A STUDY ON MIXED CONVECTION FLOW IN DIFFERENT CHANNELS

By

Sulaiman ABBAS

DEPARTMENT OF MATHEMATICS

FACULTY OF SCIENCE

AHMADU BELLO UNIVERSITY, ZARIA

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Sulaiman ABBAS

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DECLARATION

I declare that the work in this thesis titled A STUDY ON MIXED CONVECTION FLOW IN DIFFERENT CHANNELS has been carried out by me in the Department of Mathematics under the supervision of Professor B. K. Jha and Dr. A.O Ajibade. The information derived from the literature has been duly acknowledged in the text and the list of references provided. No part of this thesis was previously presented for another degree or diploma at any University.

_____ _____

Sulaiman ABBAS Date

CERTIFICATION

This thesis titledA STUDY ON MIXED CONVECTION FLOW IN DIFFERENT CHANNELSbySulaiman Abbas meets the requirements governing the award of the degree of Master of Science of the Ahmadu Bello University, Zaria and is approved for its contribution to knowledge and literary presentation.

_____ _____

Prof B.K Jha Date
Major Supervisor

_____ _____

Dr A.O. Ajibade Date
Minor Supervisor

_____ _____

Dr B. Sani Date
Head of Department

_____ _____

Prof. A. Z. Hassan Date
Dean of Postgraduate Studies

DEDICATION

I sincerely dedicated this research work to Almighty Allah for He is Beneficent and Merciful, my family, friends and colleagues for their priceless support.

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All praises are due to ALLAH (S.W.T) Who out of His mercy, this thesis is completed.

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ABSTRACT

This research work titledA STUDY ON MIXED CONVECTION FLOW IN DIFFERENT CHANNELS, is divided into three parts.The first part analytically simulates the mixed convection flow in the steady, laminar fully developed region of a parallel porous plate channel filled with porous material .The Darcy-Brinkman model and Boussinesq are employed. While the second part investigates analytical solutions for fully developed mixed convection flow in a vertical channel formed by two infinite vertical parallel plates partially filled with porous material.The third part studies the steady mixed convection flow in an annulus partially filled with porous material. In the second and third problems, the Brinkman-extended Darcy model is used to simulate momentum transfer in the porous region and the Navier-Stokes equation is used to simulate the momentum transfer in the clear fluid region. The clear fluid and porous regions are coupled by equating the velocity, temperature, heat flux and by considering shear stress jump condition at the interface.In the first part, the effect of suction/injection on mixed convection flow between vertical porous plates filled with porous material is investigated. The second part investigates the relative significance of the governing parameters on the velocity,interface velocity and temperature in the channel. While the third part studies the impact of the Darcy number and the adjustable coefficient in the stress-jump condition at the interface of fluid/porous layer on flow formation inside the annulus. Moreover,a dimensionless temperature θ_0 is introduced which is related to the usual $\frac{Gr}{\mathrm{Re}}$ parameter. The exact solutions for momentum and energy equations obtained are presented graphically and discussed.

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NOMENCLATURE AND GREEK LETTERS

Nomenclature

g = acceleration due to gravity

Gr = Grashof number

Pr = Prandtl number

S = suction/injection parameter

T' = dimensional temperature of the fluid

T_o = reference fluid temperature

u' = dimensional velocity of the fluid

u =dimensionless velocity of the fluid

V_0 = constant suction/injection velocity

y' =dimensional coordinate perpendicular to the plate

y= dimensionlesscoordinate perpendicular to the plate

d = porous layer thickness

Greek symbols

α = thermal diffusivity

β = thermal expansion coefficient

ΔT= reference temperature difference

θ = dimensionless temperature

θ_0 = reference temperature

$(\theta_0)_c$ = critical value of θ_0

μ = dynamic viscosity

ρ = mass density

ν = fluid kinematic viscosity

k = thermal conductivity

CHAPTER ONE

GENERAL INTRODUCTION

INTRODUCTION

Fluid flow occurs in a wide range of practical applications which include crude oil extraction, thermal insulation, chemical catalytic convertors, storage of grains, pollutant dispersion in aquifers, buried electrical cables, food industry, chemical transport simulations and many other biological, geophysical, engineering and environmental applications.

