RELIABILITY-BASED RATING OF THE BS 8110 DESIGN CRITERIA FOR REINFORCED CONCRETE COLUMNS

BY

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A thesis submitted to the Postgraduate School Ahmadu Bello University, Zaria in partial fulfillment of the requirements for the degree of Master of Science (M.Sc) in Structural Engineering,

JANUARY, 1997

DECLARATION

I hereby declare that this thesis is my own research work and has been composed by myself and it has not been previously presented for a higher degree.

All sources of information which are not my own original work are specifically acknowledged by references.

Abongwa Clement

January, 1997

CERTIFICATION

This project thesis entitled: **"RELIABILITY-BASED RATING OF THE BS8110 DESIGN CRITERIA FOR REINFORCED CONCRETE COLUMNS"** by Abongwa Clement, meets the regulations governing the award of the degree of Master of Science (Structural Engineering) of Ahmadu Bello University, Zaria and is approved for its contribution to knowledge and literary presentation.

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DEDICATION

To the Mumah and Otchere families.

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ABSTRACT

A reliability-based rating of the BS8110 (1985) design criteria for reinforced concrete columns is attempted in this thesis. The rating is based on the First-Order Second Moment Integration Technique. Limit state equations for axially loaded short columns, short columns with bending, slender columns and short columns subjected to biaxial bending are derived and their implied reliability levels evaluated.

All the basic design variables involving cross-sectional resistance and loading are treated as random variables with their statistical characters adopted from literature. The results show that the BS8110 (1985) design requirements are highly inconsistent and very conservative having a range of reliability index, 0, from 1.04 to 8.03. Considering varying practical values of the decision variables, the code provides an average 0-value of 4.35 (i.e. p_f = 6.79 x 10^{-6}) for the different columns studied. It is also found that the criterion for the design of short columns in biaxial bending is the most inconsistent and conservative while short columns with bending have the least indicating a better capacity prediction. Sensitivity analyses show that the 0-values for axially loaded short columns are influenced most by cross-sectional dimensions and load while for short columns with bending and slender columns the effective depth is the most influential. The 0-values for short columns in biaxial bending is influenced

substantially by all design variables except concrete strength and concrete cover. The work concludes by providing some sample charts suitable for designing to specific reliability levels.

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NOTATIONS

A_c	Area of concrete section
A_s	Area of steel (reinforcement) under tension
A_{sc}	Area of steel under compression
a_u	Deflection of column due to buckling
b	Overall depth of column section in the bending plane about y-y axis
d	Effective depth of column section
d'	Concrete cover
E	Young modulus of the concrete material
E[ ]	Expected value (mean)
e_{add}	Eccentricity caused by additional moment
$F_R(s)$	Cummulative distribution fuction for R
F_{cu}	Characteristic concrete strength
$F_S(s)$	Probability density function for S
f_y	Characteristic strength of steel
G	Concentrated dead load
g	Uniformly distributed dead load
g( )	Safety Margin
h	Overall depth of the section in the bending plane about x-x axis.
I	Second moment of inertia of the column section
J	Number of floors
K	Reduction factor for deflection
L	Length of column
L_{ex}	Effective length of column along the x-x axis

L_{ey}	Effective length of column along the y-y axis
L_0	Clear distance between the column restraints
M_{add}, M_a	Additional column end moments
M_c	Column moment
M_i	Moment transferred from the beam to the column
M_{uz}	Ultimate moment of resistance of column section
M_x	Moment about the x-x axis
M_y	Moment about the y-y axis
M'_x	Increased moment about x-x axis
M'_y	Increased moment about y-y axis
N	Applied vertical force
N_{bal}	Axial load at balanced failure
N_{uz}	Ultimate axial resistance of column section
P_f	Probability of failure
Q	Concentrated live load
q	Uniformly distributed live load
R	Strength capacity
R_a	Load ratio
S	Load effect
s	Depth of compressive stress block
V	Coefficient of variation
Var[ ]	Variance
σ_R	Standard deviation of strength capacity
σ_S	Standard deviation of load effect
ρ	Reinforcement ratio
ρ_0	correlation coefficient

Φ	Standard deviation function
μ	Mean of g( ).

CHAPTER ONE

INTRODUCTION

1.1 General

Columns are vertical shafts designed to transmit compressive vertical loads and moments from beams, slabs, other columns and wind loads. They form an integral part of a structure as they help to transmit almost all and in some cases all of the loads from the superstructure to the foundation. This implies that the design of columns must be carried out with extreme caution because their failure might lead to serious consequences like total collapse of the structure and loss of lives. Hence it is necessary to devote particular attention to the evaluation of the level of safety of the design criteria for columns as prescribed by any code of practice.

The design of reinforced concrete columns has always been carried out based on limits and specifications laid down by codes of practice such as the British Standard BS8110 (1985) and other recommended codes which have been found to be very conservative [e.g. Ellingwood, 1977]. The BS8110 (1985), just like many others, is deterministic in nature and prescribes very unique conservative factors of safety and therefore does not accurately take into consideration the non-deterministic behaviour of real life structural problems.

Uncertainties exist in the design of reinforced concrete columns just as in most areas of Civil and Structural Engineering. Rational decisions and results cannot therefore be achieved without modelling and taking the uncertainties into account. Uncertainties include human errors, negligence, poor workmanship, neglected loadings, prediction of the failure events and modelling. Underestimation of these uncertainties sometimes leads to adverse results such as collapse. Events that have been unimaginable and have let to structural failure include (Melchers, 1987) the collapse of the Tay bridge in 1879 due mainly to underestimation of wind loads in a stormy condition, the Tacoma bridge in 1940 due to wind load excitement of the deck (Melchers, 1987) and the collapse of a bridge in south Korea in 1994 due to overloading.

In order to evaluate the effect of uncertainties, advanced principles of reliability are employed. The reliability based rating reported in this thesis does not only examine the uncertainties and consequences of failure but also brings out the level of safety associated with the BS8110 design criteria for reinforced concrete columns.

1.2 Problem Statement and Justification of Study.

The BS8110 (1985) which is prescriptive and deterministic, does not consider explicitly the variability inherent in the design variables usually considered during design. Unique values known as safety factors are assigned to these variables of the design equations with inadequate consideration given to their uncertain characteristics. This is well noted in their lack of invariance of the safety measures introduced by the code of practice. For example, partial factors of safety depend on the limit state being considered, hence on the definition of the loads (live and dead) and the strengths. However, even for a given limit state, loads and strengths considered are not necessarily unique and therefore the partial factors may not be unique either (Melchers, 1987).

It is unfortunate that in spite of the incorporation of these so called safety factors, the current deterministic design methods cannot provide consistent safety levels. A recent investigation (Afolayan, 1990) showed that safety levels implicit in two international codes viz: AISI (1968) and ECCS (1987) for cold formed structural steel members are not consistent. Similar results for other engineering materials can be found in literature (e.g. Bjorhorde, 1972; Chapius and Galambos, 1982). These investigations confirm the fact that the applied safety factors cannot adequately compensate for the randomness exhibited by experimental data on material strengths and applied loads (e.g., Tall and Alpstein, 1969).

The inconsistency of the levels of safety and the unexpected failure of some structures due to uncertainties or any other reasons justify a code review not only to check and maintain safety level but also to properly accommodate randomness and uncertainties encountered during design.

1.3 Aim

To verify whether the safety levels associated with the BS8110 (1985) design criteria for reinforced concrete columns are adequate and consistent.

1.4 Objectives

(i) To derive the limit state equations for the various modes of failure for reinforced concrete columns.

(ii) To compute the safety indices for the various design criteria.

(iii) To provide isosafety design parameters.

CHAPTER TWO

LITERATURE REVIEW

2.1 Columns:

Columns are primarily compression members although they may also resist bending forces due to continuity of the structure and loading pattern. For analytical convenience it is necessary to group columns into two categories:

(a) Braced columns: which are columns in which the lateral loads are resisted by walls or some other form of bracing.

(b) Unbraced columns: columns in which the lateral loads are resisted by the bending action of the columns themselves.

Based on dimensions, columns are also classified as

(A) Short Columns

These are columns in which both L_{ex}/h and L_{ey}/b are

(i) less than 15 for braced columns.

(ii) less than 10 for unbraced columns.

In these cases L_{ex} is the effective length with respect to the x-x axis, L_{ey} is the effective length with respect to the y-y axis, h is the overall depth of the section in the bending plane about x-x axis and b is the overall depth of the section in the bending plane about y-y axis.

The effective lengths are calculated from the expression

$$L_e = ml_0 \tag{2.1}$$

where l_0 is the clear distance between the column restraints and m is the coefficient which depends on the degree of end restraints as specified in Tables 3.21 and 3.22 in the BS8110 (1985).

Short columns can resist a substantial amount of bending and they normally fail by crushing.

(B) Slender Columns:

They are columns whose either one or both of the ratio L_{ex}/h and L_{ey}/b are:

(i) greater than 15 for a braced column

(ii) greater than 10 for an unbraced column

Slender columns fail by buckling due to end moments causing it to deflect sideways and generate an additional moment M_{add} given by the expression.

$$M_{add} = Ne_{add} \tag{2.2}$$

where N is the axial vertical force and e_{add} is the eccentricity caused by the additional moment. This additional moment causes a further lateral deflection and if the axial load exceeds the critical value, this deflection and the moment would become self-propagating until the column buckles. The critical load for a pin-ended strut was derived by Euler as shown below

$$N_{crit} = \frac{\pi^2 EI}{L^2} \tag{2.3}$$

in which E is the Young modulus of the column material, I is the second moment of inertia of the column section and L is the length of the column. The buckling effect and the deflection of a slender column is illustrated diagrammatically in Fig. 2.1

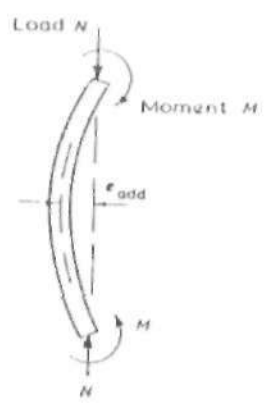

Fig. 2.1: Slender column with lateral deflection [Mosley and Bungey, 1989]

Some columns possess some properties of both short columns and slender columns. Such columns may be classified as intermediate columns.

Previous studies [e.g. Mosley and Bungey, 1989] have shown that reinforced concrete columns have the following failure modes:

(a) Crushing or material failure with negligible lateral deflection. This type of failure usually occurs with short columns and can also occur when there are large end moments on a column with intermediate slenderness ratio.
(b) Material failure intensified by lateral deflection and additional moment. This type of failure is typical with

intermediate columns.

(c) Buckling or instability failure occurs in slender columns and is liable to be proceeded by excessive deflection.

The collapse of columns in buildings is often very critical as columns usually form the main structural support. Failure of some reinforced concrete columns have been recorded. Examples include a 1092mm diameter reinforced concrete columns of the Macuto-Sheraton hotel which exemplified shear-compression failure due to the Caracas earthquake in 1967 [Park and Paulay, 1975]. Another reinforced concrete column failure was also recorded when the lower story columns of Olive View Hospital collapsed and these were as a result of the San Fernando earthquake in 1971 [Park and Paulay, 1975]. Columns of several buildings and many other structures suffered serious damages during the earthquake which hit Japan in the early period of 1995. These and many other recorded structural failures serve as an indication that a review of most if not all of the codified design for reinforced concrete columns is necessary especially in the earthquake regions.

The strength of reinforced concrete columns was studied using Linear regression analysis of column test data by Shah in 1964. However, the regression equation is not directly related to the ultimate strength theory presently used in reinforced concrete design. Moreover, the contribution of statistical variability in the individual parameters to the coefficient of variation in the column resistance was not determined.

Ellingwood (1977) examined the reinforced concrete beam-column resistance using Monté-Carlo techniques. Sensitivity of the uncertainty in resistance, measured by its coefficient of variation to changes in section geometry, strength properties, quality control and local eccentricity were determined. Ellingwood's study showed that the analysis of uncertainties plays a central role in the derivation of reliability-based design safety factors. His study also revealed that, depending on the manner in which the section resists external load, individual uncertainties may contribute disproportionately to the uncertainty in resistance.

In this study, the safety indices for the various failure criteria for reinforced concrete columns are determined using the First Order Reliability Method (Gollwitzer et al, 1988). Sensitivity tests are also carried out for geometric, strength and load parameters. Gumbel, Log-normal and normal distributions were used for load and steel strength to see how they affect the level of safety and consequently to ascertain whether the design equations lead to over or underestimation of the strength of such structural members.

2.2 The British Standard Structural Code of Practice, BS8110 (1985)

According to Thoft-Christensen and Baker (1982) structural codes are documents which by their very nature, are subjected to periodic revisions and amendments. The BS8110 (1985), which is the revised version of CP110 (1972) is a prescriptive and deterministic code of practice in which the nominal strengths of the structural members, based on the code's specification are governed by a number of partial coefficients or by an equivalent means. The BS8110 (1985) provides recommendations for structural use of concrete in buildings and structures excluding bridges and concrete made with high alumina cement.

To account for uncertainties in design, most building codes just as the BS8110 (1985) use factors of safety which are usually based on engineering judgement and previous experience with similar structures. Some buildings codes including BS8110 (1985) also apply separate partial safety coefficients to load and strength variables in an attempt to relate the contribution of each variable towards the establishment of the overall safety factor.

However, different empirical considerations, as prescribed by these codes, employed in practical design methods have their drawbacks. The use of a single-valued safety factor which connotes that the prediction of load and strength is invariant appears questionable since certain strengths and loads are more predictable than others. In addition, the adequacy of those factors giving regards for economic limitations calls for an appraisal of the conventional adoption of safety factors.

2.3 Uncertainties:

Uncertainties can be considered as an unpredicted state of nature resulting from incomplete information. Uncertainties may not be controlled nor reduced, hence the need to be properly considered during design. Identification of uncertainties in complex systems may be difficult. It may be advantageous to use a systematic scheme to help to enumerate all operations and environmental loading states and for each consider possible combinations of error or malfunction.

Melchers (1987) reviews and classifies some of these uncertainties as phenomenological, decision, modelling, physical, statistical and human error. These uncertainties can be represent schematically as shown in Fig. 2.2.

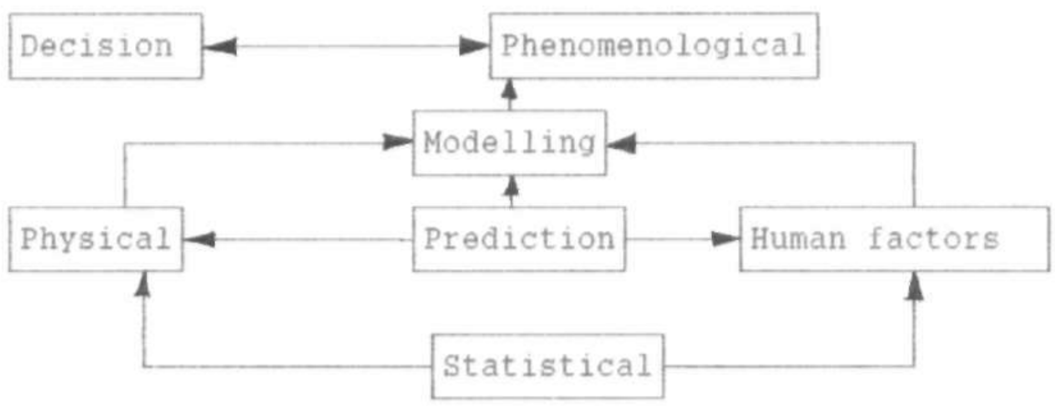

Fig. 2.2: Uncertainties is Reliability Assessment (Melchers, 1987)

2.3.1 Phenomenological Uncertainty

Sometimes an apparently unimaginable phenomenon can occur and leads to failure. The Tacoma Narrow Bridge which collapsed due to wind load excitement of the deck is an example (Melchers, 1987).

