THE EFFECTS OF THE ACCURACY OF NUMERICAL RELATIVE ORIENTATION OF AEROTRIANGULATION BY INDEPENDENT MODELS

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DEDICATED TO THE MUSTAPHA'S IMAMS AND TO ALL MY DETRACTORS AND ADVERSARIES WHERE EVER AND WHOEVER THEY MAY BE

D E C L A R A T I O N

I hereby declare that this thesis has been composed by myself and that is a record of my own research work. It has not been accepted in any previous application for a higher degree. All quotations are distinguished by quotation marks and the source of information are specifically acknowledged by means of reference.



Ademola A. Y. Mustapha

1986

C E R T I F I C A T I O N

This project/thesis/dissertation entitled THE EFFECTS OF THE ACCURACY OF NUMERICAL RELATIVE ORIENTATION OF AEROTRIANGULATION BY INDEPENDENT MODELS BY MUSTAPHA A.A.Y. meets the regulations governing the award of degree of Master of Science of Ahmadu Bello University, and is approved for its contribution to knowledge and literary presentation.

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A B S T R A C T

Aerotriangulation by independent model (AIM) was performed on Wild A-7 Autograph using wide angle photographs of average scale of 1,8000. In performing the Aerotriangulation, the relative orientation was carried out by numerical method and the model coordinates of the perspective - centre; employed for the model connections was measured.

The research seeks to establish the accuracy of AIM by using different densities and distributions of points for numerical relative orientation and twenty perspective - centre determinations.

Practical investigation reveals that cases of the same density will produce similar results irrespective of differences in distribution.

The accuracy of strip formation was found to be dependent on the distribution and density of points used for numerical relative orientation of the first model. For instance case 1 with only six points has accuracies of ± 0.1612 mm in planimetry and ± 0.163 mm in height, case 25 with fifteen relative orientation points has accuracies of ± 2.933 mm in planimetry and ± 0.505 mm in height.

TABLE OF CONTENTS

		Page
	List of Tables	xi
	List of Figures	xii
	List of Appendices	xiv
	Abbreviations, Definitions, Glossaries and symbols	xv
1	INTRODUCTION	1
1.1	Definition and scope of Aerotriangulation	1
1.2	Aerotriangulation in Nigeria	2
1.3	Aim of the research	3
2	THEORETICAL CONSIDERATION	4
2.1	Y-parallax equation	4
2.1.1	Derivation of the 3-D Conformal Coordinate Transformation equation	4
2.1.2	Derivation of Y-parallax equation	8
2.2	Model formation	14
2.2.1	Interior Orientation	14
2.2.2	Relative Orientation	16
2.3	Aerotriangulation	17
2.4	Perspective Centre determination	17
2.5	Formation of Strips from independent model	18
2.6	Block adjustment	21
3	MATERIALS/EQUIPMENT AND METHOD	22

		Page
3.1	Materials/Equipment	22
3.1.1	Photograph	22
3.1.2	Wild A7 Autograph	23
3.1.3	Computer facilities	25
3.2	Methods	25
3.2.1	Recording forms	25
3.2.2	Preliminary Data	25
3.2.3	Preliminary Settings	26
3.2.3.1	Determination/Selection of required Instrument Settings	26
3.2.3.2	Selection of the most favourable model scale	26
3.2.3.3	Selection of Gear ratios	28
3.2.3.4	Determination of projection height	29
3.2.3.5	Determination of Model base (bx)	29
3.2.3.6	Setting of the optical and mechanical fittings	30
3.3	Precautions	30
3.4	Model formation	31
3.4.1	Interior Orientation	31
3.4.2	Relative Orientation	32
3.5	Measurement of model Coordinates	33
3.6	Perspective - Centre position determination	33
3.7	Accuracy of element used	34
4.	DATA PROCESSING AND RESULTS	35

		Page
4.1	Numerical determination of values of relative Orientation elements	35
4.1.1	Results of Numerical relative orientation	39
4.2	Perspective-Centre position determination	46
4.3	Strip formation adjustment	49
4.3.1	Input data	50
4.3.2	Output data	51
4.3.3	Result of Strip fornation and adjustment	52
5	ANALYSES, SUMMARY AND CONCLUSION	54
5.1	Analyses	54
5.1.1	Numerical relative Orientation	54
5.1.2	Strip formation	58
5.1.3	Inference from figures 5.3, 5.4, 5.5, 5.6, 5.7 and computer printout in Appendix VI	65
5.1.4	Explanation for the systematic error of the result	70
5.2	Summary and Conclusion	72
6	REFERENCES	75

LIST OF TABLES

		Page
3.1	Photography detail	23
3.2	Wild A-7 technical data	24
3.3	Selection of Autograph scale and gear ratio	28
5.1	Corrections to relative orientation elements, with their standard deviation and the Root mean square of the observation for the five cases in model 1	54
5.2	Corrections to relative orientation elements, with their standard deviation and Root mean square of the observation for the five cases in model 2	55

LIST OF FIGURES

		Page
2.1	The ground, the vertical photographs and the titled photographs coordinate system	5
2.2	The primary rotation	5
2.3	The Secondary rotation	5
2.4	The Tertiary rotation	5
2.5	Relationship between model and photo plane	10
2.6	Point P in perfect model position in left hand projector	11
2.7	The effect of the displacement of P' to P"	11
2.8	The effect on XY-plane of the displacement of p' to p"	11
2.9	Formation of strips from independent model	19
4.1	Disposition of the pass points for numerical relative orientation	39
4.2	Block formation	53
4.3	Case I Formation [Example Case II]	53
5.1	Accuracy of Numerical relative Orientation with density and distribution of point [model 1]	56
5.2	Accuracy of Numerical Relative Orientation with density and distribution of points [model 2]	57

		Page
5.3	Planimetric Accuracy Vs average r.m.s of Relative Orientation	60
5.4	Planimetric Accuracy Vs average r.m.s. of Perspective-Centre position determination	61
5.5	Height accuracy Vs r.m.s. of perspective-centre position determination	61
5.6	Height accuracy Vs r.m.s. of relative Orientation of composite models	63
5.7	Graphical representation of the frequency and magnitude of planimetric and height errors from the 25 cases	64

LIST OF APPENDICES

	Page
Numerical relative Orientation observation	I
Model Coordinate observation	II
Perspective-Centre determination observation	III
Strip formation program	IV
Input Computer printout	V
Output Computer printout	VI

ABBREVIATIONS, DEFINITIONS, GLOSSARIES AND SYMBOLS

A	Matrix coefficient of observation
B	Transpose of coefficient Matrix A
bx, by, bz	Component of the base along x-, y-, z- axes
f	Focal length of the camera
H	Flying height above terrain
H_{min}, H_{max}	Minimum and Maximum elevation above datum respectively
S	Scale factor
(x^v, y^v, z^v)	Coordinate system associated with vertical photo
(x^t, y^t, z^t)	Coordinate system associated with tilt photo
(x,y,z)	Coordinate system associated with object/ ground space.
X, X'	Coordinate before and after transformation respectively
T	(T_x, T_y, T_z) Translation parameters
P_Y	Y - parallax on the model space
P_y	y - parallax on the photograph plane
S_m^o	Optimum model scale
Z_o	Optimum Projection distance
M_1A	Model 1 case A
M_2A	Model 2 case A
$\Delta x, \Delta y, \Delta z$	Small increament in x-, y-, z- axes direction
$M_w M\phi, M_k$	Rotational matrices representing w, ϕ and k rotations respectively

X_o, Y_o, Z_o	Model Coordinates of perspective centre
w, ϕ, k	rotations around x, y, z axes.

1. INTRODUCTION

1.1 Definition and Scope of Aerotriangulation:

Aerotriangulation may be defined as the method of establishing supplementary horizontal and vertical controls through the geometric relationship of adjacent aerial photographs.[1] Supplementary controls that are so established are termed photogrammetric controls. They must be tied to ground controls to provide them with geodetic significance. The most desirable location of the ground controls is along the perimeter of the project area and it has been found that the higher the number of the ground controls the better the accuracy.[30]

Auxillary data from statoscopes, Airborne Profile recorder, Inertial reference systems, Horizon camera, Solar periscope etc, were used successfully to reduce the effects of error propagation on the results of aerotriangulation and to increase the distances that can be spanned between ground controls.[1]

Aerotriangulation can be classified into (3) three broad groups:

(i) Instrumental Aerotriangulation. It is instrumental process of establishing positions and elevations from visual spatial models. Radial triangulation belong to this group.

(ii) Analytical method. It is a computational method

(iii) Semi-analytical method - It is a blend of both Instrumental and analytical methods, involving more of the latter method.

The choice of any method must always be considered in the light of the requirements for horizontal and vertical controls, accuracy needed and photogrammetric equipment available.

Aerotriangulation yields satisfactory accuracy for both horizontal and vertical photogrammetric control extension at substantial low cost. Aerotriangulation is used extensively throughout the world in various fields. A few of which are topographical mapping, Cadastral Surverying. Highway engineering and Hydrographical Surveying.

The following steps should be strictly followed for Aerotriangulation:- Model formation, Measurement of coordinate of pass point on the instrument, strip formation and Block adjustment.

1.2 Aerotriangulation Practice in Nigeria

Around the middle of late 1960s, the interest in photogrammetry became strong in Nigeria. The regional centre for training Aerial Surveying at the University of Ife was established in 1972 by the United Nations Economic Commission for Africa for the purpose of training lower and middle level manpower (technician) in photogrammetry.

In 1965 Overseas Surveys (D.O.S.S.S) on behalf of the United Kingdom Government carried out the first large scale application of Aerotriangulation in the country. But up till now not much has been done in Aerotriangulation in the country. This might be due to lack of equipment and computer facilities.

1.3 Aim and Objective of the Research

It is intended in this research to investigate the following:-

(1) The effect of the density and distribution of standard points on numerical relative orientation.

(2) The effect of accuracy of numerical relative orientation of Aerotriangulation by independent model (AIM).

(3) The effect of accuracy of perspective centre determination on Aerotriangulation by independent model.

2.0 THEORETICAL CONSIDERATION

2.1 Y - Parallax equation

Parallax can be defined as an apparent movement of a stationary point in relation to another point when viewing position is change [15]

2.1.1 Derivation of the 3-D Conformal Coordinate Transformation Equation

The derivation of the Y - parallax equation is based on the appropriate 3-D Conformal coordinate transformation derived in this section.

Assumption:

The 3-D Conformal Coordinate transformation equation will be derived on the following bases.

(i) The coordinates (x^v, y^v, z^v), (x^t, y^t, z^t) and (X, Y, Z) are associated with the vertical photograph and tilted photograph plane and the ground/object space respectively.

(ii) All Coordinate systems are right-handed systems.

(iii) Each of the rotational angles (w, ϕ and k) is regarded as positive if anti-clockwise when viewed from the positive end of the respective axis.

(iv) The sequence of rotation is about the X-, the Y- and the Z-axes.

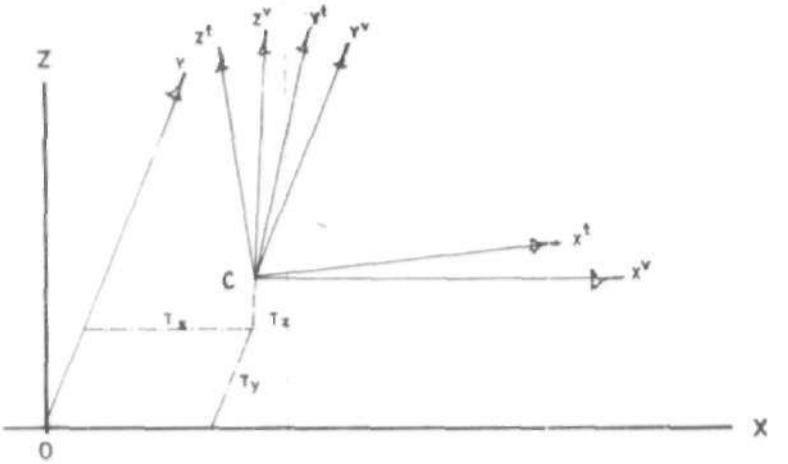


FIG: 2.1: THE GROUND THE VERTICAL-PHOTOGRAPH AND THE TILTED-PHOTOGRAPH COORDINATE SYSTEMS.

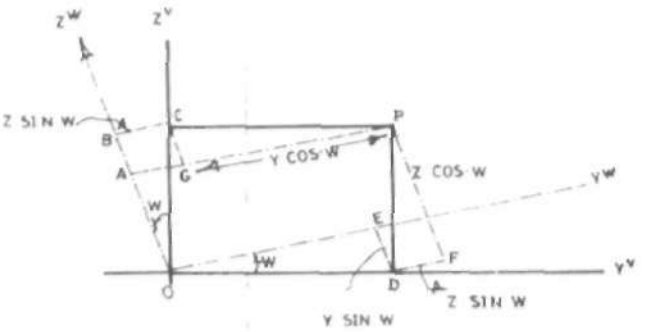


FIG: 2.2: THE PRIMARY ROTATION

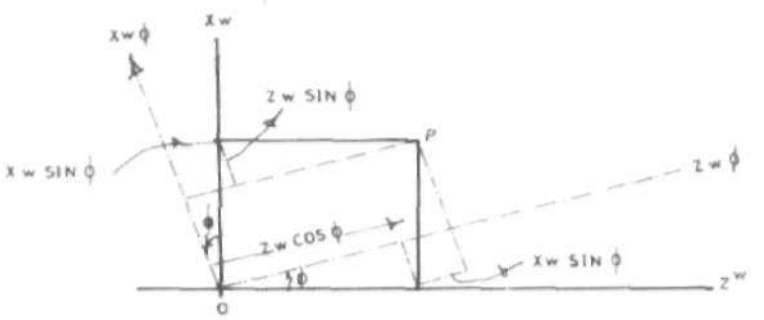


FIG: 2.3: THE SECONDARY ROTATION

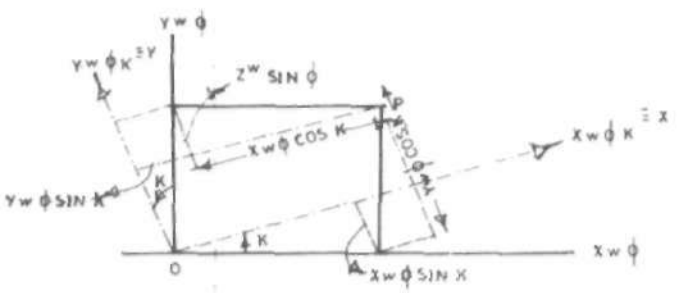


FIG: 2.4: THE TERTIARY ROTATION

(v) As shown in fig 2.2 (x^v, y^v, z^v) coordinate system is parallel to the X, Y, Z object space Coordinate System.

