THE APPLICATION OF LINEAR PROGRAMMING TECHNIQUE IN THE DISTRIBUTION OF FUEL PRODUCTS IN NORTHERN NIGERIA.

A STUDY OF ELF OIL COMPANY LIMITED

BY

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Project report submitted to the department of Business Administrative as a fulfilment of Partial requirement for the award of a Master of Business Administration (MBA).

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DEDICATION

This project is dedicated to my humble wife,

Mrs Bosede Elizabeth Garba

for her moral and financial support throughout my daily and Educational pursued.

Above all, I am greatful to God, without His support all effort would have been in vain.

CERTIFICATION

This is to certify that this project was embarked upon by Garuba Funsho Joseph of Department of Business Administrative under the close Supervisor of Mal. Nasiru Maiturari.

The project has been carefully read and approved as meeting with the required standard for the award of a Master of Business Administration.

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ACKNOWLEDGEMENT

My God! you are worthy of all my praises and thanksgiving for you have proved yourself above all circumstances in my life, through the period of my academic programme. I return all the glory and adoration to you.

I am proud of every moment I had with my supervisor, may God grant you sucess in all your endevours.

To my parents, I pray that I will never let you down in every part of my life endeavours. And to all that have contributed morally, financialy and in any form, I pray that God will regard you all.

The whole of my lecturers are a unique collection of intellectuals, the cordiality enjoy with you, the inspiring and highly educative lectures are memories for ever. Thank you all.

Mr B.A Oyedeji, I am highly indebted to you. I can never forget you in life for your fatherly advise. My bossom wife Bosede Elizabeth Garuba here I come! thank you very much. And to host of my friends especially Abiodun Osundiji for our brotherly relationship and advise, May the Lord justify the confidence we have in ourselves and grant us a reasonable future together.

GARUBA FUNSHO JOSEPH.

DECLARATION

I HEREBY DECLARE THAT THIS PROJECT, A PARTIAL REQUIREMENT FOR THE AWARD OF MASTERS OF BUSINESS ADMINISTRATION OF AHMADU BELLO UNIVERSITY ZARIA. IS A PRODUCT OF MY RESEARCH FINDINGS.

THIS RESEARCH WORK RELATES TO THE APPLICATION OF LINEAR PROGRAMMING TECHNIQUE IN THE DISTRIBUTION OF FUEL PRODUCTS IN NORTHERN NIGERIA A CASE STUDY OF ELF OIL NIGERIA LIMITED.

THE TITLE HAS NOT BEEN PREVIOUSLY PRESENTED IN ANY APPLICATION FOR HIGHER DEGREE.

ALL SOURCES OF INFORMATION COLLECTED FOR THE THESIS ARE PROPERLY ACKNOWLEDGED BY MEANS OF REFERENCES.

GARUBA FUNSHO JOSEPH ______ SIGNATURE 21/8/2000 DATE

ABSTRACT

The Applications of linear programming are so diverse and in wide used by business, industries, and organisations in advance econimies. However, the extent to which Nigerian businesses are utilising the many opportunities provided by linear programming models is anything but encouraging.

This research work surveys the present position of oil company attempting to maximise its profit. A critical examination of its current distribution method has been made and an alternative method offered as a better alternative because it is cost effective and is based on a simple, basic scientific method rather than rule of thumb or intuition.

Specifically attempt is made to show the benefit to be derived in using transportation model. The study achieves this by illustrating the mechanics of the transportation model, and lays foundation for further inquiry into potentially profitable applications of the various techniques of linear programming.

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INTRODUCTION

1.1 INTRODUCTION

The issues of Product "Distribution" has been of great concern in the business world since time immemorial. Obviously, distribution of goods and services is a very crucial issue in management in any organisation, be it private or public, manufacturing or non-manufacturing. This explains why considerable work has been done in improving physical distribution systems, particularly with regard to inventory levels, warehouse locations and transportation modes.

To the marketing manager, disribution is viewed as the various channels followed in getting products from sources to where they are needed in the right form, at the right time at the right price. The marketing manager's foresight in the cost administration or management is limited. Nonetheless, distribution involves all those elements of costs that are crucial in fulfilling the attendant requirement of availability at specified place and time. The central role of management here is to explore, with a view to arriving at an optimal programme of distribution limited resources available at a specified time. This role can be effectively achieved with the use of linear programming techniques.

Today linear programming technique is the most widely used of all the techniques of mathematical programming. This is especially true in industrialised economies, where it has helped largely in minimising total cost and / or maximising total revenue and profits, thereby improving overall effeciency.

On the other hand, the application of linear programming technique in developing economies like Nigeria is still low. For instance most business organisations choose to base their distribution system on intuition, hunches, and obsolete statistical demand forecasts, disregarding the advantages provided by various applications of scientific methods in managerial decision-making. This lack of awareness by most organisations is what has prompted the researcher to venture into the mechanics of transportation or distribution model, with the hope of laying a foundation for further inquiry and investigation, into other potentially profitable applications of the various techniques of linear programming.

The importance of the oil sector in the national economy is what has addressed the choice of emphasis by this research. ELF Oil Company, which in recent times is making in road into the Nigerian oil market, has been selected due to its attendant effort to achieve an effective and efficient distribution system.

Operational problem are involved in the distribution system of fuel products. In addition, it is believed that in future when government involvement reduces or ceases completely and competition becomes stiffer, cost consciousness will become paramount. Distribution cost being an important element that features in the income statement of organisations offers management with an important area that can reduce costs significantly if given professional attention.

The aspect of linear programming that this research is concerned with is the transportation or distribution model. How the method is employed to determine the distribution pattern that can best minimise total costs. The technicalities of linear programming will therefore be discussed only in as much as it enhances a clearer understanding of its basic concepts.

1.2 STATEMENT OF THE PROBLEM

Allocation problems arise in the distribution of resources among competing alternatives in order to minimise total costs or maximise total revenue. Such problem have the following components: a set of resources available in a given amount; a set of job to be done, each consuming a specified amount of resources; and a set of costs or return for each job and resource. In the event of more resources available than required, the solution should indicate which resources are note to be used, taking associated costs into account. Similarly, if there are more jobs that can be done with available resources the solution indicate which jobs are not to be done, again taking into account the associated costs.

In a like manner, this research seeks to identify which destination (filling station) should get what quantity of fuel from which source (Depot). And source (Depot) should transport what quantity fuel to which destination (filling station). All directed at minimising total cost of distribution.

1.3 OBJECTIVE OF THE STUDY

The objective of this research is to determine the distribution pattern that will minimise total cost of ELF oil company. The research would therefore examine the present system of distribution used by ELF Oil Company with a view of the cost to the company.

In addition, the research will determine if the use of linear programming technique will enchance cost minimisation in transportation of fuel products.

At the end of the research it is hoped that ELF Oil Company will realise the need to use the mechanics of distribution model in order to have an optional distribution pattern that best improve efficiency and cost minimisation.

1. 4 RESEARCH METHODOLOGY

The source of data for this research work will be principally through personal interview with the manager supply and distribution of ELF Oil Company Northern Regional Office, as well as through personal observations.

Invariably some books and journals would be consulted for an in-depth analysis of the data so collected. The data collected will go through the following stages:

1.4.1 Formulating the problem; by identifying the objective function and then the decision variable and constraints; and formulating an appropriate model.

1.4.2 Deriving a solution through solving of the model exactly or approximately.

1.4.3 Testing the model and solution; taking steps to ensure adequacy of the representation and comparing the solution derived from it with what would otherwise be obtained.

1. 5 HYPOTHESIS

The following hpothesis are to be tested:

H1: "The use of transportation method (linear programming technique) will enhance cost minimisation in transportation of fuel products"

H2: Designing a distribution pattern arbitrarily or by rule of thum does not result in an optimal solution.

1.6 SCOPE AND DELIMITATIONS OF THE STUDY

This research will be limited in scope to the application of linear programming technique in the transportation of fuel products, using the transportation method only. ELF oil company, which is being used as a case study, is involved in the procurement and marketing of oil products such as Diesolene, Kerosine (DPK), Petroleum (PMS), and various types of lubricants. This study will only address the issue of procurement and marketing of petroleum (PMS) product. Furthermore it will be limited to the source (Depots) and destinations (filling stations) situated within the geographical location of what is today referred to as Northern Nigeria.

Due to the complex nature of practical applications of linear programming because of the vast number of destinations spread across the region of study, only destinations identified as major by ELF Oil Company will be considered. Moreover, the distribution in this research is the physical conveyance of the product from sources to destinations. Finally, like other research works, this research will be limited by the usual limitations associated with other studies before it.

1. 7 DEFINITION OF TERMS

Within the context of this research work, some words used are scientific in their application. For that reason, it is pertinent to define such words for clarity. They include:

1.7.1 Linear Programming

This is defined as a mathematical technique designed to assist in the allocation of scarce resources in the best possible or optimal way. By linear is meant that the elements in a solution are assumed to be related that they generate straight lines when graphed.By programming is meant that a procedure is followed for analysing one feasible combination of resources after another until the best combination is attained.

1.7.2 Objective Function

This is defined as the desired result expressed in a linear form. It may be to maximise profit or to minimise cost.

1.7.3 Decision Variable

This is defined as the decision that must be made in order to specify a solution to a given problem.

1.7.4 Constraint

This is defined as the limitations on the availability of resources.

1.7.5 Model

This is defined as a particular account of a system, which in turn represents a particular part of reality as object of interest or subject of inquiry in real life. The ultimate purpose of a model is to give the manager an improved degree of Control Over His Operations And Environment.

1.7.6 Transportation Or Distribution Model

This is a special class of linear programming problem, which seeks to find the cost schedule for transporting products or services from several sources to a number of destinations. It can be successfully applied to other non-transportation situations such as product planning, machine scheduling, location analysis, workforce scheduling and media scheduling.

1.7.7 Sources

This is generally used to refer to factories/plant, origins, or supply points, but within the context of this research it will represent fuel depots or supply points.

1.7.8 Destinations

This is generally refer to as warehouses, branches, or demand points, but within the context of this research it will represent filling stations or demand points.

1.7.9 Basic Square

This is defined as an allocated square or occupied cell. That is a square that has allocation of certain units in it.

1.7.10 Open Square

This is defined as an unallocated square or unoccupied cell. That is square that has no allocation in it.

1.7.11 Exiting Square

This is basic square that is to become an open square by removal of its existing allocation to another square.

1.7.12 Entering Square

This is defined as an open square that is to become a basic square by allocation of one or more units to it.

1.7.13 Degeneracy

In transportation problems, a basic feasible solution consist of m+n-1 basic variables. This means that the number of occupied cells in a given transportation program is 1 less than the number of rows and columns in the transport matrix. Whenever, the number of occupied cells is less than m+n-1, the transport problem is said to be degenerate or degeneracy is said to occur.

1.7.14 Rim Requirements

This is defined as capacity constraints at sources and demands constraints at destinations.

1.7.15 Dummy

The transportation problem requires that supply and demand be equal; the rim requirement for the rows must be equal to the rim requirements for the columns. This is seldom possible in practical terms. The demand may be less than supply or the supply may not cover the demand. The method employed to balance this type of problem is to create a fictitious destination or source. This fictitious destination or source is referred to as dummy.

1.7.16 Donor

This is a basic square on a closed path that has minus sign in it . In other words the basic square will be reduced by the amounts of allocation transferred to an open square (square with a plus sign).

CHARPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

Many major decisions faced by a manager of a business enterprise are centred on the best best way to achieve the objectives of the firm subject to restrictions placed by the operating environment. These restrictions can take the form of limited resources, such as time, labor, energy, material or money, or they can be in form of restrictive guidelines such as storage capacity, or engineering specifications. The most frequent objective of business is to optimize profit. Alternatively the objective for organizational units within the firm is often to minimise cost. When a manager attempts to solve a general type of problem by seeking an objective that is subject to restrictions, the management science technique called Linear Programming (LP) is frequently used.

LP technique derives its name from the fact that the functional relationships the mathematical model are linear and the solution technique consists of predetermined mathematical steps that is, a programme. In applying the LP technique, first the problem must be identified as being solvable by LP, secondly the unstructured problem must be formulated as a mathematical model, and thirdly the model must be solved using established mathematical techniques. In general, LP technique can be used to strengthen the management decision making process. Its value to management can be very great if skill and intenlligence are exercised in building LP models, formulating problems for solution, and in analysing the computed results. To understand the potentials and limitations of LP, calls for a more detailed review of its basic concept.

