficacy and economics of lighting for arena illumination design.

The rising cost of energy and its negative consequences has lead to a very important need to find more reliable, cheaper and more maintainable lamps, lighting circuits, equipment and materials, which all need to be researched into extensively.

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## **Appendix I:**

### **MATLAB CODE I - RUN MONTE ANALYSIS**

**Appendix II:**  
**MATLAB CODE II- FOR ISOLUX CONTOUR OF CELL**  
**UI**

**Appendix III:**

**MATLAB CODE III – FOR ISOLUX CONTOURS OF  
FIELD QUADRANT A (RANDOM VALUES AND VALUES  
FROM XBEST OF EACH CELL)**

**Appendix IV:**  
**Typical Values of  $X_{final}$  for Cell UI**

**Appendix V:**  
**Typical Values of  $X_{best}$  for Cell UI**

**Appendix VI:**  
**Typical Values of Illumination (E) of Cell UI**

**Appendix VII:**  
**Manufacturer's Data of Mundial Floodlight**