(vi) We assumed a hypothetical pair of photographs, one absolutely vertical and the other titled, taken from the same Camera. This enforces a common orgin for both the x^t, y^t, z^t and the x^v, y^v, z^v systems.

The general transformation equation is made up of seven independent parameters consisting of a scale factor, three translation and three rotational parameters. This can be written in matrix form.

$$X' = SM^T X + T \quad [1, 13, 15, 30] \quad .. \quad .. \quad 2.1$$

where $X' = (X', Y', Z)$, the coordinate after transformation

$X = (X, Y, Z)$, the coordinate before transformation

$T = (T_X, T_y, T_X)$, the translation parameters

S = Scale factor

M = Rotational Matrix

In the case at hand, $S = 1$ and $T_x = T_y = T_z = 0$ and the general 3-D conformal transformation equation reduces to

$$X' = M^T X \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad 2.2$$

This is consequent to the afore mentioned in assumption (VI).

The rotational matrix M =

$$\begin{bmatrix} \cos\phi\cos k & \sin w\sin\phi\cos k + \cos w\sin k & -\cos w\sin\phi\cos k + \sin w\sin k \\ -\cos\phi\sin k & -\sin w\sin\phi\sin k + \cos w\cos k & \cos w\sin\phi\sin k + \sin w\cos k \\ \sin\phi & -\sin w\cos\phi & \cos w\cos\phi \end{bmatrix}$$

|1,13,15|

and

$$M^T = \begin{bmatrix} \cos\phi\cos k & -\cos\phi\sin k & \sin\phi \\ \cos w\sin k + \sin w\sin\phi\cos k & \cos w\cos k - \sin w\sin\phi\sin k & -\sin w\cos\phi \\ \sin w\sin k - \cos w\sin\phi\cos k & \sin w\cos k + \cos w\sin\phi\sin k & \cos w\cos\phi \end{bmatrix}$$

In general $w = (w) + dw$, $\phi = (\phi) + d\phi$ and $K = (K) + dk$. Where dw, $d\phi$ and dk are corrections to the approximate values (w), (ϕ) and (K).

For near vertical photograph/rotational angles are very small and the initial approximation can be taken as zero. Hence $w=dw$, $\phi=d\phi$ and $k = dk$. It follows that $\sin w = \sin((w) + dw) = dw$, $\sin\phi = (\sin(\phi) + d\phi) = d\phi$ and $\sin k = \sin((k) + dk) = dk$. And $\cos w = \cos((w) + dw) = \cos dw = 1$, $\cos\phi + \cos((\phi) + d\phi) = \cos d\phi = 1$ and $\cos k = \cos((k) + dk) = \cos k = 1$.

Substituting these values into the rotational matrix . Then since the product of two very small values tend to zero .

$$M = \begin{bmatrix} 1 & dk & -d\phi \\ -dk & 1 & dw \\ d\phi & dw & 1 \end{bmatrix} \quad \text{and}$$

$$M^T = \begin{bmatrix} 1 & -dk & d\phi \\ dk & 1 & -dw \\ -d\phi & dw & 1 \end{bmatrix}$$

2.1.2 Derivation of Y-Parallax Equations

The analysis is based on a single model point p.

Small rotations dw', dφ' and dk' cause rotation of vector 0,p to new position 0.p' introducing both x-parallax and y-parallax.

The coordinate of point p can be transformed into coordinate of point p' by means of the already derived 3-D conformed transformation | 2.2| . Recall

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = M^T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -dk & d\phi \\ dk & 1 & -dw \\ d\phi & dw & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad .. \quad 2.7$$

This situation of may be simplified by moving p' in the direction of 0p' into a new position p" with coordinates (X", Y", Z") in the horizontal plane of p.

This relationship between p" to p' is

$$\begin{bmatrix} X" \\ Y" \\ Z" \end{bmatrix} = \frac{Z}{Z} \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} \quad .. \quad .. \quad .. \quad .. \quad 2.8$$

|15|

$$X" = \frac{Z}{Z}.X' \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad 2.9$$

$$Y'' = \frac{Z}{Z}, Y' \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad 2.10$$

$$Z'' = \frac{Z}{Z}, z' = Z \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad 2.11$$

But from equation (2.7)

$$X' = X - Ydk + Zd\phi \quad .. \quad .. \quad .. \quad .. \quad 2.12$$

$$Y' = Xdk + Y - Zdw \quad .. \quad .. \quad .. \quad .. \quad 2.13$$

$$Z' = -Xd\phi + Ydw + Zdk \quad .. \quad .. \quad .. \quad .. \quad 2.14$$

Substituting equations (2.13) and (2.14) into equation (2.10) we have

$$Y'' = \frac{Z(Xdk + Y - Zdw)}{-Xd\phi + Ydw + Z} \quad .. \quad .. \quad .. \quad 2.15$$

$$Y'' = Z(Xdk + Y-Zdw)/(1 - \frac{(Xd\phi - Ydw)}{Z})Z$$

By Taylor's Series

$$\frac{a}{1-q} = q + aq + \quad - - - \quad + - - - +$$

if $q < 1$ and since the expression $\frac{Xd\phi - Ydw}{Z} < 1$

Then $Y'' = (Xdk + Y - Zdw)(1 + \frac{(Xd\phi - Ydw)}{Z})$

Since the product of two very small values tend to zero. Then

$$Y'' = Zdk + Y - Zdw + \frac{YX}{Z} d\phi - \frac{Y^2}{Z} dw + \quad .. \quad .. \quad 2.16$$

$$Y'' - Y = Xdk + \frac{YX}{Z} d\phi - Z(1 + \frac{Y^2}{Z^2} dw \quad .. \quad .. \quad 2.17$$

In order to derive the translation elements of the complete differential formulae reference should be made to figure 2.5 above. Let us consider small

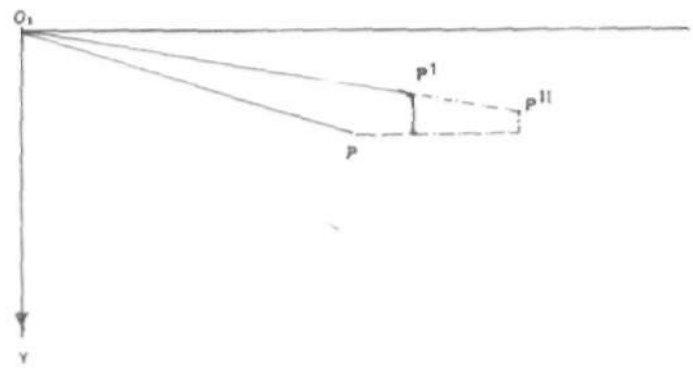


FIG-2·7B: SHOWS THE EFFECT OF THE DISPLACEMENT OF P^1 TO P^{11} ON XY-PLANE

translations dbx', dby' and dbz' all positive to the left projector and dbx", dby" and dbz" all positive to the right projector.

(i) The effect of dbx on the model

It affects the scale of the model and equally affects the X - parallax on the model

$$P_x(dbx') = dbx' \quad .. \quad .. \quad .. \quad .. \quad 2.18$$

$$P_x(dbx') = 0 \quad .. \quad .. \quad .. \quad .. \quad 2.19$$

Similarly

$$P_x(dbx") = dbx" \quad .. \quad .. \quad .. \quad .. \quad 2.20$$

$$P_y(dbx") = 0 \quad .. \quad .. \quad .. \quad .. \quad 2.21$$

The effect of dby on the model

It affects the Y - parallax equally on the model.

$$P_x(dby') = 0 \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad 2.22$$

$$P_y(dby') = dby' \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad 2.23$$

Similarly

$$P_x(dby") = 0 \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad 2.24$$

$$P_y(dby") = dby" \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad 2.25$$

The effect of dbz on the model

It affects the x - parallax and Y - parallax on the model.

$$P_x(dbz') = \frac{-x}{z} dbz' \quad .. \quad .. \quad .. \quad .. \quad 2.26$$

$$P_y(dbz') = Y/z\ dbz' \quad .. \quad .. \quad .. \quad .. \quad 2.27$$

Similarly

$$P_x(dbz") = \frac{-x}{z} dbz' \quad .. \quad .. \quad .. \quad .. \quad 2.28$$

$$P_y(dbz") = -Y/Z\ dbz" \quad .. \quad .. \quad .. \quad .. \quad 2.29$$

Therefore for left projector we have

$$Y_1" - Y = xdk' - \frac{xy}{z} d\phi' + z(1 + \frac{Y^2}{z^2})dw' - dby' + \frac{y}{z}dbz'$$

$$.. \quad .. \quad 2.30$$

Similarly for right projector

$$Y_2" - Y = (x - b)dk" - \frac{(x-b)}{z} Yd\phi" + z(1 + \frac{Y^2}{z^2})dw" -$$

$$dby" + \frac{Y}{z}dbz" \quad .. \quad .. \quad .. \quad 2.31$$

If we assume Y - parallax viz p_y to be defined as

$Y_1" - Y_2"$

Then

$$P_Y = Y_1" - Y_2" = xdk' - (x-b)dk" - \frac{xy}{z} d\phi + \frac{(x-b)Y}{z} d\phi$$

$$+ z(1 + \frac{Y^2}{z^2})dw' - z(1 + \frac{Y^2}{z^2})dw" - dby'$$

$$+ dby" + \frac{Y}{z} dbz' - \frac{Y}{z} dbz" \quad .. \quad 2.32$$

This is the general Y - parallax equation on the model plane of all right-handed plotters like the Wild A7 Autograph.

2.2 Model Formation

This is a process by which an optical model is brought into being. This can be divided into interior and exterior orientation. The latter can be further subdivided into relative and absolute orientations.

2.2.1 Interior Orientation

The purpose of interior orientation is to reconstruct cone of rays that is geometrically identical to the cone of rays that entered the camera to make the original exposure[1]. The parameters that define the actual relative relationships are called the elements of interior orientation, which normally include (a) location of principal point with respect to the fiducial marks (b) principal distance, and (c) all the distortion parameters.

2.2.2 Relative Orientation

This step is performed immediately after interior orientation determination. It is a procedure use to reconstruct photographed object inform of a three dimensional optical model similar to the photographed object[1]. In a stereo-pair this is achieved when all pairs of conjugate rays are Coplanar i.e all Y - parallaxes are elliminated.

Relative orientation method can be classified into three major groups.

(1) Empirical methods - All operations are done on

the instrument [1,9].

(2) Graphical methods - As the name implies, simple diagrams base on parallax observation are used[9].

(3) Numerical methods - Considering a vertical photograph, numerical method of orientation can be classified into two major groups (I) For flat terrain and (2) For mountainous terrain[9]

There are various methods under these groups The choice of any method depends on the type of instrument.

The numerical method of relative orientation should be done in a systematic way and the residual parallaxes are distributed over the model according to least square principle. Y-parallaxes should be elliminated in at least five symmetrically located positions in order to complete relative orientation. The dependent(1,9,15) and Independent(1,9,15) of relative orientation are general to all the three methods enumerated above.

(i) Dependent Method

All motions are applied to the right projector if it contains the new photograph. For example translation and orientation elements by', bz", k", ϕ" and W" are the used out of many combinations.

The parallax equation (2.32) derived above i

modified to suit this purpose. Thus

$$P_Y = -(x-b)dk'' + (\frac{x-b}{z})Yd\phi'' - z(1 + \frac{Y^2}{Z^2})dw'' + dbY'' + \frac{Y}{Z}dbz''$$

.. .. 2.33

It must be noted that in case of "base in" right hand camera is our projector 2. In case of "base out" left hand camera is our projector 2. The elements by" or by' is used to measure Y-parallax in all the standard points. Least squares principle is used to solve for all the corrections. The projectors are then set according to the results.

(ii) Independent Orientation

This is also refered to as swing-swing method.[1] All motions are applied to both projectors. For example orientation elements k', k", ϕ', ϕ" w" are used out of many possible combinations. In this case both bY" and bY' are used as parallax measuring device to guide against gross errors. For this purpose the derived parallax equation is modified as

$$P_Y = Xdk' - (x-b)dk'' - \frac{xY}{z}d\phi' + \frac{(x-b)}{z}Yd\phi + Z(I + \frac{Y^2}{z^2})dw'$$

.. .. 2.34

Least squares adjustment can be used to solve for the corrections to the orientation elements.

2.3 Aerotriangulation

It is defined as the process of extension of horizontal and vertical control whereby the measurement of angles and distances on overlapping photographs are related into spatial solution using perspective principles of the photographs.[1] It is an economical way of densifying controls. There are three types of Aerotriangulation methods.

(1) Analogue Aerotriangulation[30]

All operations are done on the instrument hence it is also called mechanical Aerotriangulation.[1]

(2) Analytical Aerotriangulation[1,12,30]

The main input are the measurement made on the photograph, camera constants, and some ground control points coordinate,

It is a computational method.

(3) Semi-analytical Aerotriangulation[1,12,30]

It is an intermediate method between instrumental and analytical aerotriangulation.

2.4 Perspective-Centre Position Determination

The determination of perspective-centre (pc) of each projection camera is very important in aerotriangulation by independent model in order to provide the geometrical strength required for the successive numerical connections of independent models of a strip or block. Methods of pc position determination can be classified into mechanical and computational methods.

Computation methods can be subdivided according to the principles on which they are based:-

(1) Three-Dimensional space resection methods using grid plates [3.30].

(2) Collinearity principles using general image points [2].

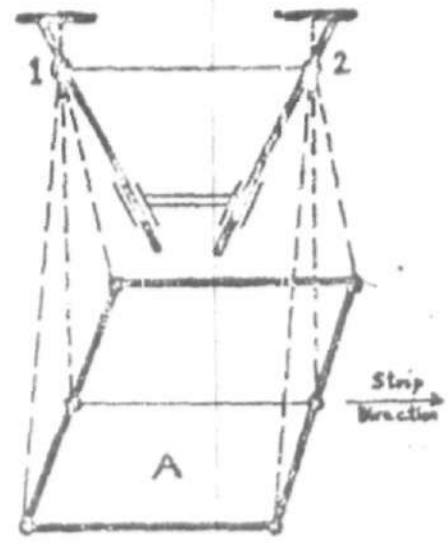
(3) Affine transformation method [3].

