DIRECT TRANSFORMATION OF NIGERIAN TRANSVERSE MERCATOR PROJECTION COORDINATES (TMP.) TO UNIVERSAL TRANSVERSE MERCATOR PROJECTION COORDINATES (UTM).

by

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Being the text of the thesis presented to the department of Surveying of the Ahmadu Bello University, Zaria in partial fulfilment of the requirements for the award of the Master 0f Science (M.Sc.) degree in Land surveying of the university.

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Supervised by Mr. N.J. Field(acting head of department of Surveying of Ahmadu Bello University, Zaria)

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covering the whole country. (Nigeria).

CHAPTER 1: INTRODUCTION

1.01 Background

Prior to the announcement of the changeover to U.T.M. coordinates, surveys in Nigeria were based on the Nigerian Transverse Mercator (3 belt) projection system.

With the award of contracts for the large scale mapping of major towns in 1976 by Federal Surveys (for the Federal Military Government) and the request that the coordinates should be supplied in U.T.M., most surveyors and survey organisations were in a dilema as to whether to convert the coordinates of controls to U.T.M. and use the tables of transformation to apply scale factor, mean sea level and other necessary corrections or to carry out the surveys and computations in Nigeria T.M.P. coordinates and then convert the coordinates to U.T.M. Many survey organisations opted for t he latter using the tables of transformation.

1.02 Transformation Via Geographical Coordinates Using Programmable Calculators:

A method was described in (1) for the conversion between Nigerian T.M.P. and U.T.M. coordinates via geographical coordinates.

The said method uses a modified formulae and allows the conversion to be made in two programmes (one programme converts from grid to geographical while the other converts from geographical to grid (U.T.M.) with small programmable calculator. The calculator used in (1) was the Hewlett Packard HP67/97 which has 224 programme steps and twenty-six stores.

1.03 Need for a Direct Method of Transformation.

Good enough the programmes discussed in 1.02 exist, but supposing programmable calculators are not available what happens?

The answer is the return to the two tier (grid T.M.P. - geographical-grid U.T.M.) method which is boring, monotonous and error prone especially when numerous number of points have to be converted. Thus the need for a direct method of transformation is established to save time, boredom and monotony.

The justification of an accuracy of (.01 metre) of the transformation by tables also is debatable when systematic, accidental and instrumental errors inherent in field observations are taken into consideration. A direct method being proposed could then serve as an alternative method of solving the T.M.P. - U.T.M. conversion problems.

1.04 METHOD

The whole country is divided into thirty-seven unequal blocks (2.07). An area of approximately one hundred and fifty square kilometres was carved out of each block, and five peripheral points were selected while the approximate latitude and longitude of these points were scaled off from a base map, to provide a set of error free geographical corrdinates. These geographical coordinates were transformed to Nigerian T.M.P. coordinates and then to U.T.M. coordinates from which a series of five equations in five unknowns were established (4.03). The solution of these equations gave the coefficients of transformation for each of the blocks.

To transform T.M.P. coordinates of any block to U.T.M., the T.M.P. coordinates and some functions of it are used to multiply the coefficients of transformation.

1.05 COMPUTING FACILITIES:

Though the CDC Cyber 72 computer of the Ahmadu Bello University, Zaria was available to the author, it was his intention to utilize to their

fullest capabilities programmable calculators within his reach.

To this end, the T159 programmable calculator (TEXAS INSTRUMENT) with 1960 programme steps and 50 stores and the Hewlett Packard HP29C pocket programmable calculator with Ninety-eight programme steps and twenty-nine stores (thirteen of which are indirect), were utilized during computations, the former has a print out attachment while the latter displays its results.

CHAPTER 2: SCOPE AND OBJECTIVES

2.01 Scope:

The thesis is intended to find a simple method of direct transformation of Nigerian Transverse Mercator projection coordinates to Universal Transverse Mercator Projection coordinates suitable for use by low grade technical staff of Survey divisions/organisations using small programmable calculators.

2.02 Objectives:

The Objectives of the thesis are:

(i) To produce transformation coefficients such that any point in Nigeria can be transformed from Nigerian TMP to UTM directly using cheap programmable calculators.

(ii) To analyse the result obtained in 2.02(i) statistically so as to determine the degree of reliability and the use to which the result can be put.

(iii) To prepare an algorithm for the application of the said result such that an average low grade staff of the states survey department or any survey organisation can utlise the results.

2.03 The Transverse Mercator Projection: Derivation on the Spheroid.

The derivation of the formulae of the Transverse Mercator Projection from the first principles of orthomorphism is found in (2). It is repeated here because it is not found in many text books.

The structure of the Transverse Mercator has the central meridian true to scale and the formulae for Northings and Eastings(linear coordinates) are such that the representative is orthomorphic (i.e. at any point in the projection, the scale along an infinitesimally short distance is the

same in all directions).

Let θ be the spheroidal latitude of a point

l the longitude reckoned from the central meridian in radians

then $N = S + Pl^2 + Ql^4 + Rl^6 + \ldots\ldots\ldots\ldots$

$E = Al + Bl^3 + Cl^5 + \ldots$

where S is the meridional distance

The coefficients A,B,....P,Q,..... are functions of the latitude and they incorporate the parameters radius and eccentricity that define the spheroid to which the survey is referred.

In fig. 2.1

θ and l are the latitude and longitude of point A

These change infinitessimally to $(\theta + d\theta)$ and $(l + dl)$ where $d\theta$ and dl imply linear shift of $pd\theta$ along the meridian and $v.\cos\theta.dl$ along the parallel respectively to point B.

p is the meridional radius of curvature

$p = a(1- e^2)(1-e^2\sin^2\theta)^{-3/2}$

v is the other principal radius of curvature

$v = a(1- e^2\sin^2\theta)^{-1/2}$

a = equatorial radius of curvature

e = eccentricity.

Any projection cam be expressed in general terms as two functions

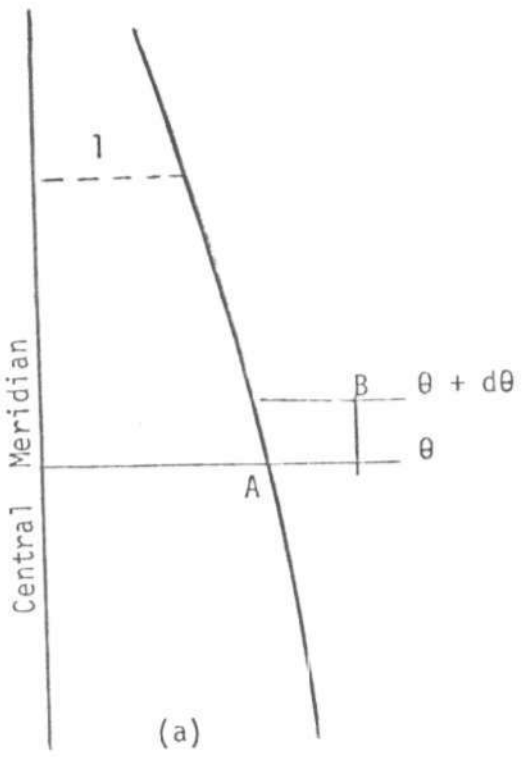
$N = f_1 (\theta, l)$

$E = f_2 (\theta, l)$

If small changes $d\theta$ and dl are made, the resulting changes of N and E are expressed by the differential formulae for two variables

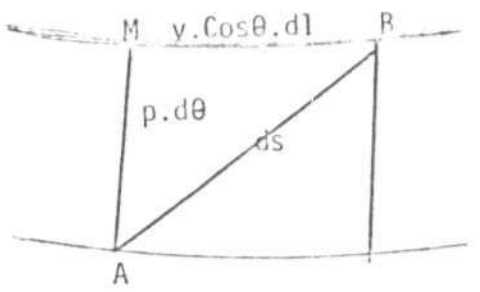
$$dN = \frac{\partial N}{\partial \theta}.d\theta + \frac{\partial N}{\partial l}.dl$$

$$dE = \frac{\partial E}{\partial \theta}.d\theta + \frac{\partial E}{\partial l}.dl$$



(a)

Fig. 2.1



(b)

Fig. 2.1

In fig. 2.2, the points A_1 and B_1 are the projection representatives of of A and B, and A_1M_1 is the projection of the spheroid meridian AM

The broken lines through A_1 are parallel to the axes of the plane coordinate system, so that

$A_1K = dN$ and $KB_1 = dE$

The infinitesimal spheroid length AB is given by

$$(dS)^2 = \rho^2(d\theta)^2 + v^2.\cos^2\theta.(dl)^2$$

The corresponding infinitesimal length on the projection is dS_1 given by $dS_1^2 = dN^2 + dE^2$

Thus the "scale factor" along A_1B_1 is $\frac{dS1}{dS}$ and on substituting the differential formulae for dN and dE we get the required formula

$$\left(\frac{dS_1}{dS}\right)^2 = \frac{\left[\left(\frac{\partial N}{\partial\theta}\right)^2+\left(\frac{\partial E}{\partial\theta}\right)^2\right](d\theta)^2+2\left[\frac{\partial N}{\partial\theta}\cdot\frac{\partial N}{\partial l}+\frac{\partial E}{\partial\theta}\cdot\frac{\partial E}{\partial l}\right](d\theta dl)+\left[\left(\frac{\partial N}{\partial l}\right)^2+\left(\frac{\partial E}{\partial l}\right)^2\right]dl^2}{\rho^2(d\theta)^2 + v^2.\cos^2\theta\cdot dl^2} \quad \ldots\ldots(1)$$

We now go orthomorphic by requesting that the above expression be independent of the direction of AB, and by satisfying the two Cauchy-Riemann conditions for orthomorphism.

If $d\theta = 0$ the scale factor is simply

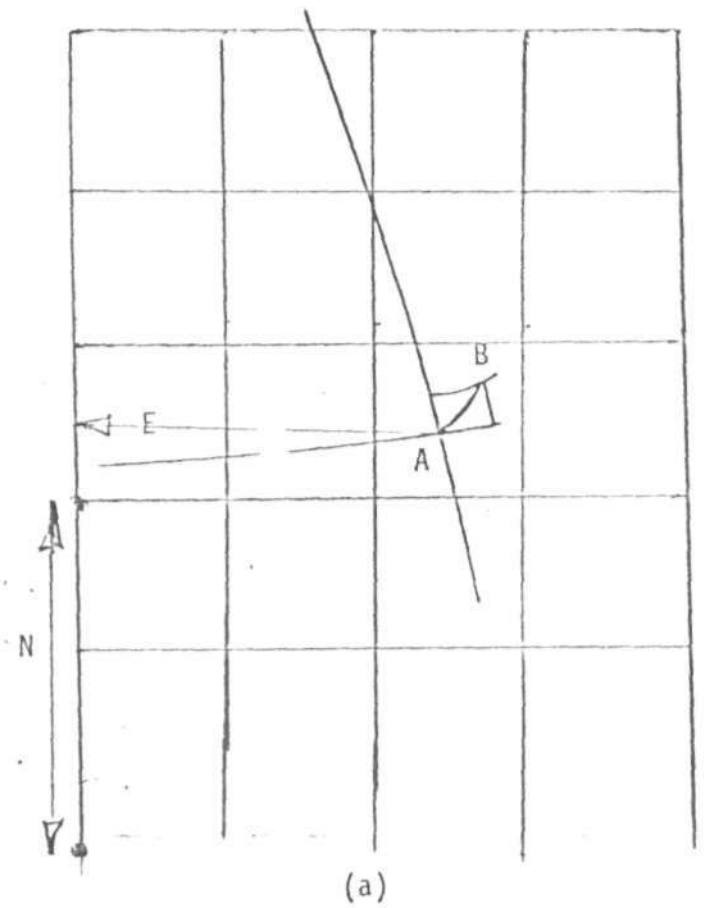
$$\left[\left(\frac{\partial N}{\partial l}\right)^2 + \left(\frac{\partial E}{\partial l}\right)^2\right] \Big/ v^2\cos^2\theta \quad \ldots\ldots(2)$$

and if $dI = 0$ the scale factor is simply

$$\left[\left(\frac{\partial N}{\partial\theta}\right)^2 + \left(\frac{\partial E}{\partial\theta}\right)^2\right] \Big/ \rho^2 \quad \ldots\ldots(3)$$

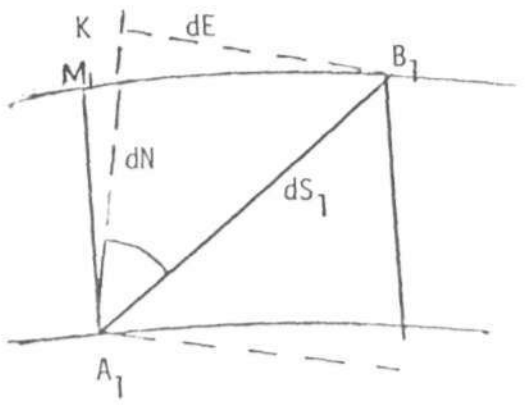
For orthomorphism, these fractions are required to be equal

The square of the scale factor must also be equal to these fractions when $d\theta$ and dl are non zeros. Which gives the first condition for orthomorphism.



(a)

Fig. 2.2



(b)

Fig. 2.2

To satisfy the first condition, the central term of the numerator of the general formula (1) must be zero. Hence we have the second condition for orthomorphism

$$\frac{\partial N}{\partial \theta} \cdot \frac{\partial N}{\partial l} + \frac{\partial E}{\partial \theta} \cdot \frac{\partial E}{\partial l} = 0 \qquad \ldots\ldots(4)$$

The first condition may also be written as

$$v^2 \cdot \cos^4\theta \left[\left(\frac{\partial N}{\partial \theta}\right)^2 + \left(\frac{\partial E}{\partial \theta}\right)^2\right] = \rho^2 \left[\left(\frac{\partial N}{\partial l}\right)^2 + \left(\frac{\partial E}{\partial l}\right)^2\right] \qquad \ldots\ldots(5)$$

These conditions are then applied to the series of N and E so that the desired formulae of the coefficients A,B,...P,Q... are revealed

The four partial differential coefficients are

$$\frac{\partial N}{\partial l} = 2P.l + 4Q.l^3 + 6R.l^5 + \cdots \qquad \ldots\ldots(a)$$

$$\frac{\partial E}{\partial l} = A + 3Bl^2 + 5C.l^4 + \cdots \qquad \ldots\ldots(b)$$

$$\frac{\partial N}{\partial \theta} = P + \frac{dP}{d\theta}.l^2 + \frac{dQ}{d\theta}.l^4 + \cdots \qquad \ldots\ldots(c)$$

$$\frac{\partial E}{\partial \theta} = \frac{dA}{d\theta}.l + \frac{dB}{d\theta}.l^3 + \frac{dC}{d\theta}.l^5 \cdots \qquad \ldots\ldots(d)$$

These are substituted into the Cauchy Riemann conditions, taking care to keep terms in each power of l together. After differentiation, the following results are obtained;

$$A = a\cos\theta(1-e^2\sin^2\theta)^{-1/2}$$

$$B = 1/6.v.\cos^2\theta\ (v/p)-\tan^2\theta)$$

$$C = 1/120.v.\cos^5\theta(5-18\tan^2\theta + \tan^4\theta)$$

$$P = 1/2.v.\ \sin\theta\cos\theta$$

$$Q = 1/24\ (v.\sin\theta\cos^3\theta(5-\tan^2\theta)$$

Thus the final formulae for northings and eastings come out to be

$$N = S + v/2\sin\theta\cos\theta l^2 + v/24\sin\theta\cos^3\theta(5-\tan^2\theta).l^4 \cdots\cdots$$

$$E = v\cos\theta.l + v/6\cos^3\theta(v/p-\tan^2\theta).l^3 + v/120\cos^5\theta(5-18\tan^2\theta+\tan^4\theta).l^5$$

in which l is in radians (i.e.) Irad = 1"/ 206264.8)

The meridional distance S is given by

$$S = a(1 - 1/4e^2 - 3/64e^4 + 5/256e^6 \ldots\ldots\ldots).\theta$$
$$- a(3/8e^2 + 3/32e^4 + 45/1024e^6 \ldots\ldots\ldots).\sin 2\theta$$
$$+ a(15/256e^4 + 45/1024e^6 \ldots\ldots\ldots\ldots).\sin 4\theta$$
$$- a(35/3072e^6 \ldots\ldots\ldots\ldots).\sin 6\theta$$

where θ is expressed in radians.