A porous medium is a material consisting of a solid matrix with an interconnected void. The interconnection of the void allows the flow of fluids through the material. The flow and the spread of fluid through random porous media such as soils, bed packing's, ceramic and concrete is an important topic of research because of its wide range of application in environmental and technological processes (Sahimi,1994). Darcy was the first to observe that under certain conditions the volume rate of water through a pipe packedwith sand was proportional to the negative of the pressure gradient. This relationship is known as Darcy's law. Darcyflow is an expression of the dominance of viscous forces applied by the solid porous matrix on the interstitial fluid and is oflimited applicability. At higher fluid velocities by increasing inertial forces, the ratio of pressure drop to velocity gradually deviates from Darcy's law. Many other results were presented on Darcy (1856) as compiled and presented by (Narasimhan ,2006). The flow through porous media capability enables engineers to simulate fluid flow through media such as ground rock, filters and catalyst beds. The simulation of underground flow through porous rock can enable engineers to predict the movement of contaminated fluid from a solid

waste landfill into a drinking water supply. Moreover, porous media can provide sites for chemical catalyst or absorption of components of the fluid. Additionally, the standard Darcy's law material model (which relates volumetric flow and pressure drop with properties of the fluid and media), the fractional power Darcy's law is also supported. This latter material model incorporates inertial effects for high Reynolds number applications.

Fluid flow in a composite channel partially filled with a porous medium and partially filled with a clear fluid, occurs in practical applications such as geophysical, biomedical, engineering and environmental applications(Kakac *et al.*;1991;, Kuznetsov; 1998;1996). If a solidifying alloy does not have a eutectic composition, the frozen part of the casting is separated from the liquid part by a mushy zone, which can be viewed as a porous medium with variable permeability (Kakac *et al*. ,1991). In addition, the use of porous substrates to improve forced convection heat transfer in chemicals, which is considered as a composite of fluid and porous layers, finds applications in heat exchangers, electronic cooling, heat pipes, filtration and chemical reactions etc. In these applications engineers avoid filling the entire channel with a solid matrix to reduce the pressure drop. The problem of fluid flow in the porous medium/clear fluid interface was first investigated by (Beavers Joseph ,1967). Comprehensive literature survey concerning this subject is amply documented in the monograph by (Nield and Bejam ,1992) and (Kaviany ,1991), as reported by (Kuznetsov ,1996).

1.1 **BASIC DEFINITIONS**

1. **Free or Natural Convection:** Is a type of heat transfer, in which the fluid motion is not generated by any external source but only by density differences in the fluid occurring due to temperature gradients.
2. **Forced Convection:** Is a heat transfer between a moving fluid and a solid surface, as a result of external source.
3. **Mixed Convection:** Is a type of heat transfer between a moving fluid and a solid surface, as a result of both natural and external source
4. **Suction/injection:** Is the increase/decrease of fluid flowing through the walls of the channel.
5. **Boussinessq approximation:** Is the assumption that the fluid flow is considered under little variations of temperature and density.
6. **Porous Media:** A porous medium is a material consisting of a solid matrix with an interconnected void. The interconnected of the void allows the flow of fluids through the material.
7. **Permeability:** Is the ability of a material to transmit fluid through it, which is the property of the porous media only and not the fluid.

1.2 DIMENSIONLESS QUANTITIES

A dimensionless quantity is a quantity without any physical units and thus a pure number. Such a number is typically defined as a product or ratio of quantities which do have units, in such a way that all the units cancel out.A quite number of these quantities were used in this thesis and brief notes of the quantities used are:

1. Grashof Number (Gr): Grashof Number is the ratio of the buoyancy force to the viscous force acting on the fluid:

$$\text{Gr} = \frac{buoyancy \;\; foeces}{viscous \;\; forces}$$

2. Reynolds number (Re): is the ratio of inertial forces to viscous forces.

1.3**Aims And Objectives of The Study**

The main aims and objectives of the work are:

1. Investigation of the effect of suction/injection on mixed convection flow between vertical porous plates filled with porous material
2. Investigation of the relative significance of the governing parameters on the interface velocity, the velocity and the temperature on mixed convection flow in a channel partially filled with porous fluid and partially filled with clear fluid.
3. Study of the effect of Darcy number and the stress jump condition at the interface of fluid/porous layer for the steady fully developed mixed convection flow in a vertical annulus partially filled with porous material.

1.4**Research Methodology**

The methodology employed in order to achieve the set objectives is that, firstly, existing literature was reviewed. Secondly, the parameters involved in the governing equations with their boundary conditions in the entire models will be non-dimensionalized. The non-dimensional models are solved analytically. Thirdly, the various boundary conditions were tested on the solutions obtained using MATLAB computer package to present the graphs of the solutions obtained. Finally, the graphs were interpreted so that the impact of the governing parameters can be discussed and draw conclusions.