Bammeke (1985) proposed a simple method, based on the space resection principle, of determination of pc position in its rational 4-D Euclidian space which yields:- pc coordinate x_o, y_o, z_o and T_o correct to within 7.24 $\pm$ 1.83μm, 6.59 $\pm$ 1.51μm, 22.58 $\pm$ 5.89μm and $\pm$ 0.00003 respectively. This is the most current development in perspective - centre determination.

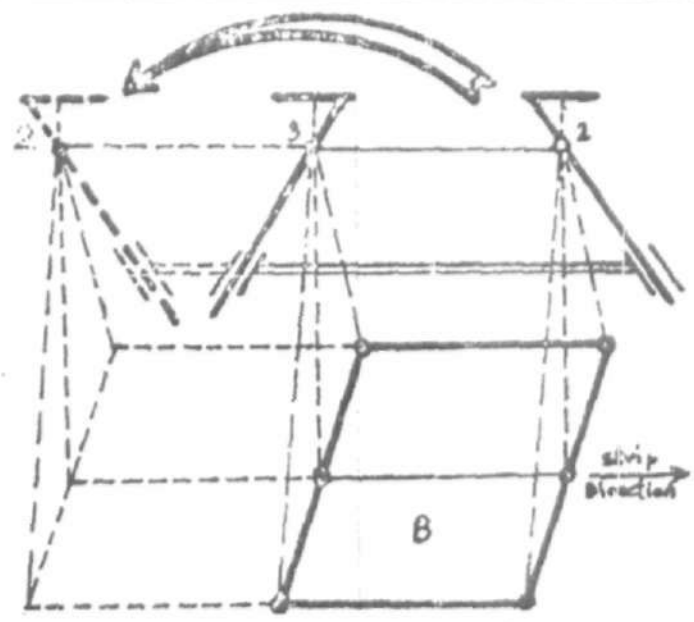
2.5 Formation of Strips From Independent Models

Figure 2.9 show diagramatically the process of strip formation from independent models. The initial formation and transformation of strips is very necessary for the detection of errors in ground control points and the points.

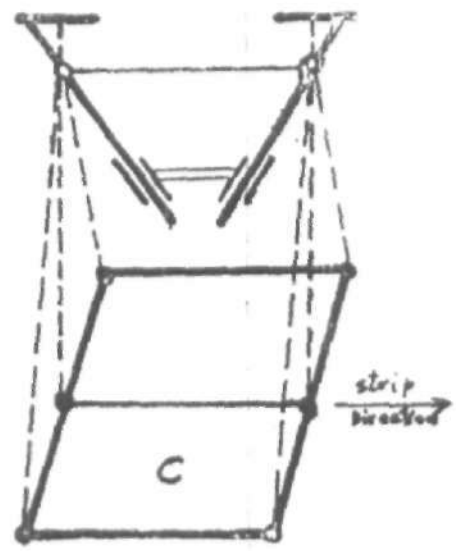
Strip formation procedure involves (i) a transtation which make the coordinate of common projection centre the same as those in the preceeding model (ii) a rotation which make the orientation of the vectors from common projection centre to the common points in the two models the same, (iii) and a scaling that make the length of the vectors in the two models the same.



Model A is formed with the instrument base inwards.



Model B is formed with the base outwards, triangulation lever and optical switch reversed. The net result is that projector 2 appears moved to the starboard position on the left



Model C is then formed with the base inwards triangulation lever and optical switch set at their normal positions.

This alternation of base-in and base-out is continued until all the models are formed into a strip

FIG 2.9
INSTRUMENTAL STRIP FORMATION ON THE A-7

The coefficient of transformation formulas (2.35) are functions of three independent parameters. The 3-parameters can be chosen in different ways.

Derivation of Transformation Fomulae

Let x, y, z be rectangular coordinate system in each of the two consective models which has the common projection centre as the origin. Let variable t be associated with the coordinates x, y, z and also let

$$D = \begin{bmatrix} d & -a & -b & -c \\ a & d & -c & b \\ b & c & d & -a \\ c & -d & a & d \end{bmatrix} \quad \text{and} \quad T = \begin{bmatrix} t & -x & -y & -z \\ x & t & -z & y \\ y & z & t & -x \\ z & -y & x & t \end{bmatrix} \qquad (23)$$

where D is the transformation matrix.

Any matrix constructed in this way has the following properties.

(i) It's column vector and row vectors are mutually orthogonal and have the same length.

(ii) It's determinant is equal to the fourth power of the length of these vectors.

(iii) The matrix is non-singular unless the parameters are equal to zero.

The transformation formula T' becomes

$$T' = DT \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad 2.35$$

This give

$$x' = at + dx - cy + bz \quad .. \quad .. \quad .. \quad .. \quad .. \quad 2.36$$

$$y' = bt + cx + dy - az \quad .. \quad .. \quad .. \quad .. \quad .. \quad 2.37$$

$$z' = ct - bx + ay + dz \quad .. \quad .. \quad .. \quad .. \quad .. \quad 2.38$$

$$t' = dt - ax - by - cz \quad .. \quad .. \quad .. \quad .. \quad .. \quad 2.39$$

Generally strip formed becomes deformed due to accumulation of systematic and accidental errors. Strip can undergo horizontal and vertical deformations [1]. Horizontal deformation includes bowing and scale deformation. Horizontal bowing is caused by a systematic kappa (k) error and results in a parabolic warping of the strip in the Y-direction, scale deformation is due to bx and bz errors and gives differential scale throughout the strip. Vertical deformations in strips are of two types; tortional and bowing.[1,18] Tortional deformation or Cross tilt results from systematic omega (w) errors and caused the strip to be twisted about the (x-axis). Vertical bowing caused the strip datum to warp up or down and is caused by systematic phi(ϕ) errors.

2.6 Block Adjustment

This is done after strip adjustment. The purpose of this adjustment is to transfer triangulated strips to ground coordinate system and to remove systematic deformation. This adjustment is an iterative process in which seven parameters are solved for. This is not done in this project as only models in a strip are involved.

3. MATERIALS/EQUIPMENT AND METHOD

3.1 Materials/Equipment

All the materials and equipment used for this project were provided by the department of surveying of the Ahmadu Bello University, Zaria. The only exception is the main frame Computer which is housed in University's computer centre.

3.1.1 Photograph

The photographs used for this research are parts of a strip block of photographs used for the purpose of mapping Akwanga area of Plateau State. The other details pertaining to the photography are as given below.

Table 3.1 Photography detail

S/NO	DETAILS	DATA	SOURCE OF INFORMATION
1.	Camera	Wild 3½"	Contact (paper) prints
2.	Format (1x1) cm^2	23 x 23 cm	- do -
3.	Focal length (fmm)	152.676 mm	- do -
4.	Flight Direction	East-West	- do -
5.	General for and aft overlap	b/w76% and 81%	Computed from overlays of paper print
6.	General lateral overlap	b/w34% sp 39%	
7.	General Photoscale (sp)	1·8000	Contact (paper) prints
8.	General average photo base (bx_o)	91 mm	Computed from paper print
9.	Date of photograph	6-6-77	Paper print
10.	Height of Highest ground control point in the block hmax	487.920m	Control coordinate list
11.	Height of lowest ground control point in the block hmin, AMSL	428.24M	- do -
12.	Average flight altitude	1221.408M	- do -
13.	Desired mapping scale	1:2,500	given

3.1.2 Wild A 7 Autograph

The departmental Wild Autograph A-7 serial number 3268 is the principal instrument used for this research.

It is a universal equipment which employs mechanical projection. Incorporated in the design is the Zeiss parallelogram device which enable the instrument to be set in the "base-in" and "base-out" modes. An optical switching lever and a parallax reversing device enable the left and right images tc be interchanged when the instrument is used in the base out mode, thus ensuring correct stereoscopic observation.

Technical data of Wild A-7 is given below.

Table 3.2 Wild A-7 Technical Data

Focal length picture size	98 to 215 mm up to 230 x 230mm (9" x 9")
Swing	0 to 400^g
Lateral tilt w', w"	70 to 109^g
Longitudinal tilt ϕ'	70 to 106^g
Longitudinal tilt ϕ"	94 to 130^g
Base components: bx	+ 280 to -280 mm
Base components: by', by"	+ 50 to -50 mm
Base Component bz', bz"	+ 27 to -27 mm
x	+ 280 to -280 mm
y	+ 350 to -420 mm
z	140 to -490 mm

3.1.3 Computer Facilities

Two types of computers were used.

1. Hewlett-Packard 9830A Computer Model 30: This was used to compute the corrections to initially assumed relative orientation elements [dk', dk", dφ', dφ" and dw'], the variance-covariance matrix, the root mean square (r.m.s) values and the respective standard deviations of each of the determined correction.

(ii) Perspective-centre coordinates of the Projection Cameras, the variance-covariance matrix, the r.m.s. values and thus standard deviation of each of the parameters determined.

2. Main Frame Computer: This is the C.D. Cyber 72 - 13 installation of University's computer centre. This was used for the strip formation.

3.2 Methods

3.2.1 Recording Forms

There are three types of forms used in this work. They are :-

1. Form for numerical relative orientation
2. Form for recording model coordinates
3. Form for perspective-centre position determination.

3.2.2 Preliminary Data

This was obtained from the aerial photograph, the Camera specification and ground control (planimetry and height) Coordinate of the project area. See

Section 3.1.1.

3.2.3 Preliminary Settings

3.2.3.1 Determination/Selection of the Required Instrument Settings

The optimum model scale (S_m^o) was determined using equation [3.1].

$$S_m^o = \frac{Z_o}{f} Sp \quad .. \quad .. \quad .. \quad .. \quad 3.1$$

where Z_o is the optimum projection distance which is equal to 490 mm

$$S_m^o = \frac{490}{152.676} \times \frac{1}{8000}$$

$$= 1/2496.669$$

3.2.3.2 Selection of The Favourable Model Scale

To arrive at the most favourable model scale S^o the following criteria were considered.

(i) Type of photography with respect to the angular field of view.

(ii) Maximum (H'max) and Minimum (H'min) relative flight altitudes.

(iii) Availability of horizontal gear combination for the desired mapping scale.

$$\tan \theta/2 = -\frac{1}{2} \sqrt{\frac{21^2}{f}} \quad .. \quad .. \quad .. \quad 3.2$$

$$= \frac{1\sqrt{2}}{2f}$$

$$\theta = 2\tan^{-1}\left(\frac{1\sqrt{2}}{2f}\right)$$

$$= 2\tan^{-1}\left(\frac{230\sqrt{2}}{2x152.676}\right) = 93^{o}.618$$

Since wide angle photographs have the range of $75^{o} \leq \theta \leq 100^{o}$, the photographs used for this study are wide angle.

$H'_{min} = H'-h_{max}$

$H'_{max} = H'-h_{min}$

Where H_{max} and H'_{min} are the maximum and minimum flying height above the terrain where H' is the average flying height above datum.

$h = \frac{1}{2}(h_{min} + h_{max})$

$= 459.080M$

$\Delta h = h_{max} - h = 28.840M$

$\Delta h = h_{min} - h = 28.840M$

$H'_{max} = 1221.408 + 28.840 = 1250.248M$

$H'_{min} = 1221.408 - 28.840 = 1192.568M$

$1192.568 \leq$ Relation flight altitude $\leq 1250.248M$

$1192.568M \leq H' \leq 1250.248M$

The computation above was done because it was not available from the flight information.

3. The selection of gear combination was done according to table 1 in operation manual of wild A-7.[29]

Table 3.3 Selection of Autograph scales and gear ratio

Normal Angle	Wide Angle				Combination	Smallest Interval	Autograph Scale	Desired Mapping Scale 1:2,500
	METRE		FEET					
	H'max	H'min	H'max	H'min				
	1200	600			Bd	0.01	3000	
	1330	670			Dc	0.1	3333	3-1-1
	1500	750			Cd	0.01	3750	1-2-1

Adopted from Wild -7 Zeiss Autograph Operation table 1.

Autograph (model) scale 1:3750 was selected because it is the closest to S^{o} for wide angle photograph whose flight altitude range from 1192m to 1251m.

3.2.3.3 Selection of Gear Ratios

There are two types of gear available on Wild A-7. They are horizontal and vertical gears.

Horizontal Gear:- Combination 1-2-1 was selected by looking up row (Autograph scale 1:3750) and column (mapping scale 1:2,500) in the table 3.3.[29]

Vertical Gear:- Similarly combination Cd with the value of the smallest/last division of the height counter of 0.01m, was selected, (see table 3.3).

3.2.3.4 Determination of the Projection Height (Z)

The projection height (Z) was determined using equation (3.3).

$$= f \cdot \frac{S_m}{S_p} \qquad .. \quad .. \quad .. \quad .. \qquad 3.3$$

where S_m is the autograph (model) scale

$\frac{S_m}{S_p}$ is the enlargement ratio.

= 152.676 x 1/3750 1/8.000

= 152.676 x 8000/3750

= 325.709 mm

Z = 325.709 mm

3.2.3.5 Determination of Model Base (x)

In this study the model base was determined empirically. This was done by clearing X-parallax of a corresponding point at point 1 and then at point 2 by Bx.

Bx at model point 1 = 195.312 mm

Bx at model point 2 = 194.670 mm

Mean = 194.991 mm

3.2.3.6 Setting of the Optical Mechanical Fittings

	NEGATIVE		POSITIVE	
	Base Inside	Base Outside	Base Inside	Base Outside
Dove prisms: indices	Inside	Outside	Outside	Inside
Triangulation lever:	back	front	back	front
Diapositive lever:	right	right	left	left
Sense of handweel rotation				
X-Collar	in	in	in	in
Y-Collar	in	in	out	out

3.3 Precautions

In order to guide against gross-errors the following precautions were taken :-

1. The orientation elements setting and the instrument fittings were always noted each time the instrument is left; and rechecked each time observation is to be done on the instrument
2. The model coordinates of pass points were read on two pointings.
3. The bY' and bY" translation elements were used for parallax measurements and on each point three readings of both bY' and bY" were taken.
4. The quick release mechanism for both x and Y motions were tape in such a way that they could not be mistakenly released.