Phenomenological uncertainty may be considered to arise whenever the form of construction or the design technique generates uncertainties about any aspect of the possible behaviour of the structure under construction, service and extreme conditions.

2.3.2 Decision Uncertainty

Decision uncertainty arises in connection with the decision as to whether a particular phenomenon has occurred. In terms of limit state it is concerned purely with the decision as to whether the limit state violation has occurred. It is a question of relative loss of structural usefulness.

2.3.3 Modelling Uncertainty:

Modelling uncertainty is caused by the use of simplified relationship between the basic variables to represent the real relationship or phenomenon of interest. In its simplest form, modelling uncertainty concerns the uncertainty of physical models such as the limit state equation. A special case of model uncertainty relates to the treatment of human error and human intervention effects. If sufficient information about these effects is known, allowance could, in principle, be mode for them in the modelling.

2.3.4 Physical Uncertainty

Physical uncertainty is that identified with the inherent random nature of a basic variable. Examples include the variation in steel yield strength, the variability of wind loading, the variability of actual floor loading and physical dimensions of a structural component.

Physical uncertainty can be reduced but not eliminated with greater availability of data or greater effort in quality control. It is a fundamental property of the basic variables. The physical uncertainty for any basic variable is generally not known and must be estimated from observation of the variable or be subjectively assessed.

2.3.5 Statistical Uncertainty

Statistical estimators such as the sample mean and higher moments can be determined from available data and then used to suggest an appropriate probability density function and associated parameters. Generally, the observations of the variable do not perfectly represent it and as a result they may be bias in the data as recorded. In addition, different sample data sets will usually produce different statistical estimators. This causes statistical uncertainty. The process of using sample statistical estimators to infer probability distribution for a variable is adequately described in standard text (e.g., Benjamin and Cornell, 1970).

Statistical uncertainty can be incorporated in a reliability analysis by letting the parameters such as means and variance (and other parameters which describe the probability distribution) themselves be random variables. Alternatively, the reliability analysis might be repeated using different values of the parameters to indicate their sensitivity.

2.3.6 Prediction Uncertainty:

The probability estimate is not only a function of the properties of the structure and forces but also influences likely to act on it. For example, if a reliability estimate for a particular structural life time is required, the analyst's uncertainty in the prediction of the life time (and the loadings which might be expected during that time) enters into the uncertainty of the reliability estimate.

2.3.7 Uncertainty due to Human factors:

The uncertainties resulting from human involvement in the design, construction, use and so on, of structures may, for convenience, be considered under two categories.

(a) Human errors

(b) Human intervention.

Human errors can be divided roughly into human variability due to natural variation in task performance and gross errors. Gross errors are errors which occur in the normal process of design, documentation, construction and the use of the structure within accepted procedures. Those errors which are a direct result of ignorance or oversight of fundamental or service requirements can also be considered as gross error.

Survey carried out by Matousek and Schneider (1976) showed that human error is involved in the majority of cases of recorded failures. It therefore follows that human error must be considered if a reliability assessment is to relate to reality. Unfortunately, understanding of human error is limited and much of the understanding is qualitative.

Human intervention is noted in the process of design, documentation and construction and to some extent in the use of structure. Some forms of intervention include design checking to obtain building approval and sanctions for violations of the law. Intervention may also be informal, such as may result from the observation that something is wrong.

The inconsistencies of many codes of practice can be attributed to the numerous uncertainties listed above. Hence absolute safety of a structure cannot be guaranteed due to these inherently random variabilities. This simply implies that engineering has to deal with the prediction and estimates of reality performed under conditions of inadequate information. If safety of an engineering structure is to be assured, then all the uncertainties mentioned above must be properly taken care of.

2.4 Structural Reliability

Reliability is commonly defined as the compliment of the probability of failure, but more properly, it is the probability of safety (or proper performance) of a structure over a given period of time. A common reliability expression for structural members is through safety indices, or betas as they are commonly known. For example, let the reserve margin of safety be redefined as g, then

$$g = \text{Resistance} - \text{load} = R - S \qquad (2.4)$$

The safety index, β is then defined as

$$\beta = \frac{\mu_g}{\sigma_g} \qquad (2.5)$$

That is, beta is the number of standard deviations for which the mean margin of safety falls on the safe side. Beta provides an exact risk (failure probability) if R and S are both normal distributions. Both R and S may depend on several random variables including material strength and loading. To improve on the beta estimate, several methods such as the First Order Reliability Method have been developed to be used in complex situations.

Recent advances in structural code development are primarily directed towards reliability based framework of limit state design or load and resistance factor design. Reliability studies have been carried out by several people [e.g., Ghosn and Moses (1986); Shin et al (1988)]. It therefore follows without too much emphasis that structural reliability holds a promising future as far as safety and prediction in structural design are concerned.

Despite the deterministic nature of many design codes, many structures have performed satisfactorily even though they have been loaded beyond capacity. This seemingly high safety can be attributed to the fact that

(a) It is rare for a structure to simultaneously experience the full design loads on every part of the structure at the same time as normally specified by the code.

(b) Conservative structural analysis methods are used.

(c) The checking methods control single member performance although structural system often have considerable reserve strength due to ductility and redundancy (Ghosn and Moses, 1986).

Nevertheless, longevity must not be considered in isolation but must go hand in hand with other design requirements such as economy. This can be achieved by dealing considerably with most of the uncertainties involved in the structural design. Structural reliability provides an avenue of accomplishing this desire.

CHAPTER THREE

THEORETICAL FORMULATION OF RELIABILITY METHOD

3.1 Safety Index Concept

In order to explicitly accommodate uncertainties in reliability, the strength capacity (R) and the loading effect (S) of a structural system or component are considered as random variables. The main objective of reliability analysis of any system or component is to ensure that R is never exceeded by S (R > S). Conversely, the probability of the complimentary event (R < S) is the measure of failure.

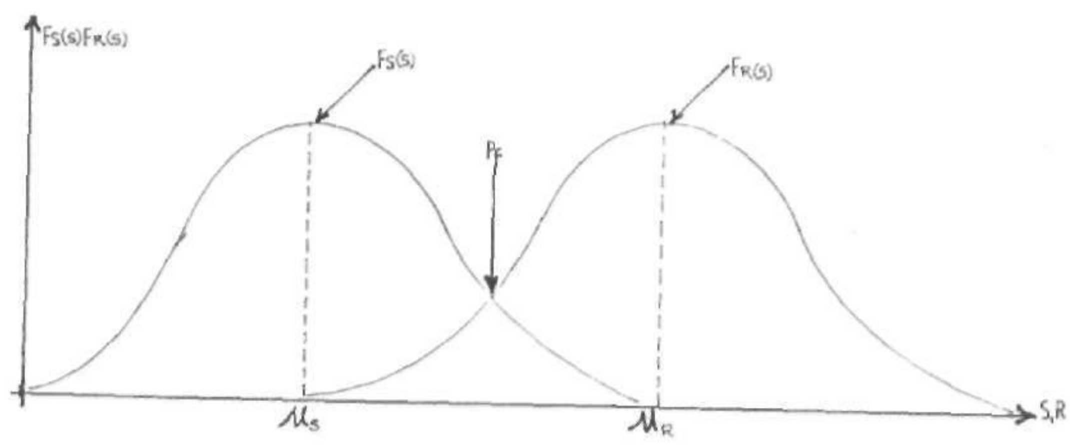

Fig. 3.1: Basic strength capacity-load effect problem.

For a simple structural member with known distribution functions for R and S in some specified mode of failure, the probability of failure, p_f is given as

$$p_f=\int_0^{\infty} F_R(s)\, f_S(s)\, ds \qquad (3.1)$$

where $F_R(s)$ is the cumulative distribution function for R and $f_S(s)$ is the probability density function of S. Equation (3.1) is considered valid when R and S are statistically independent and is best understood by plotting the density functions of R and S as shown in Fig. 3.1.

It should be emphasized that R and S must be expressed in the same dimension (e.g. loads or bending moments). R and S can also be related in terms of safety margin, M, as

$$M = R - S \qquad (3.2)$$

considering an ideal situation, if R and S are statistically independent and normally distributed, p_f may be expressed as

$$p_f = P(M \le 0) \qquad (3.3)$$

Thus

$$\mu_M = E[M] = \mu_R - \mu_S \qquad (3.4)$$

and

$$\sigma_M^2 = Var[M] = \sigma^2_R + \sigma^2_S \qquad (3.5)$$

Since R and S are normally distributed, M, a linear function of R and S would also be normally distributed, and $(M-\mu_M)/\sigma_M$ is a unit standard normal, giving:

$$p_f=\Phi\left(\frac{0-\mu_M}{\sigma_M}\right)=\Phi\left(\frac{\mu_S-\mu_R}{\sqrt{\sigma_S^2+\sigma_R^2}}\right) \qquad (3.6)$$

where Φ is the standard normal distribution function and μ_S, μ_R, σ_S and σ_R are respectively the means and standard deviations of the random variables S and R. In other words,

$$p_f = \Phi(-\mu_M/\sigma_M) = \Phi(-\beta) \tag{3.7}$$

where β is the reliability index or safety index and may be defined as the number of standard deviations by which μ_M exceeds zero.

According to Thoft-Christensen and Baker (1982), for a more general case where R and S are jointly normally distributed with correlation coefficient, ρ, in equation (3.7) still holds but σ_M is given as

$$\sigma_M = (\sigma_R^2+\sigma_S^2+2\rho\sigma_R\sigma_S)^{1/2} \tag{3.8}$$

If R and S are both lognormally distributed, p_f may be expressed as

$$p_f = \Phi\left[\frac{\ln\left(\frac{\mu_S}{\mu_R}\sqrt{\frac{V_R^2+1}{V_S^2+1}}\right)}{\sqrt{\ln[(V_R^2+1)(V_S^2+1)]}}\right] \tag{3.9}$$

where V is coefficient of variation.

Convenient analytical expressions for p_f do exist for some other combinations of distributions of R and S. However, this simple representation of the problem is rarely of much practical use in structural reliability analysis. In general equation (3.1) needs to be evaluated by numerical methods.

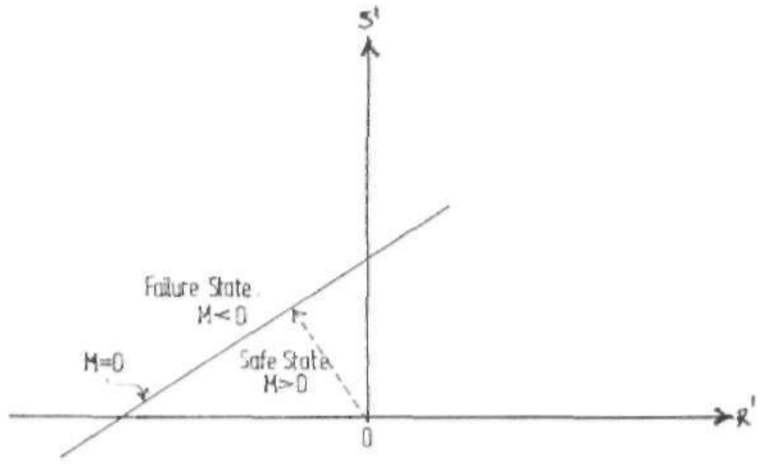

Fig. 3.2: Limit state boundary.

The straight line graph is the boundary between the safe region and the failure state. The line represents a limit state function. Representing equation 3.13 in the cartesian coordinate system gives

$$y = \frac{\sigma_R}{\sigma_S} x + \frac{(\mu_R - \mu_S)}{\sigma_S} \tag{3.14}$$

The perpendicular to line M = 0 and passing through the origin is

$$y = \frac{-\sigma_S}{\sigma_R} x \tag{3.15}$$

Solving for x and y from equations (3.14) and (3.15)

3.2 Second-Moment Formation

The computation of p_f requires knowledge of the distribution of $F_R(r)$ and $F_S(s)$. In practice, data are insufficient or not available. Even when the distributions are prescribed, the evaluation of the probabilities, generally requiring numerical integration of equation (3.1), may be impracticable. Most often, information are limited to the mean and variance of the respective random variables. As a result, the implementation of reliability concept is restricted to the first and the second moments.

The boundary separating the safe and failure state is the limit state function defined by

$$M = R - S = 0 \tag{3.10}$$

Introducing reduced variate $R' = (R - \mu_R)/\sigma_R$ then

$$R = \sigma_R R' + \mu_R \tag{3.11}$$

Similarly,

$$S = \sigma_S S' + \mu_S \tag{3.12}$$

Substituting equation (3.11) and (3.12) into the limit state equation (3.10) gives

$$\sigma_R R' - \sigma_S S' + \mu_R - \mu_S = 0 \tag{3.13}$$

Equation (3.13) is a linear equation of the reduced variables and the graphical representation is shown in Fig. 3.2.

$$x = \frac{\sigma_R(\mu_R - \mu_S)}{(\sigma_R^2 + \sigma_S^2)} \tag{3.16}$$

and

$$y = -\sigma_S \frac{(\mu_R - \mu_S)}{(\sigma_R^2 + \sigma_S^2)} \tag{3.17}$$

therefore the line distance d, from the origin to M = 0 in the general form is

$$d = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2} \tag{3.18}$$

Substituting equations (3.16) and (3.17) in equation (3.18)

$$d = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \frac{\mu_M}{\sigma_M} = \beta \tag{3.19}$$

3.3 Generalization as a Function of Basic Variables

The variables R and S are functions made up of different basic variables. In order to investigate the effect of the variables on the performance of a structural system, a limit state equation in the terms of the basic design variable is required. This limit state equation is referred to as the performance or state function and expressed as

$$g(x_i) = g(x_1, x_2, \ldots, x_n) = R - S \tag{3.20}$$

where X_i for i = 1,2,..., n represent the basic design variables. The limit state of the system can then be expressed as

$$g(x_i) = 0 \qquad (3.21)$$

Graphically, the line $g(x_i) = 0$ represents the failure surface while $g(x_i) > 0$ represents the safe region and $g(x_i) < 0$ corresponds to the failure region. Introducing the set of uncorrelated reduced variates.

$$X'_i = \frac{(X_i - \mu_{x_i})}{\sigma_{x_i}}, i = 1, 2, \ldots n \qquad (3.22)$$

In terms of the reduced variates the limit state equation is given as

$$g(\sigma_{x1}X'_1 + \mu_{x1}, \sigma_{x2} X'_2 + \mu_{x2}, \ldots, \sigma_{xn}X'_n + \mu_{xn}) = 0 \quad (3.23)$$

Similarly, just as in the case of equation (3.18), the distance from a point $X'_i = (X'_1, X'_2, \ldots, X'_n)$ on the failure surface $g(x'_i) = 0$ to the origin of X_i space is given as

$$D = \sqrt{X'^2_1 + X'^2_2 + \ldots + X'^2_n} \qquad (3.24)$$

In matrix form:

$$D = (X'_1 X'_2 \ldots X'_n) \begin{vmatrix} X'_1 \\ X'_2 \\ \cdot \\ \cdot \\ \cdot \\ X'_n \end{vmatrix} = (X'^t{}_i X)^{1/2} \qquad (3.25)$$

The point on the failure surface $(X'^{*}_1, X'^{*}_2, \ldots, X'^{*}_n)$, having the minimum distance to the origin may be determined by minimizing the function D and subjecting equation (3.24) to the constraint $g(X_i) = 0$. For this purpose, the method of langrange's multiplier may be used. Let

$$L = D + \lambda g(X_i) \tag{3.26}$$

Substituting equation (3.25) in (3.26) gives

$$L = (X'^t_i X'_i)^{1/2} + \lambda g(X_i) \tag{3.27}$$

where λ is the value of the multiplier.

