STATE SPACE CONTROLLER DESIGN

FOR A DC MOTOR

By

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A THESIS SUBMITTED TO THE DEPARTMENT OF ELECTRICAL ENGINEERING, AHMADU BELLO UNIVERSITY, ZARIA IN PARTIAL FULFILMENT OF THE REQUIREMENT FOR THE AWARD OF A MASTER OF SCIENCE (M.Sc) DEGREE IN ELECTRICAL ENGINEERING

DECLARATION

I, ABBAS ALIYU, hereby declare that this thesis titled “State Space Design of a Controller for a Dynamic System” presented to the Department of Electrical Engineering, Ahmadu Bello University, Zaria is the effort of my own research work and has never been for the award of any degree. That all literature consulted have been duly acknowledged.

_____ _____

ABBAS ALIYU DATE

CERTIFICATION

This thesis titled, "State Space Design of a Controller for a Dynamic System" by Abbas Aliyu meets the requirement governing the award of degree of Master of Science in Electrical Engineering, Ahmadu Bello University, Zaria and is approved for its contribution to knowledge and literature presentation.

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DEDICATION

This work is dedicated to my entire family for their love, encouragement and support.

ACKNOWLEDGMENTS

Bismillahi-Rahmani-Raheem. All praise is due to Allah for His grace and mercy from the beginning to the end of this program.

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ABSTRACT

Modern manufacturing systems are automated machines that perform the required task. The DC motor is perhaps the most widely used industrial energy converter whose performance decrease over time. In this work, an experimental data is used to create a mathematical model of the DC motor for performance analysis. Using state space analysis and Matlab, information on the internal behaviour of the DC motor are obtained and based on the design specification; a controller is designed to improve the performance.

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LIST OF SYMBOLS

A	State matrix
B	Input matrix
C	Output matrix
D	Transmission matrix
M_c	Controllability matrix
M_o	Observability matrix
E_a	Input voltage (V)
ϕ	Flux (Wb)
i_a	Armature current (A)
i_f	Field current (A)
T_m	Motor torque (Nm)
L_a	Armature inductance (H)
R_a	Armature resistance (Ω)

f_o	Viscous friction
J	Motor friction
K_b	Back emf constant
ω	Angular velocity (rad/s)
α	Angular acceleration (rad/s^2)
dt	Change in time
$d\omega$	Change in angular acceleration

CHAPTER ONE

INTRODUCTION TO CONTROLLER AND DC MOTOR

1.1 INTRODUCTION

Many processes ranging from complex mechanical, electrical, biological, defense, space travels, to mention a few, are dynamic in nature and can be represented mathematically by a differential equation. A dynamic system is a manifold M called the state space and a smooth evolution function Φ^t that for any element of t, a member of T, the time, maps a point of the state space back into the state space. State space control is a modern control theory that comes from the state variable method of describing differential equations. State space design is a technique in which a dynamic compensation of a system is produced by working with the state variables describing the system. The need to develop a system of mathematical representation, along with the output gives information about the state of the system variables at some predefined points along the flow of signals lead to the development of state variable approach. In the past few decades, a particular attention has been placed on how to model, simulate and guide such systems in the best way. The field of optimal control is concerned with devising the best or optimal control law that will give the best performance. Control systems are means by which any quantities of interest in a machine or other equipment are changed in a desired manner. The DC motor is used to convert electrical energy into

mechanical energy. It is widely used in many applications due to its excellent speed control characteristics. As a result of this, the control of the speed and/or position of the load on DC motor have attracted considerable research since the early decade in the last century.

Extensive use of the DC motor arises from the controllability of its speed and compatibility with new electrical, electronic and mechanical devices. The DC motors just like any other control device are evaluated based on certain performance in the time domain. Engineers and technicians must be able to evaluate the performance of dynamic systems (DC motor) quantitatively to see their response to a test signal in order to select a tuning parameter for the controller. What constitute a good response varies from one system to another. In general, a good response should display as many of these characteristics as possible:

- Constant magnitude
- It must be stable
- It must be fast
- Maximum disturbance rejection
- Minimum delay
- Limited control action (manipulation cost)

- It should be “Robust”- insensitive to process changes and model mismatch.

Design specifications are used to describe what the system should do and how it should be done. In reality, it is not possible to satisfy all these performance indices at the same time. However, understanding the process will help determine the priority of performance index. All stable control systems exhibit transient behaviour to certain degree before a steady state is reached. This is so because; inertia, mass, inductances which are all energy storage devices are inherent in physical systems. As a result, the system tends to exhibit transient, causing the output to deviate from the input instantaneously. Transient response is defined as that part of the system response that goes to zero as time goes to infinity while the steady state response indicates where the system output ends up as time approaches infinity.

The transient response of a control system is necessarily important since both the amplitude and the time duration must be kept within tolerable or prescribed limit. For a position control system, the steady state response when compared to the desired position gives an indication of the final accuracy of the system. If the steady response of the output does not agree with the input, the system is said to have a steady state error. The response of a second order system describing the

transient and steady state parameters: peak overshoot, settling time, steady state error are shown in Figure 1.1

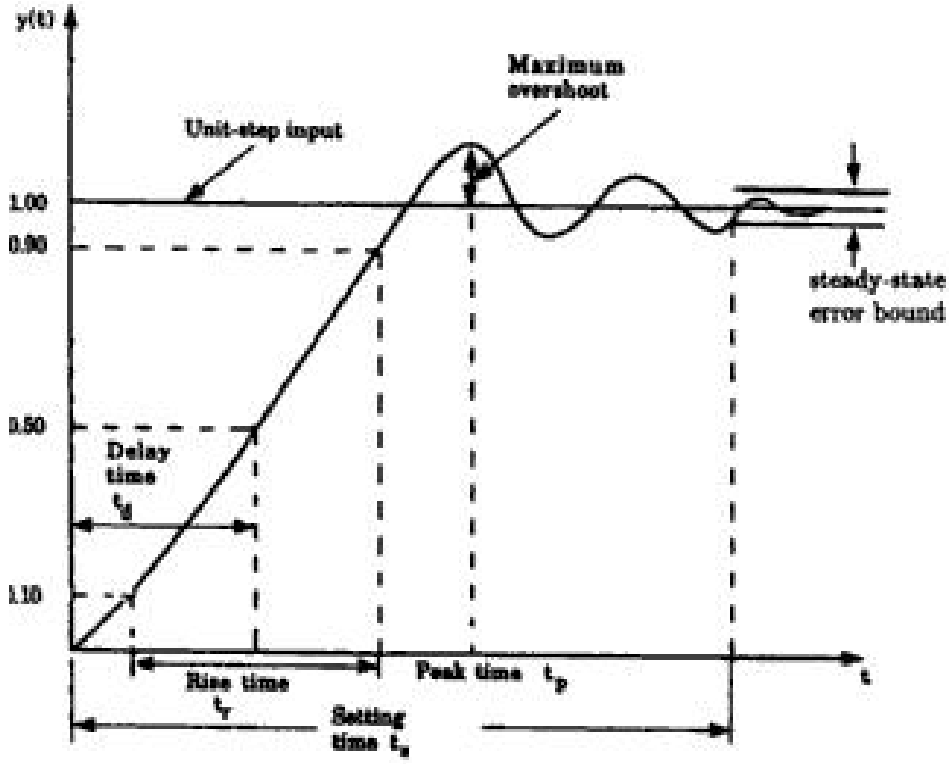


Figure 1.1 Response of a Second Order System

From the diagram of Figure 1.1, the following performance indices are defined.

- Peak overshoot: it indicates the normalized difference between the time response peak and the steady state output.
- Settling time: it is the time required for the response to reach and stay within a specified tolerance band (usually 2% or 5% of its final value).

- Steady state error: it indicates the difference between the actual output and the desired output as time goes to infinity.

Design in time domain using these performance specification mentioned above is possible analytically only for second order systems or for systems that can be approximated to second order. General design procedure using time domain specifications are difficult to establish for systems with order higher than the second.

Controllers are incorporated into the system to cause the motor output to adjust in accordance with a defined control law in order for its output to track its input.

A single controller can be used to control both the transient and steady state speed. The controller reference signal determines the desired position and/or speed at the output. The controller parameter is carefully selected so that the error between the reference signal and the output signal eventually tend to a minimal value preferably zero. The development and availability of high-powered and user friendly program and software has changed the practice of design in control systems which until recently had been dictated by historical development.

1.2 **MOTIVATION**

Control systems are designed to perform optimally. Due to friction, temperature and aging which occur over a period of time, the performance of the system

diminishes and calls for evaluation. Controllers are used to improve the performance of the systems. In designing a controller, the designers become frustrated after spending much time and energy which at the end of design does not give the desired performance. However, with a high powered tool like Matlab and its embedded function, Simulink, the designer can go through a series of design, study the behaviour of the system, carry out analysis and finally come up with the best possible value of the controller that will give a better performance of the system within a short period of time.

1.3 STATEMENT OF PROBLEM

Every control system designed for a specific application has to meet certain performance specifications. There are situations where the output of a system may look stable; yet some of the system parameters or specifications exceed or have the tendency to exceed their specified ratings. For the armature controlled DC motor under study, the variation in inertia, inductance, friction, and the resistance of the motor which are difficult to quantify, result in poor response (high overshoot, poor settling time and large steady state error) leading to a control system (DC motor) that does not give optimal performance in some defined sense. Under this circumstance, it is necessary to introduce a corrective subsystem (controller or compensator) to improve the performance of the DC motor.

1.4 METHODOLOGY

The motor used for the experiment is a shunt type armature controlled with the following name plate detail: manufacturer, NECO, frame number 5A, serial number 922564. The aim of the experiment is to study the speed characteristics of the motor and evaluate its performance. The procedures to be adopted are as follows:

I. Represent the DC Motor circuit diagram.

II. Derivation of the dynamic equation(s) of the armature controlled DC motor from the free body diagram in Figure 3.1 and calculation of the Motor parameter determined from experiment.

III. Development of the state space model of the DC Motor from the dynamic equation derived in (II)

$$\dot{X} = Ax + Bu \quad (1.1)$$

$$y = Cx + Du \quad (1.2)$$

A is the system matrix, B is the input matrix, C is the output matrix, D is the transmission matrix, u is the input vector, x is the state vector and y is the output vector.