1.5 Statement of The Problem

The study would seek to simulate analytically mixed convection flow in the steady, laminar, fully developed region of

a) parallel porous plate channel filled with porous material:
b) vertical parallel plates partially filled with porous material:
c) an annulus partially filled with porous material.

The Oberbeck-Boussinesq approximation would be employed to formulate the model and the reference temperature would be such that the pressure gradient vanishes (Rossi di Schio,2010); it involves the coupling of fluid flow and temperature flux. The one-dimensional Navier-Stokes equation with appropriate boundary conditions would describe the problem. Matlab codes would also be developed to obtain the behavior of the results.

1.6 Limitation of the Study

This study considered the analytical investigation of steady, laminar, fully developed flow of non-Darcian fluid in a porous parallel vertical plates and a vertical annulus with a dimensionless reference temperature such that the pressure gradient vanishes. The results obtained in this research may have been different if the walls of the channel are kept at a uniform temperature. A numerical approach can also be considered of the mixed convection in a porous parallel vertical plates and in an annulus partially filled with porous material.

1.7 Significance of the Study

The significance of this study are;

The solutions of natural convection obtained by Wiedman and Medina (Wiedman and Medina, 2008) can be generated using mixed convection with suction/injection in porous medium. Also, the choice of the reference temperature such that the pressure gradient vanishes can be applied in channel partially filled with porous medium.

1.8 Organization of The Thesis

This thesis is arranged and organized in five chapters, reference and appendix. Chapter one presents the general introduction, while chapter two gives the review of related literature. Chapter three presents the mathematical analysis and the non-dimensionalization of the three problems and their solutions. Chapter four presents the discussions of the results obtained in chapter three. Then chapter five presents the summary and conclusions of the research.

CHAPTER TWO

LITERATURE REVIEW

2.1INTRODUCTION

This chapter examines somerelated literature of previous research on the phenomena of fluid flow. Fluid flow has become of extensive research in recent years; the level of research in this field is due to increasing concerns in science and technology about buoyancy-induced motions in many engineering applications such as rocket nozzles, electronics cooling systems, energy storage units, chemical reactors, etc.

For engineers to avoid over heating or damage to components, a thorough investigation and knowledge of heat transfer is inevitable. This chapter therefore, explores some related literature of previous works.

2.2Mixed Convection

The study of mixed convection flow in vertical channels is motivated by its importance in wide range of engineering applications such as electronics cooling system, electrochemical processes, solar collectors etc. Ostrich (1954) solved the problem of fully developed mixed convection between vertical parallel plates with and without heat sources. Rokerya and Iqbal (1971) analyzed mixed convection in a vertical annular duct with uniform heat flux either on the internal surface or on the external surface, where the velocity and temperature profiles were obtained by the Runge-Kutta fourth-order method. An analytical study of fully developed mixed convection in vertical channel was conducted by (Aung and Worku ,1986) in which the boundary condition of uniform wall temperature was considered. Lavine (1988) presented an analytical solution for laminar mixed convection in a channel with a uniform wall heat flux. Cheng *et al*.(1990)

considered either uniform temperature or uniform heat flux at each boundary surface. Barletta (1998) has studied the fully developed mixed convection flow in a vertical channel with viscous dissipation.Hamadah (1991) presented an analysis of laminar fully developed mixed convection in a vertical channel with symmetric and asymmetric heating of the walls. Muralidhar (1990) studied mixed convection flow in a saturated porous annulus, while (Ramanaiah and Malarvizhi , 1990) presented a unified treatment of free and mixed convection on a permeable vertical cylinder in a saturated porous medium. An analytical solution for mixed convection in vertical channel with heat and mass transfer was discussed by (Boulama and Galanis ,2004). Murthy *et al*.(2004) studied the combined radiation and mixed convection from a vertical wall with suction and injection in a non-Darcy porous medium. Umavathi and Malashetty (2005) investigated magnetohydrodynamic mixed convection in a vertical channel. Grosan and Pop (2007) have investigated thermal radiation effect on fully developed mixed convection flow in vertical channel. Pop *et al*.(2009) have investigated the effect of heat generated by an exothermal reaction on the fully developed mixed convection flow in a vertical channel. Khaefinejad and Aghanajafi (2009) studied laminar flow with simultaneous effects of mixed convection and thermal radiation in a vertical channel. Rossi de Schio *et al*. (2011) analytically investigated the steady mixed convection flow in a vertical fluid saturated annulus using the Darcy-Brinkman equation model by introducing a dimensionless reference temperature which is related to the usual $\frac{Gr}{Re}$ parameter.