In scalar notation

$$L = \sqrt{X'^2_1 + X'^2_2 + \ldots + X'^2_n} + \lambda g(x_1, x_2, \ldots, x_n) \tag{3.28}$$

in which $X_i = \sigma_{Xi} X'_i + \mu_{Xi}$

Minimizing L, we obtain n+1 equations with n+1 unknown as

$$\frac{\partial L}{\partial x_i} = \frac{X'_i}{\sqrt{X'^2_1 + X'^2_2 + \ldots + X'^2_n}} + + \frac{\lambda \partial g}{\partial X'_i} = 0 \tag{3.29}$$

and

$$\frac{\partial L}{\partial \lambda} = g(X_1, X_2, \ldots, X_n) = 0 \tag{3.30}$$

The solution to equations (3.29) and (3.30) would yield the most probable failure point $(X'^*_1, X'^*_2, \ldots, X'^*_n)$. Introducing the gradient vector,

$$G = \frac{\partial g}{\partial X'_1}, \frac{\partial g}{\partial X'_2}, \ldots, \frac{\partial g}{\partial X'_n} \tag{3.31}$$

in which

$$\frac{\partial g}{\partial X'_i} = \frac{\partial g}{\partial X_i} \cdot \frac{\partial X_i}{\partial X'_i} = \sigma_{X_i} \frac{\partial g}{\partial X_i} \tag{3.32}$$

therefore, in vector form we have

$$\frac{X'}{(X'^t X')^{1/2}} + \lambda G = 0 \tag{3.33}$$

from which

$$X' = \lambda D G \tag{3.34}$$

From equation (3.25)

$$D = [(\lambda D G^t)(\lambda D G)]^{1/2} = \lambda D (G^t G)^{1/2} \tag{3.35}$$

and

$$\lambda = (G^t G)^{-1/2} \tag{3.36}$$

substituting equation (3.36) into equation (3.34) gives

$$X' = \frac{-GD}{(G^t G)^{1/2}} \tag{3.37}$$

Multiplying both sides of equation (3.37) by G^t we have

$$G^t X' = \frac{-G^t G D}{(G^t G)^{1/2}} = -(G^t G)^{1/2} D$$

implying

$$D = \frac{-G^t X'}{(G^t G)^{1/2}} \tag{3.38}$$

From equation (3.19), the minimum distance from the origin to the line representing the failure surface equals β and therefore equation (3.38) becomes

$$\beta = \frac{-G^{*t} X'^{*}}{(G^{*t} G^{*})^{1/2}} \qquad (3.39)$$

where G* is the gradient vector at the most probable failure point $(X'^{*}_1, X'^{*}_2, \ldots, X'^{*}_n)$. In scalar form equation (3.39) is

$$\beta = \frac{\sum_i X'^{*}_i \left(\frac{\partial g}{\partial X'_i}\right)_*}{\sqrt{\sum_i \left(\frac{\partial g}{\partial X'_i}\right)_*}} \qquad (3.40)$$

where the derivatives are performed at $(X'^{*}_1, X'^{*}_2, \ldots, X'^{*}_n)$ and the most probable point on the failure surface for minimum β from equation (3.37),

$$X'^{*} = \frac{-G^{*}\beta}{(G^{*t} G*)^{1/2}} \qquad (3.41)$$

and in the scalar form is

$$X'^{*}_i = \alpha^{*}_i \beta \quad ; i = 1, 2, \ldots, n \qquad (3.42)$$

where

$$\alpha_i = \frac{\left(\frac{\partial g}{\partial X'_i}\right)_*}{\sqrt{\sum_i \left(\frac{\partial g}{\partial X'_i}\right)^2_*}}$$

are direction cosines along the axes X'_i

3.3.1 First Order Interpretation:

Considering the generalized state function, g(x) expanded in a Taylor series at a point X^* which is on the failure surface

$g(x^*) = 0$ gives

$$g(X_1, X_2, \ldots, X_n) = g(X_1^*, X_2^*, \ldots, X_n^*) + \sum_{i=1}^{n} (X_i - X_i^*) \left(\frac{\partial g}{\partial x_i}\right)_*$$

$$+ \frac{\sum_{i=1}^{N} \sum_{j=1}^{n} (X_i - X_i^*)\,(X_j - X_j^*)}{\left(\frac{\partial^2 g}{\partial x_i \partial x_j}\right)_*} + \ldots \qquad (3.43)$$

where the derivatives are evaluated at $(X_1{}^*, X_2{}^*, \ldots, X_n{}^*)$. On the failure surface, $g(X_1{}^*, X_2{}^*, \ldots, X_n{}^*) = 0$, hence

$$g(X_1, X_2, \ldots, X_n) = \sum_{i=1}^{n} (X_i - X_i^*) \left(\frac{\partial g}{\partial X_i}\right)_*$$

$$+ \frac{\sum_{i=1}^{n} \sum_{j=1} (X_i - X_i^*)\,(X_j - X_j^*)}{\left(\frac{\partial^2 g}{\partial X_i \partial X_j}\right)_*} + \ldots \qquad (3.44)$$

Introducing the concept of reduced variate we have

$$X_i - X_i^* = (\sigma_{X_i} X_i + \mu_{X_i}) - (\sigma_{X_i} X_i^{1*} + \mu_{X_i}) = \sigma_{X_i} (X'_i - X'^*_i)$$

and

$$\frac{\partial g}{\partial x_i} = \frac{\partial g}{\partial X'_i}\left(\frac{\partial X_i}{\partial X_i}\right) = \frac{1}{\sigma_{X_i}}\left(\frac{\partial g}{\partial X'_i}\right)$$

Truncating equation (3.44) at the first order linear term and substituting the reduced variate into it gives

$$g(X_i) = \sum_{i=1}^{n} (X'_i - X'^*_i)\left(\frac{\partial g}{\partial X'_i}\right)_* \qquad (3.45)$$

Applying the properties of expectation and variance, the mean value of equation (3.45) is

$$\mu_g \approx -\sum_{i=1}^{n} X'^*_i\left(\frac{\partial g}{\partial X'_i}\right)_* \qquad (3.46)$$

and the corresponding variance is

$$\sigma_g^2 \approx \sum_{i=1}^{n} \sigma_{X'_i}^2\left(\frac{\partial g}{\partial X'_i}\right)_*^2 = \sum_{i=1}^{n}\left(\frac{\partial g}{\partial X'_i}\right)_*^2 \qquad (3.47)$$

Thus, the ratio

$$\frac{\mu_g}{\sigma_g} = \frac{\sum_{i=1}^{n} X'^*_i\left(\frac{\partial g}{\partial X'_i}\right)_*}{\sqrt{\sum_{i=1}^{n}\left(\frac{\partial g}{\partial X'_i}\right)_*^2}} \qquad (3.48)$$

Equation (3.48) conforms to equation (3.40), therefore

$$\beta = \mu_g/\sigma_g \qquad (3.49)$$

3.4 Transformation of Basic Variables

The prescribed probability distributions of the basic variables may be non-normal and correlation between their pairs may exists. It is necessary therefore to highlight practicable methods of their transformation into forms suitable for the estimation of the safety index and the evaluation of equation (3.7).

3.4.1 Non-Normal Variates

In a situation where the original PDF, $f_{Xi}(X_i)$ and CDF, $F_{Xi}(X_i)$ are not normal, it is necessary to transform the distribution into an equivalent normal distribution at design point (X_i^*).

Equating the CDF with the standard cumulative density function gives:

$$\Phi\left[\frac{X_i^* - \mu_{X_i}^N}{\sigma_{X_i}^N}\right] = F_{X_i}(x_i) \tag{3.50}$$

where μ_{Xi}^N, σ_{Xi}^N are respectively the mean and standard deviation of equivalent normal distribution of the basic variable X_i. Thus, the equivalent mean is

$$\mu_{X_i}^N = X_i^* - \sigma_{X_i}^N \Phi^{-1}[F_{X_i}(X_{X_i}^*)] \tag{3.51}$$

where as equating the probability densities gives

$$\frac{1}{\sigma^N_{X_i}}\varphi\left(\frac{X_i^* - \mu_{X_i}^N}{\sigma_{X_i}^N}\right) = f_{X_i}(X_i^*) \tag{3.52}$$

Substituting equation (3.51) into equation (3.52)

$$\frac{1}{\sigma_{X_i}^N}\varphi\left(\frac{X_i^* - X_i^* + \sigma_{X_i}^N \Phi^{-1}(F_{X_i}(X_i))}{\sigma_{X_i}^N}\right) = f_{X_i}(X_i^*)$$

This implies that

$$\sigma_{X_i}^N = \frac{\varphi\left(\Phi^{-1}\left[F_{X_i}(X_i^*)\right]\right)}{f_{X_i}(X_i^*)} \tag{3.53}$$

In the above equations μ^N_{Xi}, δ_{Xi}^N and the design points X_i^*

are unknowns and as a result iterative method is employed until convergence is achieved.

3.4.2 Correlated Variates

For correlated variates, transformation into a set of uncorrelated variate must be carried out. Considering a system which is a function of two variates X_1 and X_2 with expected means $E[X_1] = \mu_{X1}$, and $E[X_2] = \mu_{X2}$. The covariance of X_1 and X_2 is denoted $cov[X_1, X_2]$ and is defined by as

$$\begin{aligned} Cov[X_1, X_2] &= E[(X_1 - \mu_{x_1})(X_2 - \mu_{x_2})] \\ &= E[x_1, x_2] - E[x_1]E[x_2] \end{aligned} \qquad (3.54)$$

The correlation coefficient is given by

$$\rho_{X_1 X_2} = \frac{Cov[X_1, X_2]}{\sigma_{X_1}\sigma_{X_2}} \qquad (3.55)$$

where σ_{X1} and σ_{X2} are standard deviation of X_1 and X_2 respectively. A pair of variates is said to be uncorrelated if

$$Cov[X_1, X_2] = 0 \qquad (3.56)$$

and correlation coefficient satisfies the inequality

$$-1 \leq \rho_{X_1 X_2} \leq 1 \qquad (3.57)$$

The required transformation is necessarily dependent on the covariance or covariance matrix of the original variate and may be obtained as follows. Suppose the covariance matrix of the original variate $X_1, X_2, \ldots, X_n$ is

$$|C| = \begin{vmatrix} \sigma_{x_1}^2 & cov[x_1, x_2] & \cdots & cov[x_1, x_n] \\ cov[x_2, x_1] & \sigma_{x_2}^2 & \cdots & cov[x_2, x_n] \\ \cdot & \cdot & \cdots & \cdot \\ cov[x_n, x_1] & cov[x_n, x_2] & \cdots & \sigma_{x_n}^2 \end{vmatrix} \tag{3.58}$$

The corresponding covariance between pairs of the reduced variate $X_i{}'$ and $X_j{}'$ is

$$Cov[x'_i, x'_j] = E[(X'_i - \mu_{x_i})(X'_j - \mu_{x_j})]$$

$$= E\left[\left(\frac{X_i - \mu_{x_i}}{\sigma_{x_i}}\right)\left(\frac{X_j - \mu_{x_j}}{\sigma_{x_j}}\right)\right]$$

$$= \frac{E[(X_i - \mu_{x_i})(X_j - \mu_{x_j})]}{\sigma_{x_i}\sigma_{x_j}} = \rho_{x_i x_j} \tag{3.59}$$

Equation (3.59) shows that the covariance between a pair of reduced variates X'_i and X'_j is equal to the correlation coefficient between the corresponding pair of the original variate X_i and X_j. Therefore, the covariance matrix of the reduced variate $(X'_1, X'_2, \ldots, X'_n)$ is the corresponding correlation matrix of the original variate $(X_1, X_2, \ldots, X_n)$. Thus, the covariance matrix of $X'_1, X'_2, \ldots, X'_n$ is

$$|C'| = \begin{vmatrix} 1 & \rho_{12} & \rho_{13} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & \rho_{23} & \cdots & \rho_{2n} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \rho_{n1} & \rho_{n2} & \rho_{n3} & \cdots & 1 \end{vmatrix} \tag{3.60}$$

The required set of (uncorrelated) transformed variates can be obtained from X' through the following orthogonal transformation.

$$Y = T^t X' \tag{3.61}$$

in which

$$X' = \{X'_1, X'_2, \ldots, X'_n\} \tag{3.62a}$$

and

$$Y = \{Y_1, Y_2, \ldots, Y_n\} \tag{3.62b}$$

are the required set of uncorrelated transformed variate. T^t is the transpose of an orthogonal transformation matrix which is composed of the eigenvectors corresponding to the eigenvalues of the correlation matrix [C]. Specifically T is such that

$$T^t [C] T = [\lambda] \tag{3.63}$$

in which $[\lambda]$ is a diagonal matrix of eigenvalues of [C]. It may be emphasized that the matrix [C'] is real and symmetric as $\rho_{ij} = \rho_{ji}$. With the orthogonal transformation, the safety index becomes.

$$\beta = \frac{G^{*t} X^*}{(G^{*t} [C'] G')^{1/2}} \tag{3.64}$$

In brief, when the basic variables X_i are correlated, the first step is to obtain a set of uncorrelated variables Y_i. The next step is to normalize the uncorrelated variables and thereby a set of normalised and uncorrelated variables $Z_i = (Z_1, Z_2, \ldots, Z_n)$ is obtained. Finally the safety index can be defined in the Z coordinate system as previously explained. Fig. 3.3 shows a flow chart of the numerical algorithm for the computation of β.

3.4 FORM5

FORM5 is a computer package which uses the algorithm depicted in Fig. 3.3 and is used in this study to compute the implied safety levels for the different limit states, outlining the design criteria for R.C columns using BS8110 (1985).

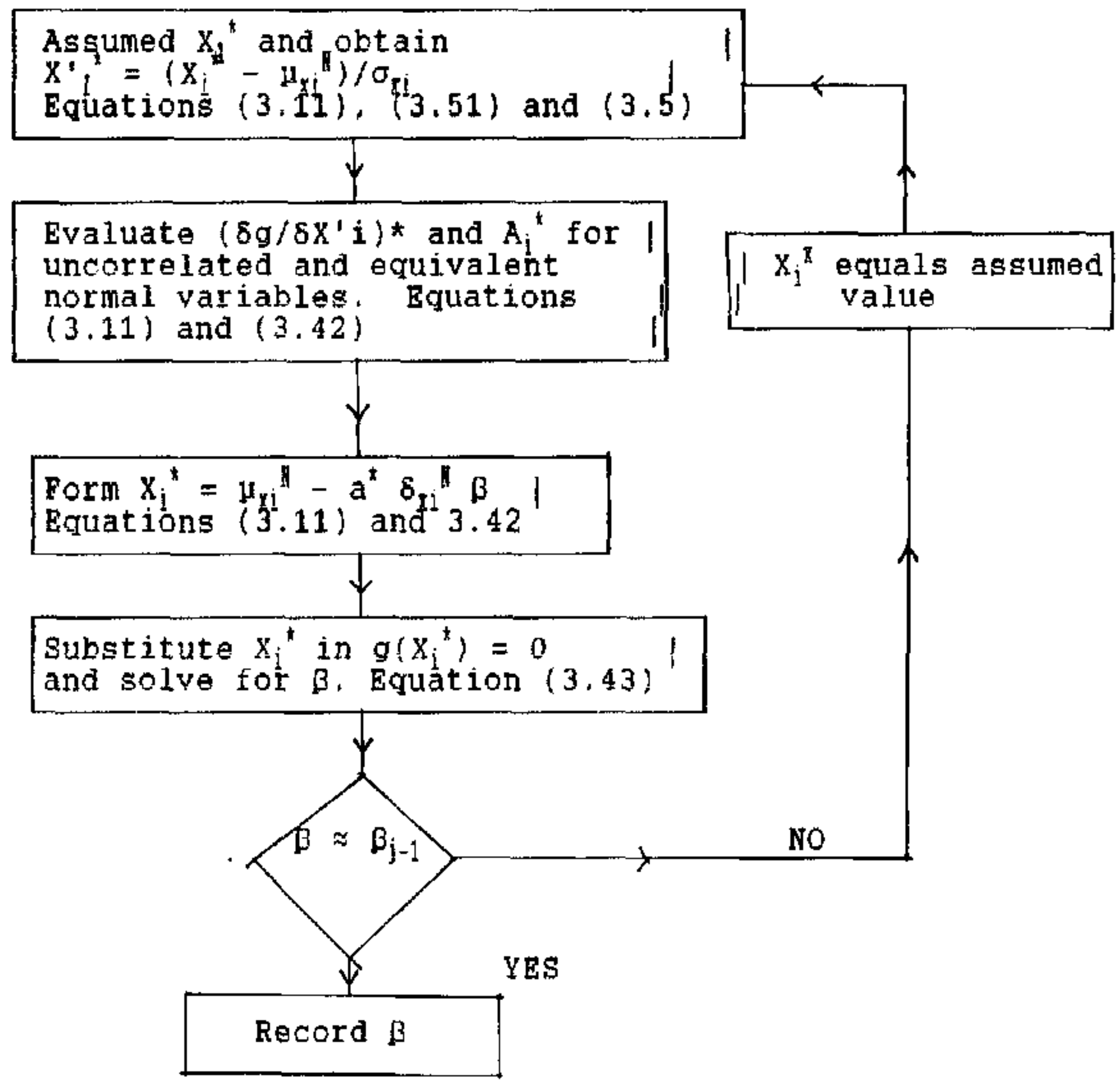

Fig. 3.3 Flow chart for the computation of β

FORM5 requires the user to specify the input parameters of the limit state function and writing a subroutine on the stochastic models of the distributions of the various design parameters.