5. Mis - identification of pass points, were avoided by means of sketches and description of the points.
6. Points as nearly as possible along the Y - coordinate axis of each photograph, were used as pass points.

3.4 Model Formation

3.4.1 Interior Orientation

The following procedure were carried out.

(1) The side of the photographs that contain the emulsion was determined. This was done by scratching both sides of the photographs with a pin. The side of the photograph that could be scratched by the pin is the emulsion side.

(2) The overlap area of the stereopair was then identified.

(3) The photographs fiducial marks were registered against the collimating marks on the photo carrier with the emulsion surface facing downward. The final centering was done using a magnifier on a light table.

(4) The principal distance of the taking Camera was then set on the focal length column. This was done once for both left and right cameras in this study.

Step 1, 2, 3 were repeated for the third photograph (or subsequent photographs).

3.4.2 Relative Orientation

The procedure is as follows :-

Step 1: The parallaxes PY_1, PY_2 - - - PY_{15} depending on the case considered were measured by by' and by" in turn (when for example by' was used by" was set at it's zero value and vise visa). Both by' and by" were used at each point to guide against blunders. Three sets of reading were taken.

Step 2: The parallax at a point is the difference between the mean of by' and by" at that point and the mean of by' and by" of all points observed on the model.

Step 3: The model coordinates of the points were also recorded.

Step 4: The corrections dk', dk", dϕ' and dϕ" and dw' were computed using equation (2.35) and final values (i.e. initial plus correction) were set on the instrument.

Step 5: Steps 1,2,3 and 4 were repeated until the residual parallaxes were cheared.

Step 6: When there exist very small residual parallaxes, steps 1,2,3 and 4 were repeated and the mean of by' and by" for all the points was set on by'.

For each model five cases of numerical relative relative orientation were considered. The cases are as follows: case A involves the use of 6 standard

points, cases B, C and D involves the use of 9 points and case E involves the use of 15 points. See section 4.1.1 for the distribution of the points.

Appendix I contains the observation and the result for ease case.

3.5 Measurement of Model Coordinates

After relative orientation the coordinate of all the model points required were read. These points include transfer points and control points. Two pointings were made and the difference between the two readings did not exceed 0.01 mm. The co-ordinates of all points were measured by visual inspection of the drums and recorded manually of forms prepared for the purpose.

3.6 Perspective-Centre position determination

The principle on which the determination is based was discussed in section 2.5. This was the last operation done on the instrument. Perspective-Centre position determination was done for all the cases because the translation elements by' varies from one case to the other as it was used for numerical relative orientation.

The procedure for the determination is as follows:-

Step 1: For each case the translation and rotational elements as recorded for numerical relative orientation was set on the instrument.

Step 2: Z-Counter was set at 4900 mm

Step 3: The x, y - coordinates of the six standard points in both left and right camera were obserbed monocularly are recoreded .

Step 4: The Z-Counter was set at 5100 mm

STep 5: Step 3 was repeated.

The appendix III contains the observation.

3.7 Accuracy of Elements Use

(i) All translation movement can be read directly to an accuracy of ± 0.01 mm

(ii) All angular movement can be read directly to an accuracy of ± 0.01 grads

(iii) Focal length can be set correctly within ± 0.01 mm.

4. DATA PROCESSING AND RESULTS

4.1 Numerical Determination of Values of Reltive orientation Elements

Recall equation (2,3,4)

$$PY = xdk' - (x-b)dk'' - \frac{xY}{z}d\phi' + \frac{(x-b)}{z}Yd\phi'' + Z(1 + \frac{Y^2}{Z^2})dw'$$

Let us consider that paralax observations were at i standard location in the model. The parallax equation becomes

$$PY_i = x_i\, dk' - (x_i - b)dk'' - \frac{x_i Y_i}{z}d\phi' + (\frac{x_i - b}{z})Y_i\, d\phi'' + Z(1 + \frac{Y_i}{Z_i^2})dw'$$

Each standard location in the model generates an observation equation. That is for points 1 to i we have

$$PY_1 = x_1 dk'(x_1-b)dk'' - \frac{x_1 Y_1}{z}d\phi' + (\frac{x_1-b}{z})Y_1 d\phi'' + Z(1 + \frac{Y_1^2}{z^2})dw'$$

$$PY_2 = x_2 dk' - (x_2-b)dk'' - \frac{x_2 Y_2}{z}d\phi + (\frac{x_2-b}{z})Y_2 d\phi'' + Z(1 + \frac{Y_i^2}{Z^2})dw$$

$$PY_i = x_i dk' - (x_i-b)dk'' - \frac{x_i Y_i}{z}d\phi' + (\frac{x_i-b}{z})Y_i d\phi'' + Z(1 + \frac{Y_i^2}{z^2})dw'$$

This can be expressed in matrix notation

$AX = b$ or $b = AX$

$$\begin{bmatrix} py_1 \\ py_2 \\ \vdots \\ py_i \end{bmatrix} = \begin{bmatrix} x_1 - (x_1-b) - (\frac{x_1 Y_1}{z}) & \frac{x_1-b}{z} Y_1 & Z(1+\frac{Y_1^2}{z^2}) \\ x_2 - (x_2-b) - (\frac{x_2 Y_2}{z}) & (x_2-b/z)y_2 & Z(1+\frac{Y_2^2}{z^2}) \\ \text{- - - - - - - - - - - - - - - - - - - -} & & \\ x_i - (x_i-b) - (x_i Y_i)/z & (x_i-b/z)Y_i & Z(1+\frac{Y_i^2}{z^2}) \end{bmatrix} \begin{bmatrix} dk' \\ dk'' \\ d\phi' \\ d\phi'' \\ dw' \end{bmatrix}$$

This equation lends itself to least squares adjustment.

A least squares program, listed below was written to compute the most probable value (MPV) of relative orientation corrections, the root mean square and the standard deviation on the departmental Hewlett-Packard 98 30A computer model 30.

The programs are as follows:-

```
10   Dim A[6,5], B[5,6], C[5,5], N[5,5], P[ 6 ], X[ 5 ],
         R[ 5 ], T[ 6 ], V[ 6 ], S[5,5], Q[ 5 ].
20   For I = 1 to 6
30   For J = 1 to 5
40   For A(I,J)
50   Next J
```

```
 60   Next I
 70   Data
 71   :
 :
 :    :
 75   Data
 90   For I = 1 to 6
100   Read P(I)
110   Next I
120   Data
130   Mat B = TRN(A)
140   Mat C = B*A
150   Mat N = INV(C)
160   Mat R = B*b
170   Mat X = N*R
180   H     = 200/π
190   Mat Q = (H)*X
200   Mat T = A*X
210   Mat V = T - b
220   I = (V(1)↑ 2+V(2)↑ 2+V(3)↑ 2+V(4)↑2+V(5)↑ 2+V(6)↑ 2)
230   L     = I ↑ 0.5
240   Mat S = [I]*N
250   END
```

A is the matrix of the coefficient

X is the matrix of the corrections

V is the matrix of residuals

L is the root mean square

S is the variance-covariance matrix

The number of observation equation generated for each case are as follows: 6 for case A, 9 for case B, C and D, 15 for case E.

4.1.1 Result of Numerical Relative Orientation

Five case were studied. The cases are illustrated with the aid of diagrams in figure 4.1

3+		+4
1+		+2
5+		+6

Case A

3+	a+	+4
1+	b+	+2
5+	c+	+6

Case B

3+	g+	+4
1+	h+	+2
5+	i+	+6

Case C

3+	d+	+4
1+	e+	+2
5+	f+	+6

Case D

3+	a+	d+	g+	+4
1+	b+	e+	h+	+2
5+	c+	f+	i+	+6

Case E

Figure 4.1 Disposition of the standard points for numerical relative orientation for the five cases

The results are as follows below:

Model 1 Case A

First Iteration

$dk' = -0.02646455 \pm 2.14877 \times 10^{-5}$

$dk'' = -0.1938288898 \pm 2.11881 \times 10^{-5}$

$d\phi' = -1.16603 \times 10^{-3} \pm 5.76752 \times 10^{-10}$

$d\phi'' = -5.83688 \times 10^{-5} \pm 4.48842 \times 10^{-10}$

$dw' = 9.45219 \times 10^{-4} \pm 6.737709 \times 10^{-10}$

R.M.S = 0.890512454

Second Iteration

$dk' = -0.039638023 \pm 5.98754 \times 10^{-6}$

$dk'' = 0.196738580 \pm 5.90404 \times 10^{-6}$

$d\phi' = 5.16491 \times 10^{-4} \pm 1.60712 \times 10^{-10}$

$d\phi'' = 5.08097 \times 10^{-4} \pm 1.25069 \times 10^{-10}$

$dw' = -7.14367 \times 10^{-4} \pm 1.87728 \times 10^{-10}$

R.M.S = 0.470076926

Third Iteration

$dk' = -0.211563266 \pm 9.19829 \times 10^{-8}$

$dk'' = 0.043115429 \pm 9.07001 \times 10^{-8}$

$d\phi' = 5.43579 \times 10^{-4} \pm 2.46891 \times 10^{-12}$

$d\phi'' = -6.68790 \times 10^{-4} \pm 1.92136 \times 10^{-12}$

$dw' = 5.75883 \times 10^{-4} \pm 2.88396 \times 10^{-12}$

R.M.S = 0.08826371

Fourth Iteration

$dk' = -1.24478 \times 10^{-3} \pm 1.45093 \times 10^{-7}$

$dk'' = -0.021024933 \pm 1.43069 \times 10^{-7}$

$d\phi' = -2.27472 \times 10^{-4} \pm 3.89444 \times 10^{-12}$

$d\phi'' = -2.37547 \times 10^{-4} \pm 3.03074 \times 10^{-12}$

R.M.S = 0.073175845

Model 1 Case B

First Iteration

$dk' = 0.277256381 \pm 5.03656 \times 10^{-6}$

$dk'' = -0.568831248 \pm 6.06671 \times 10^{-6}$

$d\phi' = -3.69794 \times 10^{-4} \pm 1.4751 \times 10^{-10}$

$d\phi'' = -4.69066 \times 10^{-4} \pm 1.51219 \times 10^{-10}$

$dw' = 9.84906 \times 10^{-4} \pm 1.68249 \times 10^{-10}$

R.M.S = 0.098156124

Second Iteration

$dk' = -0.211008464 \pm 2.4887 \times 10^{-8}$

$dk'' = -0.045759084 \pm 2.70884 \times 10^{-8}$

$d\phi' = 5.31752 \times 10^{-4} \pm 6.58650 \times 10^{-13}$

$d\phi'' = -6.73022 \times 10^{-4} \pm 6.75205 \times 10^{-13}$

$dw' = -5.45142 \times 10^{-4} \pm 7.51247 \times 10^{-13}$

R.M.S = 0.035788942

Model 1 Case C

First Iteration

$dk' = -0.2610042 \pm 1.3847 \times 10^{-8}$

$dk'' = 0.052734035 \pm 2.50814 \times 10^{-8}$

$d\phi' = 6.21701 \times 10^{-4} \pm 7.07640 \times 10^{-13}$

$d\phi'' = -6.02111 \times 10^{-4} \pm 8.701 \times 10^{-13}$

$dw' = 4.45042 \times 10^{-4} \pm 6.01231 \times 10^{-13}$

R.M.S = 0.034619345

Model 1 Case D

$dk' = -0.021156883 \pm 3.11776 \times 10^{-8}$

$dk'' = 0.043900597 \pm 2.69680 \times 10^{-8}$

$d\phi' = 5.48568 \times 10^{-4} \pm 9.145445 \times 10^{-13}$

$d\phi'' = -6.62721 \times 10^{-4} \pm 6.54047 \times 10^{-13}$

$dw' = 6.00843 \times 10^{-4} \pm 8.60359 \times 10^{-13}$

R.M.S = 0.038251713

Model 1 Case E

First Iteration

$dk' = -0.201761201 \pm 2.15881 \times 10^{-7}$

$dk'' = 0.026604175 \pm 1.91572 \times 10^{-7}$

$d\phi' = 5.08425 \times 10^{-4} \pm 6.84928 \times 10^{-12}$

$d\phi'' = -6.60173 \times 10^{-4} \pm 5.36477 \times 10^{-4}$

$dw' = 5.18403 \times 10^{-4} \pm 5.71704 \times 10^{-12}$

R.M.S = 0.120997036

Second Iteration

$dk' = 0.026675203 \pm 8.36321 \times 10^{-8}$

$dk'' = 0.024333017 \pm 7.42145 \times 10^{-8}$

$d\phi = -1.03181 \times 10^{-4} \pm 2.65340 \times 10^{-12}$

$d\phi'' = -1.22298 \times 10^{-4} \pm 2.07830 \times 10^{-12}$

$dw' = -2.28994 \times 10^{-4} \pm 2.21477 \times 10^{-12}$

R.M.S = 0.075310197

Model 2 Case A

First Iteration

$dk' = 0.018705213 \pm 2.65301 \times 10^{-8}$

$dk'' = -1.00608 \times 10^{-3} \pm 2.52665 \times 10^{-8}$

$d\phi' = -5.06164 \times 10^{-6} \pm 6.68396 \times 10^{-13}$

$d\phi'' = -2.30191 \times 10^{-5} \pm 6.95066 \times 10^{-13}$

$dw' = -6.34897 \times 10^{-5} \pm 1.00332 \times 10^{-12}$

R.M.S. = 0.033343076

Model 2 Case B

First Iteration

$dk' = 8.80960 \times 10^{-3} \pm 1.96874 \times 10^{-8}$

$dk'' = -8.96870 \times 10^{-3} \pm 2.25608 \times 10^{-8}$

$d\phi' = -8.0394 \times 10^{-6} \pm 5.29015 \times 10^{-13}$

$d\phi'' = 1.72909 \times 10^{-5} \pm 6.98526 \times 10^{-13}$

$dw' = 3.90369 \times 10^{-6} \pm 7.10046 \times 10^{-13}$

R.M.S 0.24983354

Model 2 Case C

$$dk' = 0.019351146 \pm 6.91284 \times 10^{-9}$$

$$dk'' = 1.44406 \times 10^{-4} \pm 5.25463 \times 10^{-9}$$

$$d\phi' = -9.04892 \times 10^{-6} \pm 2.05922 \times 10^{-13}$$

$$d\phi'' = 7.57142 \times 10^{-6} \pm 1.40118 \times 10^{-13}$$

$$dw' = -6.81429 \times 10^{-5} \pm 2.14633 \times 10^{-13}$$

R.M.S = 0.018806060

Model 2 Case D

$$dk' = 0.016249187 \pm 8.79733 \times 10^{-9}$$

$$dk'' = -3.52761 \times 10^{-3} \pm 8.18560 \times 10^{-9}$$

$$d\phi' = 8.41155 \times 10^{-6} \pm 2.46262 \times 10^{-13}$$

$$d\phi'' = 3.81237 \times 10^{-5} \pm 2.50376 \times 10^{-13}$$

$$dw' = -5.11245 \times 10^{-5} \pm 2.95375 \times 10^{-13}$$

R.M.S = 0.022028924

Model 2 Case E

First Iteration

$$dk' = 0.057087559 \pm 3.94897 \times 10^{-7}$$

$$dk'' = 0.121141610 \pm 3.65553 \times 10^{-7}$$

$$d\phi' = -1.25238 \times 10^{-5} \pm 1.1532 \times 10^{-11}$$

$$d\phi'' = -6.15824 \times 10^{-4} \pm 1.22402 \times 10^{-11}$$

$$dw' = 1.61715 \times 10^{-4} \pm 1.01642 \times 10^{-11}$$

R.M.S = 0.180695166

Second Iteration

$dk' = -5.00644 \times 10^{-3} \pm 4.29051 \times 10^{-7}$

$dk'' = 0.128471936 \pm 3.97170 \times 10^{-7}$

$d\phi' = 2.16265 \times 10^{-4} \pm 1.25350 \times 10^{-11}$

$d\phi'' = -6.33149 \times 10^{-4} \pm 1.32989 \times 10^{-11}$

$dw' = 4.24710 \times 10^{-4} \pm 1.10433 \times 10^{-11}$

R.M.S = 0.188347286

Third Iteration

$dk' = 0.047937325 \pm 1.09507 \times 10^{-6}$

$dk'' = -0.0247930144 \pm 9.82106 \times 10^{-7}$

$d\phi' = 2.416223 \times 10^{-3} \pm 3.49135 \times 10^{-11}$

$d\phi'' = -9.66021 \times 10^{-4} \pm 3.13458 \times 10^{-11}$

$dw' = 9.00749 \times 10^{-4} \pm 3.13881 \times 10^{-11}$

R.M.S = 0.029643133

Fourth Iteration

$dk' = 0.010700477 \pm 8.59293 \times 10^{-9}$

$dk'' = -5.95627 \times 10^{-3} \pm 7.70649 \times 10^{-9}$

$d\phi' = 4.29067 \times 10^{-6} \pm 2.73963 \times 10^{-13}$

$d\phi'' = -1.46613 \times 10^{-5} \pm 2.45967 \times 10^{-13}$

$dw' = -1.75737 \times 10^{-5} \pm 2.46299 \times 10^{-13}$

R.M.S = 0.026259390

4.2 Perspective - Centre Position Determination

The afore listed Least Squares program was used. Each monocular observation of the standard point model

coordinate generates three observation equations. Hence there are eighteen equations from the six standard points. Therefore four unknowns are to be solved from eighteen observation equations

4.2.1 Results of Perpective-Centre Position Determination

The results are as follows :-

Model 1 Case A

Projection Centre Coordinates

	LEFT	RIGHT
X_o	4999.264 ± 0.070	5194.307 ± 0.040
Y_o	4992.526 ± 0.059	4992.612 ± 0.035
Z_o	7573.074 ± 0.966	7573.085 ± 0.623
T_o	13.365 ± 0.005	13.365 ± 0.003
	R.M.S. 0.15176336	R.M.S. = 0.078460064

Model 1 Case B

Projection Centre Coordinates

	LEFT	RIGHT
X_o	4999.420 ± 0.082mm	5194.057 ± 0.044mm
Y_o	4992.677 ± 0.069mm	4992.781 ± 0.033mm
Z_o	7571.089 ± 1.190mm	7572.623 ± 0.501mm
T_o	13.355 ± 0.006	13.363 ± 0.003
	R.M.S 0.137899964	R.M.S. = 0.07451704

Model 1 Case C

Projection Centre Coordinate

	LEFT	RIGHT
X_o	4999.382 ± 0.072mm	5194.347 ± 0.051mm
Y_o	4992.412 ± 0.068mm	4992.422 ± 0.050mm
Z_o	7572.783 ± 1.074mm	7573.204 ± 0.767
T_o	13.379 ± 0.005	13.366 ± 0.004
	R.M.S = 0.15177532	R.M.S = 0.10865399 2

Model 1 Case D

Projection Centre Coordinate

	LEFT	RIGHT
X_o	4999.470 ± 0.038mm	5194.122 ± 0.033mm
Y_o	4992.668 ± 0.044mm	4992.497 ± 0.040mm
Z_o	7567.208 ± 0.675mm	7571.497 ± 0.040mm
T_o	13.336 ± 0.003	13.356 ± 0.004
	R.M.S = 0.058562404	R.M.S = 062201420

Model 1 Case E

Projection Centre Coordinate

	LEFT	RIGHT
X_o	4999.077 ± 0.087mm	5194.221 ± 0.066mm
Y_o	4992.510 ± 0.074mm	4992.566 ± 0.072mm
Z_o	7570.922 ± 1.189mm	7572.440 ± 1.193mm
T_o	13.355 ± 0.006	13.362 ± 0.006
	R.M.S = 0.10721201	R.M.S = 0.12754987

Model 2 Case A

Projection Centre Coordinate

	LEFT	RIGHT
X_o	4999.316 ± 0.057mm	5194.875 ± 0.006mm
Y_o	4991.687 ± 0.058mm	4991.697 ± 0.043mm
Z_o	7576.597 ± 1.149mm	7571.561 ± 0.074mm
T_o	13.383 ± 0.006	13.358 ± 0.004
	R.M.S = 0.09113680	R.M.S. = 0.106886378

Model 2 Case B

Projection Centre Coordinate

	LEFT	RIGHT
X_o	4991.470 ± 0.071mm	5194.082 ± 0.040
Y_o	4991.796 ± 0.060mm	4991.656 ± 0.036
Z_o	7573.861 ± 0.956mm	7570.728 ± 0.561
T_o	13.369 ± 0.005	13.354 ± 0.003
	R.M.S = 0.146871200	R.M.S = 0.0923461 9

Model 2 Case C

Projection Centre Coordinate

	LEFT	RIGHT
X_o	4999.448 ± 0.035mm	5194.213 ± 0.066mm
Y_o	4991.290 ± 0.034mm	4991.736 ± 0.058mm
Z_o	7573.606 ± 0.483mm	7571.646 ± 0.058mm
T_o	13.368 ± 0.002	13.358 ± 0.005
	R.M.S = 0.075863804	R.M.S = 0.14307330

Model 2 Case D

Projection Centre Coordinate

	LEFT	RIGHT
X_o	4999.325 ± 0.096mm	5194.267 ± 0.086
Y_o	4991.652 ± 0.082mm	4991.696 ± 0.082
Z_o	7573.340 ± 1.339mm	7572.630 ± 1.334
T_o	13.367 ± 0.007	13.363 ± 0.007
	R.M.S = 0.1225491.95	R.M.S = 0.187245130

Model 2 Case E

Projection Centre Coordinate

	LEFT	RIGHT
X_o	4999.254 ± 0.061mm	5194.456 ± 0.072mm
Y_o	4991.821 ± 0.048mm	4991.788 ± 0.057mm
Z_o	7575.019 ± 0.775mm	7575.079 ± 0.918mm
T_o	13.375 ± 0.004	13.375 ± 0.005
	R.M.S = 0.093232409	R.M.S = 0.14001623

4.3 Strip Formation Adjustment

The program used for this research is a N.R.C. program of April 6, by Schut G.R. (1967), and the modified edition dubbed from Bammeke,A.A. This program is contained in Appendix (IV).

4.3.1 Input Data

For each case study, a file called Aba was created. In all 25 data files were created for the research. The Input was typed directly into the terminal.

The coordinate cards were typed as follows:-

Field	Column	Description
1	1-4	In the card with model coordinate, this field must contain strip - or model identification [numeric, non-zero and positions]. In the card with ground-control co-ordinates this field will be left blank.
2	5-9	Numeric point Identification.
3	10-18	X-Coordinate in the model (or easting as the case may be).
4	19-27	Y-Coordinate or Northing
5	28-36	Z-Coordinate or height

Two types of control cards were used.

They are -1 and -2

The data was strictly arranged in this order.

(i) A Coordinate Card containing the ground control coordinate.

(ii) A control Card with -1 in field 1.

(iii) The card with model coordinate of all points that are not used as tie point (model coordinate of ground control points are first arranged before other points).

(iv) A Control Card -2 in field 1.

(v) A Card with the model coordinates of the projection centre, that the model has in common with the following one.

(vi) The Cards with model coordinates of the tie points common to the following model.

(vii) A Control Card with -2 in field 1.

(viii) A Card with model coordinates of the projection Centre that the model has in common with the preceeding one.

(ix) The Cards with model coordinates of the tie points common to the preceeding model.

Appendix (V), contain this. This arrangement is general for a strip that contain two models. In case there are more than two models in the strip the arrangement changes. [25]

4.3.2 Output Data

The output was stored in files created for the purpose. There are twenty-five of such files. The result of each case was stored in a file. The file was assigned a name called paq.

The Output has the same form as Input with the following addition. For each ground control points the residuals were printed (transformed coordinate minus ground) and for each tie point the deviations from the means were printed (Coordinates in the second

model minus mean) as follows:

Field 6: X - residual and deviation from mean

Field 7: Y - residual and deviation from mean

Field 8: Z - residual and deviation from mean

This is contain in the appendix (vi).

4.3.3 Results of Strip Formation and Adjustment

These are contained in the 25 computer Printouts one for each of the 25 strips which resulted from combining each case of model 1 to each of the five cases of model 2. For example case 1 is shown dragramatically in figure 4.3.

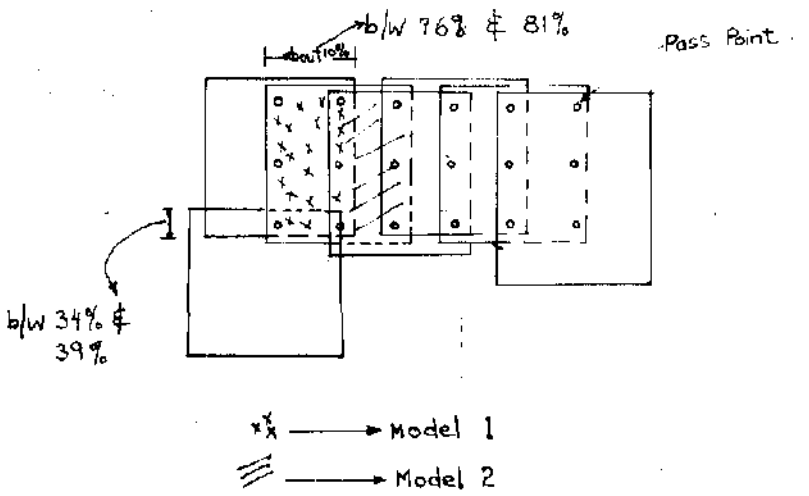
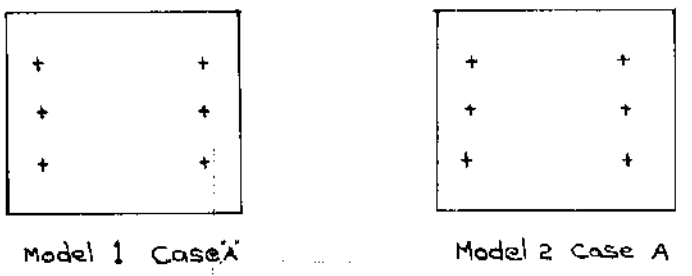
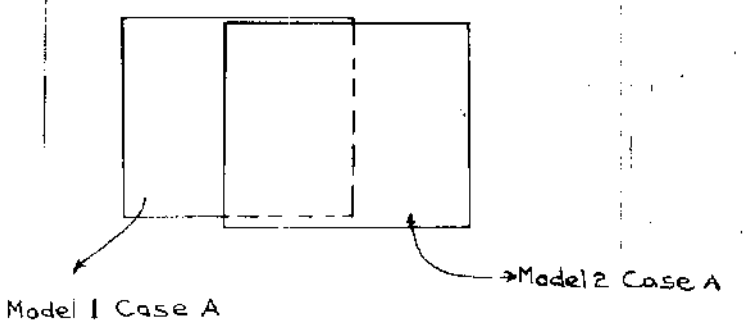


Figure 4·2 : BLocK formation.