The projection that has been described so far has the 'true' scale at the central meridian which is not always the case in practice. In practical application it is common practice to apply an overall reduction in scale at the central meridian so that the exaggeration at the boundaries of the zone being used is reduced by fifty percent. This practical application will be observed in the next two sections on Nigerians Transverse mercator projection and the Universal Transverse Mercator projection.

2.04 Nigerian Transverse Mercator Projection:

This is a modified form of the transverse mercator projection which was adapted for the whole of Nigeria. Generally referred to as the three belt system, it is based on the Clarke 1880 spheroid. The properties of this projection as adapted to Nigeria is explained in (3) and summarised in the foregoing section.

2.04.1 Belts:

The whole country is divided into three belts each of which is 4^{O} in width. The belts are labelled

(i) West belt

(ii) Mid belt

(iii) East belt, and the belts have longitudes

$04^{O}30'$, $08^{O}30'$ and $12^{O}30'$ as their respective central meridian

The belts cut each other on the parallel of 9^{O}N which passes through the middle of the country as shown in fig.2.3. The X-coordinate at the exact junction of two adjacent strips is therefore the same for both strips and the Y - coordinate is the same but with opposite sign.

2.04.2 Origin:

In order to avoid negative sign, the X - coordinate of the middle belt was called 2,200,000.00 Geodetic feet (670,553.984 metres) (where 1 metre is equivalent to 3.28086133 geodetic feet) and the Y coordinates of the adjacent strips were computed to comform with this figure giving coincidence at 9^{O}N latitude. The coordinates for the central meridian of the west and east belts are 230,738.266 metres and 1110,369.702 metres respectively.

The longitude of the central meridian is assumed to be the origin of each belt, while the latitude of the origin is 04^{O} which is given a false northing of 0 metres.

2.04.3 Scale Factor:

To reduce the errors in scale at the extremeties of the belts, a scale factor of 0.99975 was applied at the central meridian of each belt.

2.04.4 Spheroid:

The fundamental definning parameters of the Clarke 1880 Spheroid on which the Nigerian TMP is based are-

Equatorial semi-axis a = 6378,249.145metres b = 6356514·870

flattening f = 1/293.465

2.05 Universal Transverse Mercator Projection:

2.05.1 Zoning:

This projection is world wide apart from the polar zones. It divides the whole world into sixty zones each of width 6^{O}. Nigeria falls within zones 31,32 and 33 of this projection and the zones have longitude 03^{O}, 09^{O} and 15^{O} respectively as their central meridian. (see 4)

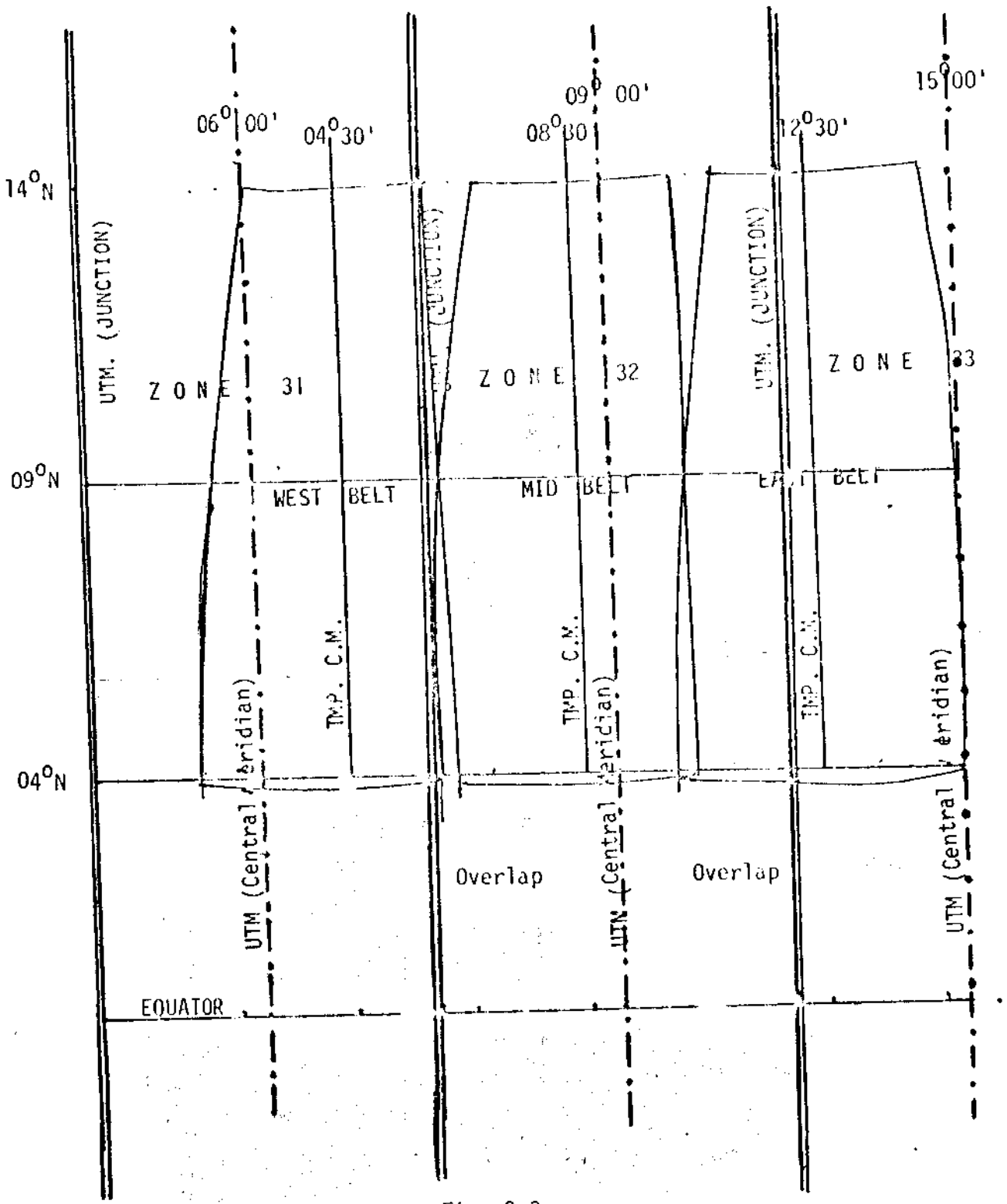


Fig. 2.3

Rectan ular grid superimposed on the Nigerian TMP. and UTM. projection systems, showing the zones and belts, central meridians and the junctions of each of the projection systems.

2.05.2 Origin:

The longitude of the central meridian of each zone is assumed to be the longitude of the origin for that zone., while the equator is assumed to be the latitude of the origin.

A false northing of 0 metre(10,000,000.00 for the southern hemisphere) and a false easting of 500,000.00metres are assumed for the central meridian of each zone.

2.05.3 Scale factor:

The scale factor of each of the central meridian is taken to be 0.9996.

2.05.4 Spheroid:

The definning parameters of the spheroid are the same as that of the Nigerian transverse mercator projection which implies that for the operation of the UTM in Nigeria the Clarke 1880 spheroid is used..

		Nigerian TMP		UTM	
	West belt	$02^{O}30'$			$00^{O}00'$
		$06^{O}30'$	Zone	31	$06^{O}00'$
Belt/zone	Mid belt	$06^{O}30'$	Zone	32	$06^{O}00'$
		$10^{O}30'$			$12^{O}00'$
Junction		$10^{O}30'$			$12^{O}00'$
	East Belt	$14^{O}30'$	Zone	33	$03^{O}000'$
Central	West belt	$04^{O}30'$	Zone	31	$03^{O}00'$
Meridian	Mid belt	$08^{O}30'$	Zone	32	$09^{O}00'$
	East belt	$12^{O}30'$	Zone	33	$15^{O}00'$

Table 2.1

Table showing the longitude of junctions and central meridian of Nigeria n TMP and the UTM projection systems.

2.06 Junctions of Belts and Zones:

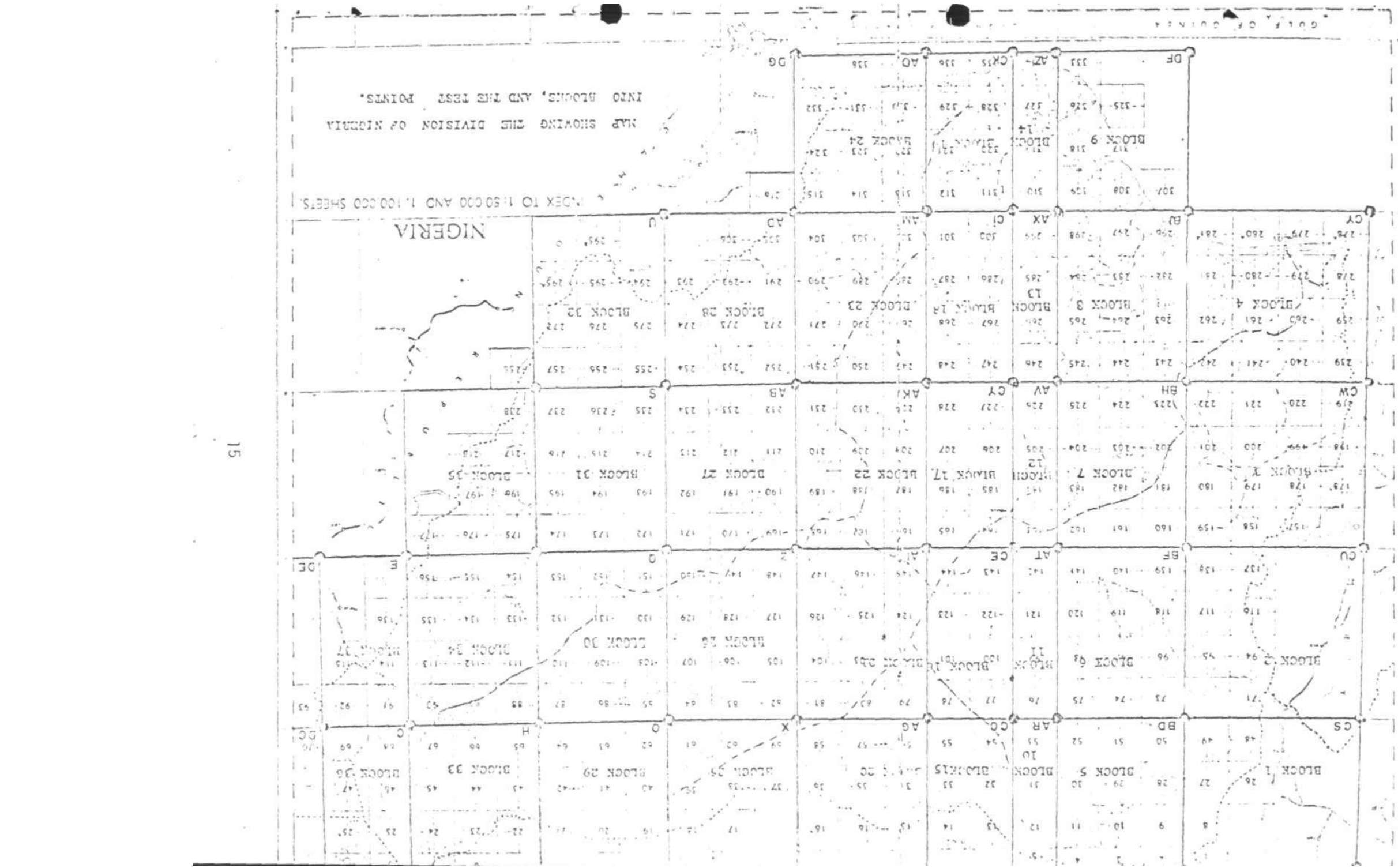
The Nigerian TMP. has an unavoidable overlap and discontinuity at the junctions of its belts(3). Fig.2.3 explains this diagramatically. A maximum discontinuity of 7700.778Metres is experienced at latitude 14° while a maximum overlap of 4371.64 Metres is experienced at latitude 04° (Appendix 1 explains how these figures were arrived at.)

2.07 Junction of Blocks:

By virtue of the overlap and discontinuity at the junctions of belts of the TMP, and the different values assumed for the false origin, central meridian which differ from that of the UTM,an inference is drawn that the junctions of each belt and zone must necessarily be the junction of the proposed block divisions to be used for the direct transformation. A few other additions are made to these for convenience, and a final table of the longitude ranges which mark the block divisions is shown in table 2.4

From	To	Long. Range	Number of Blocks.
2°30'	4° 30'	2° 00'	4
4°30'	6° 00'	1° 30'	5
6°00'	6° 30'	0° 30'	5
6°30'	7° 30'	1° 00'	5
7°30'	9° 00'	1° 30'	5
9°00'	10°30'	1° 30'	4
10°30'	12°00'	1° 30'	4
12°00'	13°30'	1° 3o'	3
13°30'	14°30'	1° 00'	2

Table 2.4 Showing the latitude ranges and the number of blocks within each range.


INDEX TO 1:50 000 AND 1:100 000 SHEETS.
MAP SHOWING THE DIVISION OF NIGERIA
INTO BLOCKS, AND THE TEST POINTS.
NIGERIA
GULF OF GUINEA
BLOCK 1
BLOCK 2
BLOCK 3
BLOCK 4
BLOCK 5
BLOCK 6
BLOCK 7
BLOCK 8
BLOCK 9
BLOCK 10
BLOCK 11
BLOCK 12
BLOCK 13
BLOCK 14
BLOCK 15
BLOCK 16
BLOCK 17
BLOCK 18
BLOCK 19
BLOCK 20
BLOCK 21
BLOCK 22
BLOCK 23
BLOCK 24
BLOCK 25
BLOCK 26
BLOCK 27
BLOCK 28
BLOCK 29
BLOCK 30
BLOCK 31
BLOCK 32
BLOCK 33
BLOCK 34
BLOCK 35
BLOCK 36
BLOCK 37

For each block, the latitudinal range is two degrees. The first four blocks have a longitudinal size of two degrees which is larger than the planned size of one and half degrees. This increase is due to the western boundary of the country which extends slightly to the west of longitude 3^{o}. The last two blocks (36 and 37) have longitudinal size of 1^{o} which is smaller. This is due to the western boundary of Nigeria which terminates at around longitude $14^{o}30'$.

CHAPTER 3: THE CONCEPT OF TRANSFORMATION.

3.01 Introduction: There are a number of survey problems where a coordinate transformation may give a convenient solution. Examples of these are; the transformation of known coordinates from onesystem to another; the transformation of coordinates from local origin to national trig., the transformation of photo coordinates to ground coordinates in photogrammetry and many other examples.