CHAPTER FOUR

ANALYSIS OF STRENGTH AND LOADING

4.1 Strength Analysis

The carrying capacity of columns depends on material properties and the cross-sectional dimensions of the columns. Columns are loaded either axially or axially combined with bending. In order to resist these loads, the cross-sectional dimensions and the materials must be carefully chosen. It therefore follows without too much emphasis that the selection of the cross-sectional and material properties is essential. This selection does not only involve the derivation of the ultimate strength equations of the column section but also aids in the building up of the limit state equations of the various failure modes. For convenience the strength analysis is treated under two groups: (a) section subjected to purely axial loads, (b) sections with combined bending and axial loads.

(a) Sections Subjected to Purely Axial Loads

These are usually sections of short braced columns which transmit axial compressive loads only. This type of columns can occur in precast concrete construction where there is no continuity between the members. They can also occur when the columns support a symmetrical and very rigid structure.

Considering the section of column in Fig. 4.1 carrying axial loads only and having a symmetrical arrangement of steel.

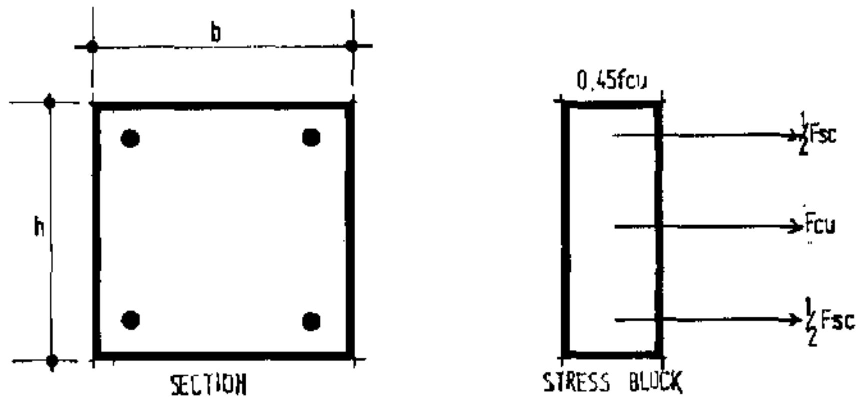

Fig. 4.1: Rectangular column section carrying axial loads only.

A rectangular cross-section with a symmetrical arrangement of steel is considered for convenience and it is the type commonly used although other sections and steel arrangement would yield identical results (Ellingwood, 1977).

When the load is perfectly axial, the ultimate resistance is given as

$$N_{uz} = F_{cu} + 0.5F_{sc} + 0.5F_{sc} = F_{cu} + F_{sc}$$
$$= 0.45f_{cu}\,A_c + 0.87f_y\,A_{sc} \qquad (4.1)$$

where

N_{uz} is the ultimate resistance in (N/mm^2)

A_c is the net area of concrete (mm^2)

A_{sc} is the area of longitudinal reinforcement (mm^2)

f_{cu} is the characteristic strength of concrete (N/mm^2)

f_y is the characteristic strength of steel (N/mm^2)

F_{cu} is the ultimate compressive force of resistance of concrete (N)

F_{sc} is the compressive force of resistance of steel (N)

Perfect conditions never exist in any structure, hence to allow for a small eccentricity, the ultimate load is calculated from (Mosley and Bungey, 1989):

$$N_{uz} = 0.4f_{cu}\,A_c + 0.75\,f_y\,A_{sc} \qquad (4.2)$$

In order to take care of the area of concrete displaced by the longitudinal reinforcement, equation (4.2) is further modified to

$$\begin{aligned} N_{uz} &= 0.4f_{cu}\,A_c + (0.75f_y - 0.4f_{cu})\,A_{sc} \\ &= 0.4f_{cu}\,bh + (0.75f_y - 0.4f_{cu})\,A_{sc} \qquad (4.3) \end{aligned}$$

Equation (4.3) is the expression used for estimating the ultimate axial load carrying capacity of a short column section subjected to purely axial loads.

(b) Columns Subjected to Combined Bending and Axial Load

Combined bending and axial load is found not only in slender columns but also in intermediate and short column resisting both moments and axial loads. Fig. 4.2 shows the cross-section of a column, strain variation and stress distribution for various positions of the neutral axis. The cross-section is subjected to a moment, M, and an axial load N. The direction of the moment is such that it causes compression on the upper part of the section and tension in the lower part.

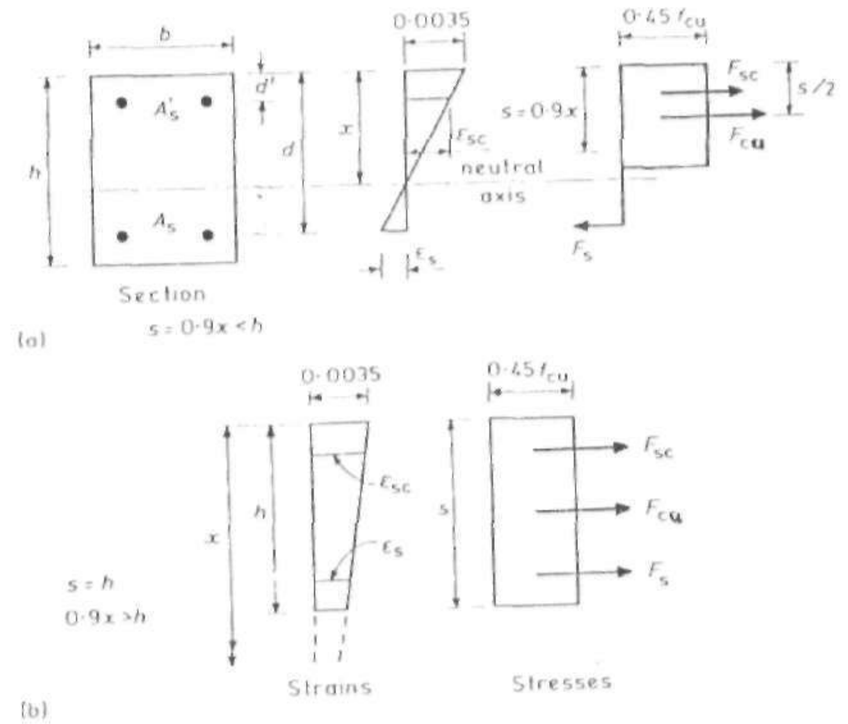

Fig. 4.2: Axial load plus bending with varying position of the neutral axis. (Mosley and Bungey, 1989).

In Fig. 4.2, F_s is the tensile or compressive force in the reinforcement area A_s and acting through the centroid of A_s. F_{sc} is the compressive force in the reinforcement area A'_s and acting through the centroid of A'_s. Considering Fig. 4.2(a), the applied force N, must be balanced by forces developed within the cross-section. Hence,

$$N_{uz} = F_{cu} + F_{sc} - F_s \qquad (4.4)$$

In equation (4.4), the reinforcement As is in tension and this would occur only when the depth of the stress block of concrete, S, is less than the effective depth (i.e., $s < d$). Further simplification yields

$$N_{uz} = 0.45f_{cu}\, bs + f_{sc}A'_s - f_sA_s \qquad (4.5)$$

where f_{sc} is the compressive stress of reinforcement A'_s (N/mm^2)

f_s is the tensile stress of reinforcement A_s (N/mm^2)

To allow for the concrete displaced by steel, equation (4.5) is modified to

$$N_{uz} = 0.45f_{cu}bs + (f_{sc} - 0.45f_{cu})\, A'_s - f_sA_s \qquad (4.6)$$

At balanced failure $f_{sc} = f_s = 0.87f_y$ and since $A'_s = A_s$, it therefore follows that equation (4.6) simplified to

$$N_{uz} = 0.45f_{cu}\, (bs-A'_s) \qquad (4.7)$$

The design moment must also be balanced by the moment of resistance developed within the cross-section. Taking moments about the mid-depth of the section,

$$M_{uz} = F_{cu}(h/2 - s/2) + F_{sc}\, (h/2 - d') - F_s\, (d-h/2)$$
$$= 0.45f_{cu}bs(h/2-s/2) + f_{sc}A'_s(h/2-d') + f_s\, A_s(d-h/2) \qquad (4.8)$$

Taking care of the concrete displaced by steel, equation (4.8) becomes

$$M_{uz} = 0.45f_{cu}bs(h/2-s/2) + (f_{sc}-0.45f_{cu})(h/2-d')A'_s + f_sA_s(d - h/2) \qquad (4.9)$$

Equation (4.9) represents the expression of the ultimate moment of resistance by a column subjected to combined bending and axial load.

It is worthwhile noting that when the depth of the neutral axis is such that $0.9x > h$ as in Fig. 4.2(b), then the whole section is subjected to uniform compressive stress of $0.45f_{cu}$. In this case the concrete provides no contribution to the moment of resistance and the first term in equation (4.9) turns to zero.

4.2 Load Analysis

In order for any design to be meaningful, a proper analysis of not only the member strength but also a comprehensive analysis of loads must be carried out. Loads are transmitted to the columns directly from slabs or through beams. Whatever the mode of transmission, there are two types of loads involved:

(i) imposed (live) loads which involve all the loads on the structure which are short lived (temporal) in nature. As a result of uncertainties involved in their computation in design, a partial factor of safety of 1.6 has been prescribed by the British standard code of practice (BS 8110, 1985),

(ii) dead loads include the selfweight of the structure and all loads which are permanently fixed on the structure. The BS8110 (1985) prescribes a partial safety factor of 1.4. Dead loads are usually calculated on a slightly conservative bases so that a member will not need redesigning because of a small change in dimensions.

In this study it will be considered that the loads are transmitted to the columns through the beams. This is because it is the most commonly used method of load transfer to the columns and the analysis and computation of the various relevant parameters are made much easier.

(a) <u>Pure Axial Load on Short Columns</u>

(i) Considering concentrate load

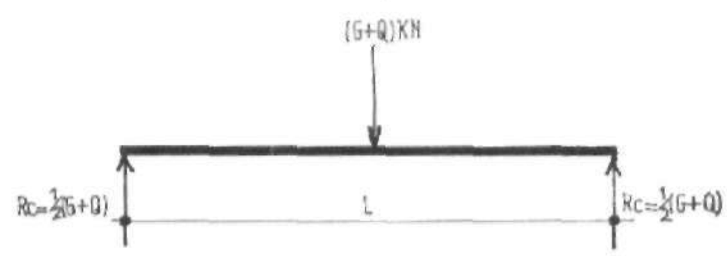

Fig. 4.3: Computation of the column reactions from a simply supported beam with a concentrated load at midspan (for purely axial loads on columns).

A simply supported beam was considered so that the loads on the columns would be purely axial. All loads have been factored according to the British standard BS8110 (1985). G is a concentrated dead load while Q is a concentrated imposed load.

$$R_c = 0.5(G + Q) \tag{4.10}$$

where R_c is the column reaction.

(ii) Considering Uniformly distributed load

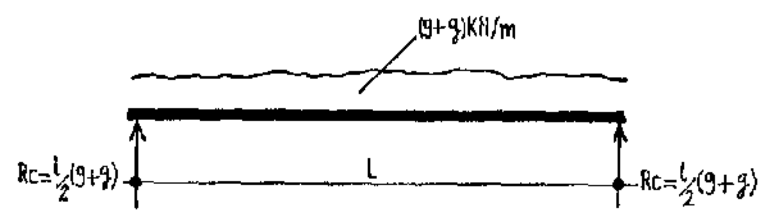

Fig. 4.4: Computation of the column reaction from a simply supported beam with a uniformly distributed load (for purely axial loads on columns).

In Fig. 4.4, g is the uniformly distributed dead load and q is uniformly distributed imposed load. Hence,

$$R_c = 0.5l(g+q) \qquad (4.11)$$

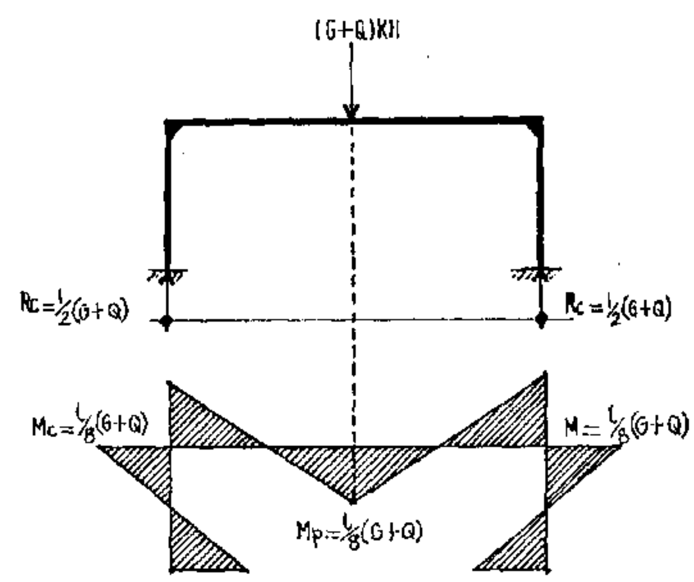

Fig. 4.5: Computation of axial loads and bending moments on short columns from beams subjected to concentrated load at midspan.

(b) Short Columns with axial Loads plus Bending:

(i) Considering concentrated load (Fig. 4.5), the value of R_c is same as in equation (4.10) so that

$$M_c = 1/8(G+Q) \qquad (4.12)$$

where M_c is the moment at column and L is the span of the beam.

(ii) Considering uniform distributed load as shown in Fig. 4.6, we can derive the following equations:

$$R_c = (g+q)L/2 \qquad (4.13)$$

and

$$M_c = (g+q)L^2/12 \qquad (4.14)$$

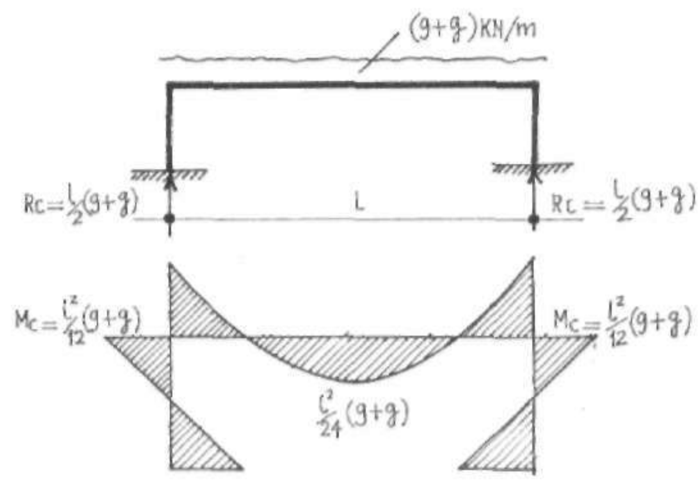

Fig. 4.6: Computation of axial load and bending moment on short columns from beam subjected to uniformly distributed load.