CASE 1

Figure 4·3 : Case A formation [EXAMPLE CASE 1]

5. ANALYSES, SUMMARY AND CONCLUSION

5.1 Analyses

5.1.1 NUMERICAL RELATIVE ORIENTATION

TABLE 5.1: NUMERICAL RELATIVE ORIENTATION

MODEL 1 (PHOTOGRAPHS - LEFT 77906 - 76 RIGHT 77906 - 75)						
CASES	dk'	dk"	d ϕ'	d ϕ"	dw	Root Mean Square (RMS)
A	$-1.24478\times10^{-3}\pm3.8091\times10^{-4}$	$-0.0210249933\pm3.78244\times10^{-4}$	$-2.27472\times10^{-4}\pm1.97343\times10^{-6}$	$-2.37547\times10^{-4}\pm1.7409\times10^{-6}$	$2.44749\times10^{-6}\pm2.13286\times10^{-6}$	0.073175845
B	$-0.211008464\pm1.49962\times10^{-4}$	$0.045759084\pm1.64585\times10^{-4}$	$5.31752\times10^{-4}\pm8.11572\times10^{-7}$	$-6.73022\times10^{-4}\pm8.21708\times10^{-7}$	$5.45142\times10^{-4}\pm8.66745\times10^{-7}$	0.035786942
C	$-0.2610042\pm1.3847\times10^{-8}$	$0.052734085\pm2.508\times10^{-8}$	$6.2170\times10^{-4}\pm70764\times10^{-13}$	$-6.02111\times10^{-4}\pm8.701\times10^{-13}$	$4.45042\times10^{-4}\pm6001231\times10^{-13}$	0.034619345
D	$-0.21156883\pm1.76571\times10^{-4}$	$0.043900597\pm1.64219\times10^{-4}$	$5.48568\times10^{-4}\pm9.5632\times10^{-7}$	$-6.62721\times10^{-4}\pm8.08727\times10^{-7}$	$6.00843\times10^{-4}\pm9.27555\times10^{-7}$	0.038251713
E	$-0.026675203+2.89191\times10^{-4}$	$0.024333017\pm2.72423\times10^{-4}$	$-1.03181\times10^{-4}\pm1.62892\times10^{-6}$	$-1.22298\times10^{-4}\pm1.44163\times10^{-6}$	$-2.28994\times10^{-4}\pm1.4882\times10^{-6}$	0.075310197

TABLE 5.2: NUMERICAL RELATIVE ORIENTATION

MODEL 2 (PHOTOGRAPHS - LEFT 77906-75 RIGHT 77906-74)

CASES	dk'	dk"	dΦ'	dΦ"	dw	Root Mean Square (R.M.S)
A	$0.018705213 \pm 1.6288 \times 10^{-4}$	$-1.00608 \times 10^{-3} \pm 1.58954 \times 10^{-4}$	$-5.0614 \times 10^{-6} \pm 8.17554 \times 10^{-7}$	$2.30191 \times 10^{-5} \pm 8.33706 \times 10^{-7}$	$6.34897 \times 10^{-5} \pm 1.00165 \times 10^{-6}$	0.033343076
B	$8.80960 \times 10^{-3} \pm 1.40311 \times 10^{-4}$	$-8.96870 \times 10^{-3} \pm 1.50202 \times 10^{-4}$	$-8.0394 \times 10^{-6} \pm 7.27334 \times 10^{-7}$	$1.72909 \times 10^{-5} \pm 8.35778 \times 10^{-7}$	$3.9036 \times 10^{-6} \pm 8.4264 \times 10^{-7}$	0.02498334
C	$0.019351146 \pm 8.31434 \times 10^{-5}$	$1.44406 \times 10^{-4} \pm 7.24888 \times 10^{-5}$	$-9.04892 \times 10^{-6} \pm 4.53785 \times 10^{-7}$	$7.57142 \times 10^{-6} \pm 3.74323 \times 10^{-7}$	$-6.81429 \times 10^{-5} \pm 4.63285 \times 10^{-7}$	0.018806060
D	$0.016249187 \pm 9.3794 \times 10^{-5}$	$-3.52761 \times 10^{-3} \pm 9.04743 \times 10^{-5}$	$8.41155 \times 10^{-6} \pm 4.96247 \times 10^{-7}$	$3.81237 \times 10^{-5} \pm 5.00375 \times 10^{-7}$	$-5.11245 \times 10^{-5} \pm 5.43484 \times 10^{-7}$	0.022028924
E	$0.010700477 \pm 1.04645 \times 10^{-3}$	$-5.95627 \times 10^{-3} \pm 9.91012 \times 10^{-4}$	$4.29067 \times 10^{-6} \pm 5.90876 \times 10^{-6}$	$1.46613 \times 10^{-5} \pm 5.59873 \times 10^{-6}$	$-1.75737 \times 10^{-5} \pm 5.60249 \times 10^{-6}$	0.026259390

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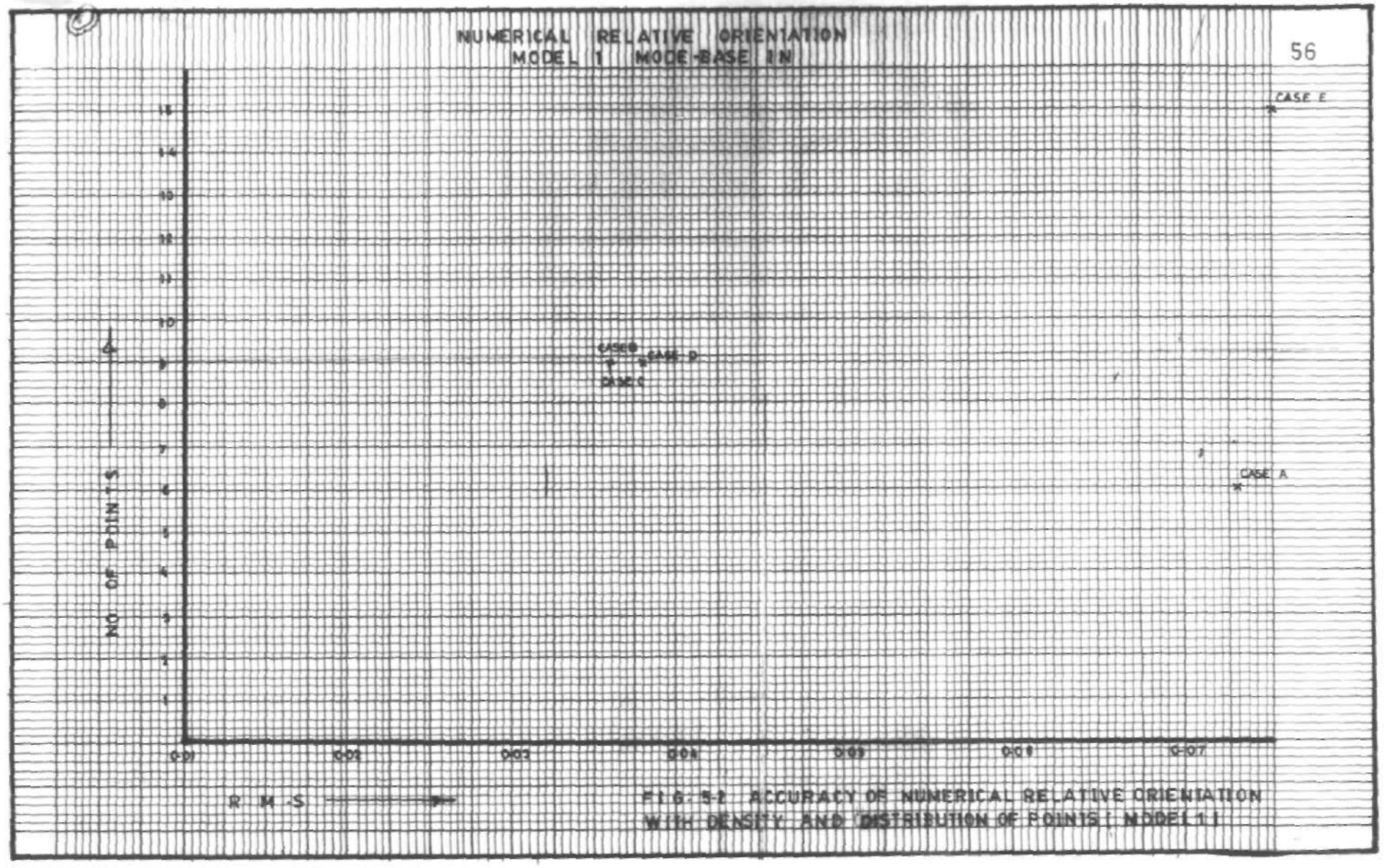


FIG. 5-2 ACCURACY OF NUMERICAL RELATIVE ORIENTATION WITH DENSITY AND DISTRIBUTION OF POINTS (MODEL 1)

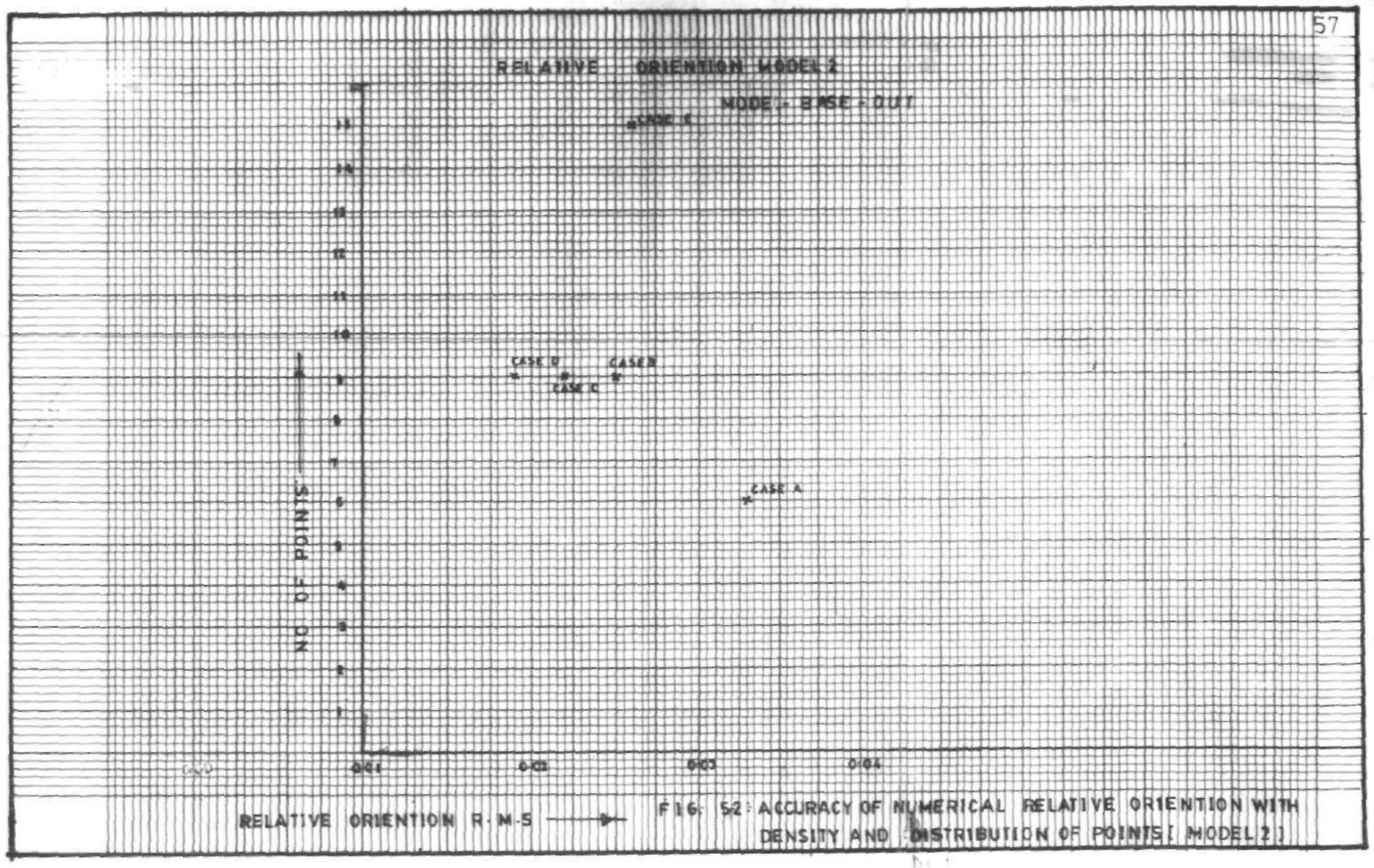


FIG. 5.2 : ACCURACY OF NUMERICAL RELATIVE ORIENTION WITH DENSITY AND DISTRIBUTION OF POINTS [MODEL 2]

Figures 5.2 and figure 5.3 were used to analyse the accuracy of numerical relative orientation with respect to the distribution and density of points.

In figure 5.2 Cases B, C and D have about the same accuracies. Figure 5.3 follows a similar trend. This implies that irrespective of the base - mode or the distribution of points, cases of the same density will produce similar results.

5.1.2 Strip Formation

Twenty-five cases were considered under this study. The cases are as follows :-

Case 1 - M_1A/M_2A [Model 1 Case A /Model 2 Case B]
A strip consisting Model 1 and Model 2. Model 1 was numerically relative oriented by means of Case A while Model 2 was by Case A.

Case 2 - M_1A/M_2B [Model 1 Case A/Model 2 Case B]
A strip consisting Model 1 and Model 2. Model 1 was numerically relative oriented by means of Case A while Model 2 was by Case B.

Case 3 - M_1A/M_2C

Case 4 - M_1A/M_2D

Case 5 - M_1A/M_2E

Case 6 - M_1B/M_2A

Case 7 - M_1B/M_2B

Case 8 - M_1B/M_2C

Case 9 - M_1B/M_2D

Case 10 - M_1B/M_2E

Case 11 - M_1C/M_2A

Case 12 - M_1C/M_2B

Case 13 - M_1C/M_2C

Case 14 - M_1C/M_2D

Case 15 - M_1C/M_2E

Case 16 - M_1D/M_2A

Case 17 - M_1D/M_2B

Case 18 - M_1D/M_2C

Case 19 - M_1D/M_2D

Case 20 - M_1D/M_2E

Case 21 - M_1E/M_2A

Case 22 - M_1E/M_2B

Case 23 - M_1E/M_2C

Case 24 - M_1E/M_2D

Case 25 - M_1E/M_2E

Where the figure 1 and 2 immediately after M represent Models 1 and 2 respectively and letters A, B, C, D and E indicate the cases used for the relative orientation. This is illustrated in section 4.1.1.