There are several types of transformation which can be employed. The choice made depends on the problem at hand. In the next section, the different types of transformation are discussed viz;

Similarity or Linear conformal transformation.

Projective transformation.

Affine transformation

Second order conformal transformation.

3.02 Similarity or Linear Conformal Transformation:

This transformation is also referred to as "Helmert" in (6). It consists of four parts;

Translation in the X direction.

Translation in the Y direction.

Rotation of axes.

Change of scale.

The two coordinate systems being related by transformation will be called

(i) Nigerian transverse mercator projection system TMP (e,n).

(ii) Universal transverse mercator projection system UTM (E,N)

In fig. 3.1 $E-b = e.\cos\theta - n.\sin\theta$

$N-a = e.\sin\theta + n.\cos\theta$

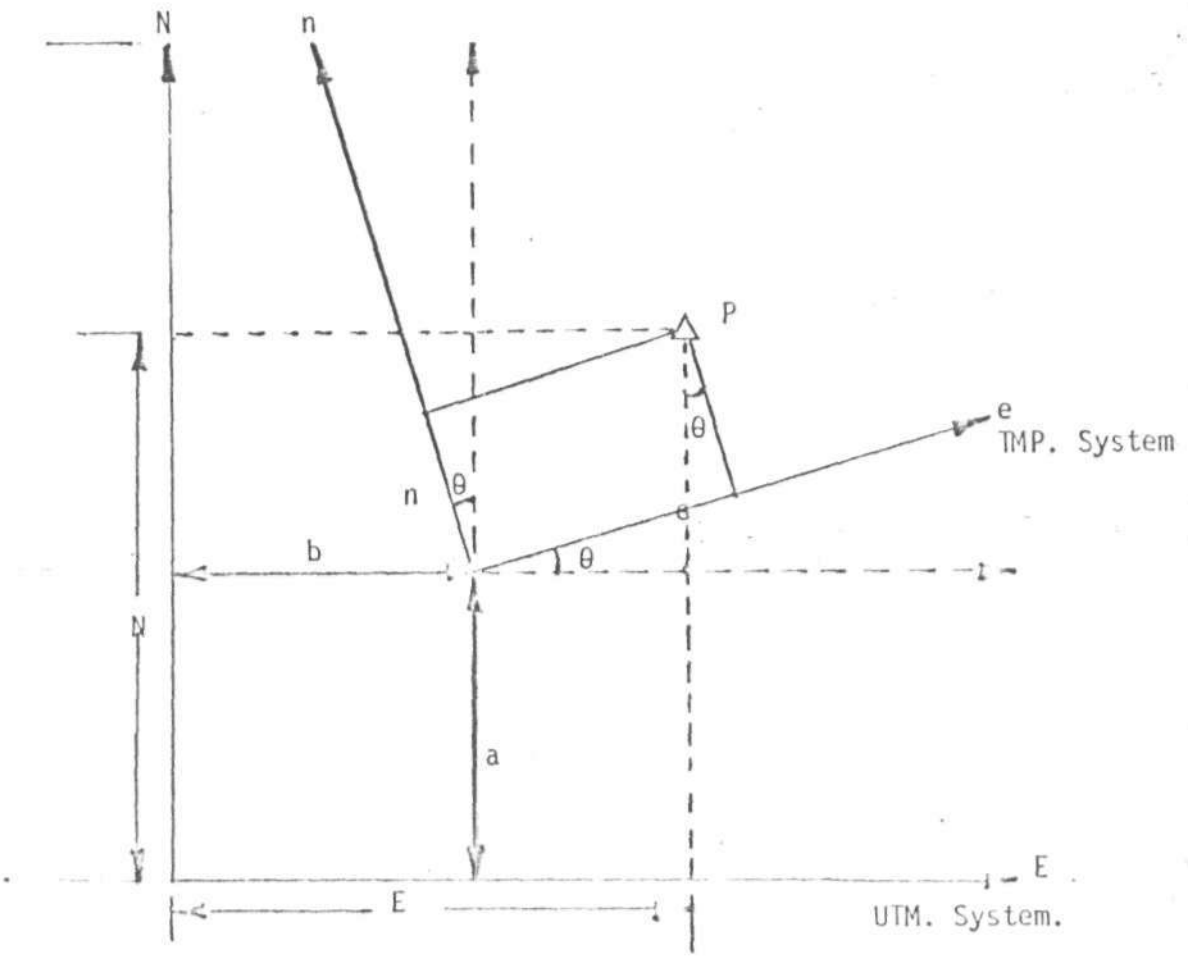


Fig. 3.1

Diagram showing the two projection systems being related by a transformation.

Suppose there is magnification of λ in each direction, then

$$E = b + \lambda e.\text{Cos}\theta - \lambda n.\text{Sin}\theta = b + P.e - Q.n$$

$$N = a + \lambda e.\text{Sin}\theta + \lambda n.\text{Cos}\theta = a + Q.e + P.n$$

$$\begin{pmatrix} E \\ N \end{pmatrix} = \begin{pmatrix} b \\ a \end{pmatrix} + \begin{pmatrix} P & -Q \\ Q & P \end{pmatrix} \begin{pmatrix} e \\ n \end{pmatrix} \quad \text{.....3.001}$$

where $P = \lambda\text{Cos}\theta$, $Q = \lambda\text{Sin}\theta$

Note: b and a measure the translation in the eastings and northings respectively.

3.03 Projective Transformation:

Projective transformation is defined in (7) as

$$X' = \frac{a_1x + a_2y + a_3}{c_1x + c_2y + c3}$$

$$Y' = \frac{b_1x + b_2y + b_3}{c_1x + c_2y + c_3} \quad \text{.....3.002}$$

Where $c_1x + c_2y + c_3 \neq 0$

X' and Y' are here defined as fractional functions of x and y with the important condition that both have the same denominator which is not equal to zero.

$$\text{if} \quad c_1x + c_2y + c_3 \Longrightarrow 0$$

$$X', Y' \longrightarrow \text{Infinity} \quad \text{.....3.003}$$

Equation 3.003 implies that some finite points could be transformed to infinity If the denominator is divided by c_3, then the minimum parameters needed for a solution is reduced from nine to eight.

3.04 Affine Transformation:

Affine transformation is defined in (7) by the formula

$$X = a_1x + b_1y + c_1$$

$$Y = a_2x + b_2y + c_2 \quad \text{.....3.004}$$

Affine transformation involve translation, rotation and change of scale and preserves ends of space (i.e) infinitely distant points are preserved after transformation. However, affine transformation gives homogeneous deformations which implies that shapes are not preserved. This statement is confirmed in (5) where affine transformation is said to transform a circle into an ellipse (page 38). Collinearity is preserved in affine transformation (i.e) a line transforms into a line but non linear deformations are not catered for. The conclusion drawn from this is that affine transformation will work well for linear cases only.

3.05 Second Order Conformal Transformation:

Conformal transformation has the property of preserving shape. A conformal transformation between plane rectangular systems is achieved by the complex polynomial formulation (discussed in (8))

$$E + iN = \sum_{k=0}^{n-1} (a_k + ib_k)(e + in)^k$$

which when expanded becomes

$$E + iN = (a_0 + ib_0) + (a_1 + ib_1)(e + in) + (a_2 + b_2)(e + in)^2 \ldots\ldots (a_n + b_n)(e + in)^2$$

for k=0

$E + iN = a_0 + ib_0$

(i.e) $a_0 = E$

$b_0 = N$ and this is simple translation.

for k = 1

$E + iN = a_0 + ib_0 + a_1 e - b_1 n + ib_1 e + ia_1 n$

Equating real and imaginary parts we have;

$E = a_0 + a_1 e - b_1 n$

$N = b_0 + b_1 e + a_1 n$ 3.006

which compares with the linear conformal transformation formula 3.001

Venturing into k = 2 to give the second order conformal transformation which can cope with the UTM's quite pronounced scale change characteristics, and equating real andimaginary parts we get;

$E = a_0 + a_1 e - b_1 n + a_2(e^2 - n^2) - b_2(2en)$ 3.007

$N = b_0 + b_1 e + a_1 n + b_2(e^2 - n^2) + a_2(2en)$ 3.008

A combined solution using these equations require at least three points whose coordinates are known in both systems while a separate solution require a re-arrangement of the equations and requires that at least five points must be known in both systems

For a separate solution of E,N, the modified equations are as follows

$E = a_0 + a_1 e + a_2 n + a_3(e^2 - n^2) + a_4(2en).$ 3.009

$N = b_0 + b_1 e + b_2 n + b_3(e^2 - n^2) + b_4(2en)$ 3.010

The negative signs in 3.007 were dropped for ease of back substitution.

CHAPTER 4 SUITABILITY OF TRANSFORMATION FORMULAE.

4.01 General: The previous chapter discussed the details of each of the Transformation formulae that can possibly be used. This chapter will consider the properties of these transformation formulae with the aim of determining the most suitable for the problem at hand.

The Transverse Mercator Projection whose formula was derived in chapte two is an orthomorphic projection. It is then implied that whatever transformation formulae to be used must be capable of preserving this orthomorphism. This automatically disqualify affine and projective transformation since the former give homogeneous deformation while the latter in some cases transform finite points into infinity. This leaves the linear conformal and the second order conformal transformations to be tested for suitability.

4.02 Linear Conformal Transformation: Test for Suitability.

To solve a transformation problem, using the linear conformal transformation formulae, the coordinates of at least three points must be known in both systems.

The equations to be used for a separate solution of E and N in matrix form are as follows

$$\begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix} = \begin{pmatrix} 1 & e_1 & n_1 \\ 1 & e_2 & n_2 \\ 1 & e_3 & n_3 \end{pmatrix} \begin{pmatrix} a_o \\ a_1 \\ a_2 \end{pmatrix} \qquad \text{.....4.001}$$

and

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} 1 & e_1 & n_1 \\ 1 & e_2 & n_2 \\ 1 & e_3 & n_3 \end{pmatrix} \begin{pmatrix} b_o \\ b_1 \\ b_2 \end{pmatrix} \qquad \text{.....4.002}$$

The equations 4.001 and 4.002 were used to compute the coefficients for the northing and the eastings and the coefficients were tested at two edges of the block. Below is the result of the test, for block 3a

Eastings Coefficients	Northings Coefficients
a_0= 4.36484682	b_0= 4.41720135
a_1= 0.99940652	b_1= 0.00406009
a_2= -0.00410116	b_2= 0.99941773

Point	N_c	E_c	N_d	E_d	N_c-N_d	E_c-E_d
C_1	1105312.304	500000.000	1105301.830	500022.800	10.48	-22.80
C_m	994761.700	500000.000	994761.010	499998.660	0.69	1.34
C_n	884217.180	500000.000	884226.076	500024.431	-8.90	24.00

Table 1

N_c,E_c are the results obtained using the tables of transformation

N_d,E_d are the results obtained by the direct method using linear equations.

N_c-N_d and E_c-E_d are the differences in coordinates betwwen the two methods of transformation.

4.03 Second Order Conformal Transformation:

Test for Suitability.

To solve a transformation problem using a second order conformal transformation formula requires that the coordinates of at least five points must be known, in both system involved in the transformation.

For a separate solution which is favoured for reasons that would be stated later, a complete set of equations built on equations 3.0009 and 3.010 is written below:

$$\begin{pmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{pmatrix} = \begin{pmatrix} 1 & e_1 & n_1 & e_1^2 - n_1^2 & 2e_1n_1 \\ 1 & e_2 & n_2 & e_2^2 - n_2^2 & 2e_2n_2 \\ 1 & e_3 & n_3 & e_3^2 - n_3^2 & 2e_3n_3 \\ 1 & e_4 & n_4 & e_4^2 - n_4^2 & 2e_4n_4 \\ 1 & e_5 & n_5 & e_5^2 - n_5^2 & 2e_5n_5 \end{pmatrix} \begin{pmatrix} a_o \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} \quad \ldots 4.003$$

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \end{pmatrix} = \begin{pmatrix} 1 & e_1 & n_1 & e_1^2 - n_1^2 & 2e_1n_1 \\ 1 & e_2 & n_2 & e_2^2 - n_2^2 & 2e_2n_2 \\ 1 & e_3 & n_3 & e_3^2 - n_3^2 & 2e_3n_3 \\ 1 & e_4 & n_4 & e_4^2 - n_4^2 & 2e_4n_4 \\ 1 & e_5 & n_5 & e_5^2 - n_5^2 & 2e_5n_5 \end{pmatrix} \begin{pmatrix} b_o \\ b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} \quad \ldots 4.004$$

E,N. represent the UTM coordinates.

e.n represent the Nigerian TMP coordinates.

a,b represent the eastings and northings coefficients of

transformation respectively.

A test computation for the same block as in 4.02 was also carried out and the results obtained are shown below.

Easting Coefficients		Northing Coefficients	
a_o	= 435860.464	b_o	= 441819.885
a_1	= 0.99924426	b_1	= 0.00183955
a_2	= -0.00183839	b_2	= 0.99923242
a_3	= 0.00020386	b_3	= -0.00000039
a_4	=-0.00000068	b_4	= 0.00020383

Point	N_c	E_c	N_d	E_d	N_c-N_d	E_c-E_d
C_1	1105312.3	500000.0	1105312.4	499999.993	-0.145	0.007
C_m	994761.6	500000.0	994761.7	500000.000	0.002	0.000
C_n	884217.2	500000.0	884217.3	500000.033	0.138	-0.033

Table 2

A comparison of tables 1 and 2 show that the second order conformal transformation formula gives a better result, thus the second order conformal transformation is chosen to be the most suitable.

4.04 Least Squares Computation.

A combined solution of the eastings and northings coefficients is possible using the least squares. Observation equations are formed from 3.007 and 3.008. These equations are pre-multiplied by the transpose to form the normal equations which are then solved to get the combined coefficients for eastings and northings. A sample computation showed that the least squares does not really give an improved solution despite the fact that

its computation takes longer time and is much more involving. Thus it was decided that the least squares computation is not really worth the troubles hence the author settled for a separate solution for the eastings and the northings

4.05 Direct Conversion by Interpolation:

The Divided Differences Method.

A non transformation method exist whereby rectangular coordinates could be converted directly between orthomorphic projection systems. The theory of the method is discussed in (9) pages 224 - 225.