2.3Porous Media

Fluid flow in channels partially filled with porous medium and partially filled with clear fluid has increased in the last few decades because of its various applications which include ceramic processing, crude oil extraction, energy storage units, thermal insulation, solidification of

casting, geothermal energy utilization, petroleum reservoirs, storage of grain, buried electrical cables etc.The fundamental nature and the growing volume of work in this area is amply documented in the books by Nield and Bejan (2006), Ingham and Pop (2005), Vafai (2005), Pop and Ingham (2001), Bejan *et al*. (2004), de Lemos (2006) and Vadesz (2008).

The problem of fluid flow at the porous medium/clear fluid interface was first studied by (Beavers and Joseph,1967) in which the Darcy law was utilized to model the flow in the porous medium. The non-Darcian effects are accounted for by using the Brinkman-Forchheimer-extended Darcy equation for the flow in the porous medium by (Vafai and Thiyagaraja ,1987). The exact solution for the fully developed steady flow in the interface region was presented by (Vafai and Kim,1990). Ochoa-Tapia and Whitaker (1995a) studied exclusively the boundary conditions at the porous medium /clear fluid in the interface region. Ochoa-Tapia and Whitaker (1995b) also shows that a discontinuity in the stress is required to match the Brinkman-extended Darcy law to the Stokes' equation, but the continuity of the velocity must be retained. Kuznestov (1996) studied the impact of the jump condition on the interface velocity. Also, (Kuznestov ,1998) treated analytically fluid flow and heat transfer in couette flow through a composite system, partially filled with fluid-saturated porous medium and partially filled with clear fluid. Paul and Singh (2002) investigated the effects of interfacial stress jump condition on free convective flow between vertical walls partially filled with porous medium. Similarly, (Singh and Gorla,2008) analyzed and presented the fully developed heat transfer to a viscous incompressible fluid between two vertical parallel walls partially filled with a porous matrix and partially with a clear fluid invoking a vertical interface. The momentum transfer in the porous medium was described by the Brinkman-extended Darcy model, and the two regions were coupled by equating the velocity and sheer stress at the interface. Also, (Pantokratoras

,2007)studied the steady laminar flow in a fluid- saturated porous medium channel bounded by two parallel plates with constant but unequal temperatures. One plate was moving with constant velocity while the other was stationary using the Brinkman-Darcy-Forchheimer model. Recently, Jha *et al.*(2011) investigated the steady-state convective couette flow of fluid in a vertical parallel-plate channel partially filled with a porous medium and partially filled with clear fluid in the presence of suction/injection. Similarly, (Kaurangini and Jha ,2010) explained the numerical and analytical solutions for the flow formation inside horizontal parallel-plate channel partially filled with a uniform porous material in the presence of suction/injection for systems with plate movement along with a constant pressure gradient. In this study, the stress jump boundary condition suggested by (Ochoa-Tapia and Whitaker,1995) is utilized at the interface to match the Brinkman-extended Darcy equation to the Stokes' equation.

Moreover, fluid flow in annulus systems containing fluid/porous layer has also attracted a lot of experimental investigation. Minkowycz and Cheng (1976) investigated the effect of permeability of the porous medium when the flow is characterized by Darcy's law. Unsteady fluid flow through a circular naturally permeable tube surrounded by a porous material has been solved by (Verma and Chauhan ,1986). While (Joshi ,1987) presented the fully developed laminar natural convection flows in vertical annuli in which one of the boundary is isothermal. Poulikakos and Kazmierczak (1987) studied forced convection in a duct partially filled with a porous material. Mixed and free convection problems in vertical porous annulus have been studied by (Muralidhar,1989). Combined effect of free and mixed convection in a vertical cylinder embedded in a porous medium is presented by (Ramanaiah and Malarvizhi,1990). Natural convection between coaxial vertical cylinders partially filled with porous material was studied by (Paul and Singh,1998), while (Bua and Torrence,1981) analytically studied the onset of

convection in the annulus between vertical coaxial cylinders filled with porous medium. Havstad and Burns (1982) analyzed the convective heat transfer in a finite concentric vertical annulus filled with a porous medium using perturbation techniques as well as numerical methods for low Rayleigh numbers. However, analytical investigation of transient response of free convective flow in a concentric tube annulus filled with a porous medium is analyzed by (Al-Nimr and Darabsch ,1995), while for forced convection by (Kuznetsov ,1996). Also, a theoretical study of fully developed forced convection in a duct partially filled with a porous material has been studied by (Poulikakos and Kuzmierczak ,1987).