The 25 cases can be subdivided into 5 base on the case use to orient the the first model. Thus we have

1. Cases 1 to 5 (model 1 was relatively oriented by Case A)

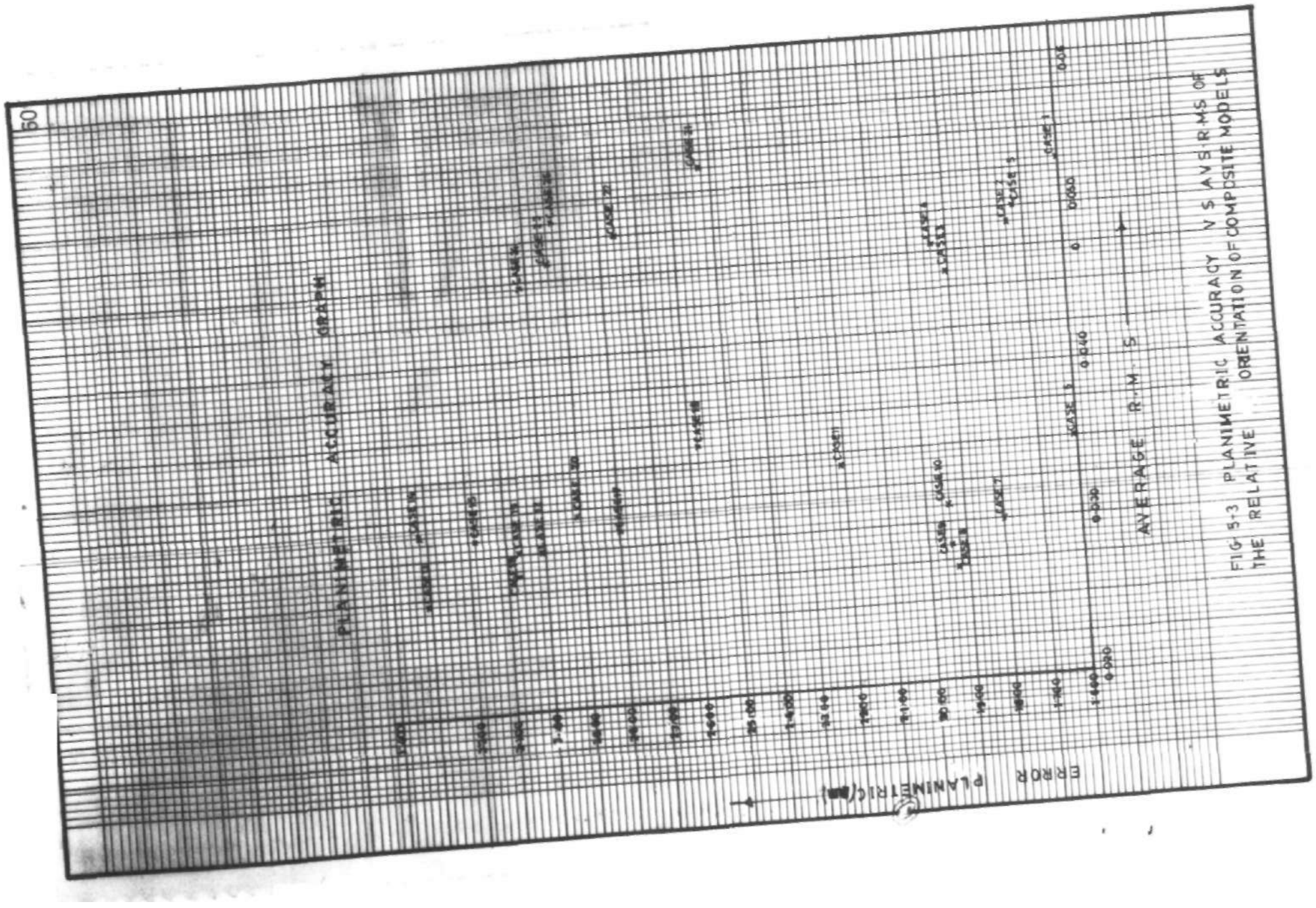


FIG 5-3 PLANIMETRIC ACCURACY V S AVS R-M-S OF THE RELATIVE ORIENTATION OF COMPOSITE MODELS

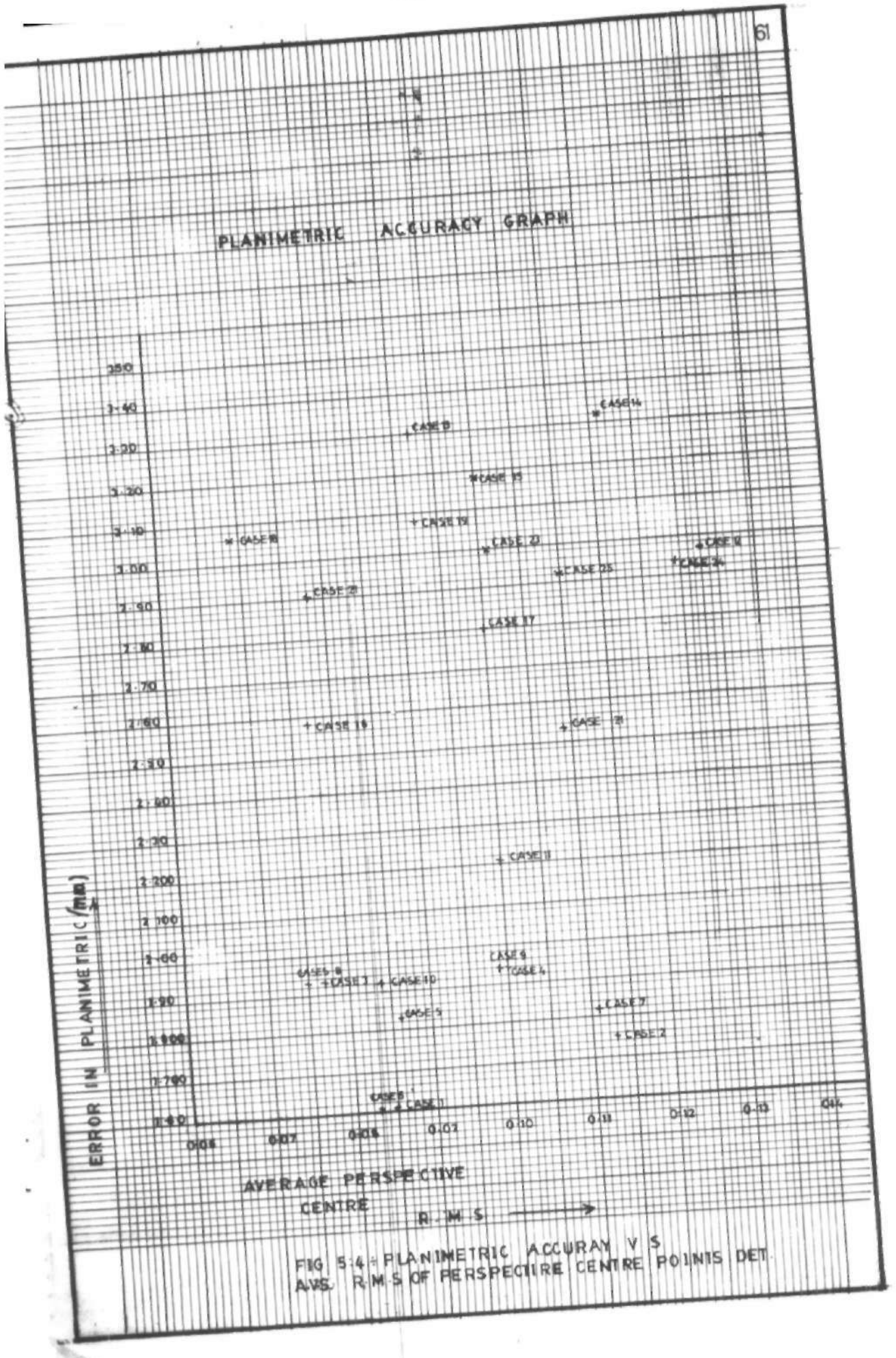


FIG 5.4 : PLANIMETRIC ACCURAY V S
AVS. R.M.S OF PERSPECTIRE CENTRE POINTS DET

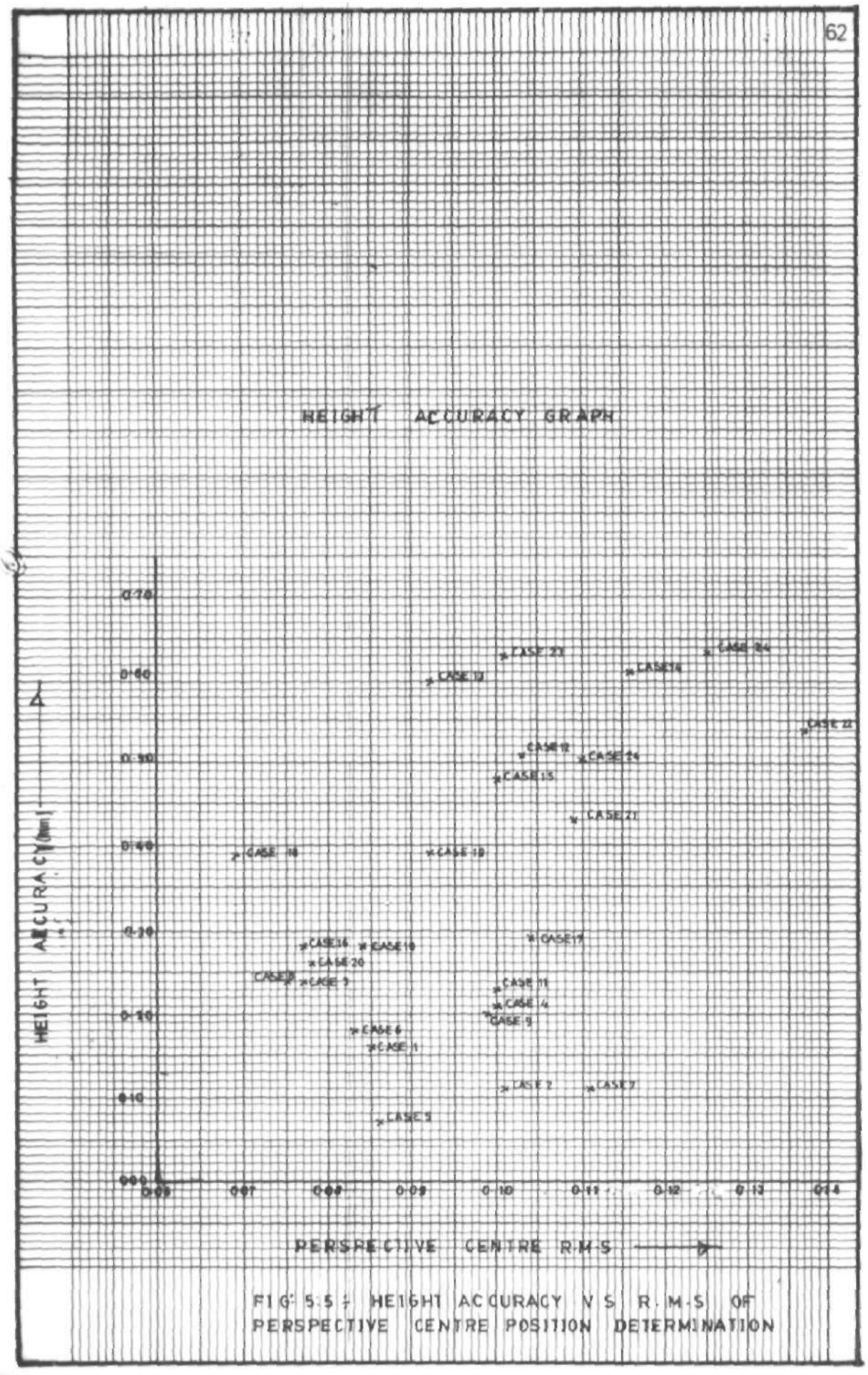


FIG: 5.5 : HEIGHT ACCURACY V S R.M.S OF PERSPECTIVE CENTRE POSITION DETERMINATION

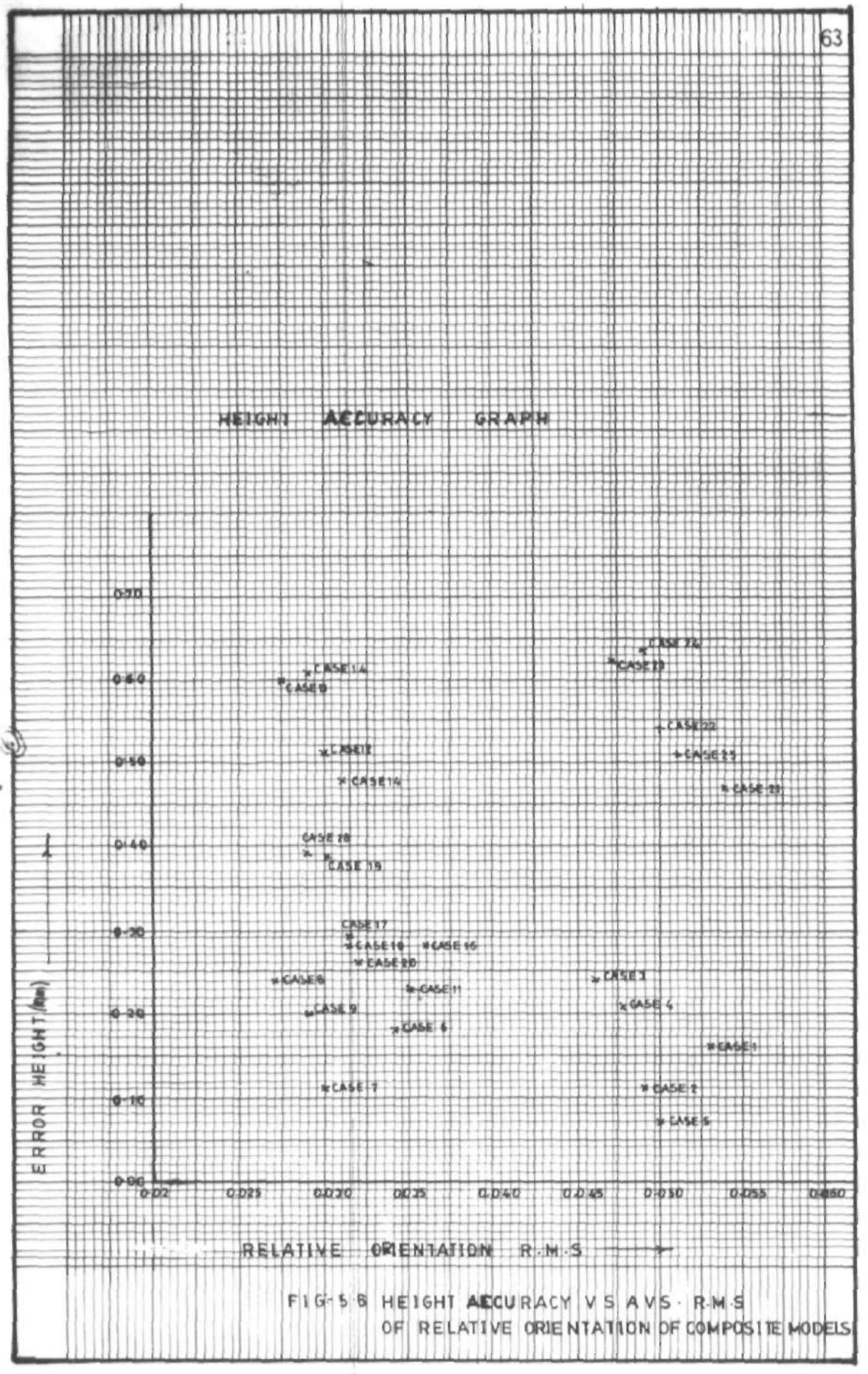


FIG-5.8 HEIGHT ACCURACY V S AVS · R·M·S
OF RELATIVE ORIENTATION OF COMPOSITE MODELS

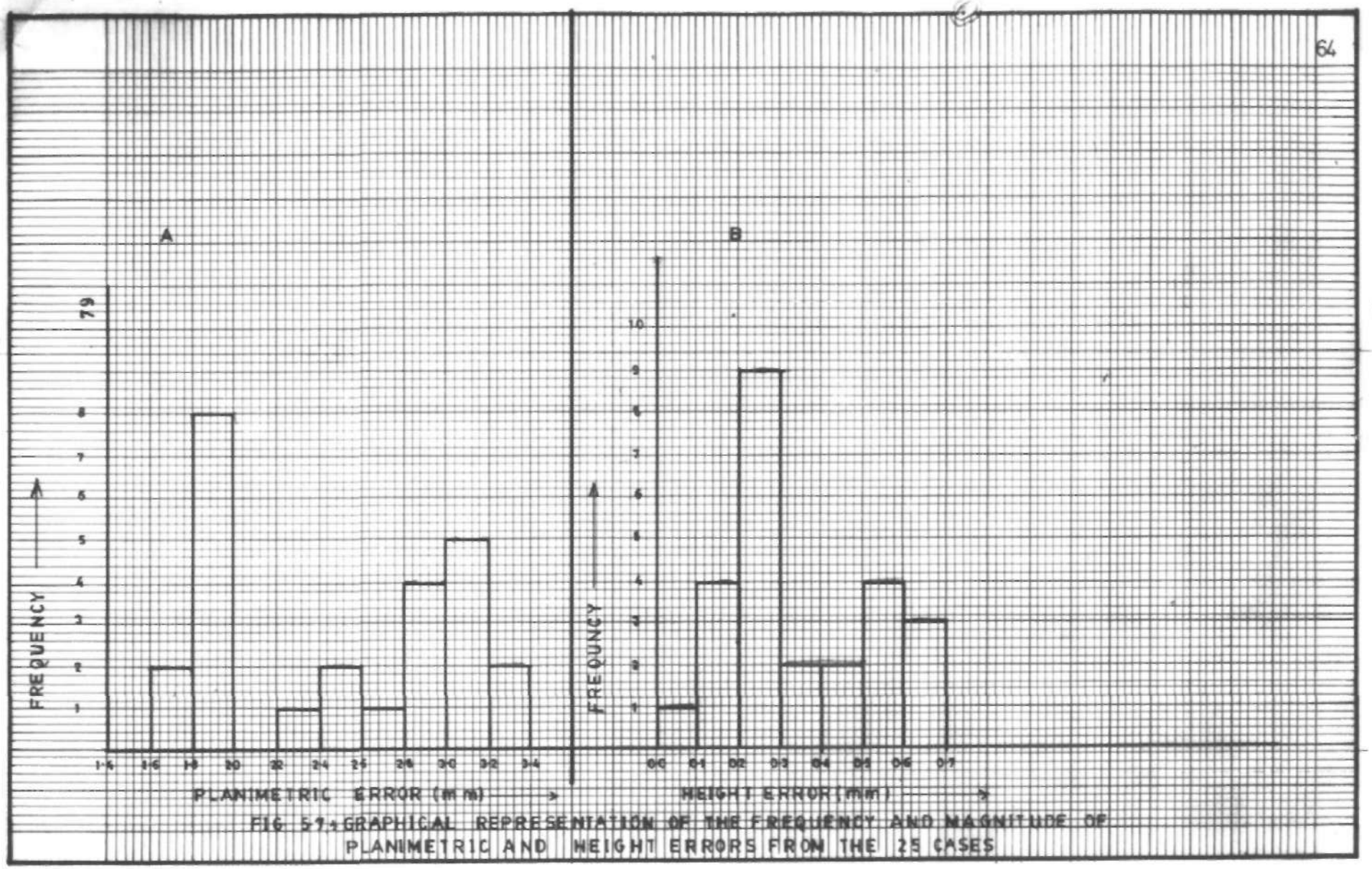


FIG. 57: GRAPHICAL REPRESENTATION OF THE FREQUENCY AND MAGNITUDE OF PLANIMETRIC AND HEIGHT ERRORS FROM THE 25 CASES

2. Cases 6 to 10 (Model 1 was relatively oriented by Case B.)
3. Cases 11 to 15 (Model 1 was relatively oriented by Case C.)
4. Cases 16 to 20 (Model 1 was relatively oriented by Case D.)
5. Cases 21 to 25 (Model 1 was relatively oriented by Case E.)

5.1.3 *Inference From figures* 5.3, 5.4, 5.5, 5.6 Cumputer output contain in Appendix VI

Figures 5.3, 5.4, 5.5 and 5.6 are scattergraph This follows that there is no corrollation between R.M.S of relative orientation and planimetric accuracy, R.M.S of relative orientation and height accuracy, R.M.S of perspective centre determination and planimetric and R.M.S of perspective centre determination and height.

Cases 1 and 6 have the highest planimetric accuracy of 1.61mm and case 5 has the highest height accuracy of 0.07mm. Case A was involved in the strip formation of cases 1, 5 and 6. This might be due to some accidental error which is well explained in section 5.1.4 below.

Based on the distribution of cases 1 to 25 in figures 5.3, 5.4, 5.5 and 5.7, cases 1 to 25 can be classified into 3 broad group.

(i) High accuracy group - This has a range of 1.61 to 1.94mm in planimetric accuracy and 0.07 to 0.028mm in height accuracy.
Cases 1 to 10 belong to this group. In cases 1 to 5 the first model was relatively oriented by means of case A while in cases 6 - 10 the first model was relatively oriented by means of case B. The accuracy exhibited by cases 1 to 5 might be due to reason stated above in paragraph. While the accuracy exhibited by cases 6 to 10 might be due to some form of systematic errors due to dby, dbz, dw, dϕ and dk.
See Section 5.1.4.

(ii) Intermediate group - This could be found in the Central region of the figures. They fall within the range of 2.21 to 2.58mm in planimetric and 0.23 to 0.43mm in height accuracy. They include cases 11, 16 and 21. In the study it can be found that anywhere case A was used for the strip formation, it tends to improve the strip formation accuracy. In cases 11, 16 and 21, case A was involved in the strip formation. This might explain the sudden improvement in their respective accuracies.
Compare with other cases in their family.
For example;

(i) Case 11 has the best accuracy in planimetry and height accuracy in

the family which includes cases 12, 13, 14, 15.

(ii) Case 16 has the best accuracy both in planimetry and height accuracy in the family which includes cases 17, 18, 19 and 20, in the group where the numerical relative orientation of the first model was done by means of case D.

(iii) Case 21 has the best accuracy both in planimetric and height compare with other cases 13 22, 23, 24 and 25 in which orientation of the first model was done by means of case E.

(iv) Low accuracy group - They fall within the range of 2.54 to 3.32mm in planimetry and 0.26 to 0.64mm in height accuracy. The include cases 12, 13, 14, 15, 17, 18, 19, 20, 22, 23, 24 and 25.

Cases that produce the worst result are cases 12, 13, 14 and 15. Their accuracy in height and planimetry falls between 1.23 to 0.60mm and 2.211 to 3.32mm respectively. The "Cases" have one thing in common. That is the relative orientation of model 1 in each strip was done by means of case C.

From figure 5.3, 5.4, 5.5 and 5.6 it was found that the accuracy of strip formed by where a model was relatively oriented by means of cases was generally poor.

To Illustrate this:

(i) Case 3 out of cases 1 to 3 where the relative

orientation of the first model in the strip was done by means of case A has the worst accuracy in the group.

(ii) Case 8 has the worst accuracy out of cases 6 to 10 where the relative orientation of the first model was done by means of case B.

(iii) Case 13 has the worst accuracy out of cases 11 to 15 where the relative orientation of the first model in the strip was done by means of case C.

(iv) Case 18 has the worst accuracy out of cases 16 to 20 where the relative orientation of the first model in the strip was done by means of case D.

(v) Case 23 has the worst accuracy out of cases 21 to 25 where the relative orientation of the first model in the strip was done by means of case E.

Cases in each family that has the second model oriented by means of case D has very low accuracy both in planimetric and height considering each family. They are cases 4, 9, 14, 19 and 24.

The figures 5.4 and 5.5 follow the same trend, this implies that the perspective - centre position accuracy and numerical relative orientation accuracy have similar effect on planimetric accuracy of the strip formation. On the other hand the common trends

in figure 5.4 and 5.5 on one hand is different from the common trend in figures 5.5 and 5.7. This implies that the effect of numerical relative orientation and perspective - centre determination accuracies on planimetric accuracy of the strip formation is different from that of the height accuracy of the strip formation.

The residual in Z - coordinate much smaller than residual in X- and Y- coordinates. See computer Output in the appendix VI.
This might be due to the pointing error which affect the planimetric accuracy more in case of fairly flat terrain and also due to rotational, translational and observation error.