Fig 4.1 represent a block. the four corners of which are labelled A,B,C,D. It is assumed that the Nigerian TMP. and the UTM. coordinates of these points are known and are

z_1, z_2, z_3, z_4 and Z_1, Z_2, Z_3, Z_4. respectively and where

$z = x + iy$

$Z = X + iY$

$x = e$ and $y = n$

$X = E$ and $Y = N$

The first order, second order and third order divided differences are defined below as

First order divided difference is defined as

$[Z_1Z_2], [Z_2Z_3], [Z_3Z_4]$.

where $[Z_1Z_2] = (Z_1 - Z_2) \div (z_1 - z_2)$

$$= \frac{(X_1 - X_2) + i(Y_1 - Y_2)}{(x_1 - x_2) + i(y_1 - y_2)} \quad \ldots\ldots\ldots\ldots 4.005$$

if $(X_1 - X_2) = A$, $(Y_1 - Y_2) = B$

$(x_1 - x_2) = a$, $(y_1 - y_2) = b$

4.005 then becomes $\frac{A + iB}{a + ib}$

The second order difference is defined as

$[Z_1Z_2Z_3]$ and $[Z_1Z_2Z_3]$ where

$$[Z_1Z_2Z_3] = \{[Z_1Z_2] - [Z_2Z_3]\} \div (z_1 - z_3)$$

$$[Z_2Z_3Z_4] = \{[Z_2Z_3] - [Z_3Z_4]\} \div (z_2 - z_4) \quad \text{.....4.007}$$

The third order difference is defined as

$$[Z_1Z_2Z_3Z_4] = \{[Z_1Z_2Z_3] - [Z_2Z_3Z_4]\} \div (z_1 - z_4)$$

The coordinate of any point in the area within the block is computed by the formula

$$Z = Z_1 + (z-z_1)[Z_1Z_2] + (z-z_1)(z-z_2)[Z_1Z_2Z_3] + (z-z_1)(z-z_2)(z-z_3)[Z_1Z_2Z_3Z_4] \quad \text{.4.009}$$

A test computation carried out using the divided differences method gave a very good result (app. 2) but the back substitution to get the coordinates for other points within the block was found to be very tedious and requires the knowledge of complex numbers which negates the objective (iii) stated in chapter 2. Thus the divided differences method was also rejected.

In the final analysis the second order conformal transformation method was declared the most suitable and the method of separate solution for the northings and the eastings was favoured.

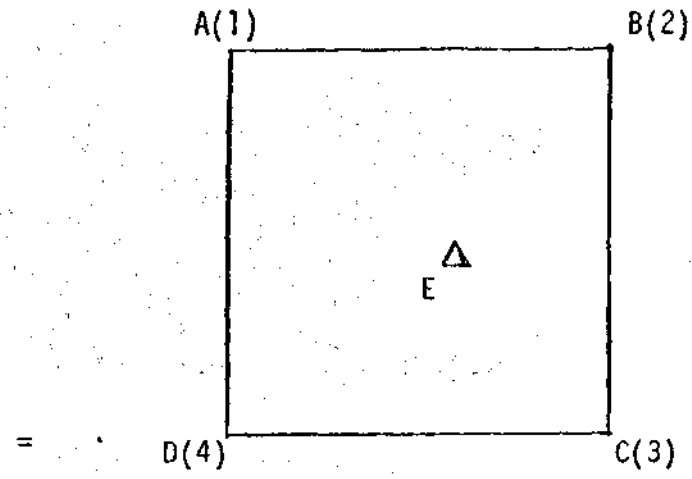


Fig. 4.1

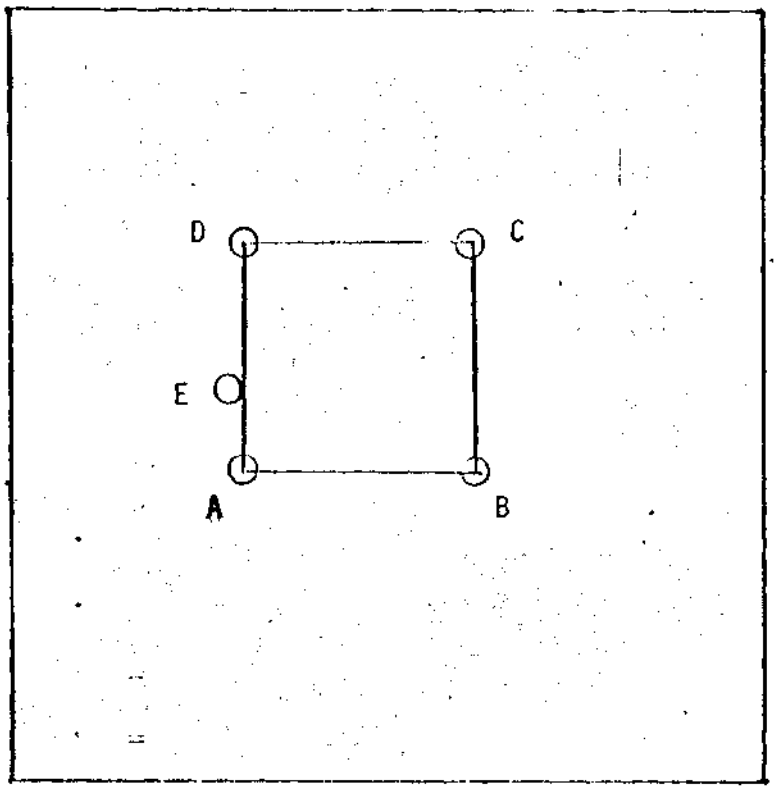


Fig. 5.

Showing the approximate location of points whose geographical coordinates were used (within each block) in the computation of the coefficients of transformation. (not to Scale).

CHAPTE 5:

COMPUTATIONAL PROCEDURE AND PROGRAMMES

5.01 Location of Points Used to Compute Coefficients.

Within the central area of each block, an area of approximately one hundred and fifty square kilometres was carved out. Five peripheral points were chosen around this area and the geographical co-ordinates of these points were scaled off from a base map (see Fig 5.1)

5.02 Computation of Transformation Cooefficients.

The scaled off coordinates were transformed to:

(a) Nigerian Transverse Mercator projection coordinates.

(b) Universal Transverse Mercator projection coordinates.

These were used in the formation of equations 4.003 and 4.004 and subsequently in the computation of the transformation co-efficients.

5.03 Transformation: Geographical to Projection.

There are thirty seven blcoks covering the whole country and five points are used to compute the transformation coefficient for each block which implies that a total of one hundred and eighty five points have to be transformed twice from geographical to grid. The need to test the results makes it necessary to even have additional points.

5.04 Programmes for Transformation: Geographical to Grid:

Two separate programmes were developed to execute the transformation from geographical to grid using the Hewlett Packard HP 29C Calculator which has ninety eight programme steps and twenty nine stores, thirteen of which were indirect.

The programmes helped to reduce the high risk of computational error. The basic formula for the transverse mercator could not be used because of restricted storage facility and programme lenght. A modified formulae which permits

programme s to be written on small programmable calculators developed in (1) was used.

The formulae used are as defined below:

$$E = FE + rJ\left[1 + \frac{J^2}{6}\left(\left(\frac{r}{\rho} - \tan^2\phi\right) + \frac{J^4}{120}\left(5 - 18\tan^2\phi + \tan^4\phi\right)\right)\right] \quad \text{---3.00}$$

$$N = S + rJ^2\tan^2\phi\left[\tfrac{1}{2} + \tfrac{J^2}{24}\left(5 - \tan^2\phi\right)\right] \quad \text{---3.00}$$

where

$$S = r.m_o\left[\phi + 0.00513100463\ \sin\phi\cos\phi\left(1 - 0.00428023\cdot\cos^2\phi\left(1 - 0.005282\cos^2\phi\right)\right)\right]$$

S = Meridional distance computed by Vincenty method.

λ_o= Longitude of central meridian

λ = Longitude of point

ϕ = Latitude of point

$\delta\lambda = \lambda - \lambda_o$

$J = \delta\lambda\cos\phi$

$r = a.m._o / (1-e^2\sin^2)^{\frac{1}{2}}$

$\frac{r}{\rho} = 1 + \varepsilon\cos^2$

m_o= scale of factor at the central meridian

r = radius of rectifying sphere for Clarke 1880 spheroid

r = 6367386.65

p,r = radii of curvature of sheroid

FE = false easting of central meridian

e = eccentricity of sheroid

ε = second eccentricity squared = $e^2/_1 - e^2$

The programmes were tested and found to be correct to within 2 mm of the results that would have been achieved by using the tables of transformation. This implies that the terms omitted in the basic formula are insignificant especially in low latitudes.

5.05 Programme Listing

Appendix 3 lists the two programmes earlier mentioned and other programmes in the following order:

(i) Geographical coordinates to northings (UTM and NTM)

(ii) Geographical coordinates to eastings (UT M and NTM)

(iii) Programme for formation of matrices from the coordinates of the five known points.

(iv) Programme for deriving UTM coordinates from NTM coordinates the newly computed coefficients of transformation.

5.06 Solution of 5 x 5 Equation

To obtain the coefficients of transformation, it was necessary to solve a series of five equations in five unknowns as defined by equations 4.003 and 4.004. These equations were solved using the TEXAS TL59 (with print out) pre-recorded (library programme 02 which solves and print out the determinant and the solution of these equations).

The results of the thirty-seven equations that were solved to get coefficient covering the whole of Nigeria is shown in Appendix 9.

The data in pages 32 - 38 are the approximate longitude and latitude of points scaled off from the base map and used in the computation of the coefficients of transformation for each of the blocks.

BLOCK	LONGITUDE	LATITUDE
1	03 28 25.185	12 55 30.981
	03 33 52.670	12 56 20.169
1	03 33 19.008	13 05 15 200
	03 27 30.102	13 04 40.733
1	03 27 40.001	12 58 28.925
2	SAME LONG. AS	10 55 30.981
	BLOCK 1	10 56 20.169
		11 05 15.200
		11 04 40.733
2		10 58 28.925
3	SAME LONG. AS	08 55 30.981
	BLOCK 1	08 56 20.169
		09 05 15.200
		09 04 40.733
3		08 58 28.925
4	SAME LONG. AS	06 55 30.981
	BLOCK 1	06 56 20.169
		07 05 15.200
		07 04 40.733
4		06 58 28.925
5	05 13 25.185	12 55 30.981
	05 18 52.670	12 56 20.169
5	05 18 19.008	13 05 15.200
	05 12 30.102	13 04 40.733
5	05 12 40.001	12 58 28.925
6	SAME LONG. AS	10 55 30.981
	BLOCK 5	10 56 20.169
		11 05 15.200
		11 04 40.733
6		10 58 28.925

BLOCK	LONGITUDE	LATITUDE
7	05 13 25.185	08 55 30.981
	05 18 52.670	08 56 20.169
7	05 18 19.008	09 05 15.200
	05 12 30.102	09 04 40.733
7	05 12 40.001	08 58 28.925
8	SAME LONG. AS	06 55 30.981
	BLOCK 5.	06 56 20.169
		07 05 15.200
		07 04 40.733
8		06 58 28.725
9	SAME LONG. AS	04 55 30.981
	BLOCK 5	04 56 20.169
		05 05 15.200
		05 04 40.733
9		04 58 28.725
10	06 13 25.185	12 55 30.981
	06 18 52.670	12 56 20.169
10	06 18 19.008	13 05 15.200
	06 12 30.102	13 04 40.733
10	06 12 @o.001	12 58 28.925
11	SAME LONG. AS	10 55 30.981
	BLOCK 10.	10 56 20.169
		11 05 15.200
		11 04 40.733
11		10 58 28.925
12	SAME LONG. AS	08 55 30.981
	BLOCK 10	08 56 20.169
		09 05 15.200
		09 04 40.733
12		08 58 28.925

BLOCK	LONGITUDE	LATITUDE
13	SAME LONG. AS	06 55 30.981
	BLOCK 10	06 56 20.169
		07 05 15.200
		07 04 40.733
		06 58 28.925
14	SAME LONG AS	04 55 30.981
	BLOCK 10	04 56 20.169
		05 05 15.200
		05 04 40.733
		04 58 08.925
15	06 58 25.185	12 55 30.981
	07 03 52.670	12 56 20.169
	07 03 19.008	13 05 15.200
	06 57 30.102	13 04 40.733
	06 57 40.001	12 58 28.925
16	SAME LONG. AS	10 55 30.981
	BLOCK 15.	10 56 20.169
		11 05 15.200
		11 04 40.733
16		10 58 28.925
	SAME LONG. AS	08 55 30.981
	BLOCK 15	08 56 20.169
		09 05 15.200
		09 04 40.733
17		08 58 28.925
18	SAME LONG AS	06 55 30.981
	BLOCK 15.	06 56 20.169
		07 05 15.200
		07 04 40.733
18		06 58 28.925

BLOCK	LONGITUDE	LATITUDE
19	SAME LONG. AS	04 55 30.981
	BLOCK 15	04 56 20.169
		05 05 15.200
		05 04 40.733
19		04 58 28.925
20	08 13 25.185	12 55 30.981
	08 18 52.670	12 56 20.169
	08 18 19.008	13 05 15.200
	08 12 30.102	13 04 40.733
20	08 12 40.001	12 58 28.925
21	SAME LONG. AS	10 55 30.981
	BLOCK 20	10 56 20.169
		11 05 15.200
		11 04 40.733
21		10 58 28.925
22	SAME LONG. AS	08 55 30.981
	BLOCK 20	08 56 20.169
		09 05 15.200
		09 04 40.733
22		08 88 88.928
23	08 13 25.185	06 55 30.981
	08 18 52.670	06 56 20.169
	08 18 19.008	07 05 15.200
	08 12 30.102	07 04 40.733
23	08 12 40.001	06 58 28.925
24	SAME LONG. AS	04 55 30.981
	BLOCK 23.	04 56 20.169
		05 05 15.200
		05 04 40.733
24		04 58 28.925

BLOCK	LONGITUDE	LATITUDE
25	09 43 25.185	12 55 30.981
	09 48 52.670	12 56 20.169
	09 48 19.008	13 05 15.200
	09 42 30.102	13 04 40.733
	09 42 40.001	12 58 28.925
26	SAME LONG. AS	10 55 30.981
	BLOCK 25	10 56 20.169
		11 05 15.200
		11 04 40.733
26		10 58 28.925
27	SAME LONG. AS	08 55 30.981
	BLOCK 25.	08 56 20.169
		09 05 15.200
		09 04 40.733
27		08 58 28.925
28	SAME LONG. AS	06 55 30.981
	BLOCK 25.	06 56 20.169
		07 05 15.200
		07 04 40.733
		06 58 28.925
29	11 13 25.185	12 55 30.981
	11 18 52.670	12 56 20.169
	11 18 19.008	13 05 15.200
	11 12 30.102	13 04 40.733
29	11 12 40.001	12 58 28.925
30	SAME LONG. AS	10 55 30.981
	BLOCK 29	10 56 20.169
		11 05 15.200
		11 04 40.733
30		10 58 28.925
31	SAME LONG. AS	08 55 30.981
	BLOCK 29.	08 56 20.169

BLOCK	LONGITUDE	LATITUDE
31	11 i3 25.185	08 55 30.981
	11 18 52.670	08 56 20.169
	11 18 19.008	09 05 15.200
	11 12 30.102	09 04 40.733
31	11 12 40.001	08 58 28.925
32	SAME LONG. AS	06 55 30.981
	BLOCK 31.	06 56 20.169
		07 05 15.200
		07 04 40.733
32		06 58 28.925
33	12 43 25.185	12 55 30.981
	12 48 52.670	12 56 20.169
	12 48 19.008	13 05 15.200
	12 42 30.102	13 04 40.733
33	12 42 40.001	12 58 28.925
34	SAME LONG AS	10 55 30.981
	BLOCK 33.	10 56 20.169
		11 05 15.200
		11 04 40.733
34		10 58 28.925
35	SAME LONG. AS	
	BLOCK33.	08 55 30.981
		08 56 20.169
		09 05 15.200
		09 04 40.733
35		08 58 28.925

BLOCK	LONGITUDE	LATITUDE
36	13 58 25.185	12 55 30.981
	14 03 52.670	12 56 20.169
	14 03 19.008	13 05 15.200
	13 57 30.102	13 04 40.733
36	13 57 40.001	12 58 28.925
37	SAME LONG. AS	10 55 30.981
	BLOCK 36	10 56 20.169
		11 05 15.200
		11 04 40.733
37		10 58 28.925

CHAPTER 6 ANALYSIS OF RESULTS.