Buhler(2003), presented as family of solutions for natural convection of clear fluid in the conduction regime. Subsequently, (Wiedman and Medina,2008), further extended the work of (Buhler ,2003) by considering the flow in a fluid-saturated porous medium. The stable laminar two-dimensional flow in a porous medium confined between parallel vertical non-porous walls is considered in this text. For cavity of high l and gap width d he define the cavity aspect ratio $A = \frac{l}{d}$. $For\ A \gg 1$. It may be accurately assumed that the flow is rectilinear and fully-developed away from capped or open ends; near capped ends there must be a recirculation region, whilst near open ends there are developing boundary layers. Rossi di schio *et al.*(2011) analytically analyze the steady free convection flow in a vertical fluid saturated porous annulus using the Darcy-Brinkman equation model by introducing a dimensionless reference temperature θ_0 which is related to the usual Gr/Re parameter.

Suction or injection of fluid through the bounding surfaces as, for example, in mass transfer cooling, can significantly change the flow formation and, as a consequence, affect the rate of heat transfer from the bounding surfaces. In general, suction tends to increase the skin-friction

and heat transfer co-efficient whereas injection acts in the opposite manner. Injection or withdrawal of fluid through porous heated or cooled surface is of general interest in practical problems involving film cooling, control of boundary layers etc. This can be lead to enhanced heating (or cooling) of the system and can help to delay the transition from laminar flow(Jha and Ajibade,2010). The novel feature of the present work is to extend the work of (Wiedman and Medina,2008) by considering suction/injection on mixed convection flow in a vertical porous channel filled with porous material, and the introduction of a dimensionless reference temperature θ_0. We also present analytical solutions of mixed convection flow and the effect of stress jump condition at the interface between a porous medium and a clear fluid bounded by two vertical non-porous plates and in a vertical annulus.

CHAPTER THREE

MATHEMATICAL ANALYSIS

3.1 INTRODUCTION

In this chapter the derivation of the momentum and energy equations on the fluid flow formation were considered. And using the appropriate non-dimensional parameters, the non-dimensional equations were obtained from dimensional equations.

3.2Role of suction/injection on mixed convection flow in a vertical porous channel filled with porous material

Role of suction/injection on Mixed Convection flow in a vertical porous channel filled with porous material is studied as in figure 3.1. The distance between the walls is d. Choose a Cartesian coordinate system with $x-$ axis vertically upwards along the direction of the flow and $y-$ axis perpendicular to it. The physical properties of the fluid are characterized by fluid viscosity μ, thermal expansion coefficient β. The plates are heated asymmetrically with one plate maintained at a temperature T_1 while the other plate at a temperature T_2 where$T_1 < T_2$. Due to this temperature difference between the plates, flow occurs in the channel. In addition, the origin of the axes is such that the channel walls are at positions $y = -\frac{d}{2}$ and $y = \frac{d}{2}$. It is assumed that fluid is being injected in to the channel with certain constant velocity (v_0) on one plate $\left(y = \frac{d}{2}\right)$, and it is sucked off from the other plate $\left(y = -\frac{d}{2}\right)$ at the same rate.

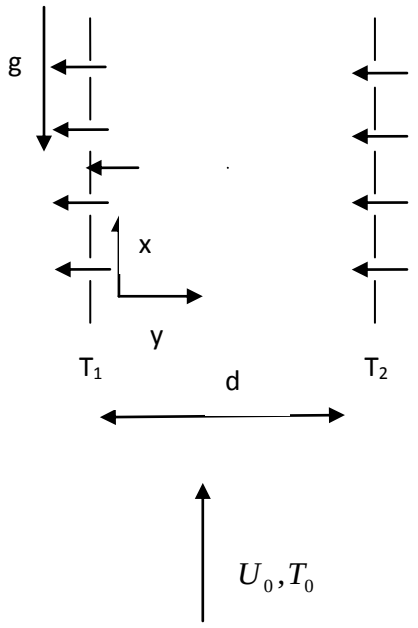


Figure 3.1, Flow Configuration and Coordinate System

The momentum and energy equations in dimensional form are:

$$-v_0 \frac{d\,u'}{d\,y'} = v_{eff}\,\frac{d^2u'}{dy'^2} - v\frac{u'}{k} + g\beta\,(T' - T_0) - \frac{1}{\rho}\frac{dp'}{dx'} = 0 \quad (3.1)$$