IV. Determination of the controllability of the system using Kalman's method to verify from the outset if a solution exists to the control problem.

$$M_c = \left[B \vdots AB \vdots \cdots \vdots A^{n-1}B \right] \tag{1.3}$$

The system is said to be controllable if the controllability matrix, M_c is of full rank (having linearly independent column) i.e. $M_c \neq 0$.

V. Determination of the observability of the system using the Kalman's method to also verify if a solution exists to the control problem.

$$M_o = \begin{bmatrix} C \\ \hline CA \\ \hline \vdots \\ \hline CA^{n-1} \end{bmatrix} \tag{1.4}$$

The system is said to be observable if the observability matrix M_o is of full rank (having n-linearly independent rows), i.e. $M_o \neq 0$.

VI. Develop a Matlab program for the system to see its behaviour to a step input.

VII. From the result of VI, a controller that will enable the DC motor achieve the set performance specifications will be designed for the system.

VIII. An observer which monitors the states that could not be measured will be designed for the system.

IX. Analyses will be carried out based on the results obtained.

1.5 THESIS OUTLINE

This work comprises five chapters. The first Chapter gives an overview of the DC motor, the state space concept and performance specifications, Chapter Two covers related works that has been done in trying to improve the performance of a DC motor while Chapter Three focuses on modeling the DC motor, representation of the model in state space form, verification using Kalman's method to ascertain whether a solution exists to the problem stated and application of a step input to see the response of the DC motor. Chapter Four presents the performance achievement based on the set of specifications stated in Chapter Three under a controller. The introduction of a reference input to remove steady state error, the design of an observer, discussion of results and analysis are also presented. Chapter Five highlights the significance of the work, limitations and recommendation for further work.

CHAPTER TWO

LITERATURE REVIEW AND THEORITICAL BACKGROUND

2.1 INTRODUCTION

This section reviews previous work that have been done on DC Motor modeling using different methods, the use of state space as well as conventional method to design controllers for dynamic systems.

Designing a controller for a dynamic system requires a model of the plant in order to have a good response. Hany et al. [1] proposed a mathematical model for a DC Motor using a method known as Bond graph. The proposed model is based on the behaviour and construction of the plant. In order to validate the model, an experiment was carried out and the result showed that the Bond graph will be one of the alternatives in modeling a plant. Umeno et al. [2] designed a robust speed controller for a DC motor based on the parameters of two degree of freedom controller. The controller can dramatically improve the characteristics of the closed loop system vis-a-viz the disturbance torque suppression performance and the robustness to system parameter without changing the command input. Also, Wai [3] in 2008 presented a paper on Analysis, Modeling and Simulation of a DC Motor and its driving system used for Wheeled Mobile Robot. The focus of the paper was on software application of Matlab/Simulink to choose DC motor

specification that can be used with the desired Wheeled Mobile Robot. Contributions from Fateh and Izadbakhsh [4] led to a robust control approach for a high speed manipulator using hybrid computed torque in state space. The high speed manipulator is driven by a permanent magnet DC motor which has the ability to track a trajectory in the presence of disturbances. Tracking problem is analyzed in state space where the completed model is considered for actuator, the proposed approach can guarantee the stability and satisfactory tracking performance. Ali and Keith [5] did a work on Integral and Proportional Integral Sliding Mode Control Technique for controlling the speed of a DC motor. At the end of the work, it was concluded that the Integral and proportional integral sliding mode controllers are the most suitable controllers for speed/position control of a DC motor. The conclusion was based on comparing the performance of the system with Integral and Proportional Sliding Mode Controller with PI/PID controller.

Neural-Networks are used effectively in identification and control of linear systems. Arif and Mazin [6] in the Journal of Engineering Science and Technology presented a suitable and generalized Neural Network that has the ability to memorize and emulate the system states. Moreover, Neural Networks are found to be more representative of system order in terms of its structural complexity. This facility is utilized efficiently to find the minimum Elman Neural Network (ENN) structure that represents the discrete –time state space of the DC motor. By

comparing the ENN weights with the well known discrete time state space equation in terms of the motor physical parameters (moment of inertia, torque constant , armature inductance), it is found that these parameters can be easily obtained. Farzan et al. [7] in a paper titled, “Applying Intelligent Controllers for Speed Regulation of DC Motor” describes the application of several new approaches in controlling the speed of DC motor: Fuzzy Auto Tuning, Gas-Based PID Controller and Gas-based Fuzzy PID Controller. All the control strategies utilize the output speed error and its derivative as feedback damping signal. Other researchers that have contributed to the use of state space technique in designing a controller for linear and nonlinear systems (magnetic levitator) using state space are Walter and John [8].

The problem associated with the magnetic levitator is its inability to control the height of a steel ball above the ground by levitating it against the force of gravity using electromagnet. Experimental result was used to compare the effectiveness of the two controllers in terms of their ability to respond to sinusoidal reference trajectory. The nonlinear controller was found to provide superior tracking performance compared to the linear one. Marius and Lucian [9] proposed a simple design method for a DC motor speed controller based on system theory concept in the continuous time domain starting from a required reference model. With very little modification, the system can be adapted to control the position or acceleration

of the motor. Another important contribution to DC motor speed control is the synthesis and design of a variable structure controller for a DC motor by Cosme [10].

The focus of the work was comparing the behaviour of a physical plant operated by a variable structure speed controller against the behaviour showed by the same plant operating under control action of a conventional controller using Matlab digital simulation. The result obtained confirms a significant diminution in response and settling time besides an attenuation of the overshoot. Glavin [12] in 2005 did a project on state space control of a magnetic suspension system in which a controller was designed for a magnetic levitator.

Nagurka [13] suggested a control design algorithm for determining PI type controllers satisfying specification on gain margin, phase margin and upper band on the sensitivity for a finite set of plants. Important properties of the algorithm developed are: it can be applied to plants of any order including plant with delay and unstable plant, it is efficient and fast, as such can be used in near real time to determine controller parameters, and it enables graphical portrayal of design trade-off in a single plot.

Siller-Alcala [14] tries to link control theory into practice in a paper titled, “How to Link Control Theory and Practice of Control Engineering: Practice in Real-time

and Simulation" at the International Journal of Education and Information Technology. The target of the paper was to enable students link theory with practice through a set of practices developed in Matlab and Simulink for a heat transfer module involving basic control concept. The variable structure controller was the subject of investigation of quite a number of researchers because of its simple construction, high reliability, fast response with minimal overshoot and insensitivity to external disturbance and parameter variation. Zoran and Goran [14] in trying to enhance the performance of a DC motor designed a variable structure controller for a DC motor position control. This controller can cause the system parameters to adjust based on sudden changes in the input signal but cannot account for a variation in parameter or sudden disturbances.

This research will focus on using experimental data to investigate the performance of a DC Motor and using the state space method, design a controller which will take into account the effect of parameter variation on the performance of a DC Motor.

2.2 THEORITICAL BACKGROUND

2.2.1 STATE

The state of a dynamic system is the minimum set of variables such that the knowledge of these variables at $t = t_o$, together with the knowledge of the input for $t \geq t_o$, completely determines the behaviour of the system for any time $t \geq t_o$.

2.2.2 **STATE VARIABLES**

The state variables of a dynamic system are the variables making up the smallest set of variables that determine the state of the dynamic system. If at least n variables $x_1, x_2, \ldots, x_n$ are needed to completely describe the behaviour of a dynamic system (so that once the input is given for t $\geq t_o$ and the initial state at $t = t_o$ is specified, the future state of the system is completely determined), then such n variables are a set of state variables.

2.2.3 **STATE VECTOR**

If n state variables are needed to completely describe the behaviour of a given system, then these n state variables can be considered the n component of a vector$x$. Such a vector is called a state vector. A state vector is thus a vector that determines uniquely the system state $x(t)$ for any time $t \geq t_o$, once the state at $t = t_o$ is given and the input $u(t)$ for $t \geq t_o$ is specified.

2.2.4 **STATE SPACE**

The n-dimensional space whose coordinate axes consist of the x_1, x_2,. . ., x_n, where x_1, x_2 , . . . ,x_n are state variables; is called a state space.

CHAPTER THREE

MODELING THE DYNAMICS OF THE DC MOTOR

3.1 INTRODUCTION

Engineering design success greatly depends on reducing the time spent in creating modules, mechanism and machines. The use of accurate mathematical models can speed up the design process and minimize the time wasted on trial and error design methods. The best engineering practices employ mathematical models early in the design process; this enables the functional requirement and design parameters of the motor system to be determined early in the design process.

3.2 MODELING THE ARMATURE CONTROLLED DC MOTOR

An armature controlled DC Motor with load inertia mounted on its shaft is shown in Figure 3.1

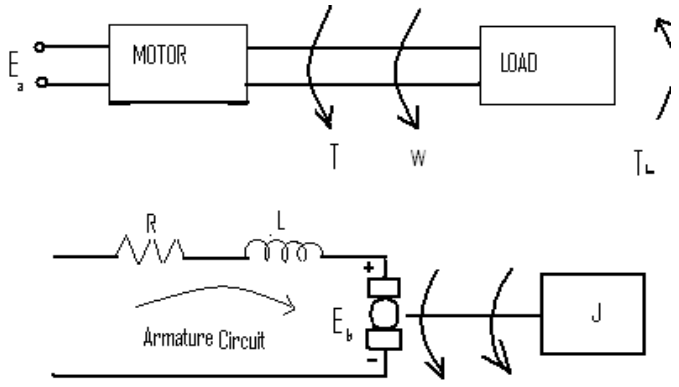


Figure 3.1 Armature Control DC Motor and Load

The input is the armature voltage E_a. The outputs are the motor torque T_m and the angular speed of the motor ω. Dependent variables are the armature current i_a and the back e.m.f of the motor E_b. R_a and L_a are the electrical resistance and inductance of the armature circuit while f_o and J are the viscous damping coefficient and load inertia. K_m and K_b are the motor and back emf constants of the motor respectively.

The DC motors are generally used in the linear range of the magnetization curve [16]. Hence, the air-gap flux ϕ is proportional to the field current.

$$\phi = K_f i_f \tag{3.1}$$

K_f is a constant.

The torque developed by the motor, T_m is proportional to the product of the airgap flux and armature current i_a:

$$T_m \propto \phi\, i_a \tag{3.2}$$

$$T_m \propto K_f i_f\, i_a \tag{3.3}$$

$$T_m = K_i K_f i_f i_a \tag{3.4}$$

K_i is a constant.