Graph 5.8(A) shows that must of the planimetric error falls between 1.8 to 2.0mm while Graph 5.8(B) show that most height error falls between 0.2 to 0.3mm. Both graphs shows that very big error are more frequent than small error contrary to low of error propagation. This shows that error in this result might be mainly systematic error.

Figure 5.8a does not follow a normal distribution but the graph could be seen as being made up of 3 graphs with each graph being normally distributed. While figures 5.8a is close to normal distribution graph.

5.1.4 Possible Explanation for the systematic Error of the Results

The systematic errors dx, dy and dz could be explained by analysing the possible.
The contributions of the different sources of error from this analysis will become evident and accordingly give credit. Systematic error causes, systematic falsification of the Aerotriangulation. The systematic falsification of the strips surface is caused by seven initial errors of the first model.

The seven errors are scale factor (ds), three translation errors (dx_o, dy_o and dz_o) and three rotational errors (dw, dϕ and dk) [1]. Error propagation equations affect the X-Coordinate most [1].

The result of the present study shows that Y-Coordinate is most affected followed by X-Coordinate. The effect of initial errors is very material in the present circumstances.
These will be discussed individually.

(1) Systematic Errors dby and dbz

The following equations express the error caused by dby and dbz.

$$\left.\begin{aligned} \Delta x &= 0 \\ \Delta y &= \frac{x}{b}(dby) \\ \Delta z &= \frac{x}{b}(dbz) \end{aligned}\right\} \quad (1) \qquad .. \quad .. \qquad 5.1$$

In this study dbz was kept fixed hence we expect Δz to be equal to zero which is not. dby is not equal to zero in this study because it was used for relative orientation. These appear to explain the big error in Y-Coordinate compared with X- and Z Coordinate.

(ii) Systematic Errors dw

The following equations express the errors caused by dw.

$$\left.\begin{aligned} \Delta x &= 0 \\ \Delta y &= xz\left[\frac{dw}{b}\right] \\ \Delta z &= -xy\left[\frac{dw}{b}\right] \end{aligned}\right\} \quad (1) \qquad .. \quad .. \qquad 5.2$$

Where b is the air base.

Systematic errors dw caused the strip to be twisted about X-axis from the above equations negative correlation exist between Δy and Δz which tends to agree with the distribution of Y-parallaxes error, See output in the appendix VI. Systematic Omega error can explain the obtained errors in the results.

(iii) Systematic Phi(ϕ) and Kappa(k) errors

Systematic phi(ϕ) errors cause vertical wraping of the strip. The x and z coordinates are affected as they are negatively correleted but y is unaffected. These might have minor effect on x and z coordinate error in the result.

Systematic Kappa errors caused the strip to wrap in the y- direction, the x and y coordinates are greatly affected but z is not much affected. This may explain the result of the study where the height accuracy is far better than the planimetric accuracy.

(iv) Propagation of Accidental Errors

Quasistematic errors in Aerotriangulation are caused by correlation of accidental co-orientation errors and by accumulation of various accidental errors.

In semi analytical aerotriangulation the accidental orientation errors will be due to omega (w), phi(ϕ) and kappa(k). Accidental omega and kappa errors both yielded discrepancies in y but with kappa having the predominant effect.

(v) Base Mode

This affects the accuracy of aerotriangulation as tie points are not symmetrically situated [7] which might be a source of the error in the result.

The results of the present study are in complete accord with the above theories.

5.2 Summary and Conclusion

A number of points have been established by the present study.

Numerical Relative Orientation

It was found that irrespective of the base mode or the distribution of points, cases of the same density will produce similar result. Root mean square of

cases B, C and D in model 1 are 0.036, 0.036 and 0.038. While in model 2 are 0.025, 0.019 and 0.022.

It was also noticed that case A and case E have about the same accuracies. The root mean square of case A and case E are 0.073 and 0.075 in model 1 respectively, while in model 2 are 0.033 and 0.026. This is contrary to the theoretical considerations where the derivation of five unknowns from more observations say 15 points should yield a better precision.

Strip Formation

The fact that height accuracy was better than planimetric accuracy agrees with the well established fact that numerical relative orientation yields better accuracy in height than Planimetric[11].

It was found that increase in density of points used for numerical relative orientation does not necessarily improve model connection accuracy. Cases in which case A was involved in it's strip formation gives better result. One would have expected that case 25 ($M_1 EM_2 E$) should have given the best result but it could only give 2.932mm in planimetric and 0.505mm in height compare with case 1 that gives 1.612mm in planimetric and 0.163mm in height.

It was found that Cases that use Case C in the formation of strips has poor accuracy with cases 12,

13, 14 and 15 having the worst range of accuracies between 0.23 to 0.605mm in height and 2.211 to 3.323mm in planimetry.

Perspective centre determination (PC)

It was found that the effect of perspective centre determination and Numerical relative orientation accuracies or on planimetric and height accuracies of strip formed are different.

In conclusion it will be proposed that the use of six standard points for numerical relative orientation should be encouraged. Points in the centre of the model should not be used for numerical relative orientation for semi analytical aerotriangulation and if at all additional points are to be used, it should be very close to the line joining standard points 3,1,5. And it must be concluded that the main attributes of this project also include the use of routine observations, trivial mathematics, straight foward data processing technique without any special treatments and minimum labour. This is of paramount practical importance, especially so to the practising photogrammetrist in production work.

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