(c) Loading On Slender Columns

Slender columns must be designed for an additional moment caused by its curvature at ultimate conditions. BS8110 (1985) (part 1, 1985) prescribes and expression for this additional moment (Mosley and Bungey 1989) as

$$M_a = Na_u \qquad (4.15)$$

where M_a is the additional moment and a_u is the deflection of the column. For rectangular (or circular) columns, this deflection is given as

$$a_u = \beta_a Kh \qquad (4.16)$$

where

$$\beta_a = \frac{1}{2000}\left(\frac{l_e}{b'}\right)^2 \qquad (4.17)$$

K is the reduction factor to allow for the fact that the deflection must be less when there is a large portion of the column section under compression. Mathematically K is given as

$$K = \frac{N_{uz} - N}{N_{uz} - N_{bal}} \ngtr 1.0 \qquad (4.18)$$

where N_{uz} is the ultimate axial resistance given in equation (4.1), N_{bal} is the axial load at balanced failure (Mosley and Bungey, 1989).

Mathematically,

$$N_{bal} = 0.25 f_{cu} A_c \qquad (4.19)$$

The ultimate moment is then computed from the expression

$$M_c = M_i + M_a$$

$$= M_i + N\left[\frac{h}{2000}\left(\frac{l_e}{b'}\right)^2\left(\frac{0.45f_{cu}bh+0.87f_yA_{sc}-N}{0.45f_{cu}bh+0.87f_yA_{sc}-0.25f_{cu}bh}\right)\right] \qquad (4.20)$$

where M_i is the moment transferred from the beam to the column. The geometric dimensions used in equation (4.20) are best illustrated diagrammatical as shown in Fig. 4.7

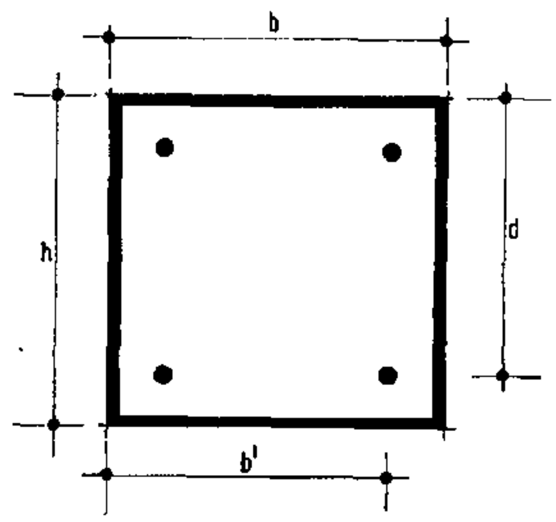

Fig. 4.7. Section of column showing some important dimensions.

(d) Biaxial Bending

According to Mosley and Bungey (1989), for most columns, biaxial bending will not govern the design. The loading patterns necessary to cause biaxial bending in a building's internal and edge columns will not usually cause large moments in both directions. Corner columns may have to resist significant bending about both axes, but the axial loads are usually small and a design similar to the adjacent edge columns is generally adequate.

The BS8110 (1985) prescribes a simplified method which specifies that a column subjected to an ultimate load N and moments M_x and M_y about the x-x and y-y axes respectively may be designed for a single axis bending but with an increased moment and subjected to the following conditions:

(a) if $M_x/h' \geq M_y/b'$ (4.21)

then the increased single axis design moment, M_x', is

$$M_x' = M_x + \beta_b\, h'/b'\, M_y \quad (4.22)$$

(b) if $M_x/h' < M_y/b'$ (4.23)

then the increased single axis design moment, M_y', is

$$M_y' = M_y + \beta_b\, b'/h'\, M_x \quad (4.24)$$

The dimensions h' and b' are best defined diagrammatically as in Fig. 4.8. The parameter β_b is a coefficient obtained from Table 3.24 in BS8110 (1985) but can also be calculated mathematically as follows:

$$\beta_b = 1 - 7/6G_a \quad \text{when } G_a < 0.6 \quad (4.25a)$$

$$\beta_b = 0.3 \quad \text{when } G_a \geq 0.6 \quad (4.25b)$$

where $G_a = N/bhf_{cu}$ (4.26)

In situations where the applied load consists of both concentrated and uniformly distributed load, their respective ultimate axial loads and moment contribution would be computed in isolation and then super imposed to get their joint effect. At this juncture, it is worthwhile to emphasize that there are numerous cases of other load combinations and load arrangements. The moments and the axial load transferred to the column can be computed using the principles of structural analysis. In this study, for convenience and for the fact that the other loading cases would yield identical results, only the simplified cases illustrated in section 4.2 are considered.

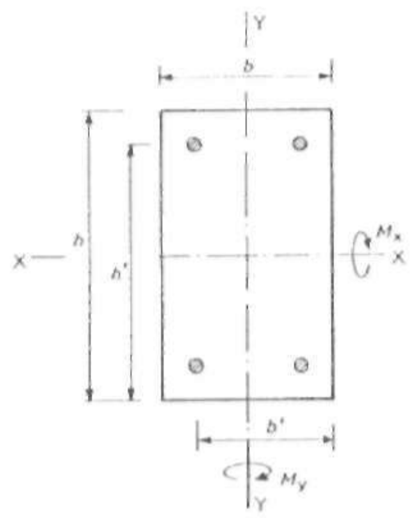

Fig. 4.8: Column Section with biaxial bending.

4.3 Limit State Equations

When an engineering structure is loaded in someway, it will respond in a manner which depends on the type and magnitude of the load, the strength and stiffness of the structure. Whether the response is considered satisfactory depends on the requirements which must be satisfied. Such requirements might include safety of the structure against collapse, limitation on damage by deflection or any range of other criteria. Each of these requirements may be termed a "LIMIT STATE" (Melchers, 1987).

Limit state equations are equations which define the limit state as accurately as possible thereby differentiating between the safe and the unsafe regions. Limit state equations are generally represented in the form

$$g(x) = R - S \tag{4.27}$$

where g(x) is the safety margin, R is the resistance and S is the load effect. When $g(x) > 0$ then the structure is considered safe while when $g(x) < 0$ the structure is considered unsafe (failed).

In the case of reinforced concrete column, the limit state requirement can generally be summarized under axial failure criterion and combined flexural and axial failure criterion.

For axial criterion

$$g(x) = 1 - N/N_{uz} \tag{4.28}$$

For combined flexural and axial criterion

$$g(x) = 1 - [M_c/M_{uz} + N/N_{uz}] \tag{4.29}$$

(a) <u>Criterion for Pure axially Loaded Short Column</u>

For concentrated load

$$g_{1}(x) = 1 - \frac{Q_j(R_d+1)}{2bh[0.4f_{cu}+(0.75f_y-0.4f_{cu})\rho]} \tag{4.30}$$

where j is the number of floors.

For uniformly distributed load

$$g_{12}(x) = 1 - \frac{qjL(R_s+1)}{2bh[0.4f_{cu}+(0.75f_y-0.4f_{cu})\rho]} \qquad (4.31)$$

(b) <u>Criterion for Combined bending and axial load in short column</u>

For concentrated load, let

$$A = 0.45f_{cu}\left(\frac{h}{2}-\frac{s}{2}\right)\frac{s}{h}+(f_{sc}-0.45f_{cu})\left(\frac{h}{2}-d'\right)\frac{\rho}{2}+f_s\left(d-\frac{h}{2}\right)\frac{\rho}{2}$$

and

$$B = 0.45f_{cu}\left(\frac{s}{h}-\frac{\rho}{2}\right)$$

then

$$g_{21}(x) = 1-\frac{Q(R_s+1)}{8bh}\left[\frac{L}{A}+\frac{4j}{B}\right] \qquad (4.32)$$

For uniformly distributed load

$$g_{22}(x) = 1-\frac{Q(R_s+1)L}{12bh}\left[\frac{L}{A}+\frac{6j}{B}\right] \qquad (4.33)$$

(c) <u>Criterion for Slender Column</u>

For concentrated load, let

$$C = \frac{h}{2000}\left(\frac{l_e}{b'}\right)^2$$

and

$$D = \frac{0.45 f_{cu} + 0.87 f_y p/2 - \frac{jQ}{2bh}(R_a+1)}{0.45 f_{cu} + 0.87 f_y p/2 - 0.25 f_{cu}}$$

then

$$g_{31}(x) = 1 - \frac{Q(R_a+1)}{8bh}\left[\frac{L+4jCD}{A} + \frac{4j}{B}\right] \tag{4.34}$$

For uniformly distributed load, let

$$F = \frac{0.45 f_{cu} + 0.87 f_y\, p/2 - \frac{hqL}{2bh}(R_a+1)}{0.45 f_{cu} + 0.87 f_y\, p/2 - 0.25 f_{cu}}$$

then

$$g_{32}(x) = 1 - \frac{qL(R_a+1)}{12bh}\left[\frac{L+6jCF}{A} + \frac{6j}{B}\right] \tag{4.35}$$

(d) <u>Criterion for Biaxial Bending</u>

For concentrated load

$$g_{41}(x) = 1 - \frac{1}{bh}\left[\frac{Q(R_a+1)L/8 + M'}{A} + \frac{Q(R_a+1)j/2}{B}\right] \tag{4.36}$$

For uniformly distributed load

$$g_{42}(x) = 1 - \frac{1}{bh}\left[\frac{q(R_a+1)L^2/12 + M'}{A} + \frac{qL(R_a+1)j/2}{B}\right] \tag{4.37}$$

M' is the increased moment due to biaxial bending.

4.4 Statistical Characteristics of Relevant Design Parameters

A thorough application, with a reasonable degree of accuracy of the principles of reliability can be achieved if the probability distribution functions that represent the various design parameters can be established. These probability distribution functions can be obtained by direct inference from observed data or by a subjective assessment or a combination of both techniques. In practice, however, direct inference is rarely possible since there is seldom sufficient data available to identify unambiguously only one distribution for a given parameter. This implies that past observations and experience for similar structure can validly be used for any structure under consideration. Probability distribution function can also be obtained from physical models.

In spite of the inavailability of sufficient data, reliability assessment has been made relatively easier by the development of advanced methods such as the first order reliability method (FORM) and Monté Carlo's method. Both methods require only the first and the second statistical moments. Prior to the application of whichever method is used, the means, variances and coefficients of variation for the individual variables must be established. This estimation is an ongoing process as additional data are continually becoming available. The values of the means, coefficients of variation and variances for the various parameters used in this study are listed in Table 4.1. This includes some recent data from studies of geometric imperfections in structure and are believed to be a representative of data for various levels of quality in fabrication and construction (Ellingwood, 1977).

TABLE 4.1: TABLE OF ESSENTIAL DESIGN PARAMETERS

i	PHYSICAL MEANING	TYPE	NUM. CODE	MEAN	COEFFICIENT OF VARIATION	STANDARD DEVIATION
1	Compressive strength of concrete, f_{cu}	Normal	2	30 N/mm^2	0.10-0.25	4.2 N/mm^2
2	Yield strength of steel, f_y	Log-normal	3	425 N/mm^2	0.09	38.25 N/mm
3	Breadth, b	Normal	2	400mm	0.04	16mm
4	Depth, h	Normal	2	400mm	0.04	16mm
5	Effective depth, d	Normal	2	350mm	0.04-0.09	22.75mm
6	concrete cover, d'	Normal	2	50mm	0.04-0.09	3.25mm
7	Live load, Q	Normal	2	30x10^4 N	02-0.5	10.5x10^{4}N
8	Rho, ρ	Normal	2	0.04	0.03	0.0012

A study carried out by Fiorato (1973) showed that all geometric variables as well as concrete strength are normally distributed while the strength of steel is assigned a log-normal distribution.

For the purpose of the application of FORM5, the following distributions have the following number code:Normal = 2, Lognormal = 3 and Gumbel = 7.

CHAPTER FIVE
ESTIMATION OF SAFETY INDICES

5.1 Introduction

The main objective of the BS8110 (1985), just as in any other structural code, is to ensure that any structure designed using the code's specification functions properly in the interest of public safety. By this, all provisions aim at reducing the probability of inadequate performance to a somewhat negligible level by prescribing acceptable routine designs.

According to Afolayan (1994), structural code provisions have generally evolved as a consequence of some reference to theoretical, experimental and judgmental activities. Undoubtedly, the aims of structural codes can be regarded as serving satisfactorily since structural failures and cases of inadequate performance have been few. Nonetheless, because of the inability to predict the real nature, it is possible that the code provisions lead to designs of varying safety levels. Further still since safety and economy are conflicting aims, the possibility that the existing code provisions lead to overly expensive structures cannot be ruled out.

The purpose of this chapter is to assess the degree of adequacy, consistency and conservatism of the BS8110 (1985) as far as the design of reinforced concrete columns is concerned.

5.2 Computation of Safety Indices and Discussion of Results

Safety indices (β) for the various design criteria were computed using FORM5 and the limit state equations derived in section 4.3. The reliability levels implicit in the various design criteria are presented in Figs. 5.1 to 5.4. On the whole, the BS8110 (1985) design criteria for reinforced concrete columns are found to be adequate with β ranging from 1.04 to 8.03 and having an average of 4.35 (P_f = 6.794 $x10^{-6}$).

It is observed from the results that for all design criteria, there is a general trend of β increasing with increasing reinforcement ratio, ρ. However, β decreases with increasing load ratio, R_a. The variation of β with ρ and R_a is uniform and the β-band is reasonably defined for all design criteria.

All the design criteria exhibit some level of inconsistency though the level of inconsistency vary from criterion to criterion. *The biaxial bending criterion has the* widest β-band thereby implying that it is the most inconsistent. This is followed by short column with purely axial load while short column with bending criterion is the least inconsistent as it has the most narrow β-band. In short columns with bending and slender columns the level of inconsistency reduces at higher values of ρ. This attribute is exhibited in Figs. 4.2 and 4.3 by the clustering of the graphs at higher values of ρ.