2.2 LINEAR PROGRAMMING (LP)

LP is a powerful mathematical technique for solving a large class of special problems. Typically, it applies to problems in which one is attempting either to maximise benefits while using limited resources or to minimise cost while keeping to certain requirements. LP was first formulated by a Russian mathematician Shri L.V Kantorovich but it was later developed in1947 by George B. Dantzig and his associates for the purpose of planning, programming and budgeting problems of the United States of America's Air Force. It was designed to assist in the allocation of resources in the best possible way. It has been employed in solving a broad range of problems in business, government, industry, hospitals, education, and many other fields. In all these areas, it has demonstrated its value to a great extent. It is thus one of the most powerful and useful technique for making managerial decisions.

2.2.1 Definition And Terminology

LP is a technique for arriving at an optimal programme from among several competing alternatives in view of limited resources available to a manager at specified period of time. Here programming means to plan for the best use of scarce resources through systematic procedures in situations where there are many alternatives uses for them and therefore more than one solution to a problem. While by linear, proportionality is implied. Simply put it means that elements in a situation are assumed to be closely related that they exhibited straight lines when graphed. To the layman this may seem limiting especially since real-life problems are characteristically non-linear. But much art and ingenuity are demonstrated in the technical literature that describes ways of reducing highly curvilinear relationships to linear forms through changing scales (normalisation) or approximations of curved relationships. For every LP problems, three terminologies are invariably involved, namely, the objective function, decision variables and constraints.

First the problem encompasses an objective, which is the goal the decision maker wants to achieve. The most frequently encountered objective for a business firm is maximising profit or minimising cost.
An LP problem requires a choice between alternatives courses of action (that is a decision) that must be made to specify a solution to the problem. The decision is represented in the model by formulating decision variables.

An LPP has inherent restrictions that make unlimited achievement of the objective function impossible. These restrictions or constraints as well as the objectives must be definable by mathematical functional relationships that are linear. A systematic approach to model formulation by which steps are taken one at a time is the best.

Formulating a 'real' problem is normally difficult because, the objective function and model constraints can be very complex requiring much time and effort to develop. It is difficult to simply ensure that all model constraints are identified and no important problem restrictions have been omitted. Finally, the problems in real life are typically much larger than abstract ones. It is not uncommon for LP model of real life problems to encompass hundreds of functional relationships and decision variables.However, the development of digital computers has eased the problem. Computerised LP packages are in widespread use, since the development of mathematical programming system (MPS) by IBM in early 1960's. There is presently an LP options of mathematical programming system extended (MPSX) and proc LP in SAS/OR (Statistical Analysis System/Operations Research), a group of programs for solving Assignment, network scheduling, and transportation problems.

2.2.2 Basic Assumptions

LP despite its versatility and usefulness as a techniques for managerial decision is limited. This is due to the fact that, to express relationship between activities, restrictions, production requirements and objectives in LP format, certain assumptions are made. Thus all solution obtained are subject or limited to such assumptions. These assumptions are as follows;

(a) Linearity - There is linear relationship between all elements in a situation. Thus they are proportional and closely related.

(b) Divisibility - There is divisibility among the decision variables. That is they can take functional values and are continous.

(c) Additivity - The objective function (measure of total effectiveness) and each of the side constraint (resource usage) can be added.

(d) Finiteness - There should be a finite number of side restrictions since LP deals with limited resources.

(e) Certainity - The coefficients of the decision variables are known with complete certainity. These coefficients are profit combination, cost per unit of production, amount of resources used per unit, technology, strategies, etc.

(f) Non-negativity - All variables must assume a non-negative value. That they must be greater than or equal to zero.

(g) Technology within the time of planning a particular programme is assumed to be fixed.

(h) Finally, since LP is static model only one decision is required for planning horizon (that is time period for which the manager is designing a programme).

2.2.3 Types Of Applications

LP model can be applied to any programme in which the objective is to optimise a linear and non-negative constraints. The model is therefore a general one, and can be applied to a wide variety of real-life problems. Apart from transportation problem, which is a special case of linear programming problem due to its special solution method, some other applications are presented below;

(1) General allocation of scare resources, including in circumstances where demands are uncertain.

(2) Scheduling; production facilities, transporting equipment, maintenance problems, and investment planning (how much money should be placed in each of general investment alternatives in order to maximise yearly return while observing constraints on the amount of investment alternative?)

(3) Production/Inventory planning; how much product should a firm plan to manufacture in each of several upcoming months in order to meet forecasted demand while minimising cost of carrying inventory, hiring, firing, over-time wages, and sub contracting?

(4) Personnel assignment; in maintenance crew utilisation over a period of time, toll collections, and worker-machine assignments.

(5) Advertising/Promotion Planning; How much money should be spent in various advertising media to promote a product with maximum effectiveness while observing certain market goals, budget limitations, and limits placed on the use of each medium.

(6) Routing of aircraft, oil tankers, etc.

(7) Structural designs.

(8) Diet Problems. From what sources should nutrients be extracted to make a food at a minimum cost while observing certain minimum nutritional requirements?

These applications certainly do not constitute a complete list of useful applications. Recently, LP has been applied to problems in waste water management, plant location, oil refining, factory assembly-line balancing, and trim loss minimisation (Example when several patterns must be cut out of a rolled sheet of material such as steel). However, for the purpose of this study the list above is satisfactory.

2.2.4 Formulating A Linear Programme

In formulating a linear programme, there are certain steps involves;

Step 1. Read the statement of the problem. Identify the essential objective that is, what is the decision maker trying go determine and what factors limit his or her choice of decisions and alternatives.

Step 2. From the initial analysis in step 1, precisely define the decision variables. The decision variables should be defined in such a way that specifying the best combination of them would solve the decision maker's problem.

Step 3. From analysis of the decision maker's objective state precisely in words what the objective function is, for example, for example, "to maximise total profit per month, to minimise total cost of transportation". Then write the stated objective as a linear relationship of the decision variables, defined in step 2. This becomes the objective function of the LP.

Step 4. From step 1, determine what resources and other factors limit the choice of permissible values of the decision variables. List these restrictions in writing. For each restriction, use the decision variables to write a corresponding linear expression and constraint. It is important to note that, certain special situations may arise in solving LP; These include:-

(a) The LP may have multiple optimal solution in which case from managerial perspective this implies that there is some flexibility in the choices open to management. This may be based on other mathematical factors involving the firms operations.

(b) The LP may have no feasible solution. If this happens the problem should be re-examine the constraints of the problem to see which constraint if any can be modified to solve the problem. This may require the firm to make more resources available (example increase the budget, increase machine hours or increase labour-hours).

(c) The LP may have an unbounded solution, such as infinite profit. In this case, the problem has been incorrectly formulated. Check the problem formulation again and make sure you understand the problem and the description is complete and accurate.

2.3 LINEAR PROGRAMMING MODELS

Basically, there are three type of models in LP. These are the general allocation, Transportation, and Assingment models.

2.3.1 General Allocation Model

In this model, to obtain output, a range of resources are critically evaluated. These resources could be machine, capital, land or labour.

The objective of the general allocation problem is to determine a programme of production or a product-mix that will maximise not the individual contribution of a given product, the total overall effectiveness of a production programme. The relationship between the elements, in a situation are inequalities. The expressions are made up of objectives functions and constraints. The objective function is the desired result in linear form. This may be to maximise profit or minimise cost. while constraints refers to the limitations or anything that will curtail the attainment of objective through the available resources.

2.3.2 Assignment Model

Assignment model is also another special case of general linear programming model. It deals with, designing an optimal assignment in which exactly one origin (for example salesman) is matched with one destination (for example territory) to attain the strategy that minimise total cost or maximise total profit. The assignment is done on a one-to-one basis. Thus the pay-off matrix in any problem must be a square matrix and the optimal assignment in a given row or column of the pay-off matrix.

It is assumed that payoffs or any given matrix are known, and it's independent of each other. The problem is OPTIMAL, if the number of assignments are equal to the number or rows (or columns). The assignment model is referred to as a special form of transportation problem. Assignment problems can be solved in any of the following ways:-

(1) View it as an LP and apply the simplest method to solve it.

(2) View it as a transportation problem and apply the NW-corner rule and stepping stone method to solve it.

(3) Consider it as an assignment problem "in its own right" and apply the assignment method.

(4) Solve the assignment problem by other specialised methods such as dynamic programming or branch-and-bound.

2.3.3 Transportation Model

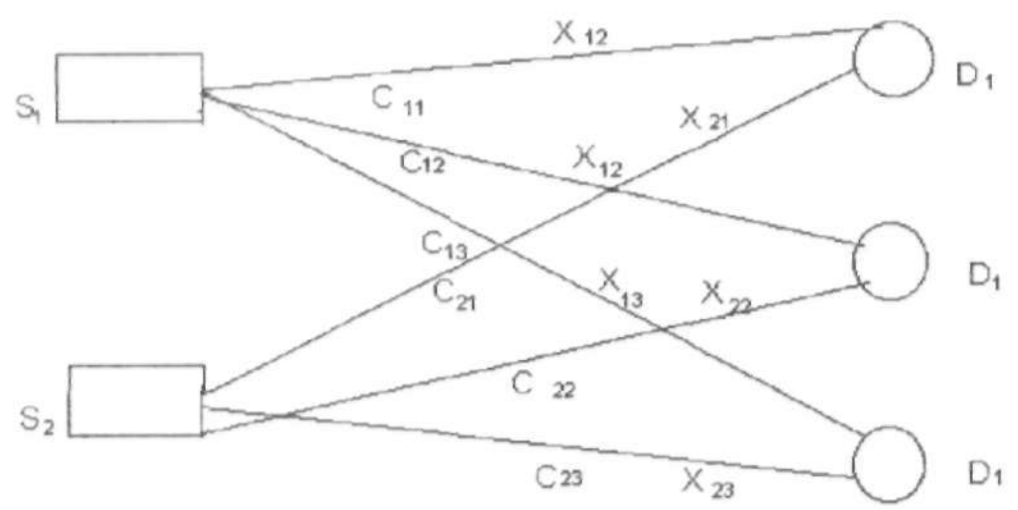
This is one of the earliest and most frequently encountered applications of linear programming. In its simplest form one product can be transported from any of the general alternative sources, or factories to each of many destinations. Transportation costs, for each feasible source-destination, combination, are known. The manufacturing or shipping volume or capacity of each sources, and demand volumes for each destination are also known. The problem is to determine which destinations are supplied by which sources, so that total transportation cost is minimised. In this study, the transportation schedule for Elf Oil Company in Northern Nigeria.

2.4 THE TRANSPORT MODEL

The transportation model refers to a specific dilemma that requires allocation of efforts to various segments of an operation. Its early usage in connection with the optimisation of transportation cost, earned it, its name. it has continued to enjoy its greatest amount of practical use, in connection with problems involving transportation and logistics in the allocation of efforts. A basic transportation problem generally involves a number of demand and supply point, where a number of sources must supply a number of destinations. Each source (for example a factory) has definite limited quantities of the goods and /or services it must supply per time period.

Each destination (for example branch or warehouse) must receive definite limited quantities of the product per time period. Linking the supply and demand points area number of possible routes, each of which has distribution cost. The objective is to determine the distribution pattern that will minimise total cost. In order words, we are trying to decide how much product should be shipped over each possible route, in ordet to satisfy all demand requirements while at the same time not exceeding the capacity limitations of any supply point. This can be represented clearly in the network model shown below:-

SUPPLY POINT	**POSSIBLE ROUTES**	**DEMAND POINTS**
(Capacity Si)	(Cost Cij)	(Requirement Dj).



(Unknown Allocation units Xij)

Fig 2.1 Network model of transportation problem.

Where, Si - capacity at supply point I

Dj - Requirement at demand point J

Cij - Cost of transfering one unit of production from supply point i to demand point j.

Z - Total shipping costs.