6.01 For the measure of precision of the direct method of transformation of Nigerian transverse mercator projection to the Universal transverse mercator coordinates (using the second order conformal transformation formula) and the suitability of the method for practical use in coordinate transformation, the standard error and mean values of the discrepancies or differences were calculated.

The differences were of course the differences $(N_c - N_d)$ $(E_c - E_d)$ between table transformed results and the results of the transformation by the direct method.

Fig. 6.1 represents a typical block for which the coefficients of transformation have been derived. It is expected and infact true that the maximum differences will occur at the corner points A,B,C,D (which happens to be the farthest points from the centre.

Working on this theory, differences were computed for points A,B,C,D for each block (appendix 4) and these differences were used to compute the mean, and the standard error (for the northings and the eastings).

$$\mu_N = \frac{\sum(N_c - N_d)}{n}$$

$$\mu_E = \frac{\sum(E_c - E_d)}{n}$$

$$\sigma_N = \sqrt{\frac{\sum(N_c - N_d)^2}{n}}$$

$$\sigma_E = \sqrt{\frac{\sum(E_c - E_d)^2}{n}}$$

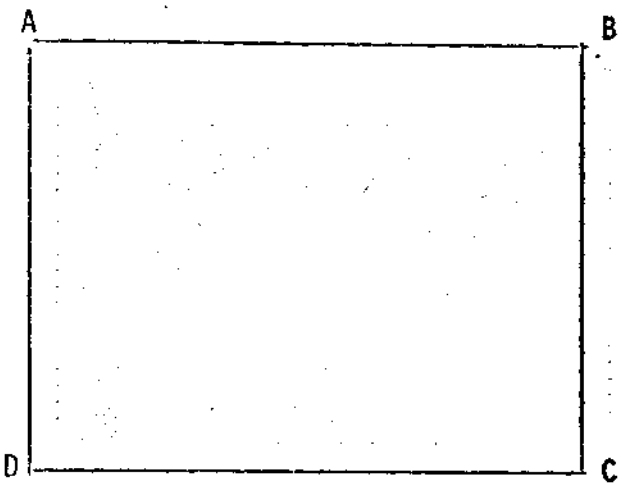


FIG. 6.1

Diagram showing the farthest points from the centre.

μ_N, μ_E denote the mean error for the northings and eastings respectively

σ_N, σ_E denote the standard error for the northings and the eastings respectively.

N_c, E_c denote the UTM coordinates obtained by the tables of transformation.

N_d, E_d denote the UTM coordinates obtained from the direct method using the second order conformal transformation formula.

n denote the total number of points tested. = 145.

On the whole, the differences for one hundred and fourty five points were computed (note that a point in some cases is common to four blocks and in such cases the point is counted as four points.) It was on the basis of this that the standard error was computed for the whole country.

The following are the results obtained

μ_E = 0.072 Metres

μ_N = 0.072 Metres

σ_N = 0.100 Metre

σ_E = 0.094 Metre

The following statistical measures were also computed

Variance $\sigma_{N_2}^2$ = 0.009

Variance σ_E^2 = 0.010

Coefficient of correlation ρ_{EN} = 0.600

A close look at (table 6.2) the tables of differences grouped for the one hundred and forty-five tested points showed that the differences followed or appeared to follow a set pattern which can be described by the following characteristics.

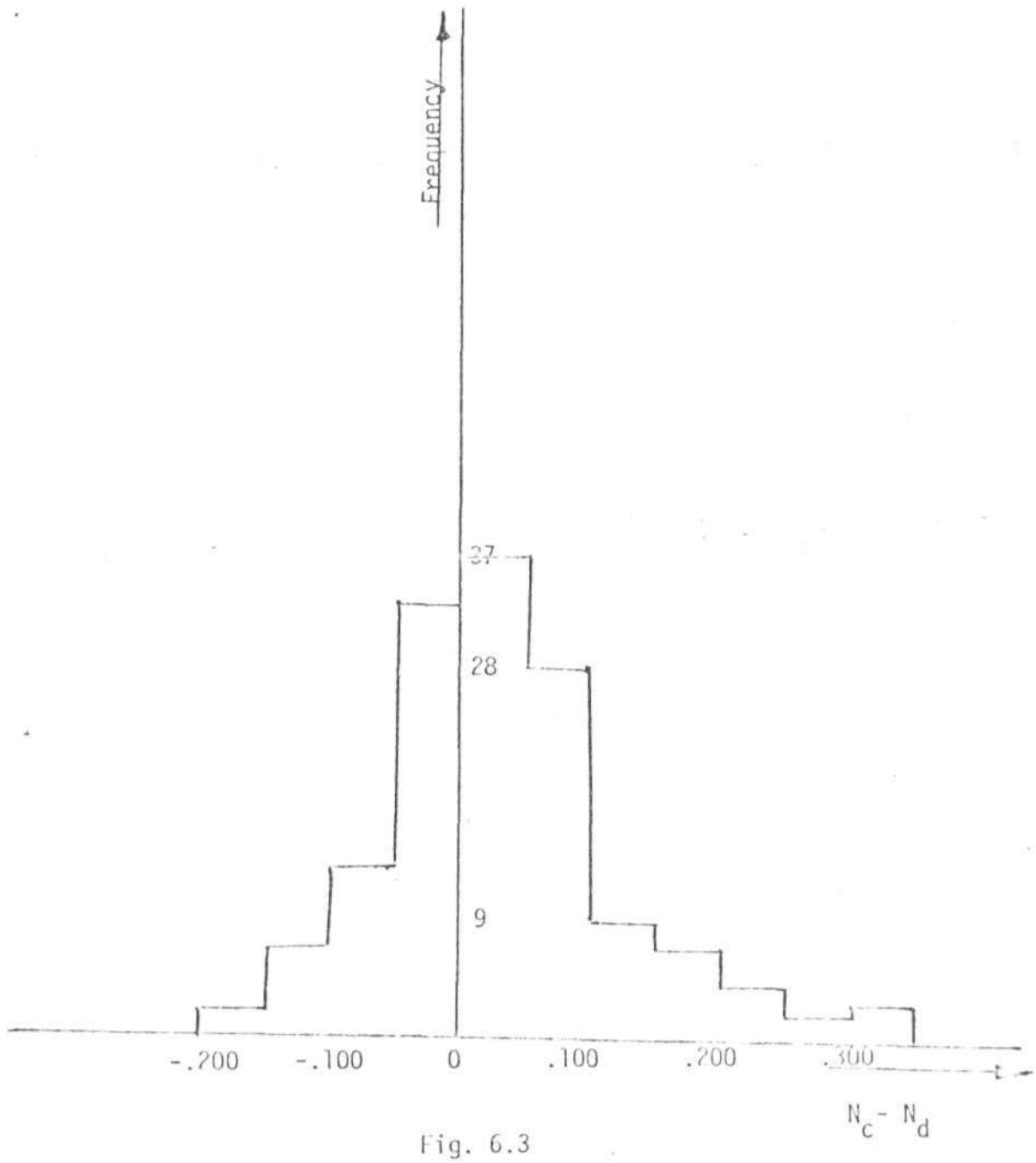


Fig. 6.3

Histogram of the grouped data showing the differences (N_c-N_d) against the frequency of occurrence of such differences.

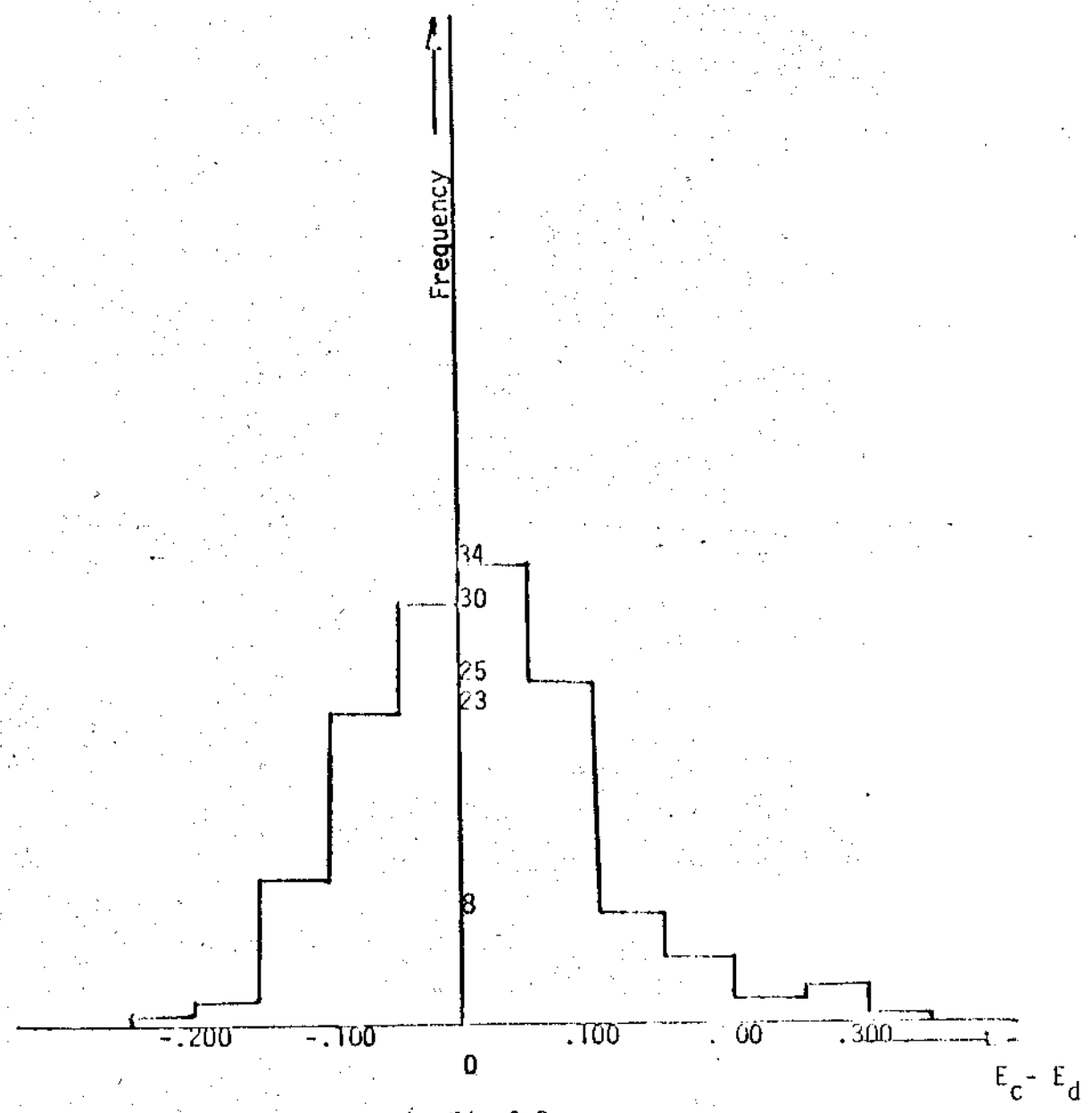


Fig 6.2

Histogram of the grouped data(the differences $E_c - E_d$, $t_c - t_d$ against the frequency of occurrence of such differences.

(i) All differences fluctuate about a central value, in this case zero which is the minimum difference $(N_c - N_d)$ or $(E_c - E_d)$ assuming that the absolute value of the differences were used.

(ii) Positive and negative deviations are almost equally frequent.

(iii) Small deviations are more common than large ones.

On the basis of these properties, the differences grouped as shown in Table 6.1 and the grouped data were used to plot the histograms shown in fig. 6.2 and 6.3.

It is observed that the smooth curve fitting the stepped lines of the histogram is similar to the normal distribution curve. If this is the case, then any deviation greater than $3\sigma_N$ and $3\sigma_E$ must be rejected. This is in accordance with the properties of normal distribution where;

(i) approximately 68.3% of the area under normal curve lies between $\mu \pm \sigma$

(ii) approximately 95.4% of the area under normal curve lies between $\mu \pm 2\sigma$

(iii) approximately 99.7% of the area under normal curve lies between $\mu \pm 3\sigma$ in this case $\mu = 0$

Range	$N_c - N_d$	$E_c - E_d$
-.300	0	0
-.300 - -.250	0	0
-.250- -.200	0	1
-.200- -.150	2	2
-.150- -.100	7	11
-.100- -.050	13	23
-.050- ..000	33	31
000- .050	37	34
.050- .100	28	25
.100- .150	9	8
.150- .200	7	5
.200- .250	4	2
.250- .300	2	3
.300	3	1

TABLE 6.2

Showing the grouped form of the data in appendix 4

6.02 Precision by Comparison of Distances and Bearings.

A further method employed in the determination of the precision of the direct method is by the comparison of bearings and distances between pairs of coordinates computed using the two available methods.

Four of the Enugu township mapping statings were used for this test, and the results are shown in table 6.3 and 6.4.

Station	E_c	N_c	E_d	N_d	$E_c - E_d$	$N_c - N_d$
Ecs 7S	7766.804	4309.135	7766.782	4309.125	.022	.010
Ecs 8S	9611.795	7066.512	9641.775	7066.504	.020	.008
Ecs 14S	2890.0633	0571.088	2890.594	0571.080	.029	.008
TBs 24	9173.079	1536.355	9173.057	1536.348	.022	.007

TABLE 6.3

		By Tables	By Direct
Ecs 7S - Ecs 8S	Bearing	$34^{o}12'55.2"$	$34^{o}12'55.3"$
	Distance	3334.474	3334.477
Ecs 14S - TB 24	Bearing	$161^{o}16'18.7"$	$161^{o}16'18.8"$
	Distance	11578.3o8	11578.3o7

TABLE 6.4

Table 6.4 shows that there is a difference of 0.1" in bearing and of 0.003 metres in distance which rpoves that the direct method is quite reliable.

6.03 Precision Measured as a Function of Distance from Centre:

The points used for the computation of the coefficients of transformation were picked close to the centre of each block. Thus if the assumption is made that the direct transformation is free of errors at the centre of each block, then it is possible to get a measure of the precision of the direct method as a ratio of the differences in coordinate and the distance from the centre of the points. The results of a test computation is shown in table 6.5.

The seven largest differences in the one hundred and fourty-five tested points were used for this test and the result in the table indicate an accuracy of .002 metres per kilometre for both the eastings and the northings. This implies a precision of 2 parts per million.

Block	Point	App. Dist. From Centre	N (N_c N_d)	E (E_c - E_d)	Misc. Per Kilometre
7	bb	272 Km.	.218	.204	.001
12	ar	272 Km.	.007	.323	.001
12	cc	272 Km.	.215	.256	.001
31	m	272 Km.	.326	.032	.001
35	c	272 Km.	.309	.007	.001
37	de	272 Km.	.349	.259	.002

Table 6.5

Showing the precision per Kilometre for the six worst cases.

CHAPTER 7 CONCLUSIONS

Since the direct method of transformation was built on the grid - geographical - grid method, it cannot be more accurate, however its relative accuracy of 2 ppm is considered to be good method enough for cadastral surveys.

The direct method of transformation is easier to operate since it requires only the basic knowledge of addition and multiplication. The method is further simplified by the provision of a chart (app. 5) which shows the 1:100,000 and (1:50,000) Nigerian map sheets numbers and the corresponding block numbers for each of these map sheets.

Nigeria being a developing country, surveyors and most survey organisations have not got direct access to computers. This direct method could serve as an alternative for surveyors who hitherto would have required the services of computer consultants to undertake the transformation of coordinates for them.