In the armature controlled DC motor, the field current is kept constant. This transforms equation 3.4 into:

$$T_m = K_m i_a \tag{3.5}$$

K_m is the DC motor torque constant.

The back electromotive force, E_b developed by the motor is proportional to the motor speed:

$$E_b = K_b \frac{d\theta}{dt} = K_b \omega \tag{3.6}$$

Applying Kirchhoff's' voltage law across the armature circuit gives:

$$E_a = L_a \frac{di_a}{dt} + R_a i_a + E_b \tag{3.7}$$

E_a is the motor terminal voltage and $L_a \frac{di_a}{dt}$ is the voltage across the inductor.

The torque equation of the DC motor is:

$$T_m = J \frac{d^2\theta}{dt^2} + f_o \frac{d\theta}{dt} \tag{3.8}$$

Combining (3.5) to (3.8) gives:

$$K_m i_a = J\frac{d\omega}{dt} + f_o\omega \quad (3.9)$$

Equation 3.9 is based on Newton's law for rotational systems.

Substituting equation (3.6) into (3.7) gives:

$$E_a - K_b\omega = L_a\frac{di_a}{dt} + R_a i_a \quad (3.10)$$

Equation 3.10 results from applying Kirchhoff's voltage law around the armature circuit.

Equations (3.9) and (3.10) give the mathematical model of the armature controlled DC motor.

3.3 STATE SPACE REPRESENTATION

In state space analysis, three types of variables are involved in modeling the dynamics of a DC Motor: input variables, output variables, and state variables. Haven derive the mathematical model of the DC motor; the next step is to find the matrices A, B, C, and D. The angular velocity ω and the armature current i_a are the state variables while E_a is the input voltage. Rearranging equation (3.9) and (3.10) gives:

$$\frac{d\omega}{dt} = \left(-f_o/J\right)\omega + \left(K_m/J\right)i_a \quad (3.11)$$

$$\frac{di_a}{dt} = -\left(K_b/L_a\right)\omega - \left(R_a/L_a\right)i_a + \left(\frac{1}{L_a}\right)E_a \tag{3.12}$$

The dynamic equation of the DC Motor in state space form is given as:

$$\frac{d}{dt}\begin{bmatrix}\omega \\ i_a\end{bmatrix} = \begin{bmatrix}\frac{-f_o}{J} & \frac{K_m}{J} \\ \frac{-K_b}{L_a} & \frac{-R_a}{L_a}\end{bmatrix}\begin{bmatrix}\omega \\ i_a\end{bmatrix} + \begin{bmatrix}0 \\ \frac{1}{L_a}\end{bmatrix}E_a \tag{3.13}$$

The output equation is given by:

$$\omega = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\omega \\ i_a\end{bmatrix} \tag{3.14}$$

The state matrices A, B, C, and D are:

$$A = \begin{bmatrix}\frac{-f_o}{J} & \frac{K_m}{J} \\ \frac{-K_b}{L_a} & \frac{-R_a}{L_a}\end{bmatrix} \quad ; \text{state matrix} \tag{3.15}$$

$$B = \begin{bmatrix}0 \\ \frac{1}{L_a}\end{bmatrix} \quad ; \text{input matrix} \tag{3.16}$$

$$C = \begin{bmatrix}1 & 0\end{bmatrix}; \text{output matrix} \tag{3.17}$$

$$D = 0; \text{disturbance matrix} \tag{3.18}$$

3.4 CALCULATING THE PARAMETERS OF THE DC MOTOR

The DC motor used for the experiment is a 200V, shunt armature controlled with frame number 5A and serial number 922564. The objective is to use experimental data to create a mathematical model of the DC motor and from the mathematical model, with the aid of Matlab plot the graphical representation of the mathematical model in order to study its performance. The procedure used is as follows:

The DC motor was connected to the supply and its terminal voltage recorded. An ammeter was connected to the armature circuit and a tachometer was inserted into the shaft. The motor was switched on, the current through the armature and the fields as well as the motor speed were recorded. The result obtained is as shown in Table 3.1.

Table 3.1 Experimental Data for the DC Motor

Armature Resistance, R_a	13.3Ω
Armature Inductance, L_a	3.0 H
Field Resistance, R_f	1.8k Ω
Field current, I_f	430 mA
Armature Current, I_a	315 mA
Transient current, I_{atr}	500 mA
Rated Motor Speed	1400 rpm
Actual Speed of Motor	1600 rpm
Transient Speed of Motor	1650 rpm
Average time to attain transient speed	5sec
Average time to attain steady speed	7sec

From the data of Table 3.1, the following calculations were obtained.

From equation 3.7, the back e.m.f E_b is given by:

$$E_b = E_a - I_a R_a \tag{3.19}$$

$E_a = 200V$, $R_a = 13.3\Omega$, $I_a = 315 \times 10^{-3} A$.

Substituting these values into equation (3.19) gives:

$$E_b = 200 - 13.3 \times 315 \times 10^{-3}$$

$$= 195.8V$$

The rated speed of the motor is 1600 rpm.

The rated speed in rad/sec is:

$$= \frac{2\pi}{60} \times 1600 \tag{3.20}$$

$$= 167.6 \text{ rad/sec.}$$

The back emf constant K_b is calculated from equation (3.6)

$$K_b = \frac{E_b}{\omega}$$

$$= \frac{195.8}{167.6}$$

$$= 1.168V/rad/sec.$$

The electrical power developed by the armature circuit is given as,

$$P = E_b \times i_a \tag{3.21}$$

The power developed by the rotor is given by:

$$P = T_m \times \omega \tag{3.22}$$

Substituting equations (3.5) and (3.6) into (3.21) gives:

$$P = E_b \times i_a = \frac{K_b \omega T_m}{K_m} \tag{3.23}$$

At equilibrium, the electrical power is equal to the power developed by the rotor.

Equating (3.22) to (3.23) gives:

$$P = \frac{K_b \omega T_m}{K_m} = T_m \times \omega \tag{3.24}$$

From equation 3.24,

$$K_b = K_m \tag{3.25}$$

From equation (3.9),

$$J \frac{d\omega}{dt} = K_m i_a - f_o \omega. \tag{3.26}$$

At steady state, $\frac{d\omega}{dt}$ approaches zero. Substituting this value into (3.26) gives:

$$0 = K_m i_a - f_o \omega \quad (3.27)$$

The coefficient of viscous friction is:

$$f_o = \frac{K_m i_a}{\omega} \quad (3.28)$$

$$= \frac{1.335 \times 315 \times 10^{-3}}{167.6}$$

$$= 0.00219 \text{ N-m/rad/sec}$$

The angular acceleration α is:

$$\alpha = \frac{d\omega}{dt} \quad (3.29)$$

$d\omega$ is the change in angular velocity and dt is the change in time.

$$d\omega = 1650 \text{ rpm}$$

$$= \frac{2\pi}{60} \times 1650$$

$$= 172.78 \text{ rad/s}$$

$$dt = 5 \text{ sec.}$$

Substituting the values of $d\omega$ and dt into equations (3.29) gives:

$$\alpha = \frac{172.78}{5}$$

$$= \quad 34.56 \text{ rad}/sec^2$$

From equation 3.5, the torque developed by the motor is:

$$T = K_m i_a$$

$$= 1.168 \times 315 \times 10^{-3}$$

$$= 0.36792 \text{ Nm}$$

From Newton second law of motion (rotational body), torque is:

$$T = J \times \alpha \qquad (3.29)$$

$$J = \frac{T}{\alpha}$$

$$= \frac{0.36792}{34.56} = 0.0106 \text{kg-}m^2$$

The motor parameters are:

Motor Inertia, J = 0.0106 Kg-m^2

Viscous friction, f_o = 0.00286 N-m/rad/sec

Resistance R = 13.3Ω

Armature Inductance L = 3.0 H

Motor constant K_m = 1.168 V/rad/sec

Back emf constant K_b = 1.168 V/rad/sec

3.5 CONTROLLABILITY AND OBSERVABILITY TEST

The concept of controllability and observability in the design of controllers for dynamic systems mainly involves verifying from the outset whether a solution exist to the control problem. If a solution does exist, it gives the designer the privilege of placing the poles anywhere along the s-plane that will give the desired performance. If any one of the state variables is independent of the control signal, there will be no way of driving the system (DC motor) from an initial state to a desired state in a finite time by the application of a control vector. Hence, the system is said to be uncontrollable. The observability of a system relates to the condition of observing or estimating the state variables from the output variables. If any one of the state variables cannot be observed from the measurement of the output, the system is said to be unobservable or not observable.

3.5.1 CONTROLLABILITY

A system is said to be completely state controllable if it is possible to transfer the system from any initial state $x(t_o)$ to any desired state $x(t)$ in specified finite time by a control vector $u(t)$.

The controllability of the DC motor is found using the controllability matrix.

$$M_c = \left[B \vdots AB \vdots \cdots \vdots A^{n-1}B \right]$$

The Matlab program used to verify the controllability of the DC motor is shown in Appendix A. The result of the program from the command window shows that the determinant of M_c = 4.40754, is not equal to zero, meaning the system is controllable. The program used to test for controllability is shown in Appendix A.

3.5.2 **OBSERVABILITY**

A system is said to be completely observable if every state $x(t_o)$ can be completely identified by measurement of the output $\mathbf{y(t)}$ over a finite time interval. The observability of the system is verified using the observability matrix.

$$M_o = \begin{bmatrix} C \\ \hline CA \\ \hline \vdots \\ \hline CA^{n-1} \end{bmatrix}$$

The Matlab program used to verify the observability of the system is also shown in Appendix A. Since the determinant of M_O = 110.1887, is not equal to zero, the

system is said to be observable. The program used to test for observability is shown in Appendix A.

3.6 APPLICATION OF STEP SIGNAL TO THE DC MOTOR

A step signal is one of the standard test signal used to carry out mathematical and graphical analysis of a control system. The step signal is used to investigate the behaviour of dynamic systems when the control problem under investigation is a function of sudden disturbance. The following Matlab program is used in arriving at Figure 3.2.

```
sys = ss (A, B, C, D) ;

  t = 0:0.01:3;
  Step (sys, t), title ('Open loop Response of the DC Motor');
   grid on.
```

Where A, B, C, D have been defined in equations 3.15-3.18 respectively.