$$-v_0 \frac{dT'}{dy'} = \alpha\,\frac{d^2T'}{dy'^2} \quad (3.2)$$

$$y' = -\frac{d}{2}\,;\; u' = 0\,;T = T_1$$

$$y' = +\frac{d}{2}\,;\; u' = 0\,;T = T_2 \quad (3.3)$$

3.3 Fully developed mixed convection flow between vertical parallel plates partially filled with porous material

Consider fully-developed laminar mixed convection flow in a vertical channel formed by two infinite vertical parallel plates containing fluid and porous layers as in figure 3.2. The thermal boundary conditions on the vertical parallel plates are T_1 and T_2 and the velocity conditions are those of impermeability and no slip. Without loss of generality, the right plate is taken to be hot relative to the left plate so that $\Delta T = T_2 - T_1$.

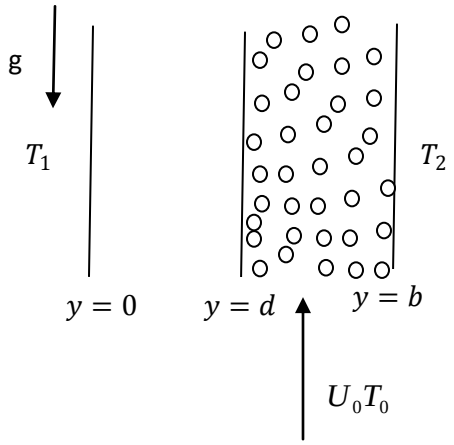


Figure 3.2: Flow Configuration

The mathematical model under the usual Boussinesq's approximation for the fluid region in dimensional form is as follows:

$$v\frac{d^2u_f^{'}}{dy^{'2}} + g\beta\left(T_f^{'} - T_0\right) - \frac{1}{\rho}\frac{dp^{'}}{dx^{'}} = 0 \quad (3.4)$$

$$\frac{d^2T_f^{'}}{dy^{'2}} = 0 \quad (3.5)$$

While for the porous region, the respective dimensional equations are

$$v_{eff}\,\frac{d^2u_p^{'}}{dy^{'2}} + g\beta\left(T_p^{'} - T_0\right) - \frac{v\,u_p^{'}}{k} - \frac{1}{\rho}\frac{dp^{'}}{dx^{'}} = 0 \quad (3.6)$$

$$\frac{d^2T_p^{'}}{dy^{'2}} = 0 \quad (3.7)$$

$$y^{'} = 0\,;\;\; T_f^{'} = T_1\,; u_f^{'} = 0$$

$$y^{'} = b;\;\; T_p^{'} = T_2\,;\; u_p^{'} = 0$$

$$at\; y = d^{'}\;;\;\; \begin{cases} T_f^{'} = T_p^{'}\,; & \frac{dT_f^{'}}{dy^{'}} = \frac{dT_p^{'}}{dy^{'}} \\ u_f^{'} = u_p^{'}\,; & v_{eff}\,\frac{du_p^{'}}{dy^{'}} - \frac{du_f^{'}}{dy^{'}} = \frac{\alpha v}{\sqrt{k}}u_p^{'} \end{cases} \quad (3.8)$$

3.4Steady fully developed non-Darcian mixed convection in an annulus partially filled with porous material and partially filled with clear fluid

The steady fully developed non-darcian mixed convective flow in anannulus are considered as in figure 3.3. One part of the channel was filled with a clear fluid while the other part was filled with fully saturated porous medium with uniform permeability. The flow formation is due to natural convection caused by the temperature difference between the vertical duct, as the wall of the clear fluid was being heated while that of the porous medium was being cooled. The interface is assumed to be permeable so that fluid can flow from one layer to the other. The direction of flow was taken along the vertical $x - axis$ while the $r^{'} - axis$ was taken along the radius of the cylinder in the horizontal direction.