Comparison shows that the biaxial bending criterion is the

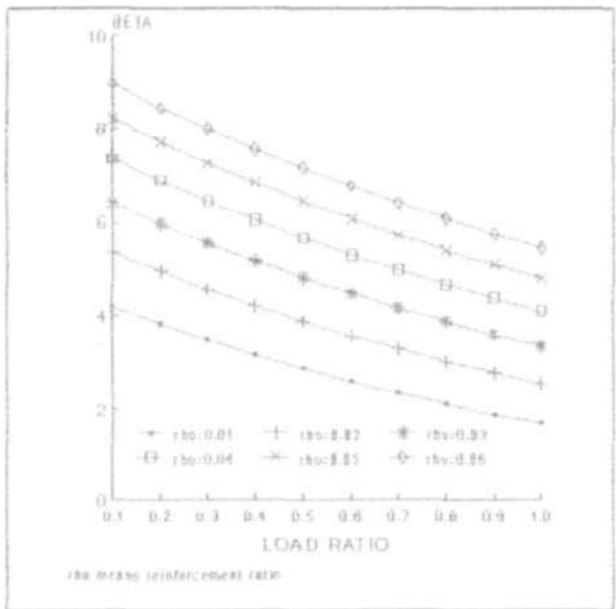

Fig. 5.1: Safety Levels for Axially Loaded Short Columns

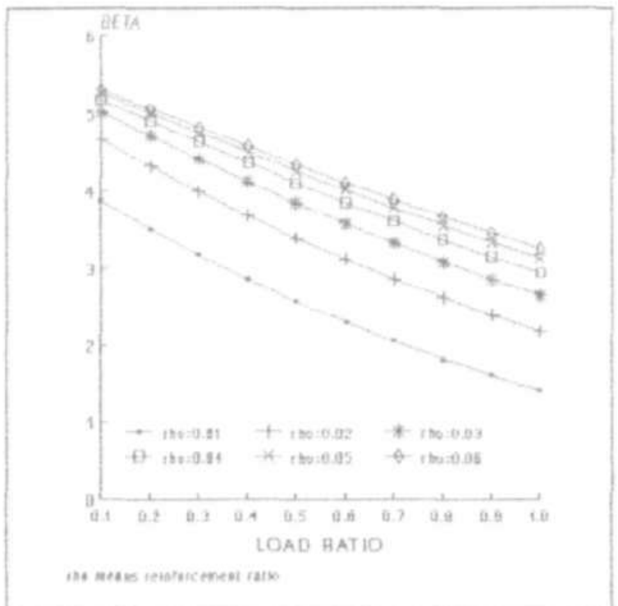

Fig. 5.2: Safety Levels for Short Columns with Bending

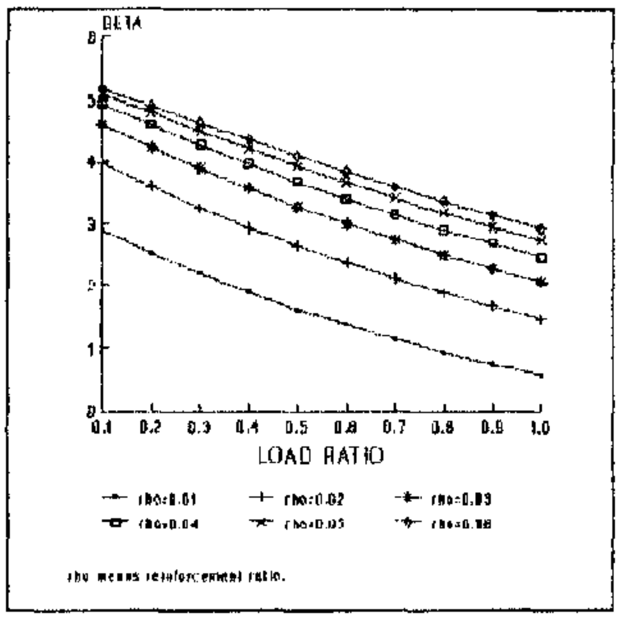

Fig. 5.3: Safety Levels for Slender Columns.

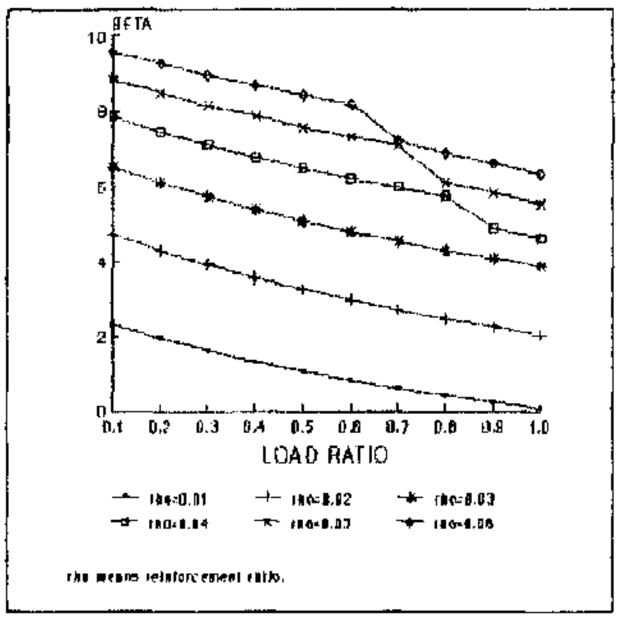

Fig. 5.4: Safety Levels for Short Columns Subjected to Biaxial Bending.

most conservative as it yields the highest values for β for different values of ρ except when $\rho = 0.01$. This is illustrated in Figs. 5.5 to 5.7 which further show that the purely axially loaded short column criterion is the second most conservative. Slender column design criterion is found to be the most critical for it yields the lowest values of β for different values of ρ except at $\rho = 0.01$.

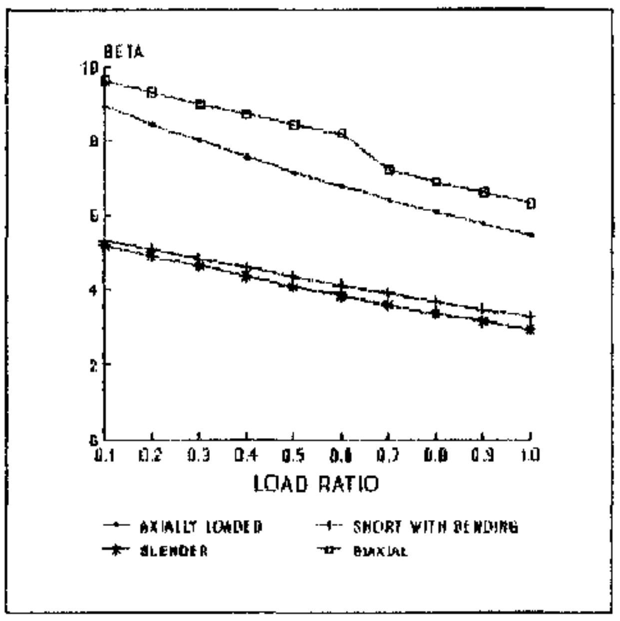

Fig. 5.5: Upper Limit of Safety Indices for all Design Criteria

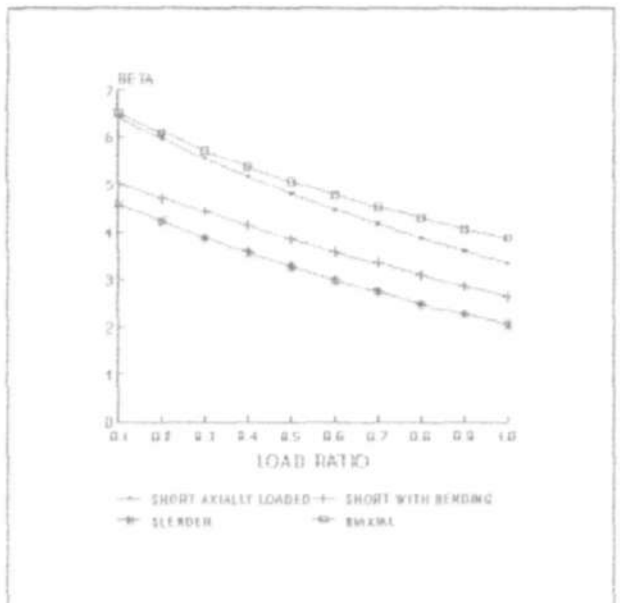

Fig. 5.6: Safety Levels for all Design Criteria when Rho = 3%.

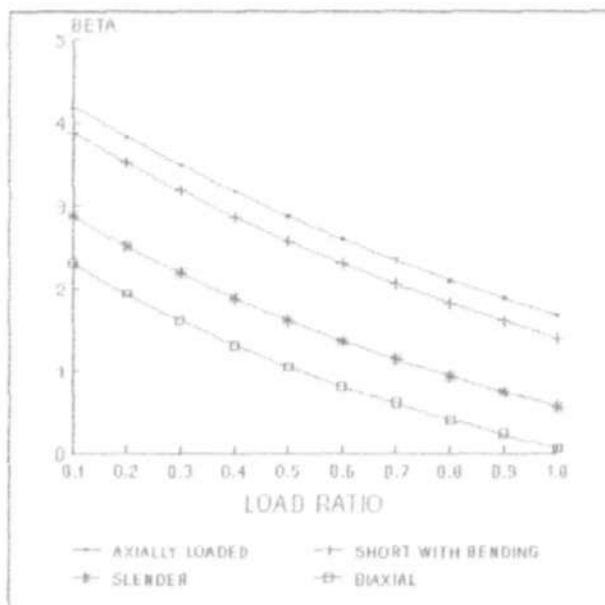

Fig. 5.7: Lower Limits of Safety Indices for all Design Criteria.

It is also observed that for most cases, the levels of inconsistency and conservatism for the various design criteria are affected when the statistical distributions of loading and steel strength are altered. The levels of inconsistency and conservatism are reduced when a short column is under purely axial load with log-normal or gumbel distribution. An average decrease of 2 in β is noted as illustrated in Figs. 5.8 and 5.9. A reduction in the inconsistency and conservation levels is also realized when the steel strength is assumed normally distributed (Fig. 5.10). On the other hand, Fig. 5.11 shows that there is an increase in the inconsistency and conservatism levels when the steel strength assumes gumbel distribution.

In short columns with bending, an average decrease of 0.81 in β and the reduction of β-band is exhibited when the load and steel strength assume distributions other than normal (see Figs. 5.12 to 5.15).

The same trends of reduction in the level of inconsistency and conservatism are also exhibited in slender columns and columns with biaxial bending. The only exception is that there is no change in β when the steel strength distribution is altered for slender columns and there is an increase in inconsistency and conservatism levels when the steel strength distribution in biaxial bending criterion assumes gumbel (see Figs. 5.16 - 5.23).

In all the four design criteria, the lowest levels of inconsistency and conservatism are established when load

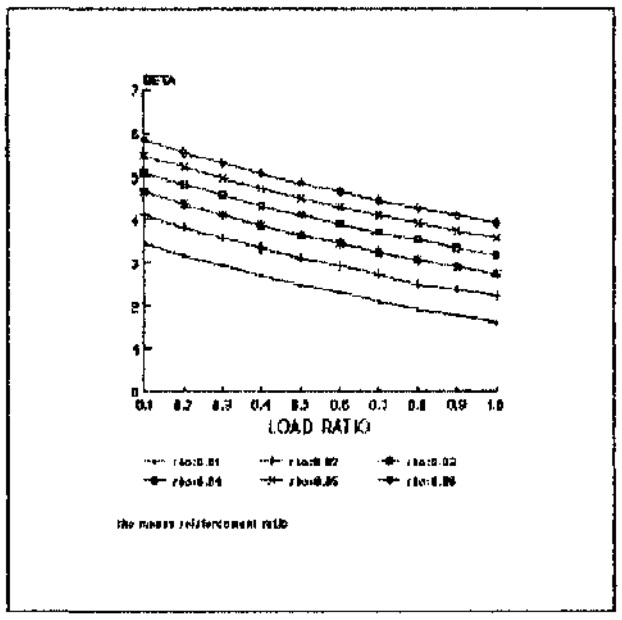

Fig. 5.8: Safety Levels for Axially Loaded Short Columns when Q and F_Y are Log-normally Distributed.

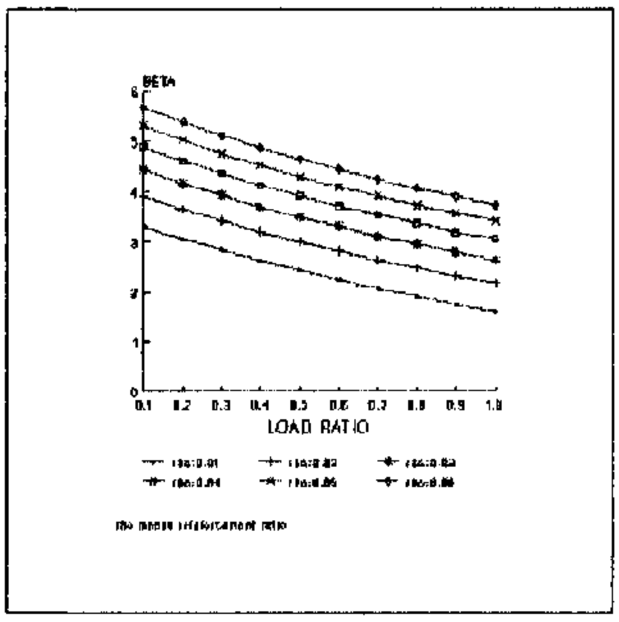

Fig. 5.9: Safety Levels for Axially Loaded Short Columns when Q is Gumbel and F_Y is Lognormally Distributed.

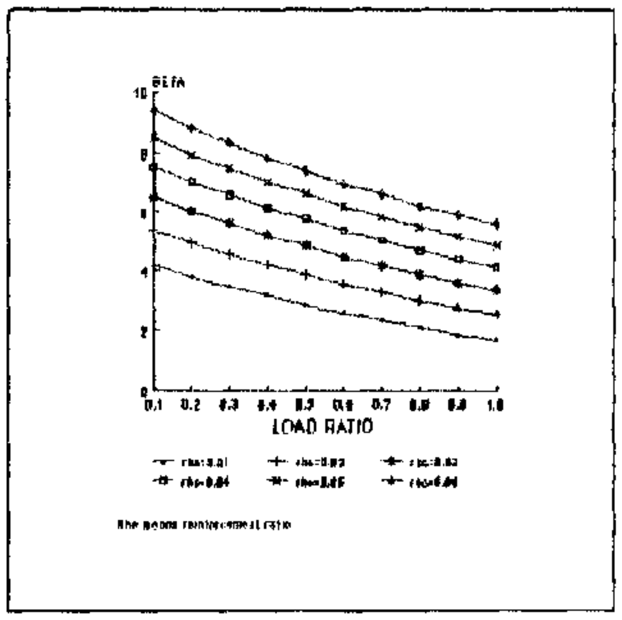

Fig. 5.10: Safety Levels for Axially Loaded Short Columns when Q is Normal and F_y is Gumbel.

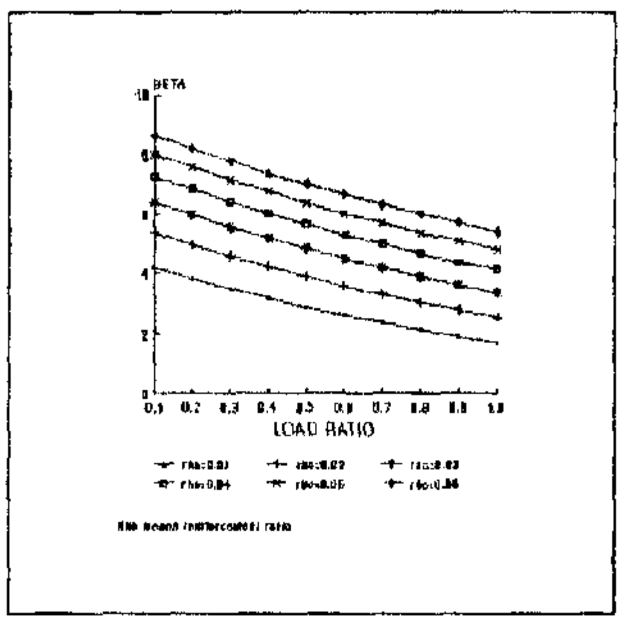

Fig. 5.11: Safety Levels for Axially Loaded Short Columns when Q and F_y are Normal.

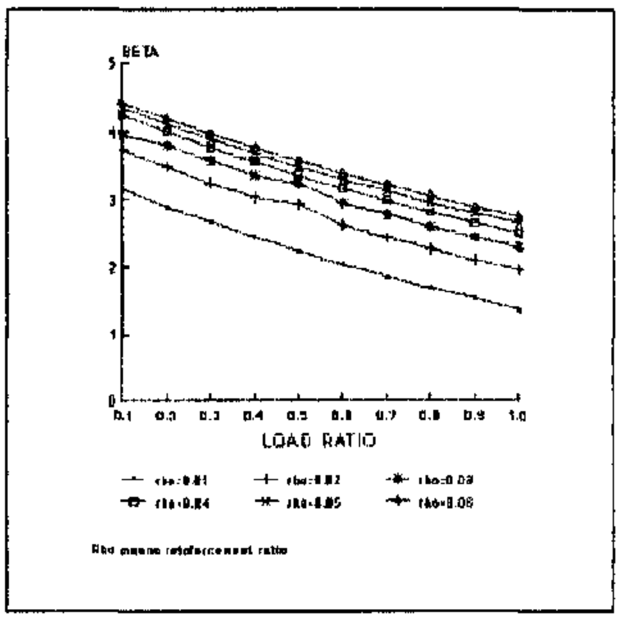

Fig. 5.12: Safety Levels for Short Columns with Bending when Q and F_y are Lognormal.