CHAPTER 8. RECOMMENDATIONS

It is hereby recommended that to use the direct method of transformation to transform Nigerian TMP. coordinates to UTM., the following procedure should be followed.

(i) The computer should find out the Nigerian map sheet number (1:100,000, 1:50,000, 1:25000, 1:10,000) on which map sheet the station whose coordinates are to be transformed appear.

(ii)The equivalent block number is then read off from the chart in appendix 5 (This is the number of the block on which the station being transformed appear.)

(iii)The coefficients of this block is then extracted from the list of coefficients and are written in columns A and B of the form designed for the computation. (See appendix 6)

(iv) The necessary multiplication and additions are then carried out to obtain the equivalent UTM. coordinates.)

A sample computation is provided in appendix 7 and can be compared with the computation using the tables of transformation.The sample computation of this is in appendix 8.
It is also recommended that an investigation into the use of this method in other developing countries be carried out. It is hoped that such an investigation will inevitably be a useful venture.

APPENDIX 1

MAXIMUM AND MINIMUM OVERLAP FOR THE TMP PROJECTION

Two points A and B located as follows are transformed from geographical coordinates to rectangular. Since the points lie on the junction between the two belts - West and East, the transformation is carried out from the two belts. The differences between the coordinates (Eastings) obtained are 7700.778 metres and 4371.64 metres and these differences mark the maximum discontinuity and maximum overlap respectively, since they are located at

Point	Latitude	Longitude
A	14^{o} 00	06^{o} 30'
B	04^{o} 00	06^{o} 30

14^{o} and 04^{o} which mark the Northern and Southern boundaries of the TMP projection system of Nigeria

APPENDIX 2

TEST COMPUTATION OF THE DIVIDED DIFFERENCES METHOD

BLOCK 5

TEST COMPUTATION USING THE DIVIDED DIFFERENCES METHOD

The following points were used to compute Z_1Z_2, $Z_1Z_2Z_3$, $Z_1Z_2Z_3Z_4$.

bb, bd, ap, ar. While the point bc was used to test the correctness of the method.

Results.

Z_1Z_2 = 1.00016005 0.00589123

$Z_1Z_2Z_3$ = 0.00000375 0.00000270 .....(i)

$Z_1Z_2Z_3Z_4$ = 0.00000006 0.00000028

Coordinates of point bc

TMP. 230738.266 E 995069.684 N .....(ii)

UTM. 662682.175 E 1437486.204 N .....(iii)

Substituting the results in (i) in the formula (4.009) the result obtained for point bc using the divided differences,method is as follows

Z = 662682.175 E 1437486.196 which compares with the result obtained by the tables of transformation as found in (iii) above.

The above result indicate that the method of divided differences is quite accurate however the back substitution is tedious and requires the knowledge of complex numbers.

APPENDIX 3

PROGRAMMES ON HP. 29C CALCULATOR.

(i) Geographical - Northings

(ii) Geographical - Eastings

(iii) Other Programes

PRESS FLEX, SWITCH TO PRGM MODE, PRESS , F PRGM, THEN KLY IN PROGRAM.

TITLE GEOGRAPHICAL → PROJECTION (NORTHINGS)

01	gLbl 2	34	×	67	+
02	1	35	Rcl 9	68	f 2
03	ST · 0	36	g R/S (1/x)	69	ST ÷ 8
04	Rcl · 5	37	f 4	70	5
05	Rcl 2	38	×	71	↑
06	g 0 (gH)	39	ST + · 0	72	Rcl 2
07	ST 2	40	Rcl · 2	73	f 6
08	f 5 (Cos)	41	Rcl 4	74	g 2
09	g 2 (x^2)	42	×	75	−
10	×	43	ST × · 0	76	Rcl 7
11	ST − · 0	44	Rcl 1	77	g 2
12	Rcl · 4	45	g 0 (g→H)	78	×
13	Rcl 2	46	Rcl 0	79	2
14	f 5	47	g 0	80	4
15	g 2	48	−	81	÷
16	×	49	g→3 (ABS)	82	.
17	ST × · 0	50	Rcl 6	83	5
18	Rcl · 0	51	×	84	+
19	CHS	52	Rcl 2	85	Rcl 8
20	1	53	f 5	86	Rcl 7
21	+	54	×	87	g 2
22	Rcl 2	55	ST 7	88	×
23	f 5	56	Rcl 5	89	Rcl 2
24	×	57	Rcl 4	90	f 6 (Tan)
25	Rcl 2	58	×	91	×
26	f 4 (Sin)	59	ST 8	92	×
27	×	60	Rcl 2	93	Rcl · 0
28	Rcl · 3	61	f 4	94	+
29	×	62	g 2	95	Rcl · 1
30	CHS	63	Rcl 3	96	−
31	ST · 0	64	×	97	R/S
32	Rcl 2	65	CHS	98	GTN
33	Rcl 9	66	1		

n

Σx

Σx^2

Σy

Σy^2

Σxy

54

STORAGE REGISTERS		
R 0	Central Merid. λ_0	0
* R 1	λ (Long.)	1
* R 2	ϕ (Lat.)	2
R 3	0.00680 3511	3
R 4	0.99995 (Nim) / 0.9996 (Utm)	4
R 5	6378249.145	5
R 6	0.01745329 3	6
R 7		7
R 8		8
R 9	3600	9
R 10		10
R 11	442153.045 (NTm) / 0.000 (UTm)	11
R 12	6367386.647	12
R 13	0.005131 005	13
R 14	0.00042 8023	14
R 15	0.005282	15
		16
	REGISTER	17
T		18
Z		19
Y		
X		

INSTRUCTIONS / COMM.

INPUT 20° 10' 15".999 AS

20.10 15799

INPUT λ PRESS STO 1

INPUT φ PRESS STO 2

THEN PRESS GSUB 2.

HP – 25. PROGRAM FORM.

PRESS FEEX, SWITCH TO PRGM MODE, PRESS , F PRGM, THEN KEY IN PROGRAM.

TITLE GEOGRAPHICAL → PROJECTION EASTINGS.

01	gLBL 1	34	f5	67	0
02	RCL 1	35	g2	68	ST ÷ .1
03	g 0 (G→H)	36	ST 9	69	RCL 9
04	RCL 0	37	RCL 3	70	RCL 2
05	g 0	38	↑	71	f6
06	—	39	1	72	g2
07	ST . 0.	40	↑	73	—
08	g3 (ABS)	41	RCL 3	74	.17
09	RCL 6	42	—	75	G→2
10	X	43	÷	76	X
11	RCL 2	44	ST X 9	77	6
12	g 0	45	1	78	÷
13	ST 2	46	ST + 9.	79	ST + .1
14	f5 (COS)	47	RCL 2	80	1
15	X	48	f6	81	ST + .1
16	ST 7.	49	g→2	82	RCL 7
17	RCL 5	50	g→2	83	RCL 8
18	RCL 4	51	ST . 1	84	X
19	X	52	RCL 2	85	ST X . 1
20	ST 8	53	f6	86	RCL . 0
21	RCL 2.	54	g→2	87	g→+ x>0
22	f4	55	1	88	GTO 2
23	g2	56	8	89	RCL .5
24	↑	57	X	90	RCL .1
26	RCL 3	58	ST — .1	91	—
26	X	59	5.	92	R/S
27	CHS	60	ST + .1	93	gLBL 2
28	↑	61	RCL 7	94	RCL .5
29	1	62	g2	95	RCL .1
30	+	63	g→2	96	+
31	f2 √	64	ST X . 1	97	R/S
32	ST ÷ 8	65	1	98	gRTN
33	RCL 2	66	2	99	55

	STORAGE REGISTERS.	
	R 0	λ_0 (CM.) 0
*	R 1	λ (LONG.) 1
*	R 2	ϕ (LAT) 2
	R 3	0.00 6803511 3
	R 4	0.99975 (TMP) | 0.9996 (UTM) 4
	R 5	6378249.145 5
	R 6	0.0174533293 6
	R 7	7.
	R 8	8
	R 9	9
n	R·0	10
Σx	R·1	11
Σx^2	R·2	12
Σy	R·3	13
Σy^2	R·4	14
ΣXY	R·5	50,000.00 UTM 15 230938.261 — WBS 670553.934 — MID 1110369.702 — EA 16
	REGISTER	17
T		18
Z		19
Y		
X		

INSTRUCTIONS / COMM.

INPUT λ ✓ PRESS STO 1
φ ✓ STO 2
PRESS GSUB 1

ASHIM IBRAHIM LIBRARY
RECEIVED

HP-25 PROGRAM ENTRY.

PRESS f PRGM, SWITCH TO PRGM MODE, PRESS f PRGM, THEN KEY IN PROGRAM

TITLE COORDINATES FROM COEFFICIENTS.

01	gLBL	34	RCL 7	67	
02	RCL 1	35	×	68	
03	g2	36	ST+9	69	
04	↑	37	RCL 0	70	
05	RCL 2	38	R/S	71	
06	+	39		72	
07	g2	40	↑	73	
08	—	41	RCL 0	74	
09	STO 8	42	×	75	
10	RCL 1	43	ST 0	76	
11	RCL 2	44	RCL 1	77	
12	×	45	RCL 1	78	
13	2	46	×	79	
14	×	47	ST+0	80	
15	STO 9	48	RCL 2	81	
16		49	RCL 2	82	
17	↑	50	×	83	
18	RCL 3	51	ST+0	84	
19	×	52	RCL 8	85	
20	ST 0	53	RCL 3	86	
21	RCL 1	54	×	87	
22	RCL 4	55	ST+0	88	
23	×	56	RCL 9	89	
24	ST+0	57	RCL 4	90	
25	RCL 2	58	×	91	
26	RCL 5	59	ST+0	92	
27	×	60	RCL 0	93	
28	ST+0	61	R/S	94	
29	RCL 8	62	GRTN	95	
30	RCL 6	63		96	
31	×	64		97	
32	ST+0	65		98	
33	RCL 4	66		99	

	STORAGE REGISTERS		
	R_0	b₀	0
*	R_1	e	1
*	R_2	n	2
N	R_3	b_0	3
N	R_4	b_1	4
N	R_5	b_2	5
N	R_6	b_3	6
N	R_7	b_4	7
	R_8		8
	R_9		9
n	R_{10}	q_0	10
Σx	R_{11}	q_1	11
Σx^2	R_{12}	q_2	12
Σy	R_{13}	q_3	13
Σy^2	R_{14}	q_4	14
Σxy	R_{15}		15
			16
	REGISTER		
	T		18
	Z		19
	Y		
	X		

E

INSTRUCTIONS/COMM.

FEED IN e, n.

Note e×n must be divided by 100000 before use.

Then press GSUB 1 to get N (VTM) and R/S to get E (VTM)

HP – 29c. PROGRAM FORM.

PRESS FEEX, SWITCH TO PRGM MODE, PRESS , F PRGM., THEN KEY IN PROGRAM.

TITLE MATRIX FORMATION

01	g Lbl 1	34		67			
02	f fix 8	35		68			
03	Rcl 1	36		69			
04	g → 2	37		70			
05	↑	38		71			
06	Rcl 2	39		72			
07	g 2	40		73			
08	—	41		74			
09	R/S	42		75			
10	Rcl 1	43		76			
11	Rcl 2	44		77			
12	X	45		78	n		
13	2	46		79	ΣY		
14	X	47		80	ΣX^2		
15	R/S	48		81	Σy		
16	g RTN	49		82	Σy^2		
17		50		83	ΣXY		
18		51		84			
19		52		85			
20		53		86			
21		54		87			
22		55		88			
23		56		89			
24		57		90			
26		58		91			
26		59		92			
27		60		93			
28		61		94			
29		62		95			
30		63		96			
31		64		97			
32		65		98			

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STORAGE REGISTERS.

R 0		0
R 1	e	1
R 2	n	2
R 3		3
R 4		4
R 5		5
R 6		6
R 7		7
R 8		8
R 9		9
R 10		10
R 11		11
R 12		12
R 13		13
R 14		14
R 15		15
	REGISTER	17
T		18
Z		19
Y		
X		

INSTRUCTIONS / COMM.

1st Display → $(e^2 - n^2)$

2nd Display → 2en.

APPENDIX 4

DIFFERENCES (E_c- E_d), (N_c- N_d) FOR THE ONE HUNDRED AND FOURTY-FIVE TESTED POINTS FOR STATISTICAL ANALYSIS.

(E_c- E_d), and (N_c- N_d) ARE THE DIFFERENCES BETWEEN THE COORDINATES OBTAINED USING THE TABLES OF TRANSFORMATION AND THAT OBTAINED USING THE DIRECT METHOD.

	POINT	$N_c - N_d$	$E_c - E_d$		POINT	$N_c - N_d$	$E_c - E_d$
1	cq	.155	-.126	34	az	-.065	.224
2	cs	.090	.071	35	ax	-.027	.007
3	bb	-.002	-.088	36	ap	-.011	.148
4	bd	-.018	.012	37	ar	-.007	.323
5	cs	.173	-.124	38	ca	.068	.079
6	cu	-.015	.083	39	cc	.215	.256
7	bd	-.078	-.038	40	ar	-.031	-.086
8	bf	.049	-.006	41	at	.023	.066
9	cu	.072	-.036	42	cc	.093	-.152
10	cw	.019	.058	43	ce	.143	.032
11	bf	-.036	-.048	44	at	-.080	-.063
12	bh	-.041	.028	45	av	.019	.075
13	cw	-.027	-.028	46	ce	.040	-.092
14	cy	.018	.035	47	cg	.012	-.048
15	bh	.056	-.051	48	av	-.076	.131
16	bj	-.132	.033	49	ax	.026	.147
17	bb	.218	.204	50	cg	.011	-.010
18	ap	-.115	.03o	51	ci	.097	.135
19	bd	-.016	.082	52	ax	-.038	.256
20	ar	.060	.188	53	az	.000	.090
21	bd	.108	-.030	54	ci	-.011	.028
22	bf	.018	.064	55	ak	.061	.044
23	ar	-.082	.015	56	ca	.098	-.078
24	at	-.023	-.002	57	cc	-.021	-.078
25	bf	-.041	.055	58	ae	.005	-.080
26	bh	.077	.060	59	ag	.111	-.106
27	at	-.031	-.007	60	cc	-.077	-.044
28	av	-.116	.035	61	ce	.023	-.177
29	bh	.053	-.087	62	ag	.054	-.133
30	bj	.072	.001	63	ai	-.073	-.056
31	av	.005	-.054	64	ce	-.093	-.113
32	ax	-.016	-.105	65	cg	-.062	-.074
33	bf	.045	-.210	66	ai	-.008	.004

	POINT	$N_c - N_d$	$E_c - E_d$		POINT	$N_c - N_d$	$E_c - E_d$
67	ak	-.047	-.102	100	o	.025	-.076
68	cg	.016	.085	101	x	.106	.067
69	de	.349	.274	102	z	.053	.023
70	ci	.036	-.003	103	o	.078	.034
71	ak	.064	.026	104	q	.169	.044
72	am	.039	.076	105	z	.027	.004
73	ci	.011	-.078	106	ab	.032	-.036
74	ck	.006	-.035	107	q	.062	-.012
75	am	.028	-.085	108	s	.055	.018
76	ao	.016	-.036	109	ab	-.030	-.050
77	ae	.005	-.145	110	ad	-.054	-.039
78	ag	.115	.027	111	x	-.030	-.038
79	v	.122	.046	112	u	-.005	-.078
80	x	.062	-.133	113	m	.326	.032
81	ag	.088	.067	114	o	.166	.154
82	ai	-.001	-.070	115	f	-.046	-.062
83	x	.034	-.079	116	h	.081	.035
84	z	.087	.032	117	o	.165	.156
85	ai	.044	.062	118	q	.104	.086
86	ak	-.018	.034	119	h	-.105	-.100
87	z	.016	.048	120	j	-.134	.090
88	ab	.076	.076	121	q	.081	.031
89	ak	-.029	.022	122	s	.244	.069
90	am	-.027	-.039	123	j	.080	-.125
91	ab	.009	-.194	124	i	-.150	-.015
92	ad	-.019	-.005	125	s	.085	-.010
93	am	-.086	.165	126	i	-.126	-.081
94	ao	.030	-.050	127	t	.059	.056
95	ad	.014	.055	128	f	.113	.115
96	dg	-.013	.041	129	h	-.027	.000
97	v	-.036	-.099	130	a	.235	-.025
98	x	.086	-.008	131	c	.309	-.007
99	m	.092	-.007	132	h	-.051	.163

	POINT	$N_c - N_d$	$E_c - E_d$
133	j	-.151	.081
134	c	.019	.075
135	e	.092	.043
136	j	-.085	.145
137	i	-.156	-.007
138	e	.005	-.008
139	a	.267	.016
140	c	.000	.057
141	da	.266	-.012
142	dc	.074	.007
143	c	.186	.140
144	e	.192	.047
145	dc	-.040	.128

APPENDIX 5

CHART OF THE 1:100,000 MAP SHEET NUMBER AND THE CORRESPONDING BLOCK NUMBER FOR THE NEW DIRECT METHOD.