Assuming there are S sources and D destinations. The network model problem can also be formulated in the following algebraic form;

$$\text{Minimise, } Z = \sum_{i=1}^{S} \sum_{j=1}^{D} C_{ij} \; X_{ij}$$

$$\text{Subject to, } \sum_{j=1}^{D} X_{ij} = S_i \text{ for } i = 1,2,\text{------}, S.$$

$$\sum_{i=1}^{S} X_{ij} = D_j \text{ for } j = 1,2,\text{------}, D.$$

and X_{ij} for all I and J

Lets now consider an example problem for purpose of clarity;

<u>AN EXAMPLE PROBLEM;</u>

Suppose Nasara industries operates two plants located in Kano and Zaria, which produce the same product. There are three product distribution centres located in Jos, Bauchi, and Abuja. the monthly production capacities of the two plants are;

Factory	Supply (Unit/Month)
Kano	60
Zaria	40
	100

The average monthly requirement in each product distribution centre is given as:-

Distribution Centre	Demand (Unit/Month)
Jos	40
Bauchi	30
Abuja	30
	100

The transportation cost per unit between each factory and distribution centre are given as:-

Factory	Distribution centre		
	Jos	**Bauchi**	**Abuja**
Kano	15	16	15
Zaria	12	15	11

The Kano factory can transport one unit of the product to Jos for N15 and to Bauchi for N16 and so on. The major transportation problem is to consider the factory capacities and the distribution centre demands as well as the costs of the shipments. This is purely a resource allocation problem. The same problem can be represented in a ntetwork model as shown below.

Factories   Distribution Centre

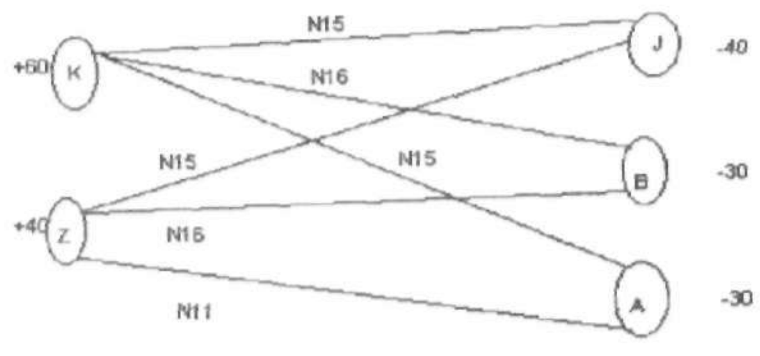


Fig 2.2 Network representation of Nasara transportation problem.

The problem can also be presented in a Matrix form especially suited for analysing transportation problems, as shown below;

Destination / Sources	JOS	BAUCHI	ABUJA	SUPPLY
KANO	15	16	15	60
ZARIA	12	15	11	40
DEMAND	40	30	30	100.

Fig 2.3 Nasara Transportation Problem.

There is one row for each factory and one column for each distribution centre. The number of units available form the Factory are written to the right of each row, and the number of units required at each centre written at the bottom of each matrix cell.

The transportation problem as mentioned earlier is a special type of linear program. To obtain an optimal solution, we first have to formulate the problem as an LP. To do so, we shall answer the following questions;

1. What is Nasara trying to decide or determine?
2. What is Nasara's objective? And
3. What are the constraints on Nasara's possible choices?

In response to the first question, Nasara must determine the number of units to ship from each distribution centre. Regarding the second question, Nasara's objective is to minimise total transportation or distribution cost.

As for the third question, the available capacities at each of the two factories cannot be exceeded and the stated demands of each of the three distribution centres must be met. To formulate the LP, use the first letter of each source and destination as an abbreviation;

Decision Variables: Let the following equal the number of units sent from the first to the second

Let, XKJ = Kano to Jos

XKB = Kano to Bauchi

XKA = Kano to Abuja

XZB = Zaria to Bauchi

XZA= Zaria to Abuja.

OBJECTIVE FUNCTION:

Minimise Z = 15XKJ + 16XKB + 15XKA + 15XZJ + 12XZB + 11XZA.

Subjeect to;

$$
\begin{array}{llllllll}
(1). & XKJ + XKB + XKA & & & & & & = 60 \\
(2). & & & & XZJ + XZB + XZA & & & = 40 \\
(3). & XKJ & & + & XZJ & & & = 40 \\
(4). & & + XKB & & & + XZB & & = 30 \\
(5). & & & + XKA & & & + XZA & = 30 \\
(6). & XKJ, XKB, XKA, XZJ, XZB, XZA & & & & & & \geq 0
\end{array}
$$

Constraint (1) says that the total supply of the Kano Factory is 60 units.

Constraint (2) similarly expresses the supplies at Zaria. Constraint (3) - (5) are demand constraints for Jos, Bauchi, and Abuja respectively.

One way to solve Nasara's problem is to apply the simplex method to the above LP. However, conceptually the transportation method is similar to the simplex method. we begin with an initial feasible solution and test for optimally. If the solution is not optimal, we improve it by changing the shipping pattern (called reallocating).

Continus checking and reallocating until an optimal solution is found,.

These steps are outlined clearly in the decision tree below:

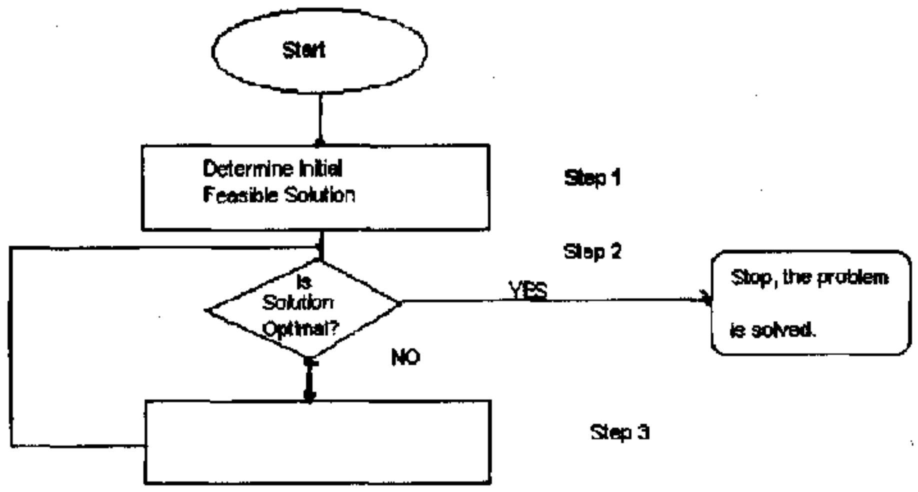


Fig:2 .4- General solution approach for the transportation problem.

One key difference between the transportation method and simplex method concerns the determination of an initial feasible solution. When simplex is used to solve a minimisation problem, we must add artificial variable in order to make the origin (Where all decision variables are zero) artificially feasible. As simplex progress from tableau to tableau the artificial variables are dropped as they become non basic. Eventually, a truly feasible solution is found, at which point all artificial variables have been dropped. The transportation method eliminates the need to use artificial variables because it is quite easy to find an initial solution that is feasible without them. Moreso where large constraints are involved it will be enormous as to require the use of LP computer programme. Thus for computational efficiency the transportation mehtod proves much more efficient than the method.

We shall delve into the solution stage for Nasara problem in due course.

2.4 1 Approaches To The Transportation Problem

According to N. Paul Loomba transportation problems are approached through some three conventionally established steps. The first step involves making the initial assignment in a manner that an initial feasible solution is obtained.This involves assigning the largest amount requirements to a destination after considering its needs and the capacity of the sources. The cells having the assignments are called OCCUPIED CELLS and those with no assignments are referred to as UNOCCUPIED OR EMPTY CELLS. The next is the test for optimality stage. Here determine whether the solution is optimal or not. This is done by evaluating the opportunity cost associated with the empty cells in a given solution. If the opportunity cost of all empty cells are positive orzero,definite be confident that an optimal solution has been obtained. However, if there appears a negative value in the empty cells, the solution is not optimal and must be improved. When the happens to be the case, determine a new and better basic feasible solution. This next step involves furhter evaluation of the empty cells, so that a better improvement in the objective function is obtained. Once this has been obtained, steps two and three are repeated until the best solution is obtained.

2.4.2 Balancing The Transportation Problem

For any trasnportation problem a solution can onl;y be obtained when amount of capacity and demand in the matrix are equal. In reality however, supply and demand are not likely to be exactly equal. Often, companies maintain greater capacity than we average demand so that they are able to meet peak demand.Other prefer to maintain capacity at average demand levels and build inventories in low demand periods to meet peak demand.

These policies inevitably lead to unequal demand and supply during some periods. When such a situation arises, the transportation method has designed a way that facilitates obtainnig a solution. This it does by forcing a balance through the introduction of "a dummy warehouse, to absorb the excess supply or a dummy plant to fill the excess demand". This dummy warehouse or plant requires exactly the amount that is needed to make total supply equal total demand. It is note worthy that the amount of product assigned to a dummy plant or warehouse will never be produced or shipped. In actual sense, products demanded by a dummy warehouse will reduce the actual production level of the plant designed to ship to it. The assignment of production to a dummy plant will determine customers who will or will not be served. Since no shipment will actually be made to or from these dummy, it does not matter what costs are assigned to the dummy routes so long as all dummy routes are assigned the same cost. Unequal costs will unfairly bias the real allocations. Nonetheless, for computational convenience the associated transportation cost for any dummy created are normally set at zero. After addition of the dummy facility the problem is the solved in the normal way.

2.4.3 Degeneracy In Transportation Problem.

For any given transportation problem, there is an exact number of occupied cells which will allow a solution. It is one less than the sum of the number of rows and columns in the tranportation problem (m+n -1) where, M - number of rows, and N=number of columns. Degeneracy problem can arise from the presence of too few occupied cells in a transportation problem, either after an initial allocation or as the result of re-allocation procedures. This happens when more than two constraints intersect at a single extreme point, resulting in one or more of the basic variables having a zero value. Moreover, it can happen when inclusion of a favourable empty cell (cell having the highest oppotunity cost) results in a simultaneous vacating of two or more of the presently occupied cells. A degenerate solution will not allow the various allocations to be assigned to all rows and columns.

Thereby causing difficulties for the transportation because, we cannot evaluate its optimality without without taking corrective action. It is thus, useful to always check after initial allocation and before solutions, to ensure that an optimal solution can be attained.

To solve degeneracy, we make an artificial allocation of some small quantity in one of the empty cells so that the number of occupied cells become m+n - 1. The small quantity could be the figure zero or could be denoted by the Greek letter E (epsilon). The quantity's addition to or subtraction from any number does not have any significant change in the number. The transportation problem is then solved in the normal way, any occupied cell with zero or E implies no shipment.

2.4.4 Making The Initial Assignment.

The Nasara transportation problem set up in figure 3e, will be used to demonstrate the various stages in solving a standard transportation problem. There are four methods of establishing an initial feasible solution. These four methods are:

(1) Northwest Corner Rule (N-W Corner Rule)

(2) Vogel's Cost Method

(3) Least cost Method.

(4) Russell's method.

Only the first three methods will be discussed in this project. Before proceeding, it is important to remember that the initial feasible solution must satisfy the following three criteria.

(a) The Supply of each Source must be used up.

(b) The demand of each destination must be met.

(c) The number of entries made in the transportation table must equal m+n -1.

We shall now proceed to make the initial assignment using three of the four methods.

2.4.4.1. North West Corner Rule.

Using the Nasara problem in fig 2 we make the initial allocation. The NW-corner rule states that we start in the North west corner of the matrix (in our case square K-J) and allocate as many units as possible to that square while observing the supply and demand constraints. Then move to the right allocating units until row 1 supply is used up. Then move to the right allocating units until row 1 supply is used up. Then move down to row 2, and allocate as many units as possible along row 2 until the row 2 supply is used up. *Continue in this manner until all row supplies are exhausted.* Note that, the NW-Corner initial solution will always look like a stair-cases. In addition it pays no attention to the relative costs of the various routes, when making the first assignment.

The initial solution is given below;

Destination Source	JOS (J)	BAUCHI (B)	ABUJA (A)	SUPPLY
KANO (K)	$\boxed{15}$ 40	$\boxed{16}$ 20	$\boxed{15}$	60
ZARIA	$\boxed{12}$	$\boxed{15}$ 10	$\boxed{11}$ 30	40
DEMAND	40	30	30	100

Fig 2.5: North west corner initial solution for the Nasara problem.