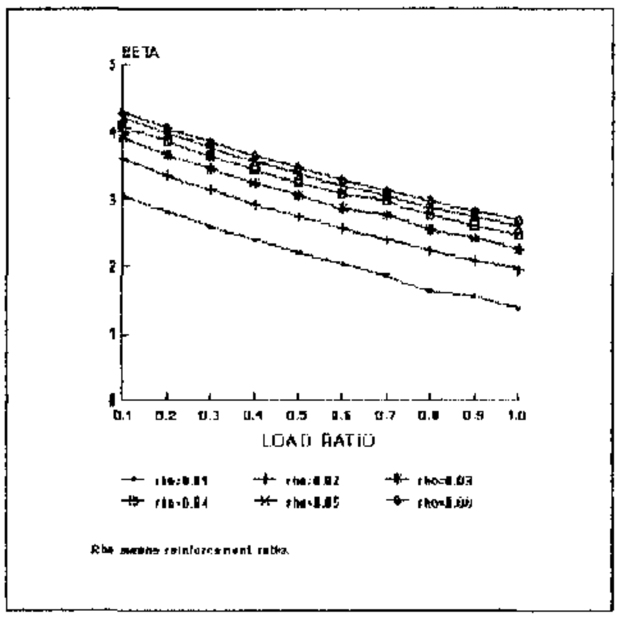

Fig. 5.13: Safety Levels for Short Columns with Bending when Q is Gumbel and F_y is Lognormal.

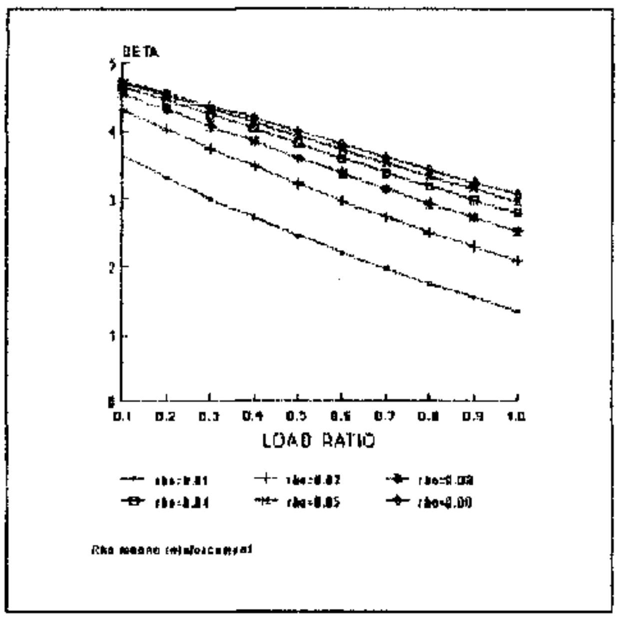

Fig. 5.14: Safety Levels for Short Columns with Bending when Q is Normal and F_y is Gumbel.

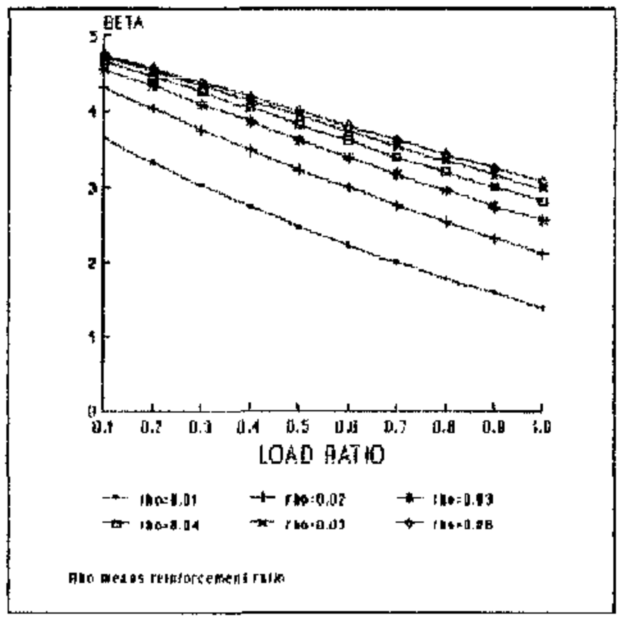

Fig. 5.15: Safety Levels for Short Columns with Bending when Q and F_y are Normal.

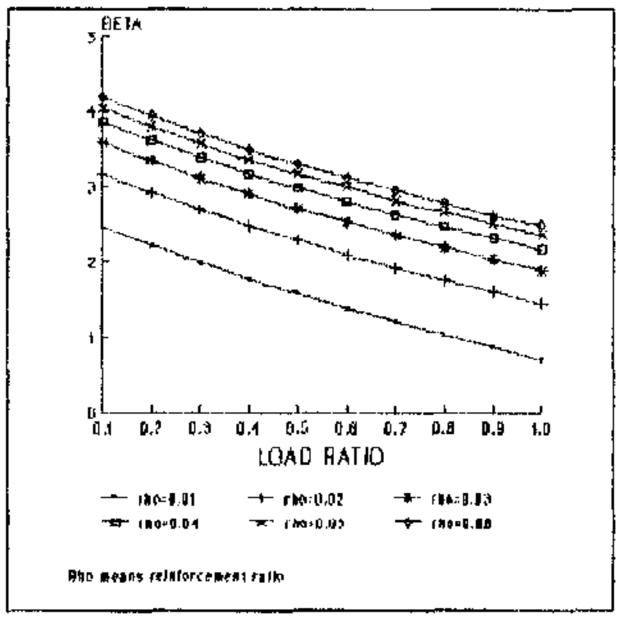

Fig. 5.16: Safety Levels for Slender Columns when Q is Gumbel and F_y is Lognormal.

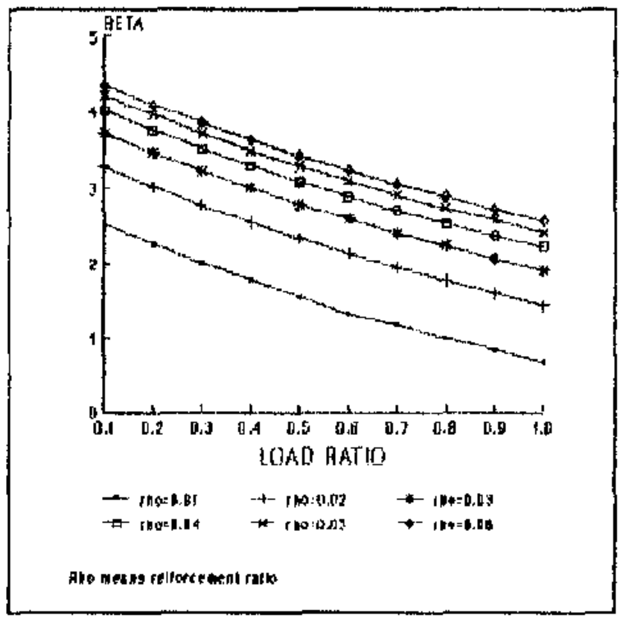

Fig. 5.17: Safety Levels for Slender Columns when Q and F_y are Lognormal.

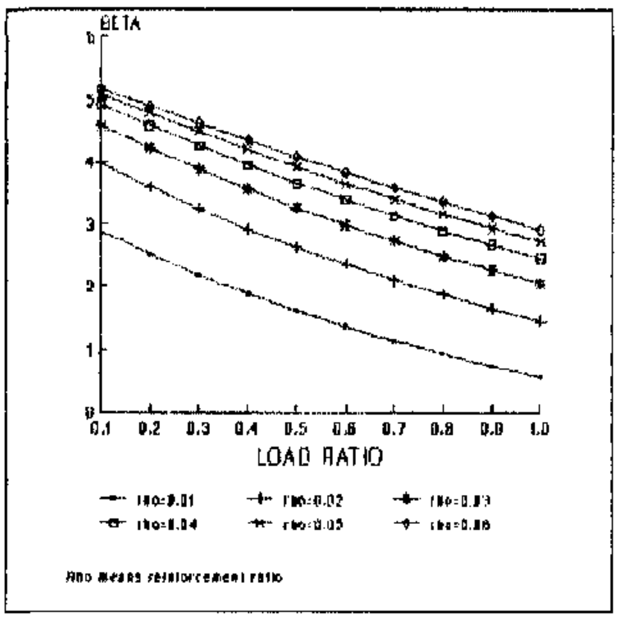

Fig. 5.18: Safety Levels for Slender Columns when Q is Normal and F_y is Gumbel.

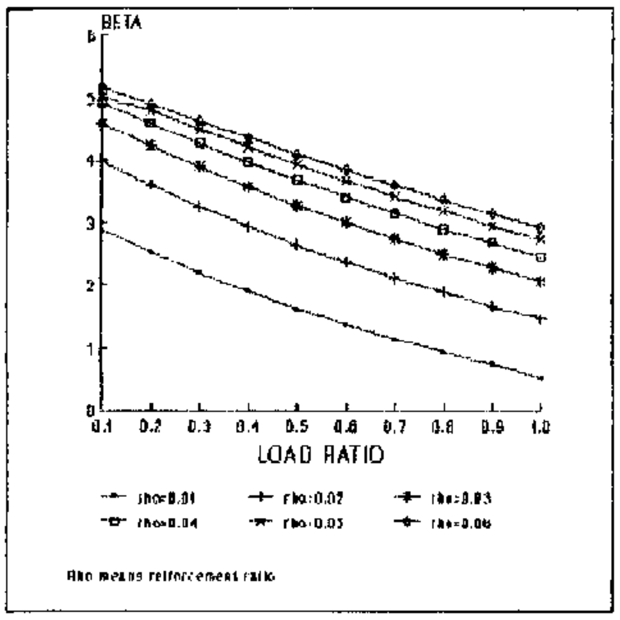

Fig. 5.19: Safety Levels for Slender Columns when Q and F_y are Normal.

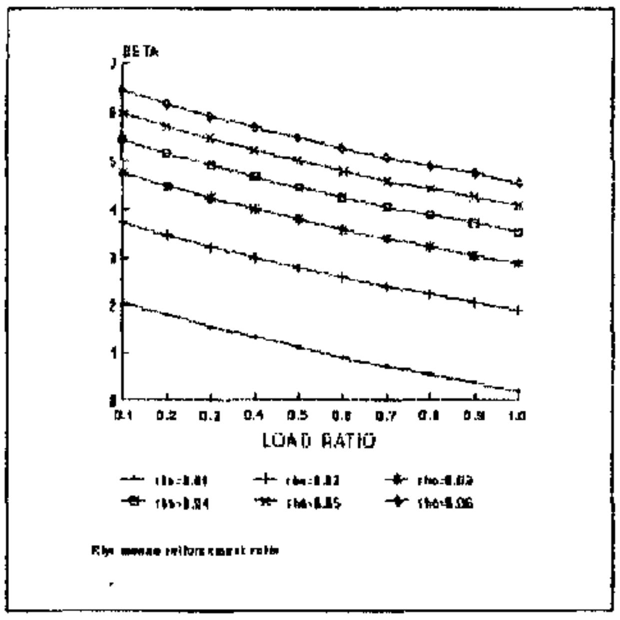

Fig. 5.20: Safety Levels for Short Columns subjected to Biaxial Bending when Q and F_y are Lognormal.

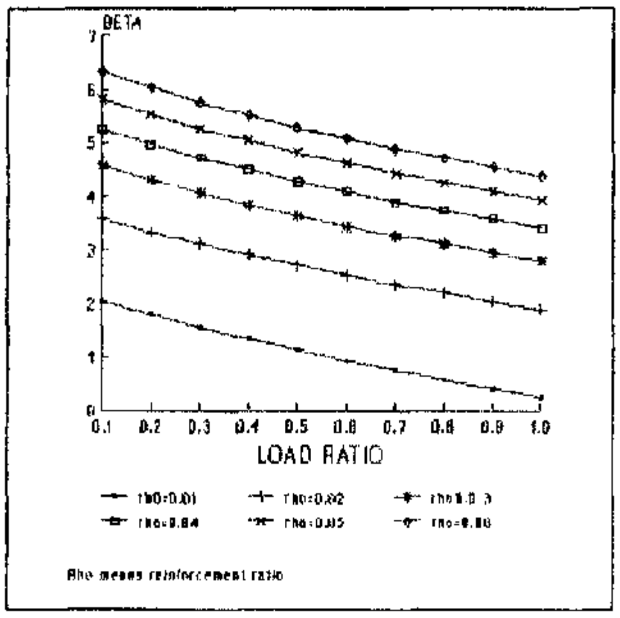

Fig. 5.21: Safety Levels for Short Columns subjected to Biaxial Bending when Q is Gumbel and F_y is Lognormal.

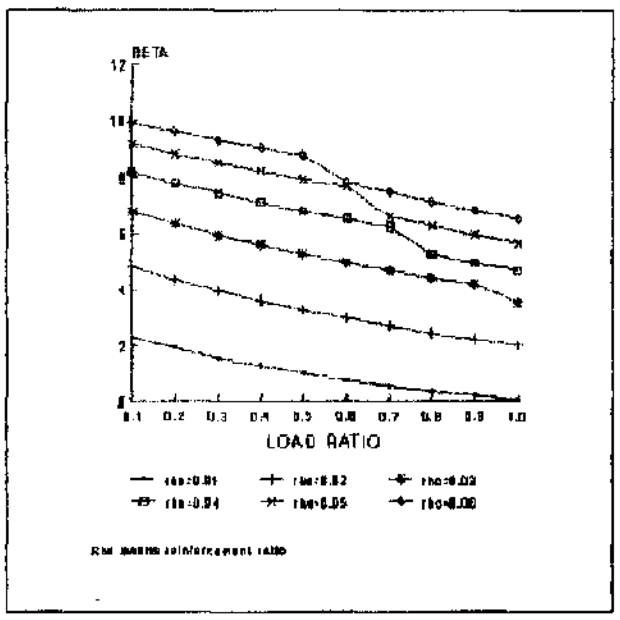

Fig. 5.22 Safety Levels for Short Columns subjected to Biaxial Bending when Q is Normal and F_y is Gumbel.

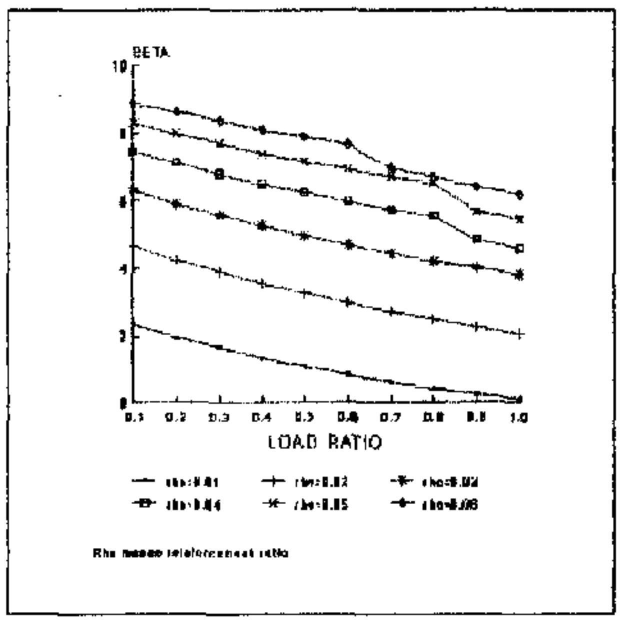

Fig. 5.23 Safety Levels for Short Columns subjected to Biaxial Bending when Q and F_y are Normal.

distribution is gumbel and steel strength distribution is log-normal thereby properly predicting the column capacity. The investigation shows that short columns with bending have their capacities well predicted, followed by slender columns. These results show the importance of properly understanding the probability distributions of the various design variables in establishing structural safety.

5.2.1 Isosafety Design Functions

Graphical plot of ρ against R_a at given β are constructed and their relationship examined. For short columns with purely axial load criterion, ρ is found to vary linearly and proportionately with R_a (see Fig. 5.24). For other design criteria, ρ also increases with R_a in a polynomial manner (Figs. 5.25 to 5.27).