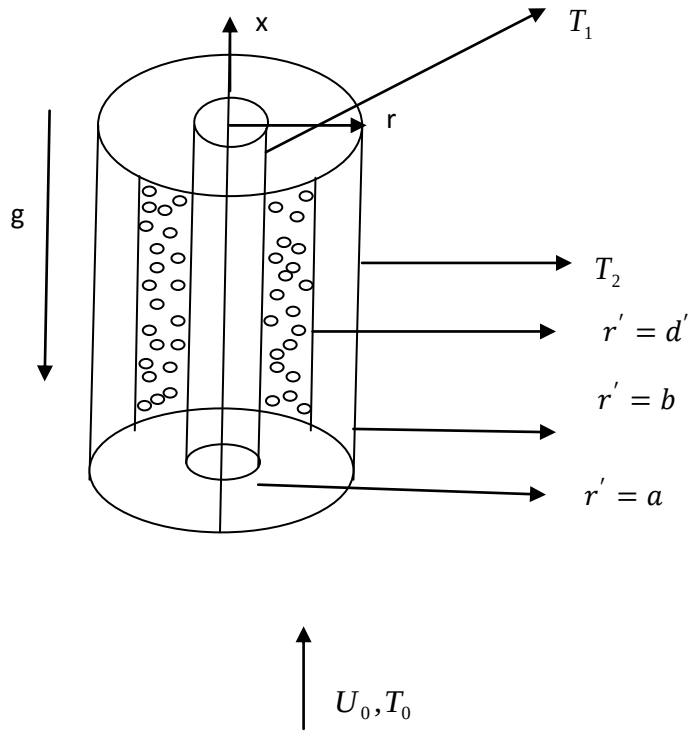


Figure 3.3 Schematic Diagram of the Annulus Problem

The mathematical model under the usual Boussinesq's approximation for the clear fluid region in dimensional form is as follows:

$$\nu \frac{d^2 u'_f}{dr'^2} + \frac{1}{r'}\frac{du'_f}{dr} + g\beta\left(T'_f - T_0\right) - \frac{1}{\rho}\frac{dp'}{dx'} = 0 (3.9)$$

$$\frac{d^2 T'_f}{dr'^2} + \frac{1}{r'}\frac{dT'_f}{dr'} = 0 (3.10)$$

While for the porous region, the respective dimensional equations are

$$\nu_{eff}\frac{d^2u'_p}{dr'^2}+\frac{1}{r'}\frac{du'_p}{dr'}+g\beta\left(T'_p-T_0\right)-\frac{1}{\rho}\frac{dp'}{dx'}=0(3.11)$$

$$\frac{d^2T'_p}{dr'^2}+\frac{1}{r'}\frac{dT'_p}{dr'}=0(3.12)$$

$$at\ r'=a \qquad u'_f=0 \ ; \ T'_f=T_1$$

$$at\ r'=b \qquad u'_0=0 \ ; \ T'_p=T_2$$

$$at\ r'=d'u'_f=u'_p\frac{dT'_f}{dr'}=\frac{dT'_p}{dr'}(3.13)$$

$$T'_f=T'_p\nu_{eff}\frac{du'_p}{dr'}-\frac{\nu du'_p}{dr'}=\frac{\alpha\nu u'_p}{\sqrt{k}}$$

3.5NON-DIMENSIONALISATION

By defining the following non dimensional parameters on equations (3.1) and (3.2)

$y=\frac{y'}{d}$and$\theta=\frac{(T'-T_0)}{\Delta T}$, where $S=\frac{v_0 d}{v}$, $R=\frac{v_{eff}}{v}$, $Da=\frac{k}{d^2}$, $u=\frac{u'v}{g\beta\,\Delta T d^2}$.

Then the equationsin dimensionless form are recast as follows:

$$R\frac{d^2u}{dy^2}+S\frac{du}{dy}-\frac{u}{Da}=-\theta(3.14)$$

$$\frac{d^2\theta}{dy^2}+SPr\frac{d\theta}{dy}=0(3.15)$$

The boundary conditions are respectively,

$$u\left(-\frac{1}{2}\right)=0, \qquad u\left(+\frac{1}{2}\right)=0;\ \ \theta\left(-\frac{1}{2}\right)=-\theta_0\ , \qquad \theta\left(+\frac{1}{2}\right)=1-\theta_0 (3.16)$$

Also, by defining the following non-dimensional parameters on the governing equations (3.4), (3.5), (3.6) and (3.7)

$$y=\frac{y^{'}}{b}\ and\ \theta_{f,p}=\frac{\left(T_f^{'};T_p^{'}-T_0\right)}{\Delta T}\ \ ,d=\frac{d^{'}}{b}\ , \qquad \Delta T=T_2-T_1\ ; \quad u_f;u_p=\frac{\left(u_f^{'};u_p^{'}\right)v}{[g\ \beta\ \Delta T\ d^2]}$$