The solution above was obtained as follows:

(a) Start with square K-J (the NW-corner square). Allocate as many units as possible to that square. Since the supply of row 1 (Kano) is 60 units and the demand of column 1 (Jos) is 40 units. The maximum number of units can be allocate to square K-J is 40, so as not to exceed the demand constraint for column 1.

(b) Since column 1 demand is used up, we move to column 2 (square K-B). The remaining supply of row 1 is (60 - 40 = 20). The demand of column 2 is 30 units, so we can allocate the remainning supply of row 1.

(c) Column 2 has a remaining demand of 10 units. Move along column 2 down to row 2. Row 2 has supply of 40 units.Thus Allocate the remaining demand of 10 units for column 2 to square Z-B. This cancels out column 2.

(d)Row 2 has a remaining supply of 40-10 = 30 units. Move to the right to square Z-A. Column 3 has a demand of 30 unit. So allocate the remaining 30 units to square Z-A. This completes the NW-corner initial feasible solution.

The total cost of the solution is easily computed as follows:-

40 (N15) + 20 (N16) + 10 (N15) + 30 (Nil) = N1400.

Determine whether or not this cost figure can be improved. Note that the total number of allocations is Four (squares K-J, K-B,Z-B, AND Z-A), which satisfies the M+n -1 criterion. M=2, n=3. M+n -1 =e+3 - 1 =4.

2.4.4.2 Vogels Approximation Method Or Penalty Method

VAM is a heuristic, that is a method which assist to produce results that are approximately optimal. In practical VAM appears to produce the optimal solution in a large percentage of problems. It proceeds by iteratively performing the following laid down steps;

STEP 1. The first compute rim values for each row and column. The rim value for a given row (or column) is the difference between the two smallest cost in that row (or column). Using the Nasara example, the rim values are computed in fig 2.6 below as follows:

Row 1, 15- 15 = 0

Row 2, 12- 11 = 1

Column 1, 15- 12 = 3

Column 2, 16- 15 = 1

Column 3, 15- 11 = 4.

Rim values are interpreted as minimum penalty costs. For example consider row 2. If we had completed choice, we would certainly like to ship all we could from Zaria to Abuja, since the unit shipping cost (N11), is the minimum for row 2. Our second best choice would be to ship to Jos since its shipping cost (12), is the second smallest cost in row 2.

Thus the minimum penalty cost for not shipping to Abuja is N12 - N11 = N1 per unit, of course, the penalty may be greater if ship instead to Bauchi.

RimValues.

	3	1	4	
DESTINATION / SOURCE	J	B	A	SUPPLY
K	15	16	15	60
Z	12	15	11	40
DEMAND	40	30	30	100

Fig 2.6: Calculation of Rim values.

STEP 2. Now select the largest rim value and allocate as many units of product as possible to the minimum cost square in the corresponding row or column. The rationale behind this rule is to make an allocation that would avoid the largest penalty cost. Should there be a tie for the highest penalty difference one can arbitrarily choose anyone of them. Another way would be to follow the reasonable rule of thumb that would select the tied rim value that has the lowest cost entry in its column (or row). Fig 2.6 above is obtained following the steps (1and2) laid down above.

STEP 3. Further deduct the amount supplied and demanded form plant capacity and warehouse demand respectively. The amount remaining will be unallocated supply and demand. If plant capacity and or a warehouse demand is totally exhausted or satisfied by the action cross out the appropriate row or column, to eliminate it from further consideration.

The maximum rim value is obtained in column 3, which has a rim value of 4. since the lowest cost in column 3 is N11. We shall allocate as much as possible to the square Z-A. The supply for row 2 is 40 units while demand for column 3 is 30 units to square Z-A. Thus "X" (cross out) column 3 from further consideration. The result of the 1st iteration is shown in fig 2.7 below;

	3	1	4	
Destination / source	J	B	A	SUPPLY
K	15	16	15	60
Z	12	15	11 30	10
Demand	40	30	30	100

Fig 2.7: Iteration 1, steps 2 and 3.

ITERATION 2. Steps 1 and 2 will be repeated in fig 2.7. STEP 1 compute the rim values as before, but this time do not use any costs that occur in rows or columns that have been deleted from consideration. The rim values computed from fig 2.7 are:

Row 1, 16 - 15 = 1

Row 2, 15 - 12 = 3

Column 1, No change = 3

Column 2, No change = 1

Column 3, Deleted.

STEP 2. The maximum rim values is 3. Row 2 and Column 1 have the same rim value of 3. Following a reasonable rule of thumb approach, we select the tied rim value that has the lowest cost entry in its columns (or row). Incidentally for both column 1 and row 1, square Z-J happens to be the lowest cost. We allocate as much as possible to the square. Since the unexhausted supply is only 10 units, we allocate all 10 units. The results of iteration 2 are shown in fig 2.8

Destination / Source	J	B	A	SUPPLY
K	$\boxed{15}$	$\boxed{16}$	$\boxed{15}$	60
Z	10 $\boxed{12}$	$\boxed{15}$	30 $\boxed{11}$	X
DEMAND	40	30	X	100

Fig 2.8 Iteration 2.

ITERATION 3. Note that since row 1 is the only row that is left no choice concerning the remaing allocation. The results are shown in fig 2.9. Now lets consider initial allocation using another method.

DESTINATION / SOURCE	J	B	A	SUPPLY
K	30 $\boxed{15}$	30 $\boxed{16}$	$\boxed{15}$	X
Z	10 $\boxed{12}$	$\boxed{15}$	30 $\boxed{11}$	X
DEMAND	40	30	X	100

Fig 2.9 Iteration 3.

2.4.4.3 Least Cost Method.

This method involves making allocation of as much product as possible to that matrix cell with the lowest shipping cost, while considering the origin capacity and destination requirement. Then we move to the next lowest - cost cell and make as much allocation as possible depending on the remaining capacity and requirement for its row and column and so on till intial feasible solution is obtained. When there is a tie for the lowest cost cell in the process of allocation, one can exercise judgement or arbitrarily choose any of the cells to break the tie. This method requires more computational effort than the NW-corner rule. However, it typically provides a better starting solution, which will then require fewer iterations. Lets now use the least cost method to establish a starting solution for the Nasara problem.

First scan the table for the least cost square. The least (Nl 1) occurs at square Z-A. Now allocate 30 units which exhausts the demand in column A and results in the following figure;

SOURCE \ DESTINATION	J	B	A	SUPPLY
K	15	16	15	60
Z	12	15	11 30	10
DEMAND	40	30	0	100

Fig 2.10 Initial allocation using least cost method.

Now with column A deleted, locate the next least square, which is square Z-J (N12). Allocate the remaining 10 units which is the maximum supply available in row 2. This results in the fig below.

DESTINATION / SOURCE	J	B	A	SUPPLY
K	30 [15]	30 [16]	[15]	60
Z	10 [12]	[15]	30 [11]	0
DEMAND	30	30	0	100

Fig: 2.11: Initial allocation (continued)

Now that column A and row Z are deleted form consideration, continue in a like manner along row K and allocate as follows;

30 units to K-J (exhausting column J)

31 units to K-B (exhausting column VB and row K)

This procedure results in the following starting solution for the Nasara problem.

Destination / source	J	B	A	Supply
K	30 [15]	30 [16]	[15]	0
Z	10 [12]	[15]	30 [11]	0
Demand	0	0	0	100

Fig. 2.12: Initial feasible solution for Nasara problem.

Note that the number of entries made is four which equals m+n-1. Now proceed to check whether the initial feasible solution obtained in the various methods is the best solution or not.

2.4.5 Test For Optimality

Finding an initial feasible solution is only the first step in solving a transportation problem. There is need to establish whether the basic feasible solution obtained is an optimal solution or not and if not a way of improving it. It is this process of checking to ascertain if an initial feasible solution is optimal that is referred to as **Test for optimality.** There are two methods for testing for optimality in transportation problems. These are;

1. The stepping stone method-which is based on directly calculating the opportunity cost of each empty cell.
2. The modified distribution method (modi) - which is based on the concept of dual variables used for evaluating the empty cells of a given program.

2.4.5.1. The Stepping Stone Method.

After each initial assignment is made, ther is need to determine if improvements can be made in the total distribution cost by shifting some of the units to be shipped to some of the opens squares in the transportation table. One method of doing this is to trace through the increases and decreases necessary to keep the solution feasible. Uder this method, test are began by evaluation the opportunity costs. For each of these empty cells the traced path should take advantage of occupied cells only at which to make a 90 angle turn. The objective is to trace a path back to the starting empty cell. Appropriate plus and minus signs are then placed alternately, with the plus sign placed at the beginnig empty cell, and negative sign in the occupied cell to which an arrow is drawn. These closed loops for each of the empty cells are made use use of to calculate the respective opportunity costs. The number of the closed loops is normally equal to the number of empty cells in the transportation matrix. The absence of any negative opportunity costs in the empty cells indicate that an optimal solution has been obtained. But if there appears any negative sign, the given problem is not optimal and should be improved. Lets now use the Nasara problem initial feasible solution obtained through NW-corner method in fig. 2.5 to illustrate the complexities in tracing this closed path.

Lets see what would happen if one unit were allocate to the open square Z-J. Look at fig 2.5 (our current NW corner intial solution) and imagine placing a 1 in square Z-J. What "chain-reaction" would this cause? If we put 1 in square Z-J. column 1 would thenadd up to 1 +40=41 units exceeding the demand of 40 units. Thus to prevent this from happening there must decrease square K-J from 40 to 39. But then row 1 would decrease down to 39 + 20=59, which is less than the supply of 60 for that row. Thus to maintain balance, add 1 unit to square K-B, giving it 1 + 20 = 21. But then column 2 will add up to 21 + 10 = 31 which exceeds the column 2 demand of 30 units. So by subtracting one unit form square Z-B giving it 10 - 1 =9 units. Now, back to where we started, namely square Z-J, and we find that row 2 adds up to the required supply of 400 units as follows:

Z - J has 1 units.

Z - B has 9 units

Z - A has 30 units.

Row 2 has 40 units.

We can summarise this chain reaction in fig below:

	J	B
K	$\boxed{15}$ 40 -	$\boxed{16}$ 20 +
Z	$\boxed{12}$ +	$\boxed{15}$ - 10

Fig 2.13: Summary showing chain reaction.

Now by how much will the total cost increase if we were to allocate one unit to the open square Z - J? To answer this, note that increasing the allocation to square Z - J by one unit (from 40 to 39) would save N15. Increasing the allocation to square K-B by one unit would cost an additional N16.

And finally, decreasing the allocation to square Z-B by one units would save N15. The net cost increase would be,

12 - 15 + 16 - 15 = N2

The negative sign means that the cost would actually decrese by N2 per unit for every unit allocate to square Z-J. This confirms that the NW-corner solution is not optimal.

To evaluate each of the remianing open squares in a similar manner. For each open square, identify the closed path of plusses, and minuses, and compute the net cost increase. The result of evaluating each open square is summarised in table 1 below;

OPEN SQUARE	CLOSED PATH.	NET COST INCREASE
Z - J	+ ZJ - KJ + KB -ZB	12 - 15 + 16 - 15 = - 2
K - A	+ KA - ZA + ZB - KB	15 - 11 + 15 - 16 = 3

_ Table 2.1: Evaluate of open squares.

It is a fact that, for each open square, there is a unique closed path. It is permissible to skip over some basic squares in constructing the closed path. The test for optimality as follows; When minimising costs, if all cost increase evaluations of open squares are positive or O, the current solution is not optimal ass in our cases. so we continue to step. 3.

STEP 3. MOVE TO A BETTER FEASIBLE SOLUTION. Here these things will be done;

a. Determine which open square is to become a basic square that is, determine the entering open square.

b. Determine which of the current basic squares is to become an open square, that is determine the exiting basic square.

c. Generate a new feasible solution.

Determining the Entering open square. This is normally the open square with the highest per unit cost decrease. (open square having the most negative net cost increase). In our example no choice to make, because there is only one open square with negative sign. The choice is obvious.