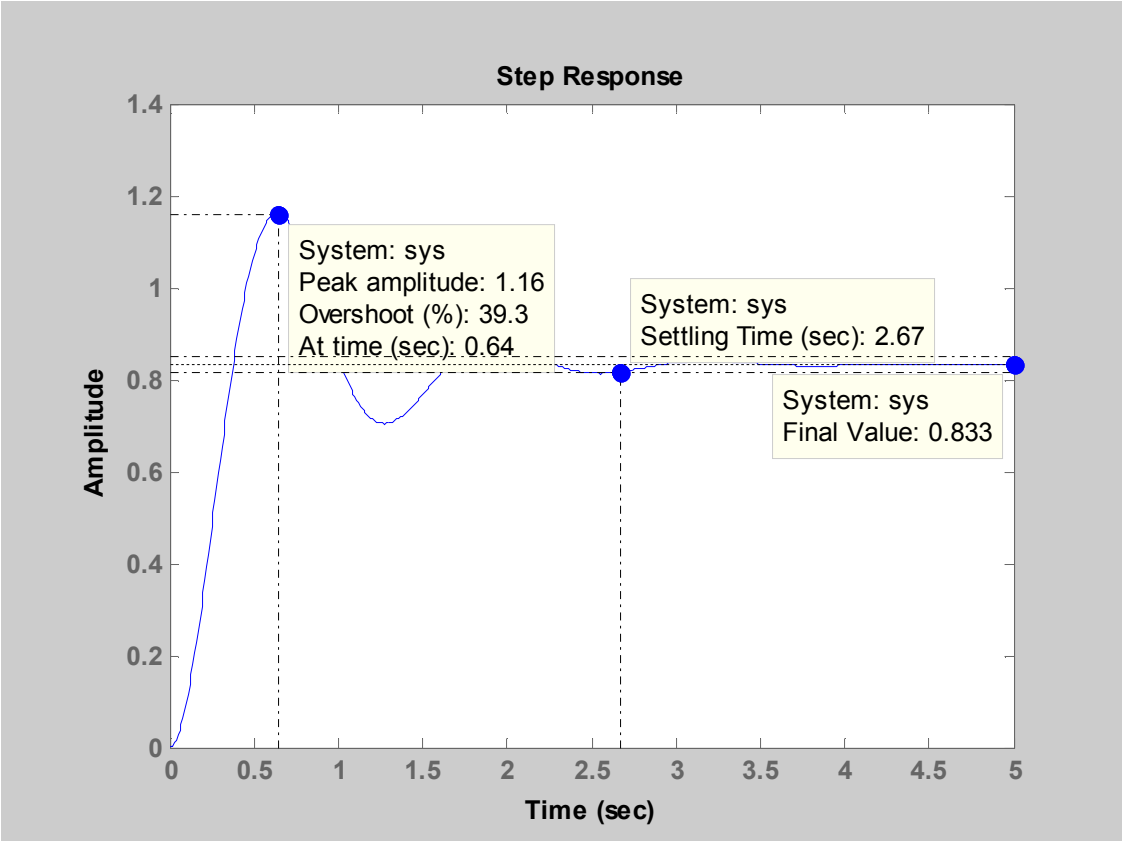


Figure 3.2 Responses to a Step Input

From Figure 3.2, the following deductions were made:

I. Peak Overshoot: Let $u(t)$ be the unit step response of the system, u_{max} be the maximum value of $u(t)$ and u_{ss}, the steady state value of $u(t)$. The peak overshoot is given as:

$$\text{Peak overshoot} = U_{max} - U_{ss} \qquad (3.30)$$

$$= 1.16 - 0.833$$

$$= 0.327.$$

The peak overshoot expressed as a percentage of the final value of the step response is:

$$\text{The \% peak overshoot} = \frac{Maximum\ overshoot}{steady\ state\ value} \times 100 \qquad (3.31)$$

$$= \frac{0.327}{0.833} \times 100$$

$$M_p = 39.3\%$$

II. Settling Time: The settling time t_s is defined as the time required for the step response to reach and stay within a specified percentage of its final value. A frequently used figure is 2% or 5%[21].

From Figure 3.1, the settling time is:

$$t_s = 2.67 \text{Seconds.}$$

III. Steady State Error (SSE): One of the objectives of control systems is that the system output response should follow a specified reference in the steady state. The difference between the output and the reference input when the steady state is reached is the steady state error.

The steady state error from Figure 3.1is:

$$SSE = \text{Reference input} - \text{steady state output} \qquad (3.32)$$

$$= 1 - 0.833$$

$$= 0.167$$

$= 16.7\%$

3.7 PERFORMANCE SPECIFICATIONS

a. The DC Motor is rotating at 0.83 rad/sec with an input voltage of 1 Volt. Since one of the most basic requirements of any control systems is that it should rotate at the desired speed, the steady-state error of the motor speed should be less or equal to 1% [20].

b. Other performance specification is that, the motor should accelerate to its steady-state speed quickly after it is turned on. In this case, the motor's settling time is chosen as 2 Seconds [20].

c. Since a speed faster than the required speed may damage the DC motor, the motor overshoot should be less than or equal to 5% [20].

CHAPTER FOUR

DESIGNING THE STATE FEEDBACK CONTROLLER

4.1 INTRODUCTION

As stated earlier that, a control system is that means by which any quantity of interest in a machine, mechanism or other equipment is maintained or altered in a desired manner [16, 24]. An automatic controller compares the actual value of the plant output with the reference input (desired value), determines the deviation, and produces a control signal that will reduce the deviation to a small value, preferably zero. A block diagram showing the structure of a state feedback system is shown in Figure 4.1.

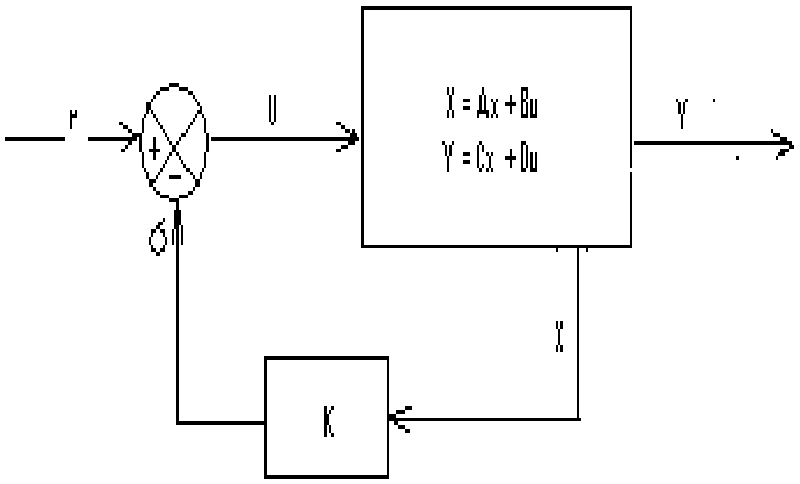


Figure 4.1 DC Machine with Controller K

r = reference input, u = motor input, y = output of the motor, k = feedback gain

The state variable feedback for the DC motor is a scalar function of the form

$$\sigma = -Kx \quad = -[K_1 \quad K_2]\, x \tag{4.1}$$

From Figure (4.1),

$$u = -\mathrm{K}x + \mathrm{r}. \tag{4.2}$$

Substituting equation 4.1 into equation 1.1 gives:

$$\dot{x} = Ax + B(-Kx + r) \tag{4.3}$$

$$\dot{x} = \quad Ax - BKx + Br \tag{4.4}$$

$$\dot{x} = (A - BK)x + Br \tag{4.5}$$

The state feedback controller has its eigenvalues (poles) assigned by solving the determinant (sI-A+BK) for the characteristic polynomial and equating its coefficient in term of K_i to the coefficient of the desired characteristic polynomial [13]. This is done in Matlab using 'place' function. For the desired eigenvalues of the system z =$-1.4649 \pm 4.9307i$; by first placing them at:

P1, P2 =$[-1 + i, -1 - i]$,

P1, P2 =$[-4 + 4i, -4 - 4i]$,

P1, P2 =$[-6 + 5i, -6 - 5i]$,

P1, P2 = $[-8 + 8i, -8 - 8i]$,

P1, P2 = $[-10 + 10i, -10 - 10i]$

using the Matlab command: K= place (A, B, [P1, P2] ;

The following graphs in Figures 4.2, 4.3, 4.4 and 4.5 were obtained.