From the plots it is possible to design to specific reliability levels. For example, to obtain a reliability level of 4.0 for a section of 400 x 400mm^2 and R_a of 0.7, a ρ of 0.028 (i.e., A_s = 4480mm^2) is required for a purely axially loaded short column, 0.05 (A_s = 8000mm^2) is required for short column with bending while it is impossible to obtain that safety level for the same R_a in a slender column and a short column with biaxial bending.

A reliability level of 5.0 and 6.0 are obtained for biaxial bending criterion for R_a = 0.7 with ρ = 0.0335 and 0.037 respectively thus showing its high level of inconsistency.

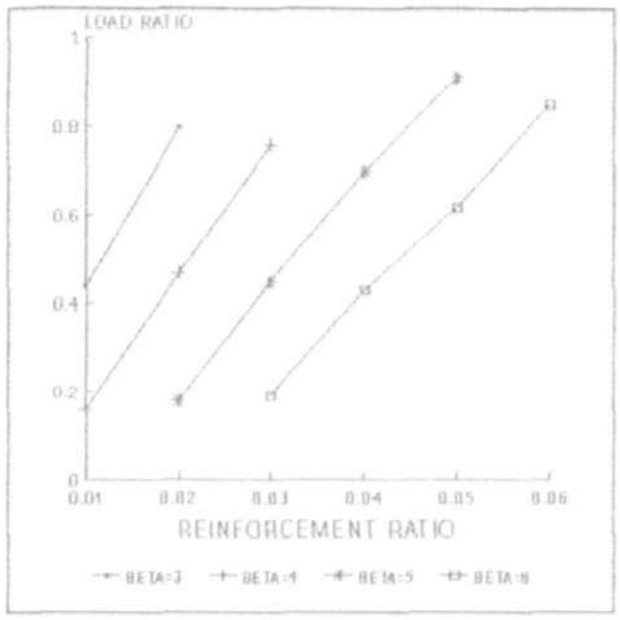

Fig. 5.24 Isosafety Design Functions for Axially Loaded Short Columns.

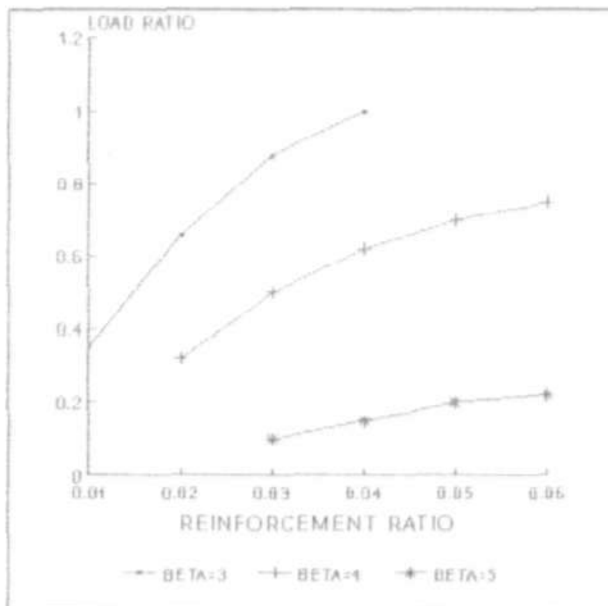

Fig. 5.25 Isosafety Design Functions for Short Columns with Bending.

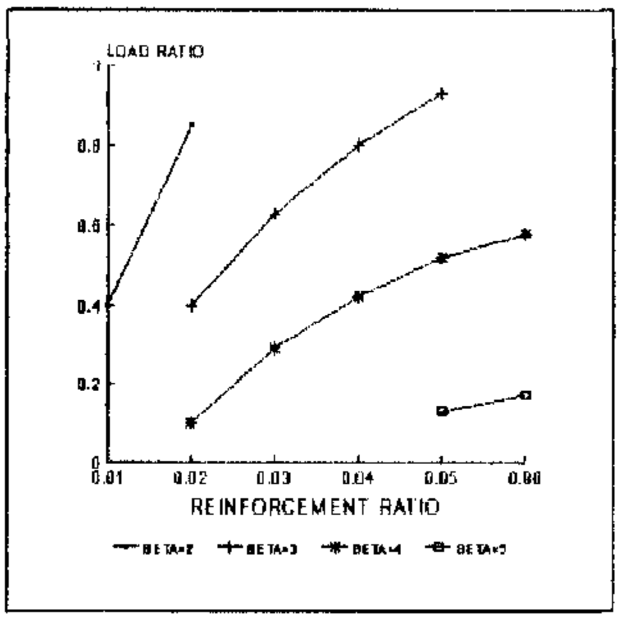

Fig. 5.26 Isosafety Design Functions for Slender Columns.

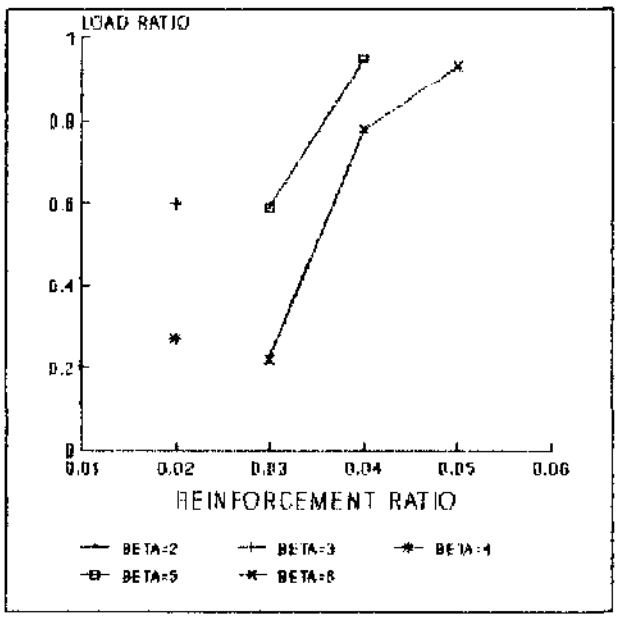

Fig. 5.27 Isosafety Design Functions for Short Columns subjected to Biaxial Bending.

This investigation further reveals that for a given section, a short column with bending requires 78.6% more reinforcement that a purely axially loaded short column to achieve equal reliability.

However these plots cannot be confidently applied to other sections nor different design variables. To design to specific reliability levels for other given design parameters, it would be expected that their plots of ρ against R_a must be carried out. This aspect is beyond the scope of this investigation and as such it is not further examined.

5.2.2 Sensitivity Tests on Design Parameters.

Sensitivity tests are carried out for each parameter of the design variables. Each parameter of the design variables is varied by a fixed amount while others are held constant and the results (i.e., the value of β) is computed for each variation. Thus, the parameters which change the output β are noted as the most critical at the variation range used and their relative importance in the design process is established (Afolayan, 1994). For this purpose, datum values of the design variables involved are those listed in Table 4.1. For these values the corresponding safety levels are denoted by β_0. Graphical representations of percentage change against β/β_0 are presented in Figs. 5.28 to 5.31.

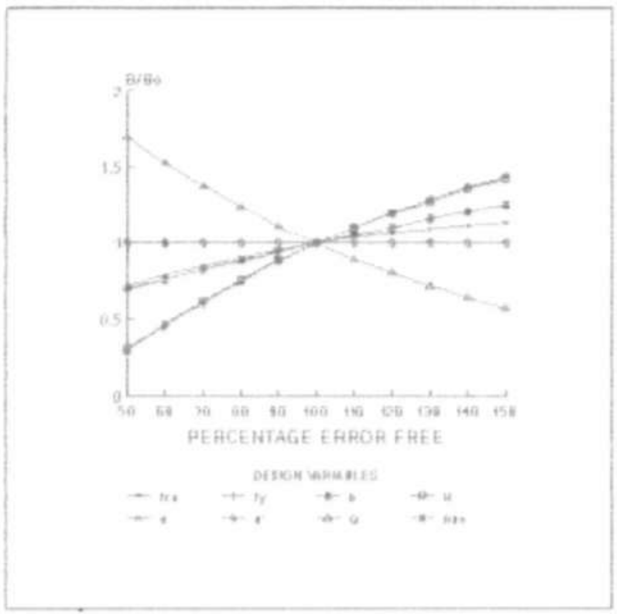

Fig. 5.28: Sensitivity Test for Axially Loaded Short Columns

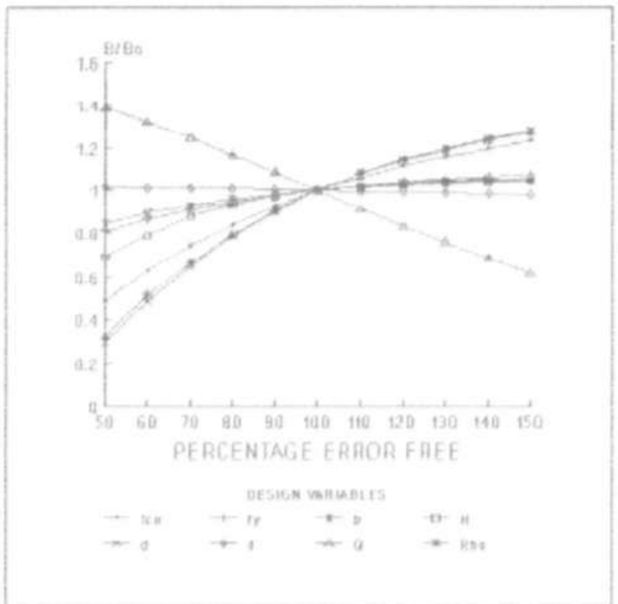

Fig. 5.29: Sensitivity Test for Short Columns with Bending.

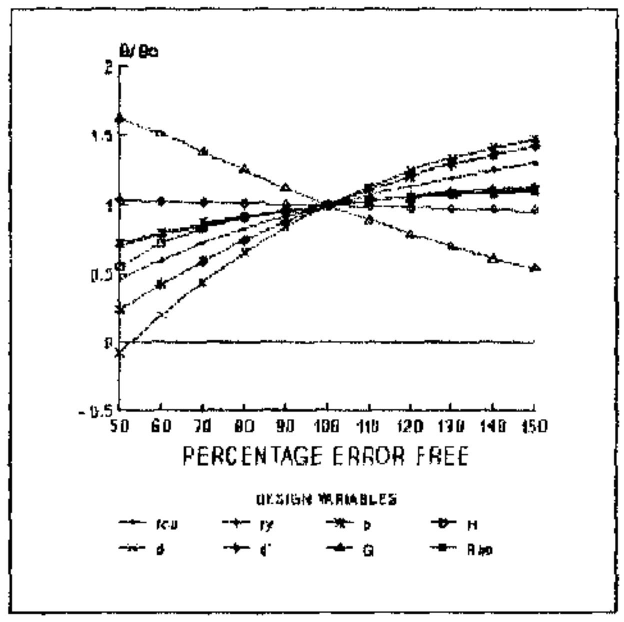

Fig. 5.30: Sensitivity Test for Slender Columns.

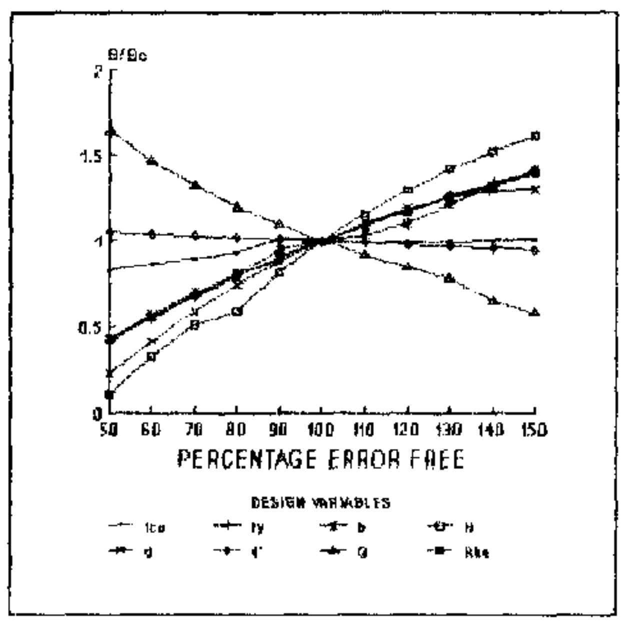

Fig. 5.31: Sensitivity Test for Short Columns subjected to Biaxial Bending.

Generally, the results reveal that the influence on β is more when the change in the design parameters is in the negative direction than in the positive direction. The trend of the influence shows a beneficial effect on β in the positive direction for all parameters except for load.

From Fig. 5.28, it can be established that for short columns with purely axial load, the most influential parameter is the load followed by the overall depth and the breadth of the section while the other parameters have minor effect with effective depth and concrete cover having no influence on β.

In short columns with bending and slender columns, as presented in Figs. 5.29 and 5.30, the most influential parameter is the effective depth which causes failure in slender column at 50% error free. The breadth, load and concrete strength also cause significant effect on β on both design criteria while a minor effect is caused by the others with β almost insensitive to concrete cover.

All parameters except concrete cover and concrete strength have a significant influence on β in biaxial bending criterion. Fig. 5.31 shows that β is influenced most by the overall depth followed by the effective depth and load. The design of a column with biaxial bending is therefore crucial since all of the design parameters except concrete cover and concrete strength significantly influence the β-value. This sensitivity analysis shows how errors in the choice of values for design variables significantly change structural safety levels. It should be understood that 100% error free corresponds to preselected design values for a given performance level.

CHAPTER SIX

CONCLUSION AND RECOMMENDATIONS

6.1 Conclusion

The BS8110 (1985) design criteria for reinforced concrete columns have been studied and the limit state equations for the various design criteria are properly derived for probabilistic assessment.

The reliability levels implicit in each design criterion are computed using FORM5. The results exhibit the level of inconsistency and conservatism involved in each criterion. An average safety index of $\beta = 4.35$ (6.794×10^{-6}) is found for the requirements prescribed by the BS8110 (1985). This value is outside the range of 2 to 4 proposed by Moses (1990) illustrating the conservatism of the code. The distribution of β around this mean value also demonstrated a high level of inconsistency. A comparison between the reliability levels of the different design criteria shows that the most inconsistent and conservative requirement is that of biaxial bending while short columns with bending have the least implying that their capacity are the best predicted.

The alteration of the statistical distributions of the load and steel strength is found to affect the levels of inconsistency and conservatism. The lowest levels of conservatism and inconsistency are established when load assumes gumbel and steel strength log-normal distributions.

An attempt is also made to present isosafety design functions using a plot of ρ against R_a. It is noted that to obtain a given β for a given cross-section short columns with bending requires 78.5% more reinforcement than that required by purely axially loaded columns.

Finally, sensitivity tests on all design variables show that the safety levels for short columns, axially loaded are governed by load and cross-sectional dimensions. The safety levels of short columns with bending and slender columns are influenced most by the effective depth, load and breadth while biaxial bending is influenced significantly by all design variables except concrete strength and concrete cover. The biaxial bending design is influenced most by the effective depth.

This investigation has highlighted the difficulties associated with the BS8110 (1985) design criteria for reinforced concrete columns to provide economical designs. Also the directions for adequate quality control have been indicated by the sensitivity analysis.

6.2 Recommendations

The concepts and methods of probability provide a proper basis for the evaluation of structural safety and performance, and the development of any design criterion. Based on these concepts and as a result of recent advances made in structural reliability it is highly recommended that probability based review of all authorized codes be enforced. This would give a better insight into the implied levels of safety of all design criteria of the code in question. A probabilistic approach also gives an avenue to properly accommodate the randomness of design variables thereby leading to an economic design.

To acquire a reasonable level of design consistency, it is strongly emphasized that the BS8110 (1985) and any other structural code should properly outline an isosafety design procedure.

Due to the effect of changes in the design values of the design variables on β through sensitivity analysis, quality control should not be compromised with.

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