Determining the Exiting Basic Square. The rule for determining which basic square is to become an open square is to examine the closed path associated with the entering square, make a list of the basic squares on this path which have minus signs on them. These squares are called <u>donors.</u> The exiting basic variable is the donor that has the smallest current allocation made to it. In our example the traced closed path showing the "+" and "-" squares is shown below;

DESTINATION / SOURCE	J	B	A	SUPPLY
K	$\boxed{15}$ 40 (−)	$\boxed{16}$ (+) 20	$\boxed{15}$	60
Z	$\boxed{12}$ (+)	$\boxed{15}$ (−) 10	$\boxed{11}$	40
DEMAND	40	30	30	100

Fig 2.14: Traced closed path showing "plus" and "minus" squares.

This makes spotting of the donors, the exiting variable easy. In our example the choice is various Z-B.

Generating a new feasible solution: To generate a new feasible solution we let the entering square become a basic square and force the exiting square to become an open square. The number of units originally allocated to the exiting square is now alloacted to the entering basic square. Each donor square is decreased by this amount.

In this case (1) allocate 10 units to Z-J, (2) make Z-B an open square having allocation, (3) decrease K-J by 10 units and (4) increase K-B by 10 units. This is summarised in table 2.2 below:-

	Square	Previous Status	Previous Value	New Status	New Value
Enter	Z - J	Open (+)	0	Basic	10
	K - J	Basic (-)	40	Basic	30 =(40 - 10)
	K - B	Basic (+)	20	Basic	30=(20+10)
Exit	Z - B	Basic (-)	10	Open	0=(10 - 10)

Table 2.2: Summary of evaluation of open squares.

The new current solution is shown in fig 2.15. Note that in step 2, the net cost increase for square Z-J was found to be N2. Since Z-J entered at 10 units, the new feasible solution has an assocated cost N2/unit x 10 units = N20 lower than the cost of the solution shown in fig. 2.11. Thus the new solution is better.

DESTINATION / SOURCE	J	B	A	SUPPLY
K	15 30	16 30	15	60
Z	12 10	15	11 30	40
DEMAND	40	30	30	100

Fig 2.15: A new improved solution.

The next step is to evaluate each open square of the current solution in fig. 2.15 for the net cost increase. The results of this evaluation are summarised in table 2.3 below;

Open square	Closed path	Net cost increase (N)
K-A	+KA-ZA+ZJ-KJ	15-11+12-15=1
Z-B	+ZB-ZJ+KJ-KB	15-12+15-16=2

Table 2.3: Summary of evaluation of open squares.

Since all evaluations are positive, the current solution is optimal. The end of algorithm is here.

Using the result of the above iterations, that Nasara should send the following units of product in order to minimise total cost:-

30 units from Kano to Jos.

30 units from Kano to Bauchi.

10 units from Zaria to Jos

30 units from Zaria to Abuja.

The total (minimum) cost to Nasara will be:-

(30x15) + (30x16) + (10x12) + (30x11) = N1380=

This compares with the NW-corner solution cost of N1400=. Thus seen how the solution was improved.

Interestingly, the Nasara optimal solution obtained through the least cost method in fig 2.12 and VAM in fig 2.9, compares equally with that obtained through the stepping stone method. Thus least cost method typically provides a better starting solution 2 which requires fewer iterations or on iteration at all as in this present case. Note that, the least cost method will not always perform so well, but it is typically much better than the NW-corner rule. On the other hand VAM's primary use is to establish an initial feasible solution for the stepping stone method. Nonetheless when compared with NW-corner rule view the NW-corner rule as a crude heuristic. It completely ingores costs, gives poor results, but is extremely easy and fast. Most management scientist would probably select VAM over the NW-corner rule, because VAM's additional computational cost seems to justify the large benefits realised. For the stepping stone method, seen that it is rather tedious and time consuming.

Modified Distribution Method (Modi)

Modi is simply an algebraic version of the stepping stone method. However, the purpose only indicate how it works to solve transportation problems, so that it may be compared with the stepping stone method.

MODI the test for optimality is conducted by the utilisation of dual variables mout having to draw closed paths for each empty cell. As a matter of fact MODI method easily permits us to identify the most favourable empty cell soon as a set of values for the dual variable has been chosen. Then a closed path through the most favourable empty cell is drawn so that a new basic feasible solution can be obtained under the guidance of the closed path.

MODI, row (R) and column (k) indexes are first computed for all rows and columns. Generally, this is done by setting index of one row or one column with the greatest number of allocation equal to Zero. The remaining matrix check values are set relative to it by equating each matrix square value (cost) which items have been allocated, to the sum of the matrix check values for rows and column (C=R+K). This is followed by the evaluation of the cells without allocations using the coefficients C-R-K. If this results in a zero or positive value for all unoccupied cells, the solution is optimal. But if it is not, the solution must be improved upon using the closed path rule. It should be remembered that in the modified distribution method, only one closed path is drawn after the highest opportunity cost cell has been identified. However, draw illustrate MODI in solving the Nasara problem.

STEP 1. For the initial feasible solution, we use the NW-corner method solution in fig 2.5.

STEP 2. Is the current solution optimal? To answer this question draw introduce the algebra of MODI. For each row of the of the transportation table, the associate a variable Ui and for each column a variable Vj and for each column a variables Vj. Since we have two rows and three columns, this produces the variables U1,U2, and V1,V2,V3 which are in corporate into the transportation table as shown in fig below;

	1	2	3	SUPPLY	Ui
1	40 $\boxed{15}$	20 $\boxed{16}$	$\boxed{15}$	60	U1
2	$\boxed{12}$	10 $\boxed{15}$	30 $\boxed{11}$	40	U2
DEMAND	40	30	30	100	
Vj	V1	V2	V3		

Fig 2.16: Creating variables Ui and Vj.

Note that we have relabeled the rows 1,2 and the columns 1,2,3. Let Cij be the unit cost associated with square (i.j). (the square in row 1, column j). For example C11 = 15, C12 = 16, = 12 etc. If square (i,j) is a basic square then.

Cij = Ui + Vj. (2-1).

You can proceed to use the above equation to solve for Ui and Vj. We select the Ui or Vj set it equal to zero and solve for the remainning values. We select row having the largest number of basic squares, we select any arbitrarily. We select Row 2 and set U2 = 0. This starts the process of solving for the other six equations corresponding to the basic squares (1,1), (1,2), (2,2,) and (2,3) in the form of EQ (2-1);

$15 = C11 = U1 + V1$

$16 = C12 = U1 + V2$

$15 = C22 = U2 + V2$

$11 = C23 = U2 + V3$

Further calculate:

If U2 = 0 then V2 = 15 (from the third equation)

If U2 = 0 then V3 = 11 (from the fourth equation)

If V2 = 15 then U1 = 1 (from the second equation)

If U1 = 1 then V1 = 14 (from the first equation)

The above results can now be recorded as in fig below;

	1	2	3	SUPPLY	Ui
1	15 40	16 20	15	60	1
2	12	15 10	11 30	40	0
DEMAND	40	30	30	100	
Vj	14	15	11		

Fig 2.17: Evaluation of open squares.

To ascertain if the current solution is optimal, we now calculate for each non basic square the value Cij- Ui - Vj. If all these values are nonnegative, the current solution is optimal. If not proceed to next step.

Let evaluate the current solution. For each open square in fig 2.17, we calculate Cij -Vj. These results are recorded in fig 2.18 below;

	1	2	3	SUPPLY	Ui
1	$\boxed{15}$ 40	$\boxed{16}$ 20	$\boxed{15}$ 3	60	1
2	$\boxed{12}$ -2	$\boxed{15}$ 10	$\boxed{11}$ 30	40	0
DEMAND	40	30	30	100	
Vj	14	15	11	=	

Fig 2.18: Evaluation of open squares continued.

For clarity all allocations basic squares are circle. Consider the open square (1,3). There are Cij-Vi-Vj = 15 - 1 -11 =3

Then for open square (2,1) we have, 12 - 0 - 14 = -2. Since these are negative result, the solution is not optimal. We move to step3.

STEP 3. Move to a better feasible solution. This step basically has the same procedure as the stepping stone method. As square (2,1) is the only square with negative value use it as the entering square while basic square (2,2) exits. Using the stepping stone method, a new feasible solution as in fig 2.19 below;

	1	2	3	SUPPLY	Ui
1	15 40	16 20	15 3	60	1
2	12 -2	15 10	11 30	40	0
DEMAND	40	30	30	100	
Vj	14	15	11	=	

Fig 2.19: New Better feasible solution.

Return to step 2 to confirm if the current solution is optimal. Using fig 2.19 above, we find Ui and Vj as before and calculate the Cij- Ui - Vj equations for the open squares. The result is in fig 2.20 (we set U2 = 0 first).

	1	2	3	SUPPLY	Ui
1	15 30	16 30	15 1	60	3
2	12 10	15 4	11 30	40	0
DEMAND	40	30	30		
Vj	12	13	11		

Fig 2.20: Evaluation of open squares.

Since all open squares are non negative, the current solution is optimal. Note that the sequence of MODI iterations corresponds to those of the stepping stone method. A check on the total costs reveals, 30 x 15 + 30 x 16 + 10 x 12 + 30 x 11 = N1380. This compares equally with the previous solutions obtained using least cost and stepping stone method.

2.4.6. Designing A New And Better Programme (comparative analysis).

After ascertainning that a given programme is not optimal one has to improve on the programme to obtain a much better Feasible solution. As already observed above, the stepping stone method achieves the design of a new and better programme by subtracting (along the closed path) the smallest quantity allocated to a basic square with a negative sign from all other basic squares at which negative signs are placed. In a like manner add this same quantity to each square, be it occipied or empty at which a positive sing is indicated. This results in reallocation if the units of products for transportation. One then examines whether it is optimal or not by repeating the determination of the opportunity cost of each empty square. Should this indicate a non optimal solution we repeat the improvement process until the optimal solution is obtained.

2.5 APPLICATIONS OF THE TRANSPORTATION METHOD

In this section, a list of both theoretical and actual management applications of the transportation problem formulation and technique will be presented. Attempt will be made to present a brief descriptive example of a given problem within each of application.

Though the terms "supply", "demand", shipment", "distribution", destination", and "source" have been used repeatedly in this write up. It should be noted that this does not signify that transportation method is contrained to situations involving the physical transportation of goods only. On the contrary many cases of allocation may require a movement in time rather than in space, such as where goods produced in one time period are sold to customsers whose demand is received in another time period. Alternatively, it may refer to the assignment of one class of items to another as in the assignment of a set of jobs or task to set of employees. Nonetheless, in subsequent paragraphs division has been made of the application into theoretical and actual.

This is purely for academic reasons. On the contrary, as will be seen all example problems are practicable and therefore could actually pass as

General Applications.

2.5.1 Theoretical Applications.

Here, a number of applications which are not actually "transportation" situations are described, although they could be solved using the transportation method. Some of these examples include;

(a) Production /Distribution Planning.

While distribution planning has already been illustrated in the Nasara problem. The intention here is to show how the distribution problem can as well be a production planing problem. This occurs where plants have diffrent production costs.

Consider the following example, Bebeji Stone Industry operates three gravel pits located in the geographically separate towns of Kiru, Bichi and Gwarzo. Gravel mined at these locations is distributed by truck to concrete plants located in the towns of Kano, Katsina, Daura and Kazaure. The weekly production capacities of the three stone pits are 18, 26 and 20 truckloads respectively. The weekly demands of the four "markets" are 19, 15, 12, and 18 truckloads respectively. The production costs differ at the three quarries and are estimated to be N50 per truckloads at Kiru, N35 per truckload between each quarry and market are given in table 2.4 below (in Naira per truckload). The costs are based on kilometer and driver wage rates.

The puzzle for Bebeji is to determine the best way to satisfy the weekly market demands for gravel.

Bebeji problem is an example of both a production as well as a production distribution problem. This stems from difference in production costs as stated earlier. When there is no difference in production costs the problem can be viewed as a distribution problem.

QUARRY	MARKET			
	KANO	KATSINA	DAURA	KAZAURE
Kiru	25	40	45	40
Bichi	20	25	35	30
Gwarzo	20	30	40	35

Table 2.4 Bebeji Production Problem.

(b) Production / Inventory Planning.