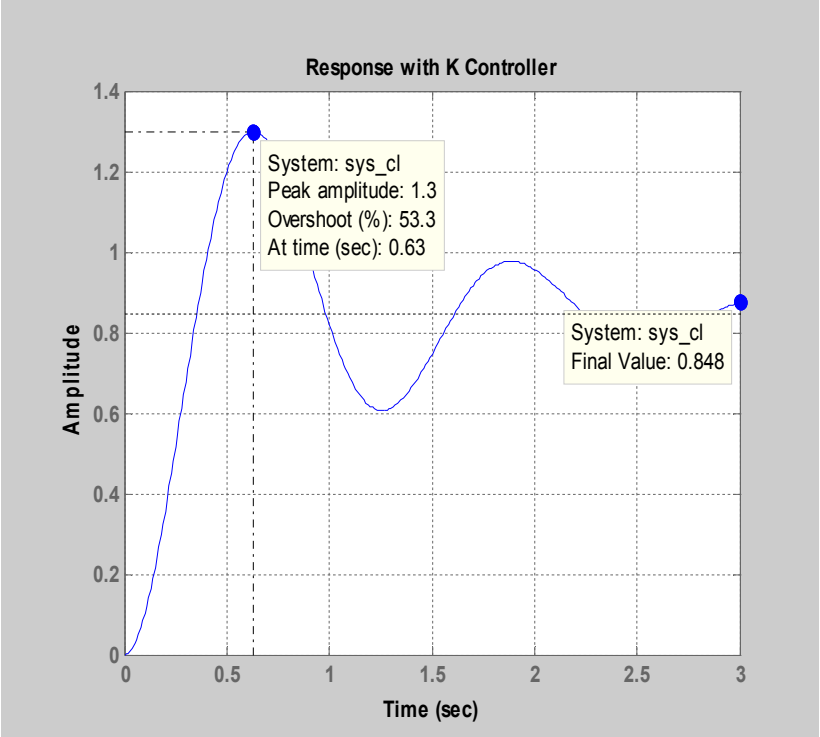


Figure 4.2 Poles at located at $-1 \pm i$

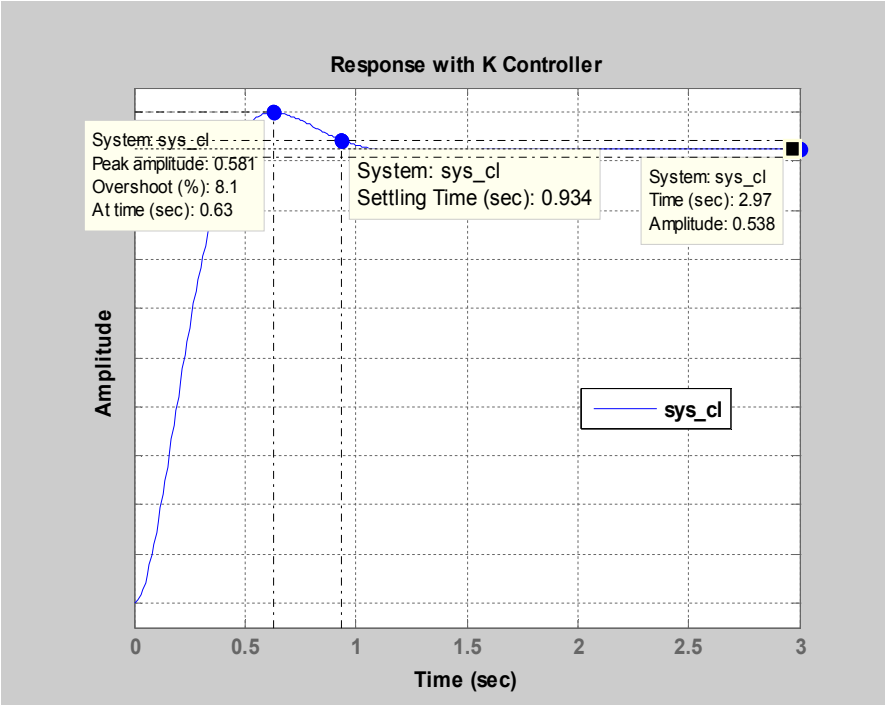


Figure 4.3 Poles located at $-4 \pm 4\,i$

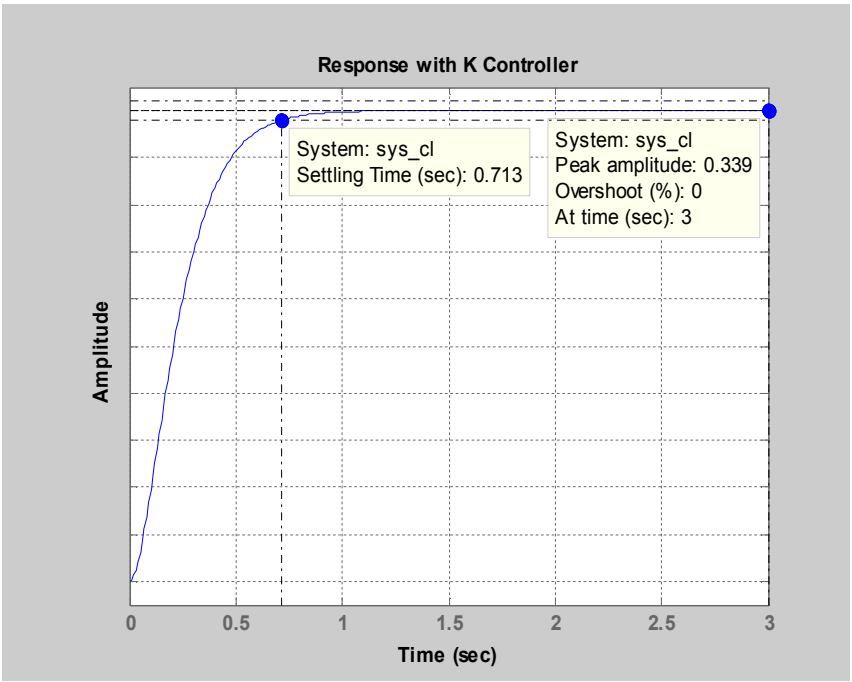


Figure 4.4 Poles at $-8 \pm 8i$

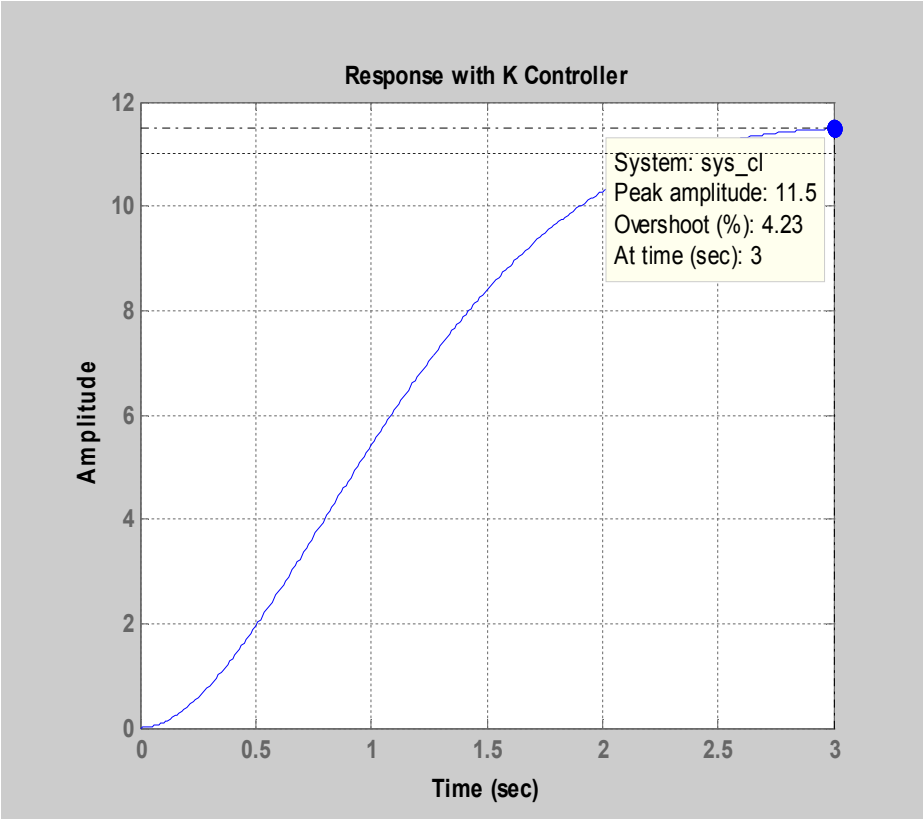


Figure 4.5 Poles at $-10 \pm 10i$

However, $-6 \pm 5i$ was chosen because it gave a better response of the system performance. This is shown in Figure 4.6. The feedback gain K at this location of the poles P1, P2 = ($-6 \pm 5i$) using the "place" command is:

K = [1.4564 45.3509]

The poles were verified by calculating the eigenvalues of (A-BK). For these values of the poles, it is assumed a full state feedback is available.

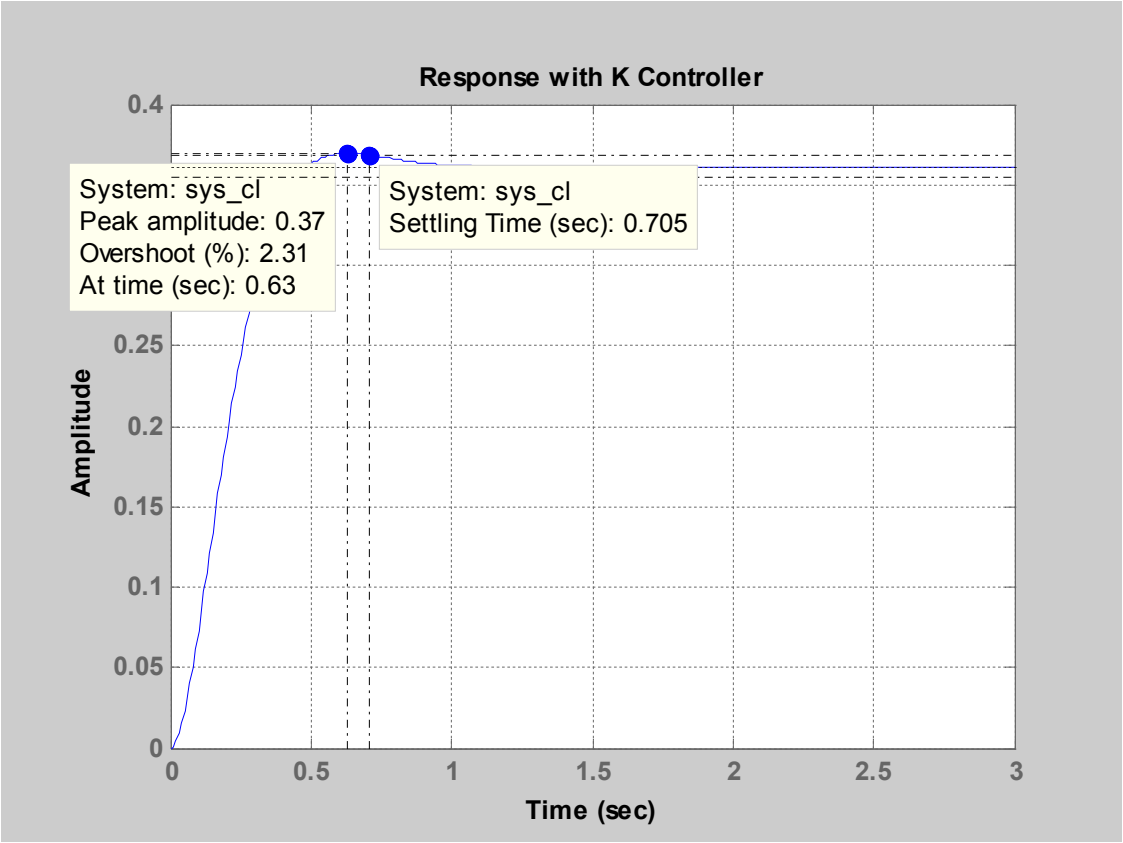


Figure 4.6 Response of the System to Controller Gain

It can be seen from Figure 4.6, that the overshoot M_p= ((0.369 −0.361)/ (0.361)) ×100 = 2.2% which is less than the design specification of 5% has been achieved. Also, the settling time t_s = 0.705 sec. which also is less than the design specification of 2 sec. has been met.