1:100,000 MAP SHEET NUMBER	BLOCK NUMBER	1:100,000 MAP SHEET NUMBER	BLOCK NUMBER	1:100,000 MAP SHEET NUMBER	BLOCK NUMBER	1:100,000 MAP SHEET NUMBER	BLOCK NUMBER
1	1	31	10	64	29	97	6
2	5	32	15	65	33	98	6
3	5	33	15	66	33	99	11
4	5	34	20	67	33	100	16
5	10	35	20	68	36	101	16
6	33	36	20	69	36	102	21
7	33	37	25	70	36	103	21
7A	36	38	25	71	2	104	21
8	1	39	25	72	2	105	26
9	5	40	29	73	6	106	26
10	5	41	29	74	6	107	26
11	5	42	29	75	6	108	30
12	10	43	33	76	11	109	30
13	15	44	33	77	16	110	30
14	15	45	33	78	16	111	34
15	20	46	36	79	21	112	34
16	20	47	36	80	21	113	34
16A	20	48	1	81	21	114	37
17	25	49	1	82	26	115	37
18	25	50	5	83	26	116	2
19	29	51	5	84	26	117	2
20	29	52	5	85	30	118	6
21	29	53	10	86	30	119	6
22	33	54	15	87	30	120	6
23	33	55	15	88	34	121	11
24	33	56	20	89	34	122	16
25	36	57	20	90	34	123	16
25A	36	58	20	91	37	124	21
26	1	59	25	92	37	125	21
27	1	60	25	93	37	126	21
28	5	61	25	94	2	127	26
29	5	62	29	95	2	128	26
30	5	63	29	96	6	129	26

1:100,000 MAP SHEET NUMBER	BLOCK NUMBER	1:100,000 MAP SHEET NUMBER	BLOCK NUMBER	1:100,000 MAP SHEET NUMBER	BLOCK NUMBER	1:100,000 MAP SHEET NUMBER	BLOCK NUMBER
130	30	164	17	197	35	231	22
131	30	165	17	198	3	232	27
132	30	166	22	199	3	233	27
133	34	167	22	200	3	234	27
134	34	168	22	201	3	235	31
135	34	169	27	202	7	236	31
136	37	170	27	203	7	237	31
137	2	171	27	204	7	238	35
138	2	172	31	205	12	239	4
139	6	173	31	206	17	240	4
140	6	174	31	207	17	241	4
141	6	175	35	208	22	242	4
142	11	176	35	209	22	243	8
143	16	177	35	210	22	244	8
144	16	178A	3	211	27	245	8
145	21	178	3	212	27	246	13
146	21	179	3	213	27	247	18
147	21	180	3	214	31	248	18
148	26	181	7	215	31	249	23
149	26	182	7	216	31	250	23
150	26	183	7	217	35	251	23
151	30	184	12	218	35	252	28
152	30	185	17	219	3	253	28
153	30	186	17	220	3	254	28
154	34	187	22	221	3	255	32
155	34	188	22	222	3	256	32
156	34	189	22	223	7	257	32
157	3	190	27	224	7	258	35
158	3	191	27	225	7	259	4
159	3	192	27	226	12	260	4
160	7	193	31	227	17	261	4
161	7	194	31	228	17	262	4
162	7	195	31	229	22	263	8
163	12	196	35	230	22	264	8

1:100,000 MAP SHEET NUMBER	BLOCK NUMBER
265 ---	8
266 ---	13
267 ---	18
268 ---	18
269 ---	23
270 ---	23
271 ---	23
272 ---	28
273 ---	28
274 ---	28
275 ---	32
276 ---	32
277 ---	32
278A---	4
278 ---	4
279A---	4
279 ---	4
280A---	4
280 ---	4
281A---	4
281 ---	4
282 ---	8
283 ---	8
284 ---	8
285 ---	13
286 ---	18
287 ---	18
288 ---	23
289 ---	23
290 ---	23
291 ---	28
292 ---	28
293 ---	28
294 ---	32
295A---	32
295 ---	32

1:100,000 MAP SHEET NUMBER	BLOCK NUMBER
296 ---	8
297 ---	8
298 ---	8
299 ---	13
300 ---	18
301 ---	18
302 ---	23
303 ---	23
304 ---	23
305 ---	28
306 ---	28
307 ---	9
308 ---	9
309 ---	9
310 ---	14
311 ---	19
312 ---	19
313 ---	24
314 ---	24
315 ---	24
316 ---	28
317 ---	9
318 ---	9
319 ---	14
320 ---	19
321 ---	19
322 ---	24
323 ---	24
324 ---	24
325 ---	9
326 ---	9
327 ---	14
328 ---	19
329 ---	19
330 ---	24
331 ---	24

1:100,000 MAP SHEET NUMBER	BLOCK NUMBER
332 ---	24
333 ---	9
334 ---	14
335 ---	19
336 ---	19
337 ---	24
338 ---	24

APPENDIX 6

COMPUTATION FORM DESIGNED TO BE USED FOR THE DIRECT METHOD

DIRECT TRANSFORMATION OF COORDINATES

NIGERIAN TMP. TO UTM.

AREA..... STATION.....

LOCATION..... MAP SHEET NO.....

COMPUTER..... BLOCK NO.....

DATE .....

TMP. COORDINATES: N = ... n = N x 0.00001

E = ... e = E x 0.00001

PRODUCT B x C	B	C	A	PRODUCT C x A
	b_o=	1.0000000	a_o=	
	b_1=	e=	a_1=	
	b_2=	n=	a_2=	
	b_3=	e^2-n^2=	a_3=	
	b_4=	2en=	a_4=	

.....=SUM SUM.....

x100000 x100000

UTM (N) UTM (E)=

FINAL UTM. COORDINATES: N = ...

E = ...

APPENDIX 7

SAMPLE COMPUTATION OF THE DIRECT METHOD

DIRECT TRANSFORMATION OF COORDINATES

NIGERIAN TMP. TO UTM.

AREA. KADUNA.....
LOCATION..NIGERIA.
COMPUTER. ARINOLA..L.L.

STATION. B6 (NATIONAL TRIG.)
MAP SHEET NO....144....
BLOCK NO.....16.....
DATE .....1|10|80.....

TMP. COORDINATES: N = 698543:424 n = N x 0.00001
E = 555886:291 e = E x 0.00001

PRODUCT B x C	B	C	A	PRODUCT C x A
4·42514914	b_o=4·42514914	$\frac{1}{}$	a_o=-2·26289549	-2·26289549
-0·00329999	b_1=-0·00061580	e=5·35886291	a_1=1·00080214	5·36316147
6·99104452	b_2=1·00080314	n=6·98543424	a_2=0·00061361	0·00428633
-0·00000763	b_3=0·00000038	$e^3 n^2$=20·07887943	a_3=0·00006868	0·00136898
-0·00509926	b_4=-0·00006811	^{2}en 74·86716812	a_4=-0·00000026	-0·00001947
11:40778678=SUM			SUM 3:10590182	
x100000			x100000	
1140778·678 UTM (N)			UTM (E)=310590·182	

FINAL UTM. COORDINATES: N = 1140778:678
E = 310590:182

APPENDIX 8

SAMPLE COMPUTATION USING THE METHOD OF THE TABLES OF TRANSFORMATION.

TRANSVERSE MERCATOR PROJECTION

Nigerian System — Ellipsoid Clarke 1880 — UNITS: metres

PAGE

AREA: KADUNA

LOCATION: NIGERIA

COMPUTER: ARINOLA.2.L

DATE: 17/9/80

STATION: B6

BELT: 32

Central Meridian	False Easting (Fs)
West belt 4°30'	230 738.27 m
Mid belt 8°30'	670 883.98 m
East belt 12°30'	1 110 369.70 m

1 foot = 0.3047972654 metres
1 metre = 3.280840426 feet

Transformation of coordinates

geographical to grid

LATITUDE Φ	10° 18' 58".215	p	0.622850	LONGITUDE λ		09° 16' 13".150	
Tabular (IV) p¹ (From minute of φ)	304 154 458	p²	0.3877167	Central Meridian λ₀		09°	
Interpolation for seconds of φ	− 15 510	p³	0.2414377	Δλ		0° 43' 46".850	
Δ² (IV) from graph	+ 001	p⁴	0.150339 8	Δλ (seconds)		6226".850	
(IV)	304 138 949	p = 0.0001 Δλ					
Tabular (V) (From minute of φ)	112 296	Tabular (I) (From minute of Φ)				1 138 478 767	
Interpolation for seconds of φ	− 030	Interpolation for seconds of Φ				+ 1 787 782	
(V)	112 266				(I)	1140 266 551	
(IV) p	189 382 792	Tabular (II) (From minute of Φ)	1318.293				
(V) p²	+ 27 105	Interpolation for seconds of Φ	+ 1 980				
From graph B₀	+ .008	(II)	1320 273	(II) p²		511 918	
+ East − West of Central Meridian E'	189 409 905	(III)	1 258	(III) p³		189	
False Easting Fs	500 000 000	From graph		A₀		000	
Em	310 590 095	N = (I) + (II) p² + (III) p³ + A₀		Nm		140 778 658	
Eft		E' = (IV) p + (V) p³ + B₀		Nft			

SECOND STEP

Convergence

Grid azimuth = Geodetic azimuth − C

Tabular (XII) (From minute of Φ)			(XII) p	
Interpolation for seconds of Φ	+		(XII) p³	
(XII)		From graph	C₀	
C = (XII) p + (XIII) p³ + C₀			C	
(XIII)		+ East − West of Central Meridian	C	

Transformation of coordinates

grid to geographical

Eft		Φ = Φ' − (VII) q² + (VIII) q⁴ − D₀		Nft		543	
Em	535 886 291	Δλ = (IX) q − (X) q³ + E₀		Nm		698 339 424	
Fs	−670 553 984	q always positive (+)		Tabular (I) (From minute of Φ'₀)		698 339 439	
E'		q	0.1346769	Difference		203 985	
q = 0.000001 E'		q²	0.1813583	Tabular (IX) (From min of Φ'₀)	32 874 827		
		q³	0.002442	Interpolation for seconds of Φ'₀	+ 192		
		q⁴	0.000329	Δ² (IX) from graph	+ 000		
Interpolating N in (I), Φ'		10° 19' 06".641		(IX)	32 875 019		
Tabular (VII) (From min of Φ'₀)	464 655			Tabular (X) (From min of Φ'₀)	144 541	(IX)	+ 4427 1203
Interpolation for seconds of Φ'	+ 065			Interpolation for seconds of Φ'	004	(X) q³	− 353
(VII)	464 720			(X)	144 545	E₀	+ 000
(VII) q² (in sec)	8 428	(VII) q²	− 08 428	Δλ (seconds)			4426".850
(VIII)	4 86	(VIII) q⁴	+ 002	East when E' is (+) West when E' is (−) Δλ		01° 13' 46".850	
From graph	D₀	− 000		Central Meridian, λ₀		08° 30'	
LATITUDE Φ		10° 18' 58".215		LONGITUDE λ		09° 16' 13".150	

FIRST STEP

Convergence

Grid azimuth = Geodetic azimuth − C

Tabular (XV) (From minute of Φ)			(XV) q	+
Interpolation for seconds of Φ			(XV) q³	−
(XV)		From graph	F₀	+
C = (XV) q − (XVI) q³ + F₀			C	
(XVI)		+ East − West of Central Meridian	C	

Vector

Grid distance = Geodetic distance × K.

	1 000 000	Tabular (XVIII)	
+ (XVIII) q²		K = K₀ [Z] = 0.99975 [1 + (XVIII) q² + 0.00003 q⁴]	
+ 0.00003 q⁴			

APENDIX 9

COEFFICIENTS OF TRANSFORMATION FOR THE THIRTY-SEVEN BLOCKS COVERING THE WHOLE COUNTRY, (NIGERIA).