This area of application is generally included in the broader area of production planning "or" master scheduling". These are otherrealistic situations that can be handled conveniently in the transportation format. Consider the following example. DeltaPlast Industries must schedule production of a 3000 gallon water tank over the next four months. Demand for this tank is expected to be 30units in Jan, 20units in Feb,25units in March, and 28units in April. Regular time capacity is 19units per month. By using overtime an additional 5units can be produced per month. A tank produced on regular time cost N450 to manufacture, on overtime the same tank cost N650 to manufacture. The holding cost to carry a tank in inventory from one month to the next is N50. The problem here is to determine the optimal production /

inventory plan for the next four months. This requires planning production and inventory on order to minimise the cost of meeting the forecasted demand over the four month planning horizon.

The problem can be formulated and solved as a transportation problem. One can take each of the four months to represent destinations with corresponding demands. There are also eight possible sources of supply corresponding to regular time and overtime production in each of the four months. After nothing the effect of the holding cost of N50 per tank per month, one can then solve the problem using the transportation method.

(c) Vendor Selection.

Solutions to vendor selection problems, when vendors specify a limit on the quatities they are able to supply in the short-run, can be obtained using transportation method. Consider the following example, Ladi Kwali pottery uses three types of clay (C1,C2, and C3) for its pottery class instruction. Four wholesale vendors (V1, V2,V3 and V4)offer the clay. Each month Ladi Kwali uses 1500, 1800 and 900 pounds of C1, C2 and C3 respectively. The vendors have indicated to Ladi Kwali that they can supply the following monthly quantities of all three types of clay (in pounds);

Vendors	V1	V2	V3	V4
Supply	1800	800	1500	1200.

Vendor prices for each type of clay vary due to variations in production costs and in transportation costs to ship the clay to Ladi Kwali workshop. The issue is for Ladi Kwali to determine from which vendors it should purchase the various clays.

Vendor

Clay	V1	V2	V3	V4
C1	25	10	15	22
C2	45	15	39	*
C3	60	30	36	50

Ladi Kwali's problem can be formulated and solved as a transportation problem. The objective will be to minimise the monthly cost of purchasing the clay. In these each of the three clays is viewed as a destination and each vendor as a source (or Vice versa). This can subsequently be solved in the normal transportation method.

(d) Schedulling Resource Requirements.

Here the versatility of the transportation algorithm is demonstrated as seen in the following example. Standard Construction Limited has been awarded a contract to do part of the concrete work on a large auditorium in Bayero University Kano. Standard has scheduled a series of five large concrete pours in the upcoming week, to begin on Sunday and go through to Thursday. For these pours, standard must use a new kind of concrete-forming method that requires special concrete form panels. Standard currently does not own any of thse panels and does not anticipate any immediate use of them once this special job is completed. Standard anticipates it will need the following quantities of these panels: 100 on Sunday, 150 on Monday, 70 on Tuesday, 120 on Wednesday, 140 on Thursday. New panels can be purchased at the cost N50 = per panel. A panel that is used on a given day can be on the following day, because the used panel can be remived as soon as the concrete has set. The cost of reusing a panel is estimated at N20. However, due to some labour-scheduling problems only half of the panels used on Tuesday could be prepared for reuse on Wednesday , the other half could be ready on Thursday. Otherwise panels used on a given day can ready for reuse the following day.

In order to maximise profit, standard need to decided how to obtain the required panels at minimum cost for the week's work. In this example, the destination can be represented by each of the five days of the upcoming week, Sunday, Monday, Tuesday, Wednesday and Thursday. The sources of panel supply are of two general types; either brand new panels or used panels (those used the previous day of the week). Using this as a base, the problem can be solved as a transportation problem.

(e) Assignment Problems.

Another major class of problems that can be modelled as transportation problem is in the assignment of tasks or jobs to machines, employees, or work centres. The key to treating this type of problemas a transportation problem is ensuring that the capacities of the machines and the requirements of the jobs are expressed in common units, such as standard hours.

Also included in this class of problems are such applications as in contract awards to bidders, assignment of sales - people to territories, strategic military assignments, workforce planning, media scheduling for advertising and traffic routings.

2.5.2 Actual Applications

The consideration here is essentially about distribution - type problems. In these kind of problems the objective is to "transport" a homogeneous commodity from various "origins" to different "destinations" at a minimum total cost. This is purely what this project is centred on. Since the following chapter is devoted to this class of problems, further discussion on it will be withheld at this point.

CHAPTER THREE

BACKGROUP INFORMATION AND DATA COLLECTION

3.1 INTRODUCTION

The history of petroleum development in Nigeria goes back to around 1937 when shell BP Development Company started prospecting for crude oil over the country. Up to 1955 by virtue of the home government position as the colonia master, shell was the only company that had licence to search for oil in any part of the country. However other companies have joined in prospecting for oil and among them are; ELF petroleum, Gulf oil company, Mobil production, Philips, Ashland and Henry Stephens elz.

3.2 BACKGROUP INFORMATION ON ELF OIL COMPANY

ELF Oil Nigeria Limited is formerly known as ELF Marketing Nigeria Limited, Nigeria's youngest major petroleum marketing company was born in February 1983 as a joint venture between ELF Acquitaine of France and the Ibru Organisation.

ELF Oil's first target is to develop her image in Nigeria to the level of the respect and reputation it enjoys in France and Europe where the group has a substantial share of the market. ELF Acquitaine has an international reputation for its production and sales of high quality lubricant for marine, automobile and industrial purpose. ELF products are carefully formulated with efficient and modern additive combinations to meet the latest international standards. ELF is presently supply 6% of Nigeria's internal requirement petroleum ELF Oil marketing policy and ensure the efficient distribution of its products ELF operates four marketing divisions called Regions - North, East, Mid - west, and West. Under each region there are marketing areas called locations.

The company's products are directed to three main petroleum market segments namely, Automotive, Industrial and Domestic markets. The products reach the market through the retail, industrial and domestic product sales outlets.

3.2.1 Automotive / Domestic Users

Automotive and Domestic market segment is serviced through the company's retail outlets comprising of over 200 service stations through the country. In addition to ensuring that laid down standards are maintained, the company takes full responsibility for the efficient operation of these stations. Training and development programs are organised for services station personnel to enable them improve efficiency and increase sales at the company's outlets.

3.2.2 Industrial Users

The industrial market segment includes both major and minor consumers within the heavy industries, civil engineering companies textiles and other government corporations and institutions which are with supplied with automotive gas oil (AGO)and special grades of lubricant.

3.2.3 Management

The company's general policy is determined by a board of directors. About 100 employees carryout the company's business activities throughout the country. This figure comprises management both senior and junior staff including personnel of some retail outlets which are run directly by the company. Below is the organisational chart of ELF oil Nigeria;

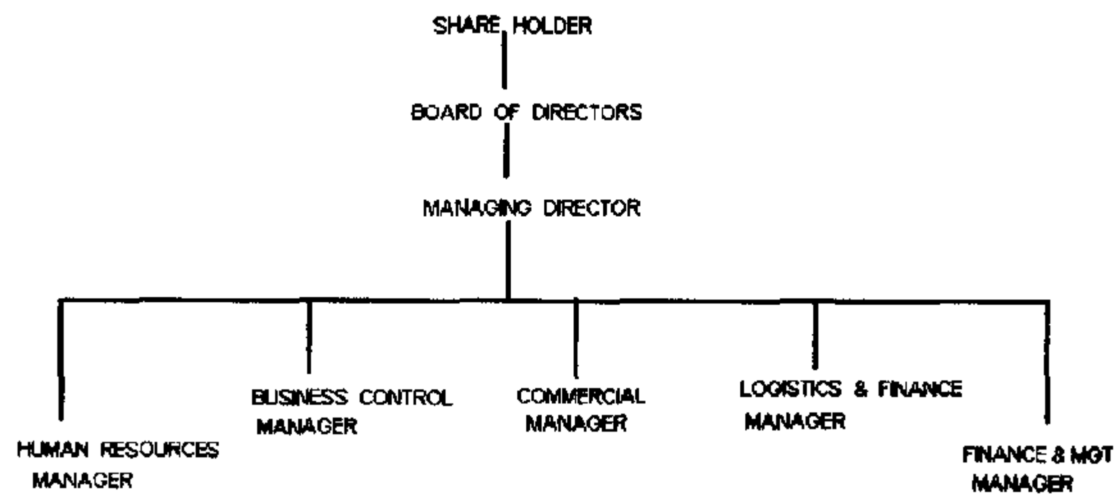


FIG 3.1 The organisation chart for ELF Nigeria Limited.

Each authority is briefly discussed in the following section. Five major lines of authority stand distinct with the managing director directing the affairs of the company. He is responsible for ensuring that the company's policy on sales, operations, finance and administration are implemented, through the regional officers which are responsible for distribution of the product within fixed areas. While some responsibilities are decentralised certain degree of central control is maintained in the head office.

3.2.3.1 Logistics and engineering Department

The department arranges for the most economic movement of products from supply points, that is depot to services stations and other sales outlets. It is also responsible for the designing, construction, maintenance and repairs of all the company's own storage points and service station.

3.2.3.2 Commercial Department

This department formulates policies designed to promote the sales of the company' products by the regions and location offices.

3.2.3.3 Finance And Management Services.

This department is responsible for all matters concerning finance, trading results, cost of the company's activities involving both capital, revenue and provision of management support services.

3.2.3.4 Business Control Department

This department ensures that procedures laid down by management for day to day activities are followed in order to achieve set goals and objectives.

3.2.3.5 Human Resources Department

This department is responsible for employee services, training and welfare.

3.2.4 System Of Distribution.

This section examines the general system of distribution which exist. It is usual for the products to be refined hear the oil fields and transfferred in ocean going tankers to large installations strategically cited. Products were then moved by coasted tankers railcar or large road tankwagons to smaller installations like inland depots, service stations and it was from these points that majority of the supplies were delivered to the consumers. However, presently, there is a growing tendency to refine products near the main areas of consumption. One of the golden rules of distribution is that the handling of product en-route to the consumer must be kept to an absolute minimum, since all handling means an increase in the cost of distribution.

Wherever possible, deliveries are made direct to consumers from refineries and installations using the most economic mode of transport available. The extent to which direct deliveries are possible depend on the reception facilities

at the customers site. In any way, it is essential when planning a network of distribution to ensure that the installations and depots are so located that bulk supplies can be transported to them by cheapest method, be it road, rail or water. At the same time be in close proximity to the area of consumption.
To effect the maximum economy in distribution cost, products are moved in the largest possible quantities. It is easier to achieve this when transporting supplies from one company's point to another, than when making delivery to consumers since their storage capacity is a controlling factor.

3.2.5 Installations and Depots:

This will examine the physical aspect of distribution as represented by the siting or installations and depots with a view to make distribution of oil products more convenient. For the purpose of maintaining uniform price throughout the country the federal government established depots which are strategically located all over the country from where most marketing companies such as ELF lift the products direct to their stations.
These depots are fed by the nations three refineries (Warri, Kaduna and Port-Harcourt). Moreover, during periods of scarcity road tankers are used to bring products from depots where the supply is surplus to other depots with greater requirement. The cost of such bridging is paid by the federal government through the petroleum equalisation fund(PEF).

ELF Oil has its location officers near PPMC depots all over the country. The regional office and their location offices are as follows:

REGIONAL	NORTH	WEST	MIDWEST	EAST
(a)	(b)	(c)	(d)	(e)
OFFICE	KADUNA	APAPA	WARRI	PORT-HARCOURT
LOCATION OFFICE	Kaduna Kano Gusau Jos Yola Abuja Suleja Gombe	Apapa Ilorin Shagamu Ibadan	Warri Benin Ondo	Port-Harcourt Enugu Aba Calabar Makurdi

3.2.6 Transportation pattern

Due to the importance of transportation in petroleum distribution in the country, ELF oil (Nig) Ltd has designed a two way system of distribution. The single transporter and dedicated transporter system.