Achieving the first two performance specification has however caused a large steady state error into the system. This is shown in Figure 4.7.

In order to solve this problem, a reference input must be introduced into the system.

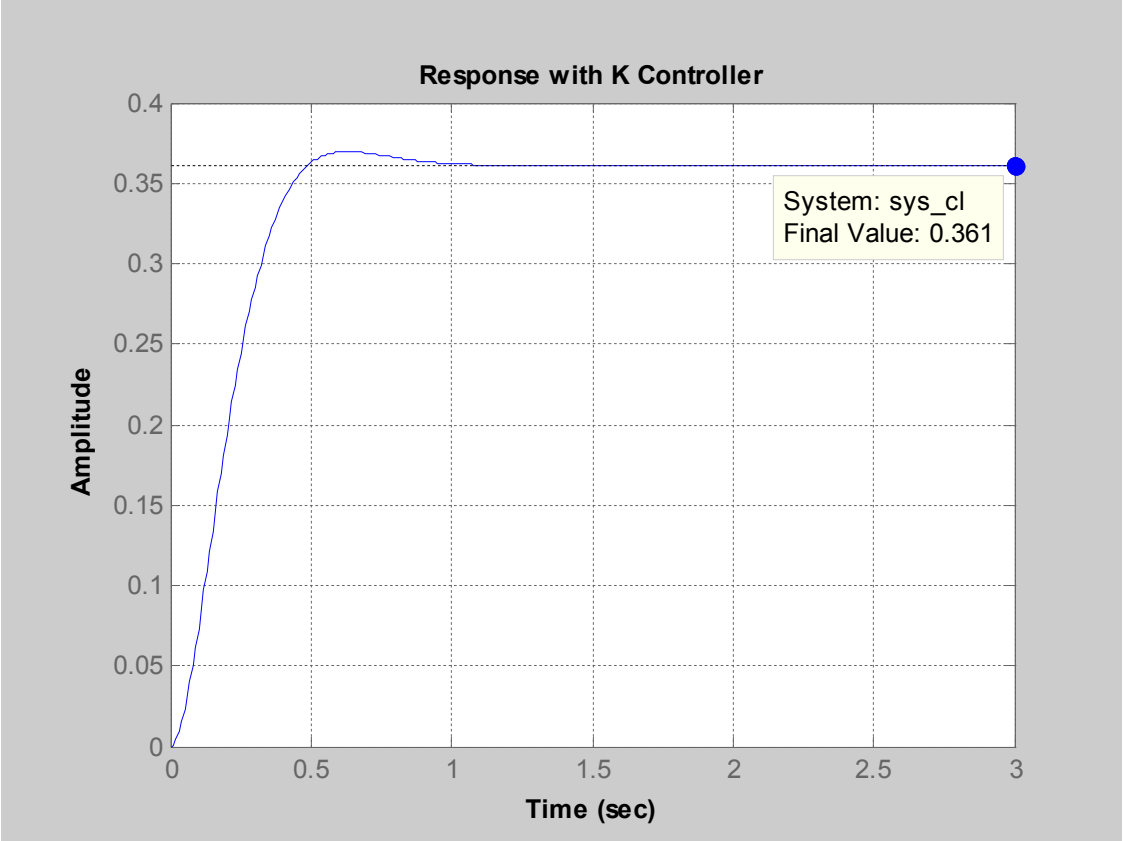


Figure 4.7 Output Response showing Steady State Error

From the diagram of Figure 4.7, the steady state error SSE is:

$$SSE = 1-0.361$$

$$= 0.639$$

$$= 63.9\%$$

4.2 INTRODUCING THE REFERENCE INPUT

In finding the state feedback matrix K of the system, the reference input was taken as zero. This caused the output signal to deviate from the input as seen in Figure 4.7. In order for the output signal to track the input signal, a reference input must be introduced into the system.

A block diagram showing the feedback gain K and the reference input is shown in Figure 4.8.

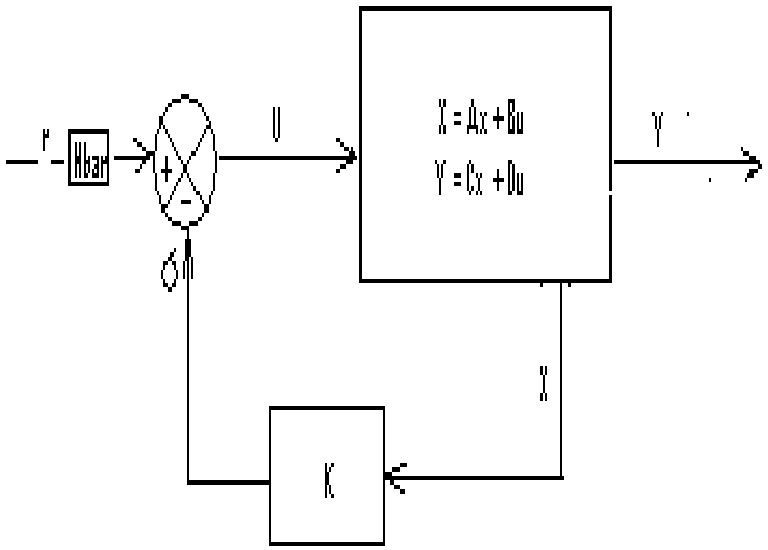


Figure 4.8 DC Motor with Reference Input

In determining the reference input, the steady states are used since the actual output is not fed back into the system and compared to a reference; only the states multiplied by the feedback gain K are fed back into the system. This is achieved by computing what the steady state value should be, multiply it by the chosen gain

and use the value obtained as a reference for computing the input. This can be done using a Matlab function called rscale.

Let the reference input Nbar be denoted as $\bar{N}$. The value of $\bar{N}$ is computed by writing the following Matlab command:

function $\bar{N}$ = rscale (A, B, C, D, K).

This function rscale finds the scale factor N which eliminates the steady state error to a step reference input for a continuous-time, single-input system with full-state feedback as in Figure 4.8.

The value of Nbar [$\bar{N}$] when run on the command window is:

$\bar{N}$ = 2.7680

The response of the system with the inclusion of $\bar{N}$ is shown in Figure 4.9.