BLOCK NO.	NORTHINGS COEFFICIENTS	EASTINGS COEFFICIENTS
1	b_o = 4.41794616	a_o = 4.35882326
	b_1 = 0.00187496	a_1 = 0.99926625
1	b_2 = 0.99928878	a_2 = -0.00189183
	b_3 = 0.00000289	a_3 = 0.00020055
1	b_4 = 0.00020125	a_4 = -0.00000167
2	b_o = 4.41810942	a_o = 4.35862022
	b_1 = 0.00184647	a_1 = 0.99925677
2	b_2 = 0.99925966	a_2 = -0.00184556
	b_3 = 0.00000170	a_3 = 0.00020315
2	b_4 = 0.00020306	a_4 = -0.00000153
3	b_o = 4.41812925	a_o = 4.35858824
	b_1 = 0.00183322	a_1 = 0.99924837
3	b_2 = 0.99925860	a_2 = -0.00183301
	b_3 = 0.00000201	a_3 = 0.00020428
3	b_4 = 0.00020406	a_4 = -0.00000113
4	b_o = 4.41816698	a_o = 4.35857478
	b_1 = 0.00182955	a_1 = 0.99924287
4	b_2 = 0.99924527	a_2 = -0.00182422
	b_3 = 0.00000091	a_3 = 0.00020552
4	b_4 = 0.00020470	a_4 = -0.00000085

BLOCK NO.	NORTHINGS COEFFICIENTS	EASTINGS COEFFICIENTS
5	b_o= 4.41796071	a_o = 4.35857681
	b_1= 0.00185411	a_1 = 0.99927337
5	b_2= 0.99929226	a_2 = -0.00184257
	b_3= 0.00000330	a_3 = 0.00020286
5	b_4= 0.00020205	a_4 = -0.00000260
6	b_o= 4.41806541	a_o = 4.35858618
	b_1= 0.00183799	a_1 = 0.99927513
6	b_2= 0.99927492	a_2 = -0.00184242
	b_3= 0.00000280	a_3 = 0.00020298
6	b_4= 0.00020323	a_4 = -0.00000272
7	b_o= 4.41812758	a_o = 4.35855472
	b_1= 0.00183176	a_1 = 0.99925452
7	b_2= 0.99925974	a_2 = -0.00182252
	b_3= 0.00000205	a_3 = 0.00020487
7	b_4= 0.00020403	a_4 = -0.00000206
8	b_o= 4.41816817	a_o = 4.35856353
	b_1= 0.00181858	a_1 = 0.99925670
8	b_2= 0.99925227	a_2 = -0.00182854
	b_3= 0.00000224	a_3 = 0.00020417
8	b_4= 0.00020529	a_4 = -0.00000185

BLOCK NO.	NORTHINGS COEFFICIENTS	EASTINGS COEFFICIENTS
9	b_o = 4.41816532	a_o = 4.35863895
	b_1 = 0.00183041	a_1 = 0.99919984
9	b_2 = 0.99924010	a_2 = -0.00181792
	b_3 = -0.00000016	a_3 = 0.00021212
9	b_4 = 0.00020580	a_4 = 0.00000038
10	b_o = 4.44791840	a_o = -2.31887339
	b_1 = -0.00581736	a_1 = 1.00575814
10	b_2 = 1.00577102	a_2 = 0.00583468
	b_3 = 0.00001744	a_3 = -0.00060236
10	b_4 = -0.00060332	a_4 = -0.00001720
11	b_o = 4.44768492	a_o = -2.31826685
	b_1 = -0.00569933	a_1 = 1.00577262
11	b_2 = 1.00576755	a_2 = 0.00569486
	b_3 = 0.00001476	a_3 = -0.00060898
11	b_4 = -0.00060861	a_4 = -0.00001487
12	b_o = 4.44752911	a_o = -2.31803433
	b_1 = -0.00562479	a_1 = 1.00577407
12	b_2 = 1.00576741	a_2 = 0.00562369
	b_3 = 0.00001221	a_3 = -0.00061296
12	b_4 = -0.00061260	a_4 = -0.00001254

BLOCK NO.	NORTHINGS COEFFICIENTS	EASTINGS COEFFICIENTS
13	b_0 = 4.44741931	a_0 = -2.31789521
	b_1 = -0.00557514	a_1 = 1.00575949
13	b_2 = 1.00577017	a_2 = 0.00557525
	b_3 = 0.00000959	a_3 = -0.00061492
13	b_4 = -0.00061569	a_4 = -0.00000898
14	b_0 = 4.44734801	a_0 = -2.31787903
	b_1 = -0.00554428	a_1 = 1.00576961
14	b_2 = 1.00577550	a_2 = 0.00554375
	b_3 = 0.00000710	a_3 = -0.00061708
14	b_4 = -0.00061770	a_4 = -0.00000686
15	b_0 = 4.42521897	a_0 = -2.26290095
	b_1 = -0.00065083	a_1 = 1.00080962
15	b_2 = 1.00080621	a_2 = 0.00061445
	b_3 = 0.00000132	a_3 = -0.00006819
15	b_4 = -0.00006667	a_4 = -0.00000071
16	b_0 = 4.42514914	a_0 = -2.26289549
	b_1 = -0.00061580	a_1 = 1.00080214
16	b_2 = 1.00080314	a_2 = 0.00061361
	b_3 = 0.00000038	a_3 = -0.00006818
16	b_4 = -0.00006811	a_4 = -0.00000026

BLOCK NO.	NORTHINGS COEFFICIENTS	EASTINGS COEFFICIENTS
17	b_o = 4.42515184	a_o = -2.26292423
	b_1 = -0.00061323	a_1 = 1.00081382
17	b_2 = 1.00079984	a_2 = 0.00061013
	b_3 = 0.00000001	a_3 = -0.00006890
17	b_4 = -0.00006811	a_4 = -0.00000056
18	b_o = 4.42513846	a_o = -2.26290724
	b_1 = -0.00061715	a_1 = 1.00080104
18	b_2 = 1.00080981	a_2 = 0.00061550
	b_3 = 0.00000070	a_3 = -0.00006795
18	b_4 = -0.00006847	a_4 = -0.00000019
19	b_o = 4.42512821	a_o = -2.26294649
	b_1 = -0.00061222	a_1 = 1.00081890
19	b_2 = 1.00080927	a_2 = 0.00060965
	b_3 = 0.00000029	a_3 = -0.00006964
19	b_4 = -0.00006868	a_4 = -0.00000025
20	b_o = 4.42510497	a_o = -2.26295247
	b_1 = -0.00065197	a_1 = 1.00081118
20	b_2 = 1.00083213	a_2 = 0.00062174
	b_3 = 0.00000225	a_3 = -0.00006793
20	b_4 = -0.00006733	a_4 = -0.00000082

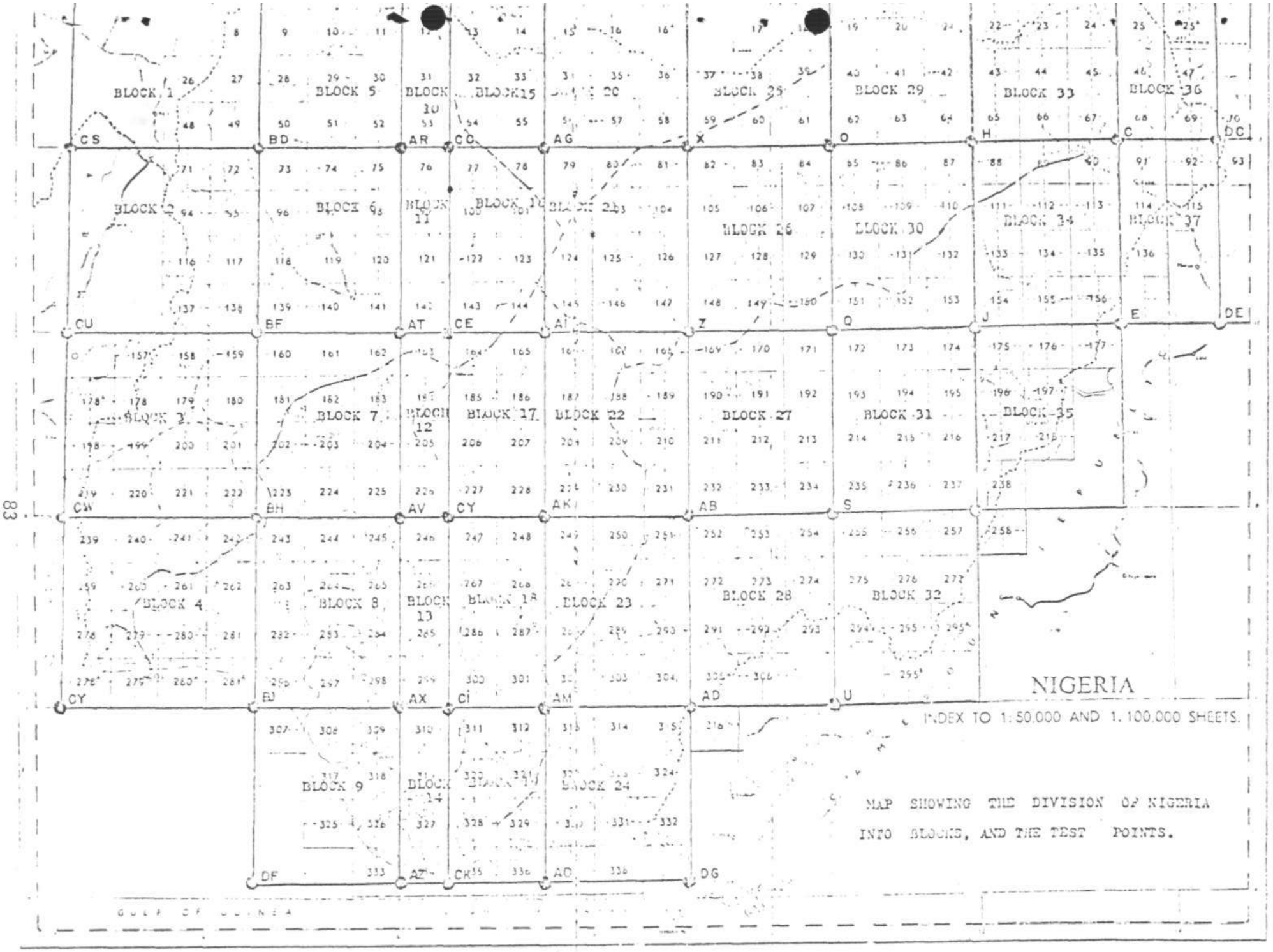
BLOCK NO.	NORTHINGS COEFFICIENTS	EASTINGS COEFFICIENTS
21	b_o = 4.42519612	a_o = -2.26288298
	b_1 = -0.00063463	a_1 = 1.00078935
21	b_2 = 1.00080521	a_2 = 0.00062035
	b_3 = 0.00000101	a_3 = -0.00006745
21	b_4 = -0.00006741	a_4 = 0.00000006
22	b_o = 4.42517320	a_o = -2.26289281
	b_1 = -0.00062615	a_1 = 1.00078886
22	b_2 = 1.00080621	a_2 = 0.00062512
	b_3 = 0.00000087	a_3 = -0.00006713
22	b_4 = -0.00006786	a_4 = -0.00000016
23	b_o = 4.42511222	a_o = -2.26292276
	b_1 = -0.00060384	a_1 = 1.00081090
23	b_2 = 1.00080148	a_2 = 0.00060773
	b_3 = -0.00000046	a_3 = -0.00006886
23	b_4 = -0.00006841	a_4 = -0.00000008
24	b_o = 4.42508394	a_o = -2.26283058
	b_1 = -0.00059815	a_1 = 1.00078042
24	b_2 = 1.00080893	a_2 = 0.00061131
	b_3 = -0.00000080	a_3 = -0.00006651
24	b_4 = -0.00006883	a_4 = 0.00000018

BLOCK NO.	NORTHINGS COEFFICIENTS	EASTINGS COEFFICIENTS
25	b_0 = 4.42515622	a_0= -2.26289941
	b_1 = -0.00065553	a_1= 1.00079555
25	b_2 = 1.00082845	a_2= 0.00061967
	b_3 = 0.00000203	a_3= -0.00006761
25	b_4 = -0.00006732	a_4= -0.00000015
26	b_0 = 4.42523354	a_0= -2.26286175
	b_1 = -0.00065771	a_1= 1.00077895
26	b_2 = 1.00082272	a_2= 0.00062664
	b_3 = 0.00000221	a_3= -0.00006692
26	b_4 = -0.00006737	a_4= 0.00000014
27	b_0 = 4.42517456	a_0= -2.26288991
	b_1 = -0.00062431	a_1= 1.00080326
27	b_2 = 1.00080654	a_2= 0.00060627
	b_3 = 0.00000071	a_3= -0.00006841
27	b_4 = -0.00006802	a_4= 0.00000014
28	b_0 = 4.42509185	a_0= -2.26293322
	b_1 = -0.00059884	a_1= 1.00081248
28	b_2 = 1.00079807	a_2= 0.00060692
	b_3 = -0.00000071	a_3= -0.00006887
28	b_4 = -0.00006829	a_4= -0.00000003

BLOCK NO.	NORTHINGS COEFFICIENTS	EASTINGS COEFFICIENTS
29	b_o= 4.38135389	a_o = -2.17698767
	b_1= 0.00415742	a_1 = 0.99131598
29	b_2= 0.99133485	a_2 = -0.00418788
	b_3= 0.00001191	a_3 = 0.00046992
29	b_4= 0.00047022	a_4 = -0.00001067
30	b_o= 4.38186315	a_o = -2.17657994
	b_1= 0.00416270	a_1 = 0.99120179
30	b_2= 0.99121551	a_2 = -0.00415387
	b_3= 0.00000880	a_3 = 0.00047407
30	b_4= 0.00047341	a_4 = -0.00000874
31	b_o= 4.38219356	a_o = -2.17630612
	b_1= 0.00413201	a_1 = 0.99114436
31	b_2= 0.99116464	a_2 = -0.00415291
	b_3= 0.00000865	a_3 = 0.00047616
31	b_4= 0.00047583	a_4 = -0.00000740
32	b_o= 4.38219670	a_o = -2.17604348
	b_1= 0.00417598	a_1 = 0.99109253
32	b_2= 0.99107098	a_2 = -0.00416005
	b_3= 0.00000493	a_3 = 0.00047818
32	b_4= 0.00047892	a_4 = -0.00000600

BLOCK NO.	NORTHINGS COEFFICIENTS	EASTINGS COEFFICIENTS
33	b_0 = 4.46080474	a_0 = -8.93075999
	b_1 = -0.00332000	a_1 = 1.00828642
33	b_2 = 1.00831007	a_2 = 0.00328383
	b_3 = 0.00000750	a_3 = -0.00033564
33	b_4 = -0.00033544	a_4 = -0.00000608
34	b_0 = 4.46028669	a_0 = -8.93063052
	b_1 = -0.00326030	a_1 = 1.00829706
34	b_2 = 1.00835480	a_2 = 0.00324394
	b_3 = 0.00000699	a_3 = -0.00033668
34	b_4 = -0.00033807	a_4 = -0.00000531
35	b_0 = 4.45968096	a_0 = -8.93080035
	b_1 = -0.00315724	a_1 = 1.00837739
35	b_2 = 1.00836200	a_2 = 0.00316025
	b_3 = 0.00000382	a_3 = -0.00034083
35	b_4 = -0.00034033	a_4 = -0.00000419
36	b_0 = 4.46116263	a_0 = -8.93038349
	b_1 = -0.00333479	a_1 = 1.00824469
36	b_2 = 1.00826372	a_2 = 0.00325938
	b_3 = 0.00000685	a_3 = -0.00033511
36	b_4 = -0.00033414	a_4 = -0.00000464

BLOCK NO.	NORTHINGS COEFFICIENTS	EASTINGS COEFFICIENTS
37	b_0 = 4.46028320	a_0 = -8.93023274
	b_1 = -0.00323851	a_1 = 1.00826681
37	b_2 = 1.00832943	a_2 = 0.00319465
	b_3 = 0.00000573	a_3 = -0.00033681
37	b_4 = -0.00033784	a_4 = -0.00000339


BLOCK 1
BLOCK 5
BLOCK 10
BLOCK15
BLOCK 20
BLOCK 25
BLOCK 29
BLOCK 33
BLOCK 36
BLOCK 2
BLOCK 6
BLOCK 11
BLOCK 16
BLOCK 21
BLOCK 26
BLOCK 30
BLOCK 34
BLOCK 37
BLOCK 3
BLOCK 7
BLOCK 12
BLOCK 17
BLOCK 22
BLOCK 27
BLOCK 31
BLOCK 35
BLOCK 4
BLOCK 8
BLOCK 13
BLOCK 18
BLOCK 23
BLOCK 28
BLOCK 32
BLOCK 9
BLOCK 14
BLOCK 24
CS
BD
AR
CC
AG
X
O
H
C
DC
CU
BF
AT
CE
AI
Z
Q
J
E
DE
CW
BH
AV
CY
AK
AB
S
CY
BJ
AX
CI
AM
AD
U
DF
AZ
CK
AO
DG
NIGERIA
INDEX TO 1:50.000 AND 1.100.000 SHEETS.
MAP SHOWING THE DIVISION OF NIGERIA
INTO BLOCKS, AND THE TEST POINTS.
GULF OF GUINEA

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