3.2.6.1 Single Transporter For Each Depot:

In this system a transporter is selected for each depot. In the northern region for example, there are nine single transporters for the nine depots namely;

<u>Transporter</u>	<u>Depot of Operation</u>
ATB	Kaduna refinery (KR)
AAI	Gombe Depot (GOD)
ARY	Kano Depot (KD)

ABM	Gusau	(GUD)
ARI	Jos Depot	(JD)
AAR	Suleja Depot	(SD)
AHJ	Yola Depot	(YD)
HNL	Maiduguri Depot	(MAD)
AUD	Minna Depot	(MID)

Each of these transporters provides three trucks daily for loading of products from these depots. The truck of the following capacity.

33000 ton for PMS

13620 ton for AGO

9080 ton for DPK

3.2.6.2 Dedicated Transporter System:

These are transporters that carry the company's lubricants from Kaduna office which serves as the supply point to all other locations within the Northern region. They deliver to only industrial customers who are the highest consumers of the company's lubricant.

Payment is normally made in relation to the number of trips taken in a month, blanket rate of Ten trips for 85000 = Naira. This system of transportation will not concern this study.

3.2.7 Method of Payment For Single Transporter

Payment of transporters depend on the distance covered and capacity of product lifted from the NNPC. The payment rate is fixed on basis of kilometer bracket. Details of this will be presented in later parts of this chapter.

3.2.8 Filling Stations

The company has more than 200 stations nation-wide. However there are 27 stations (representing demand points) within the Northern region (i.e area under study).

Consumption or demand by filling stations is determined by product availability and sales area.

(a) product availability- This concerns the quantity of product available at the depot. If the product is available, the demand by various stations depending on consumption rate will likely increase.

(b) Sales area: The location of the station determines quantity of the product supplied to the station. A station located in the centre of the town will enjoy more product supply than one at the outskirts of the town.

3.3 DATA COLLECTION

Data collected was primarily obtained through personal interview of personal at the regional office (North). Observation of supply and demand pattern of the company was also conducted, resulting in the following data:

3.3.1 Filling Stations and their Requirements.

The supply to various filling stations considered here is assumed to be under normal situation. In other words periods of petroleum scarcity are not recognised.

3.3.1.1 Kaduna Location.

In Kaduna location there are six stations. Two of the six stations received their supply from Minna depot.These two stations are Lagos road and Yauri road Kontagora. Each receive 66000 litres of pms, 9080 its of KPK twice weekly.

The four remainning stations receive supply from Kaduna refinery in the following manner:

- Kachia road station - 33000 litres of pms, daily except weekend.
- Ibrahim Taiwo road station - 3 trucks of 33000 litres of pms weekly.
- Hanwa junction and hospital - road station Zaria - 66000 litres of pms each weekly.

3.3.1.2 Abuja Location.

In Abuja location there are two stations which receive their supply from Suleja depot as follows:

- Sultan Abubakar station - 33000 litres of pms five times weekly.
- Herbert Macaulay road station - 33000 litres of pms, five times weekly.

3.3.1.3 Jos Location.

There are four stations in Jos location.Out of these two Stations Yakubu Gowon way and Bauchi road stations receive their supplies from Jos Depot while Jos road station and Bauchi road station Gombe, receive their supply from Gombe Depot. Quantity received is as floows:

- Yakubu Gowon way and Bauchi road station Jos - 3 trucks each of 33000 litres of pms weekly.
- Bauchi road station Gombe and Jos road station Bauchi -1 truck of 33000 litres of pms twice weekly.

3.3.1.4 Kano Location.

Kano location has five stations under it. These stations receive supplies from Kano depot as follows:

- Kano Coop station - 33000 litres of pms, five times weekly.
- Km2 Hotoro station - 33000 litres of pms, three times weekly.
- Dawanaw station - 33000 litres of pms, three times weekly.
- Wudil station and Kofar Katsina - 33000 litres of pms, each twice weekly.

3.3.1.5 Maiduguri Location.

Three stations receive their supplier from Maiduguri depot. The stations are:

- Airport road Maiduguri - 33000 litres of pms, three times weekly.
- Bama road Maiduguri - 33000 litres of pms, three times weekly.
- Gamboro road Maiduguri - 33000 litres of pms, three times weekly.

3.3.1.6 Yola Location.

There are two stations under Yola. They receive their supplies from Yola depot as follows:

- Mararaban - mubi station - 33000 litres of pms, twice weekly.
- Yola - jimeta road station - 33000 litres of pms, twice weekly.

3.3.1.7 Gusau Location.

There are five stations under Gusau. They receive their supplies from Gusau depot as follows:

- Elf organic station Sokoto - 33000 litres of pms twice weekly
- Bala Maina station Gusau - 33000 litres of pms twice weekly
- Elf organic station shinkafi - 33000 litres of pms weekly
- Elf organic station Jega - 33000 litres of pms weekly
- Elf organic station Kamba - 33000 litres of pms weekly.

3.3.2 Supply available at Various Depots

The following figures represent supplies available at the various depots all things being equal of course these is purely for statistical convenience as during scarcity period the supplies could be virtually zero at the depot. In any case the following represent normal supply schedules:

	REFINERY / DEPOT	PRODUCT	SUPPLY
1.	Kaduna refinery	pms	800.000
2.	Yola Depot	pms	200.000
3.	Suleja Depot	pms	600.000
4.	Minna Depot	pms	320.000
5.	Maiduguri Depot	pms	170.000
6.	Kano Depot	pms	650.000
7.	Jos Depot	pms	400.000
8.	Gusau Depot	pms	100.000
9.	Gombe Depot	pms	400.000

3.4 PROBLEM FORMULATION

ELF Oil has total of nine depots (sources) and twenty - seven filling stations (destinations) in Northern region. All these are administered by the Northern regional office in Kaduna.

In order to be able to formulate the problem as a transportation problem it is necessary to know clearly the quantities available from each source and that required from each source to each destination. The depots and filling stations are presented below.

SOURCE	DESTINATION
1. Kaduna Refinery	Kachia road station, Ibrahim Taiwo road station, Hanwa junction Zaria, Hospital zaria.
2. Minna Depot	Lagos road station and Yauri road station Kontagora.
3. Suleja Depot	Sultan Abubakar station Abuja and Herbert Macaulay road Abuja.
4. Jos Depot	Yakubu Gowon way and Bauchi road station Jos.
5. Gombe Depot	Jos road station Bauchi and Bauchi road station Gombe.
6. Kano Depot	Kano coop station, Kano Hotoro station, Dawanaw station, Wudil station and Kofar Kwaya station Katsina.
7. Maiduguri Depot	Airport road station, Bama road station, Gamboro road station Maiduguri.
8. Yola Depot	Mararaban - mubi station and Yola- jimeta road station Yola.
9. Gusau Depot	ELF organic station Sokoto, Bala maina station Gusau ELF organic station shinkafi, ELF organic station jega and ELF organic station Kamba.

Let the sources be designated by Xi and the destinations be designated by Yj as follows:

X1 = Kaduana refinery

X2 = Minna Depot

X3 = Suleja Dpot

X4 = Jos Depot

X5 = Gombe Depot

X6 = Kano Depot

X7 = Maiduguri Depot

X8 = Yola Depot

X9 = Gusau Depot

While the destinations are:

Y1 = Kachia road station Kaduna

Y2 = Ibrahim Taiwo road Kaduna

Y3 = Hanwa junction Zaria

Y4 = Hospital road Zaria

Y5 = Lagos road Kontagora

Y6 = Yauri road Kontagora

Y7 = Sultan Abubakar Abuja

Y8 = Herbert Macaulay Abuja

Y9 = Yakubu Gowon way Jos

Y10	=	Bauchi road Jos
Y11	=	Jos road Bauchi
Y12	=	Bauchi road Gombe
Y13	=	Kano Coop Kano
Y14	=	Km2 Hotoro Kano
Y15	=	Danawa station Kano
Y16	=	Wudil station Kano.
Y17	=	Kofar kwaya Kastina
Y18	=	Airport road Maiduguri
Y19	=	Bama road Maiduguri
Y20	=	Gamboru road Maiduguri
Y21	=	Mararaban-mubi Yola
Y22	=	Yola - jimeta road Yola
Y23	=	ELF organic station Sokoto
Y24	=	Bala maina station Gusau
Y25	=	ELF organic station Shinkafi
Y26	=	ELF organic station Jega
Y27	=	ELF organic station Kamba.

The following represent the product availability and demand figures for various sources and destinations:

1. XI - Capacity = 800,000

Destination	Quantity shipped (litres per week)	Transport cost
Y1	5x33000=165000	.293
Y2	3x33020=99000	.293
Y3	2x33000=66000	.356
Y4	2x33000=66000	.356

2. X2 - Capacity = 320000

Destination	Quantity shipped (litres per week)	Transport cost
Y5	33000x2 = 66000	.788
Y6	33000x2 = 66000	.788

3. X3 - Capacity = 600,000

Destination	Quantity shipped (litres per week)	Transport cost
Y7	33000x2 = 165000	.293
Y8	33000x5 = 165000	.293

4. X4 - Capacity = 400,000

Destination	Quantity shipped (litres per week)	Transport cost
Y9	33000x3 = 99000	.293
Y1O	33000x3 = 99000	.293

5. X5 - Capacity = 400,000

Destination	Quantity shipped (litres per week)	Transport cost
Y11	33000x2 = 66000	.678
Y12	33000x2 = 66000	.293

6. X6 - Capacity = 650,000

Destination	Quantity shipped (litres per week)	Transport cost
Y13	33000x5 = 165000	.293
Y14	33000x5 = 99000	.293
Y15	33000x5 = 99000	.293
Y16	33000x2 = 66000	.293
Y17	33000x2 = 66000	.678

7. X7 Capacity = 170,000

Destination	Quantity shipped (litres per week)	Transport cost
Y18	33000x2 = 99000	.293
Y19	33000x2 = 99000	.293
Y20	33000x2 = 99000	.293

8. X8 Capacity = 200,000

Destination	Quantity shipped (litres per week)	Transport cost
Y21	33000x2 = 66000	.293
Y22	33000x2 = 66000	.293

9. X9 Capacity = 100,000

Destination	Quantity shipped (litres per week)	Transport cost
Y23	33000x2 = 66000	.678
Y24	33000x2 = 66000	.293
Y25	33000 = 33000	.677
Y26	33000 = 33000	.356
Y27	33000 = 33000	.788

From the above information it is possible to come up with the following transportation problem:

The tableau above is reffered to as Initial Transportation Tableau because it is assummed that the decision to transport from any of the depots to any of the filling stations is yet to be made. From the problem the quantities that could be received from each depot are given, while the requirements for each filling station is also given. In addition the twice at which a litre of PMS could be transported from one depot to any filling station is also given.

From the problem tabular, it is clear that the demands of the filling stations are not equal to the supply available from the various depots. Supply has far outweighed demand thus making it necessary to introduce a dummy filling station (YD) to absord the excess supply available. This ensures that an important assumption of the transportation model of demand being equal to supply is satisfied. the transportation cost from each depot to each filling station has been determined on basis of distance bracket in kilometres per litre. This is presented below:

DISTANCE (IN KM)	RATE (IN NAIRA PER LITRE)
0 - 50	.293
51 - 75	.356
76 - 125	.415
126 - 175	.677
176 - 225	.677

226 - 275	.897
276 - 325	1.09
326 - 375	1.15
376 - 425	1.20
426 - 475	1.30
476 - 525	1.40
526 - 575	1.49
626 - 675	1.59
676 - 725	1.69
676 - 725	1.79
726 - 775	1.89
776 - 825	1.99
826 - 875	2.09
876 - 925	2.19
926 - 975	2.29
976 - 1025	2.39
1025 Above	2.89

CHAPTER FOUR

DATA ANALYSIS

4.1 INTRODUCTION TO METHOD USED FOR ANALYSIS IN THIS STUDY.

This chapter attempts to analyse the transportation problem encountered by ELF Oil Nigeria Limited presented in chapter 111 using the Vogel's Approximation method.

As seen earlier in chapter 11, VAM is a method for establishing an initial feasible solution. It is an efficient method which produces initial solutions that are near the optimal one. In other words, VAM appears to produce the optimal solution with minimal number of interactions. It is necessary to point out some peculiar issues that are encountered in this particular problem. These issues concern the size of the data involved which gives two hundred and fifty variables. This makes it highly difficult if not impossible to solve manually. It is therefore necessary to employ the assistance of computer software. Unfortunately the available computer package cannot accommodate the number of variables in the problem (252 variables). These have further restricted the depth of analysis that could be made because only the manual method with all its attendant drawbacks will be utilised. As a result, the analysis will be based on only the initial allocation stage. Lets first view the current system of transportation being used by ELF oil Nigeria Limited.