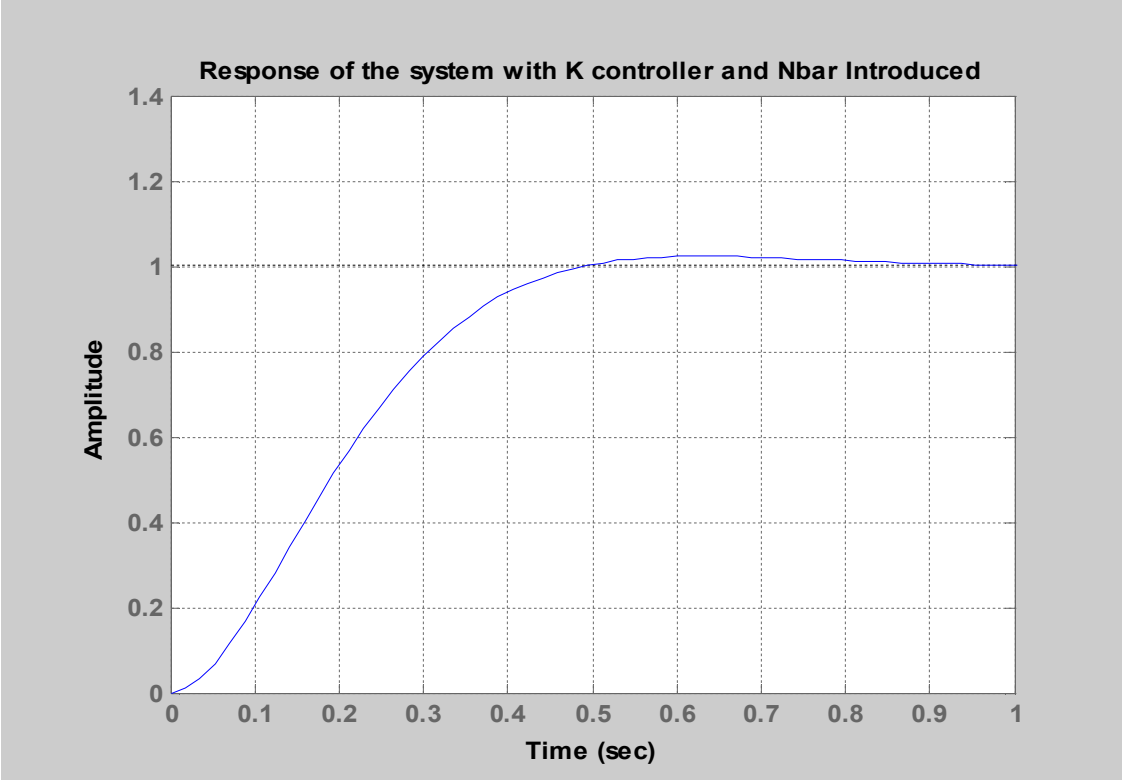


Figure 4.9 Responses of Controller K and Reference Input

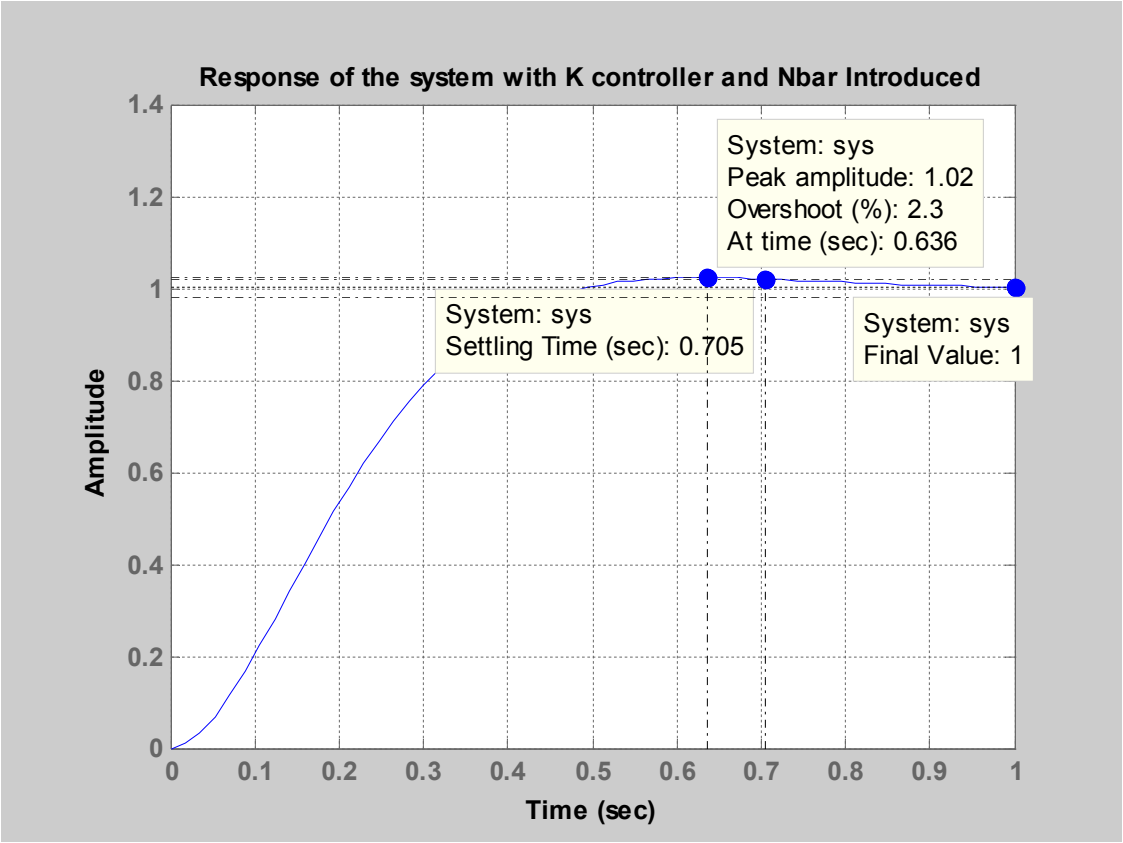


Figure 4.10 Design Specifications Achieved

In Figure 4.10, all the design specification (Peak Overshoot = 2.3% which is less than 5%, settling time of 0.705Second which also is less than the design specification of 2 Seconds and a steady state error of less than 1%) have been achieved.

4.3 OBSERVER DESIGN (STATE ESTIMATOR)

Controller design methods such as state feedback are designed with the assumption that all state variables are accessible for measurement. However, in many practical applications, it may not be economical or convenient to measure all the state

variables. An alternative approach is to use an estimation technique. From equation 3.14, since the output matrix of the state space model is not all ones, and hence does not allow for direct monitoring of the internal state of the system, an observer (state estimator) is needed to provide an estimate of the actual state for use in the state feedback.

An observer is merely a copy of the system that computes an estimate of the states and output, and compares the estimated output to the real output. Since the observer is only generating an estimate of the states $\hat{x}$, it is important that the error in the estimated state go to zero faster than the actual system dynamics need to converge to their steady state value [14].

A block diagram showing the schematic of a state feedback system with the full observer is shown in Figure 4.11.

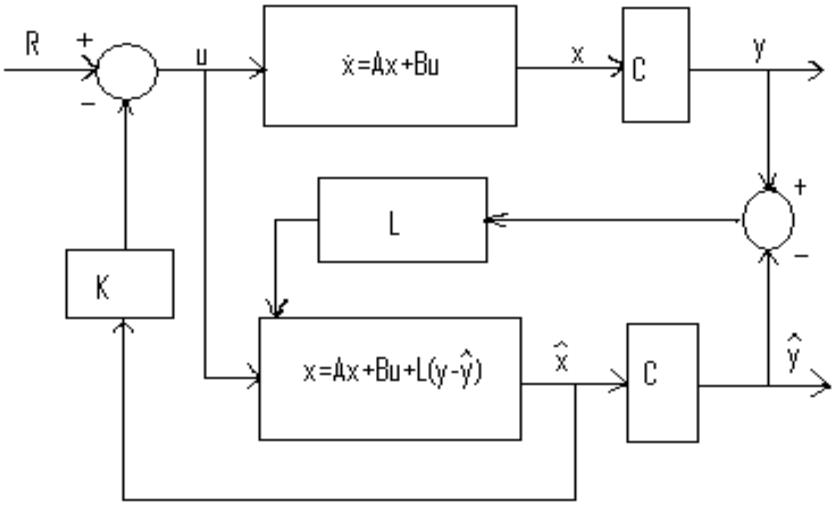


Figure 4.11 State Feedbacks with Observer

The observer model is represented thus:

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y-\hat{y}) \quad (4.6)$$

$$\hat{y} = C\hat{x} \quad (4.7)$$

Subtracting equation 4.6 from equation (1.1) and equation (4.7) from equation (1.2) gives equation (4.8) and (4.9) respectively.

$$(\dot{x} - \dot{\hat{x}}) = A(x - \hat{x}) - L(y - \hat{y}) \quad (4.8)$$

$$(y - \hat{y}) = C(x - \hat{x}) \quad (4.9)$$

Let the error signal be $\dot{e} = \dot{x} - \dot{\hat{x}}$ (4.10)

Rearranging equation 4.8 and representing $(\dot{x} - \dot{\hat{x}}) = \dot{e}$ as the error gives:

$$\dot{e} = (A - LC)e \quad (4.11)$$

$$(y - \hat{y}) = Ce \quad (4.12)$$

From equation 4.11, the matrix equation of the observer is given by the expression, A– LC. The observer poles can be arbitrarily assigned just as the poles of the

controller. This is achieved using the Matlab command "place" as used earlier in finding the controller gain. However, matrix B is replaced with matrix C and its transpose taken [10]. The observer must have "faster" poles (smaller magnitude) than the controller poles [3]. The observer poles must be placed at least two or five times farther to the left of the s-plane than the dominant poles of the system; this is to ensure that the observation error converges to zero as quickly as possible. Such faster decay of the observer error compared with the desired dynamics makes the controller poles to dominate the entire system. The observer pole was chosen to be:

$$op1 = -6 + 4.930i$$

$$op2 = -6 - 4.930i$$

$$L = \text{place } (A', C' \ [op1 \ op2]' \qquad (4.13)$$

The observer feedback gain L at this location of the pole using the Matlab function "place" is:

$$L = [10.0698 \ 0.3674] \qquad (4.14)$$

4.4 STATE FEEDBACK WITH ESTIMATED STATES

Having designed the state feedback controller and observer separately, the system is now lumped together using the estimated states from the observer for the state feedback controller by replacing u= r $-Kx$ with u = r$-K\hat{x}$ where r is the

reference input after scaling by Nbar. A block diagram of the controller-estimator configuration is shown in Figure 4.11.

For the state feedback controller based on the observed state $\hat{x}$ as shown in Figure 4.11,

$$u = -K\hat{x} \tag{4.15}$$

Subtracting equation 4.15 from equation 1.1 gives:

$$\dot{x} = Ax - BK\hat{x} = (A - BK)x + BK(x - \hat{x}) \tag{4.16}$$

The difference between the actual state $x(t)$ and the estimated state $\hat{x}$(t) has been defined earlier in equation (4.10) as the error $e(t)$. Substituting the error vector into equation 4.16 gives:

$$\dot{x} = (A - BK)x + BKe \tag{4.17}$$

Recall that the observer error equation was given by equation 4.11, and repeated here as:

$$\dot{e} = (A - LC)e \tag{4.18}$$

Combining equation 4.17 and 4.18, the controller-estimator system can be represented as:

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} r \qquad (4.19)$$

Equation 4.19 describes the controller-observer state feedback system.

Simulating equation 4.19, a plot showing the actual states, the estimated states and the state error is shown in Figure 4.12. The Matlab program used in plotting Figure 4.12 is shown in Appendix A.

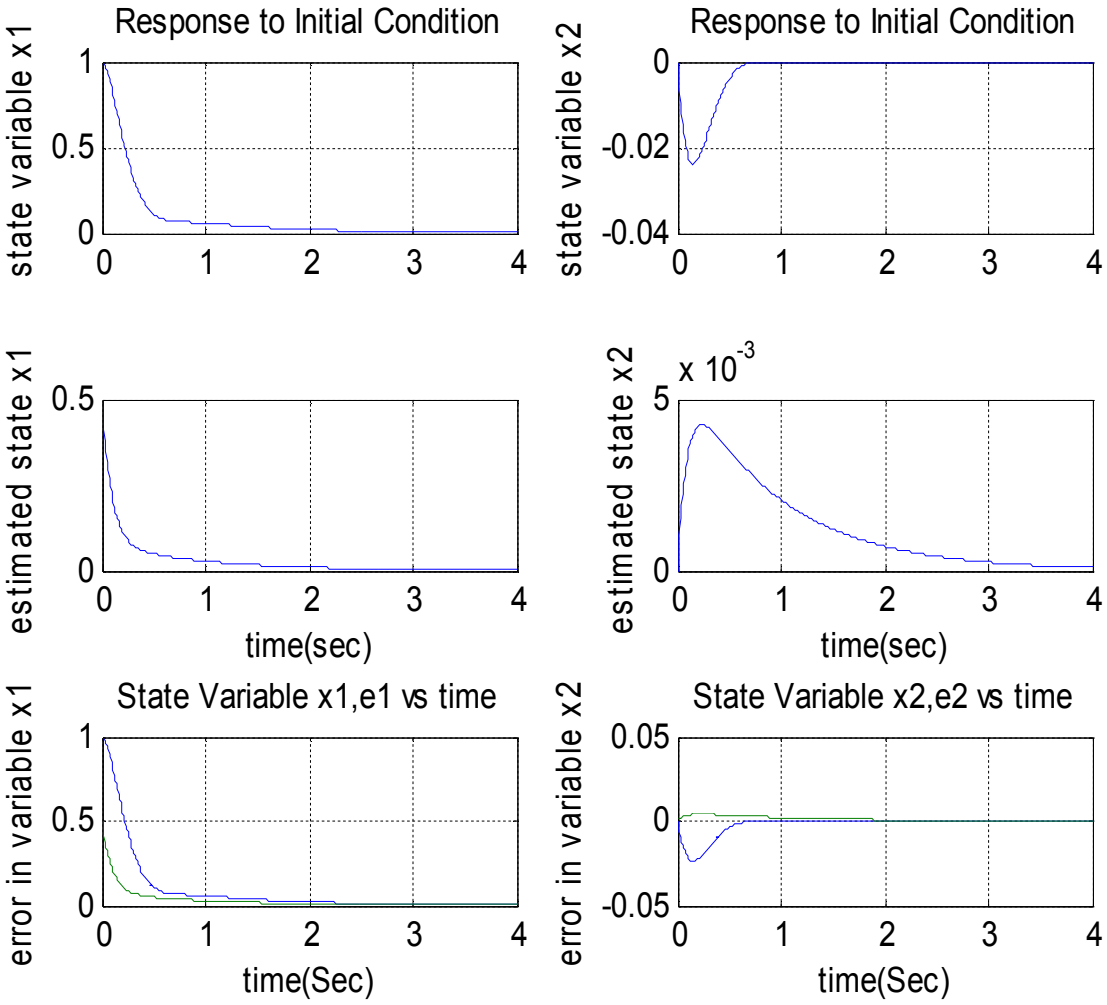


Figure 4.12 Actual State, Estimated States and State Error

At the beginning of the simulation in Figure 4.11, the state and state estimates are not identical but quickly converge and then goes to zero.

4.5 **RESULTS AND ANALYSIS**

In studying the performance of an armature controlled DC Motor, an experiment was carried out, Table 3.1, to determine the motor parameters. The parameters are as shown on page 23-24. Representing these parameters in state space form, a Step input was applied. The behaviour of the system is as shown in Figure 3.5. From the analysis of Figure 3.5, three control issues were raised.

- The DC motor has an overshoot of 39.3%,
- A settling time of 2.76 Seconds,
- A steady state error of 16.7%.

It was mentioned in Section 3.7a, b, and c respectively that:

The basic requirement of a dc motor is that:

- It should accelerate at the desired speed;
- that a speed faster than design speed can cause a damage to the system;
- That the motor should accelerate to its steady state speed quickly as soon as it is turned on.

On the basis of this, the following specifications were set to be achieved with the aid of a controller whose value will be computed from a Matlab function.

- ❖ An overshoot of less than or at most 5%.
- ❖ A settling time of less than or equal to 2 second
- ❖ A steady state error of less or equal to 1%

Designing a controller K for the DC Motor using equation 4.5, several poles locations:

$$\text{P1, P2} = [-1 + i, -1 - i],$$

$$\text{P1, P2} = [-4 + 4i, -4 - 4i],$$

$$\text{P1, P2} = [-6 + 5i, -6 - 5i],$$

$$\text{P1, P2} = [-8 + 8i, -8 - 8i],$$

$$\text{P1, P2} = [-10 + 10i, -10 - 10i]$$

were selected and inserted into: K = place (A, B, [P1, P2])

The graphs shown in Figures 4.2, 4.3, 4.4 and 4.5 respectively were obtained.

4.6 ANALYSIS OF GRAPHS

a) Figure 4.2: Poles at $-1 \pm i$

The system overshoot at this location is 53.3% which is worse than the actual system response as shown in Figure 3.2 and its still oscillating.

b) Figure 4.3: Poles at $-4 \pm 4i$

At this point, the system overshoot is 8.1% which is close to designed specification, a settling time of 0.934 second. The first condition is not achieved as 8.1% is more than 5%.

c) Figure 4.4: Poles at $-8 \pm 8i$.

At -8 ± 8, the system overshoot is 0% and has a settling time of 0.713second which is not realizable because all dynamic systems exhibit transients

d) Figure 4.5: Poles at $-10 \pm 10i$.

The system overshoot is 4.23% yet the system is unstable.

e) Figure 4.6: poles at $-6 \pm 5i$.

The system has an overshot of 2.3% and a settling time of 0.701Second.

From the analysis above, poles at $-6 \pm 5i$ meet the designed specification and was chosen because it gives a better response of the system performance. This is shown in Figure 4.9. The feedback gain K at this location using the "place" command is:

K = [1.4564 45.3509]

The poles were verified by calculating the eigenvalues of (A-BK). For these values of the poles, it was assumed a full state feedback is available.

Using poles located at -6 ± 5 to achieve the first two indices has however, introduced a steady state error of magnitude 63.9%. This is shown in Figure 4.7. To eliminate the error, a Matlab function rscale was used to compute the reference input that that will cause the output (speed) to follow the input. Its value is 2.7680. Figure 4.10 shows performance specification has been met.

In chapter four sections 4.3, an observer was designed for the DC Motor using Matlab command "place" as used in calculating the controller gain. The observer value was computed using poles -6 ± 4.930. Substituting this value into 4.13 gives:

$$L = [10.0698 \quad 0.3674].$$

From the discussions so far, the following deductions were made

i. A controller K [1.4564, 45.3509] has caused the peak overshoot to move from 39.3% to 2.3% and,
ii. A settling time from 2.76 second to 0.701 Second.
iii. A reference input Nbar= 2.7680 has caused the output signal to track the input signal.

iv. The introduction of the observer L [10.0698 0.3674] helps to estimate the states and report variation to the controller causing it to adjust the input thereby keeping the output in phase with the input.

In conclusion, it can be stated that the state feedback DC motor with state controller K as shown in Figure 4.9 has a better response and hence a better performance than the uncompensated system shown in Figure 3.2

CHAPTER FIVE

CONCLUSION

5.1 INTRODUCTION

This chapter highlights the major achievement of this work, suggestions for further work and the difficulties encountered in the cause of carrying out this research.

5.2 SIGNIFICANCE OF THE WORK

Performance of a dynamic system is an important aspect of control processes. This is so because, the performance of this devices decrease with usage over a period of time due to factors like: environmental condition, friction, power fluctuations to mention a few. In order for the quality of the process in which the dynamic system is used to control be maintained, the performance of the system must be checked. The state space approach that has been used in designing a controller for a DC Motor so far discussed will be a very good guide in evaluating the performance of these devices.

5.3 LIMITATION

The success of any experimental setup depends fundamentally on the data collected. The data obtained is a function of the accuracy of the equipment used in taking the readings. As such, the reliability of the equipment used in this

experiment such as: ammeter, tachometer, voltmeter, weight cannot be guaranteed. Hence this may affect the result obtained.

5.4 CONCLUSION

At the end of this research, an experimental data was used to create a mathematical model of a DC Motor. The controllability and observability of the system was confirmed using Kalman's method to verify whether a solution exist to the control problem. With the aid of a Matlab function, a step signal was applied to the derived mathematical model represented in state space to give information about the internal behaviour of the DC Motor. From the information revealed, a controller was designed for the DC Motor, causing the Motor to meet set performance specification. A reference input, Nbar, which causes the output to track the input signal, was also designed. An observer, whose function is to estimate the variation in the state variables (speed and current) and initiates a signal, causing the controller to adjust the input signal was also designed.

5.5 SUGGESTION FOR FURTHER WORK

I. Future studies could be carried out on the same topic by converting the continuous time approach used in the work into discrete time model and implemented using a digital processor.

II. A variable structure controller could be designed for the DC Motor and compared with the state feedback controller used in this project to see which gives a better performance.

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APPENDIX A

MATLAB PROGRAM USED TO TEST FOR

% CONTROLABILITY, OBSERVABILITY AND ALL

PLOTS OF THE DC MOTOR

```
Clear;

clc;

J = 0.0106; % motor inertia

f = 0.00286; % coefficient of viscuous friction

k1= 1.168; % back emf constant

k2= 1.168; % motor constant

R = 13.3; % resistance of the motor

L= 5.0; % inductance of the motor

A = [-f/J k1/J

     -k2/L -R/L]; % state matrix

B = [0
     1/L]; % input matrix

C= [1 0]; % output matrix

D = 0; % disturbance matrix

%.....TEST FOR CONTROLLABILITY OF THE DC MOTOR...%

Mc = [B A*B] %controllability matrix

if (det(Mc)~= 0) %find the determinant of controllable matrix

    'the system is controllable' %if it is not equal to zero

else

    'the system is not controllable'

end

%....TEST FOR THE OBSERVABILITY OF THE DC MOTOR...%
```

```
Mo = [C
       C*A] %observability matrix
if (det (Mo)~= 0)  %find the determinant of the observable matrix
    'the system is observable' %if the determinant is not equal to zero
else
    'the system is not observable'
end
sys = ss (A,B,C,D);
t = 0: 0.01:5;
step (sys, t);
x = eig (A)
y = det (Mc);
z = det (Mo);
sys = ss(A,B,C,D);
P1 =-1.4649 + 4.9307i;
P2 =-1.4649 - 4.9307i;
K = place (A, B, [P1 P2]);
t= 0:0.01:3;
sys_cl=ss (A-B*K, B, C, D);
step (sys_cl,t);
Nbar=rscale (sys_cl, K)
t=0:0.01:10;
step (sys_cl, t,sys*Nbar),title('step response with K Controller and Nbar')
%FINDING THE CONTROLLER MATRIX K
%that helps to meet the design specifications
P1, P2= [-6+5i, -6-5i]
```

```
K=place (A, B, [P1 P2]);%Finding the Feedback Gain Matrix K

%A Matlab program that helps to compute the

%observer Matrix L

OP1, OP2 = [-6 + 4.930i, -6 - 4.930i];

L1= place (A',C',[op1,op2])'

sys=ss([A-B*K B*K;zeros(2,2) A-L1*C],eye(4),eye(4),eye(4));

t=0:0.01:4;

%A program that plot the Actual State, the estimated
                    %state and the error
z=initial (sys, [1; 0; 0.5; 0], t);

x1=[1 0 0 0]*z';

x2= [0 1 0 0]*z';

e1= [0 0 1 0]*z';

e2= [0 0 0 1]*z';

subplot (2, 2,1);plot(t,x1),

grid on

title ('Response to Initial Condition')

ylabel ('state variable x1')

subplot (2,2,2);plot(t,x2),grid on

title ('Response to Initial Condition')

ylabel('state variable x2')

subplot (2, 2,3);plot(t,e1),grid

xlabel('time(sec)'),ylabel('error state variable e1')

subplot(2,2,4);plot(t,e2'),grid on

xlabel('time(sec)'),ylabel('error state variable e2')
```

THE RESULT OF THE PROGRAM AS RUN ON THE COMMANDWINDOW

```
A =

   -0.2698 110.1887

   -0.2336   -2.6600

B =

         0

    0.2000

C =

     1     0

D =

    0

Mc =

         0   22.0377

    0.2000   -0.5320

ans =
```

the system is controllable

Mo =

1.0000 0

-0.2698 110.1887

ans =

the system is observable

eig (A) =

-1.4649 + 4.9307i

-1.4649 - 4.9307

Nbar = 2.7680