4.2 PRESENT METHOD USED BY ELF OIL NIGERIA LIMITED

Presently, ELF Oil Nigeria Limited uses rule of thum in deciding what quantity is to be shipped from which depot to which destination. In order to cut its cost, ELF established zones according to the number of depot scattered around the Northern Region. Thus this gave rise to Nine Zones and in each zone it established a distance bracket of 0-50, 50-99km etc, and attached costs to each bracket. The cost bracket is used to pay transporters for shipping the company's product. It is this arrangement that has given rise to what was obtained in section 3.5 of each chapter 11, as ELF transportation schedule. Presented below as table 4.2.

Table 4.2 Current ELF Distribution Method.

Now view the transportation schedule used by elf as presented in table 4.2:

ELF 's cost will be:-

Current cost to ELF, TCI =[165000(.293) + 99000(.293) + 66000(.356)

+66000(.356) + (66000(.788) +(66000(.788)]

+[165000(.293) +165000(.293)] +[99000(.293)

+99000(.293)] +[66000(.678) +66000(.293)]

+[165000(.293) + 99000(.293) +99000(.293)

+66000(.293) +66000(.678) +(99000(.293)

+99000(.293)+99000(.293)+(66000(.293)

+66000(.293)]+[66000(.678)+66000(.293)

+33000(.677)+33000(.356)+3000(.788)]

=(48345+29007+23496+23496)+(52008+

52008)+(48345+48345)+(29007+29007)

+(44748+19338)+(48345+29007+29007

+19338+44748)+(29007+29007+29007)

+(19338+19338)+(44748+19338+22341

+11748+26004)

=124344+104016+96690+58014+64086

+170445+87021+38676+124179.

TC1 =**<u>N867,471.</u>**

However, a closer look at the current ELF distribution system reveals the following issues:-

(a) ELF has not been meeting up the requirements of Y23, Y24, Y26 and Y27 from X9, and Y18, Y19 as well as Y20 from X7 by a total shortfall of 258000 litres per week, while keeping a lot of excess fuel in some depots.

(b) ELF did not explain from which depots it makes up for the shortfall.

(c) ELF does not seem to consider the costs associated with storage of its excess supply.

With these three issues in mind, lets not analyse the same problem using Vogel's Approximation method.

4.3 DATA ANALYSIS BY TRANSPORTATION USING VAM

For the transportation schedule in 3.4 to be presented as a transportation problem, it needs to undergo some slight modifications as follows:-

Supply = 800,000+320,000+600,000+400,000+400,000+650,000 + 170,000 +200,000 +100,000.

= **3640,000 litres.**

Demand = 165000+99000+66000+66000+66000+66000+165000

+165000+99000+99000+66000+66000+165000+99000

+99000+66000+66000+99000+99000+99000+66000

+66000+66000+66000+33000+33000+33000
=**<u>2343000 litres.</u>**

Thus, supply = 3640000

Demand = <u>2343000</u>

Excess = **<u>1297000.</u>** litres

Since, supply has exceeded demand, we need to create a dummy filling station (YD) to absord the excess supply. This will satisfy one of the conditions in transportation method of demand being equal to supply. Note that this condition is not satisfied in the case of current ELF problem solution in 4.2 above. The zero cost in each cell of the dummy column (YD) indicates that it will cost nothing to transport fuel to non existent filling station. The adjusted initial transportation table is presented as table 4.3 (see page 91).

4.3.1 Initial Problem

After the slight adjustment, the ELF transportation matrix will now be solved using vogel's Approximation method. Table 4.3 represents the initial problem. It shows the transportation cost from each depot to each fillingstations respectively. The question is now, "How many units of the product should be moved from each source to each destination, so that transportation cost will be as low as possible? The section following will attempt to provide an answer to this question.

4.3.2 Initial Feasible Solution

Vogel's Approximation Method requires that, for an initial feasible solution to be obtained, a specific procedure to be followed in placing quantities to be shipped from a source to a destination. What cell and how to put in it is determined through laid down steps as discussed in section 2.4.2. following the steps in 2.4.2, our initial feasible solution. (see page 92).

Initial cost, Tc2 = [165000(.293)+99000(.293)+66000(.356)+66000(.356)
+66000(1.40)+33000(1.5)+305000(0)]+[66000(.788)+
66000(.788)+32000(1.87)+156000(0)]+[165000(.293)
+165000(.293)+270000(0)]+[99000(.293)+99000(.293)
+66000(.415)+136000(0)]+[66000(.293)+28000(1.20)
+99000(1.2)+66000(1.09)+141000(0)]+[165000(.293)
+99000(.293)+99000(.293)+66000(.293)+66000(.678)
+155000(0)]+[99000(.293)+71000(.293)]+[66000(.293)
+134000(0)]+[66000(.293)+1000(.677)+33000(.356)]
=(48345+29007+23496+23496+92400+49500+0)+(52008
+52008+59840+0+(48345+48345+0)+(29007+29007+27390+0)+
(19338+33600+118800+71940+0)+(48345+29007+29007+19338
+44748+0)+(29007+20803)+(19338+0)+19338+677+11748).
=266244+163856+96690+85404+243678+170445+49810
+19338+31763.

TC2 = **<u>N1,127228.</u>**

What will follow next is to ask ourselves if this total transportation cost represents the optimal solution. The answer can be provided conducting a test for optimality. Nonetheless, even before testing for optimality, These means that the cost does not represent an optimal solution considering the size of the problem.

Nevertheless, lets at this stage attempt to compare the current ELF solution and the initial feasible solution obtained through VAM. Before the comparison, however, there is need to make some addition assumptions for ELF. These assumptions are that:-

(a) To meet up the requirements of the filling stations in 4.2(a), ELF has to ship fuel from other depots that have excess.

(b) There is additional marketing costs in terms of losses in market share to competitors should ELF fail to meet up these requirements.

(c) The cost of transportation will have to be born by ELF because it is a locally arranged bridging.

(d) The nearest depot that can make up the shortfall in X7 is likely to be X5 due to shorter distance and excecc of about 26800 litres. The transportation cost involved will be at least N1.20 per litre. This is obtained from the initial transportation problem tableau table 4.1.

(e) The nearest depot that can make up the shortfall in X9is either of X1 or X6 which have excesses of 40400 litres and 155000 litres respectively. For all the two depots the least transportation cost involved will be an average of Nl .60 per litre. This is obtained from the initial transportation problem tableau table 4.1. The average of the two transportation costs 1.20 + 1.60 =2.8/ 2 =N1.4 per litre. Then we multiply this average by the total shortfall we have 258000 x 1.4 =N361200:00. This means the current ELF transportation cost will increase by the same amount thus, N867471 + 361200=N 1,228671:00. This cost (Nl,228671) is greater than the initaial feasible solution (Nl 127228) obtained using VAM by N101443.00.

4.3.3 Test for Optimality

Test for optimality examines each of the open squares in the transportation table to determine if improvements can be affected in the total transportation cost. The modified distribution method as discussed in 2.4.5.2 could be used. However, due to the large size of the variable involved in this particular problem it will be manually impossible to undertake without the assistance of a computer software, which is not available. For this reason no further test for optimality will be conduced on ELF initial feasible solution. All deductions will be drawn from initial feasible solution which incidentally happens to be better than what ELF is presently incurring.

4.3.4 Optimal Solution.

This will not be discussed further for reasons advanced in 4.3.3 above.

4.4 TEST OF HYPOTHESIS.

The hypotheses formulated for this work postulates that "The use of transportation method (LP technique) will enhance cost minimisation in transportation of fuel products, "and that "designing a distribution pattern arbitrarily or by rule of thum does not result in an optimal solution."

These hypotheses are valid, because the existing rule of thum system of distribution being practiced by the ELF and the scientific method presented by transportation method (VAM) have all been tested. Though ELF claims to be spending N867471 presently, we have seen that if ELF corrects some of the observations raised, the amount should be N1,228671. However, the same quantities of fuel could be transported from the same depots to same warehouses at much less than N1,127228, if transportation method is utilised. The difference between using the method and not using it is more than N101443. Remember this is the result of initial allocation only. In the final analysis, therefore, it could be said that the transportation method is by far superior and it will benefit the company to a great extent.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS.

5.1 SUMMARY AND CONCLUSION

This research work attempts to show that linear programming is a very versatile tool that can be used by managers to solve a large class of special problems. Indeed we have seen that a wide range of practical management problems can be formulated as linear programming problems. Despite its restricting assumptions which are viewed by many as being limiting on the ultimate solutions which it always provides, it still offers important opportunities for scientific solutions.

One of the major areas of success in the application of Linear Programming method is the transportation or distribution problem. Like other Linear Programming situations the transportation problem involves resource allocation issues. For a typical transportation problem the allocation decision relates to physical movement of goods from one set of locations called supply point or sources to another set of locations called demand points or destinations. The decision variables concern the quantity of goods to be moved from each source to the destination. The problem objective is to ship the goods needed to each destination using the available stock at the sources, while minimising the total shipping cost.

In general we saw that solving a basic transportation problem involves, three iteratives steps:

a. Making the initial assignment in order to obtain the initial feasible solution.

b. Testing for optimality to confirm if what is obtained as the initial feasible solution is optimal.

c. Designning a new and better program if necessary. This continues until no further improvement can be found.

Different methods of making initial allocation in transportation problems such as Vogel's approximation methods (VAM) and least cost method were discussed. As observed VAM leads to an optimal solution more rapidly with less manual effort than other solution techniques.

The modified distribution method (MODI) and stepping-stone methods wewre used in testing for optimality in the study. This test involved evaluating all open squares of the transportation tableau after each allocation to determine if the solution can be improved by shifting units from occupied cells to any unoccupied cell. This process continues till all unoccupied cells yield a positive sum.

How special situations such as degeneracy, unbalanced problems and multiple optimal solutions are resolved in a transportation problem were also discussed. Analysis of the ELF transportation problem was conducted. The cost obtained as the initial feasible solution using VAM was found to be higher than what ELF is currently incurring. Apparently, this current under stated cost was explained by the observations raised in section 4.2. Important among those observations was the fact that ELF was not meeting up the requirements of its filling station while maintaining excess fuel in some depots. And that if ELF is to reap the fruits of a scientific system such as VAM it should ship fuel from some depots that have excess, which will inevitably increase the cost to a level higher than what was obtained as the initial feasible solution using VAM. Thereby confirming that the application of transportation model in solving transportation problems does minimise transportation cost thus making distribution as efficient as possible.

It can therefore be said that, transportation method offers a modern or scientific tool that can be used in minimising cost. This is very important as the need for businesses especially petroleum marketers to discover ways of reducing their cost, increasingly becomes paramount. Apparently the selling price of petroleum is the same irrespective of geographical location or distance from depot, within the country. So profit maximisation by individual firms will depend squarely on the ability of the company to reduce its cost to the best minimum. For obvious reasons, present economic circumstances, and the general trend towards privatisation and commercialisation makes it more necessary for ELF and other companies to opt for scientific ways and means of minimising total costs.

5.2 RECOMMENDATIONS

To reduce their cost and thereby increase their profits the management of ELF Oil Nigeria Limited should consider the following recommendation:

(a) ELF Oil Nigeria Limited should correct the observations raised regarding the shortfall in supply to Y18, Y19, Y20, and Y23, Y24,Y25 as well as Y26 from the depots suggested in 4.3.2 (d) and (e) above.

(b) In view of (a) ELF should explore services of experts to analyse other aspects of its operations with a view to finding other areas for improvement.

(c) ELF should establish a computer department or utilise the services of a computer specialist to benefit from the many scientific options available through various Linear Programming packages.

(d) ELF should opt for the distribution schedule arrived at using VAM in the analysis conducted in chapter four. ELF should however, utilise the appropriate software package to obtain an optimal solution.

Moreover, it is equally important to note that as the future unfolds, new filling stations and their requirements as well as changes in edepot capacity may entail new distribution patterns. Attention should therefore be paid to these development.

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