Then the equations in dimensionless form are recast as follows:

$$\frac{d^2u_f}{dy^2}+\theta_f=0 (3.17)$$

$$\frac{d^2\theta_f}{dy^2}=0 (3.18)$$

And for the porous region ;

$$\gamma\frac{d^2u_p}{dy^2}+\theta_p-\frac{u_p}{Da}=0 (3.19)$$

$$\frac{d^2\theta_p}{dy^2}=0 (3.20)$$

The boundary and matching conditions in dimensionless forms are:

$$y=0\ ;\ \ \theta_f=-\theta_0\ ;u_f=0$$

$$y=1;\ \ \theta_p=1-\theta_0\ ;\ u_p=0$$

$$at\ y = d\ ;\ \begin{cases} \theta_f = \theta_p\ ; & \dfrac{d\theta_f}{dy} = \dfrac{d\theta_p}{dy} \\ u_f = u_p\ ; & \gamma\ \dfrac{du_p}{dy} - \dfrac{du_f}{dy} = \dfrac{\alpha}{\sqrt{Da}} u_p \end{cases} \quad (3.21)$$

Furthermore, by defining the following non-dimensional parameters on the governing equations $(3.9), (3.10), (3.11)\ and\ (3.12)$

$$r = \frac{r'}{a}\ , \lambda = \frac{b}{a}\ , d = \frac{d'}{a}\ , \Delta T = T_2 - T_1\ , \theta_f\ ; \theta_p = \frac{\left(T'_f\ ; T'_p - T_0\right)}{\Delta T}\ , \theta_0 = \frac{(T_0 - T_1)}{(T_2 - T_1)},$$

$$T_0 = \theta_0 \Delta T + T_1\ , \left(u_f\ ;\ u_p\right) = \frac{\left(u'_f\ ; u'_p\right)\nu}{[g\beta\Delta T a^2]}\ ;\ \gamma = \frac{\nu_{eff}}{\nu}$$

Then the equations in dimensionless form for the clear fluid region are recast as follows:

$$\frac{d^2 u_f}{dr^2} + \frac{1}{r}\frac{du_f}{dr} + \theta_f = 0 (3.22)$$

$$\frac{d^2 \theta_f}{dr^2} + \frac{1}{r}\frac{d\theta_f}{dr} = 0 (3.23)$$

While for the porous region we ;

$$-\frac{u_p}{Da} + \frac{d^2 u_p}{dr^2}\gamma + \frac{1}{r}\gamma\frac{du_p}{dr} + \theta_p = 0 (3.24)$$

$$\frac{d^2 \theta_p}{dr^2} + \frac{1}{r}\frac{d\theta_p}{dr} = 0 (3.25)$$

The boundary and matching conditions in dimensionless form are:

at $r = 1 \quad u_f = 0; \quad \theta_f = -\theta_0$

$$r = \lambda u_p = 0; \quad \theta_p = 1 - \theta_0$$

$$r = d \quad \begin{cases} u_f = u_p \ ; \ \dfrac{d\theta_f}{dr} = \dfrac{d\theta_p}{dr} \\ \theta_f = \theta_p \ ; \ \gamma \dfrac{du_p}{dr} - \dfrac{du_f}{dr} = \dfrac{\alpha}{\sqrt{Da}} u_p \end{cases} \quad (3.26)$$

3.6SOLUTION OF PROBLEMS

The dimensionless equations derived in thischapter are solved by the method of undetermined coefficient. In this also, the mass flux within the channel is determined.

3.7Solution of problem 3.1

The analytical solution of equations (3.14) subject to the boundary conditions (3.16) is

$$\theta(y) = c_1 + c_2 exp(-ySPr) (3.27)$$

Where:

$$c_1 = -\theta_0 + \frac{exp\left(\frac{SPr}{2}\right)}{2sinh\left(\frac{SPr}{2}\right)}$$

$$c_2 = -\frac{1}{2sinh\left(\frac{SPr}{2}\right)}$$

Substituting equation (4.1) in to equation (3.14) subject to the boundary conditions (3.16) will give

$$u(y) = [(\theta_0 f_9 + f_{10})cosh(\delta y) + (\theta_0 f_{11} + f_{12})sinh(\delta y)]exp\left(-\frac{Sy}{2R}\right) + C_1 Da - \frac{C_2}{F_1 R}exp(-SPry) \quad (3.28)$$

Where:

$$\delta = \sqrt{\left(\frac{S}{2R}\right)^2 + \left(\frac{1}{RDa}\right)}$$

$$F_1 = S^2 Pr^2 - \frac{S^2 Pr}{R} - \frac{1}{R\,Da}$$

$$F_2 = C_1 Da - \frac{C_2}{RF_1}exp\left(\frac{SPr}{2}\right)$$

$$F_3 = C_1 Da - \frac{C_2}{RF_1}exp\left(\frac{-SPr}{2}\right)$$

$$F_4 = \frac{exp\left(\frac{SPr}{2}\right)}{2sinh\left(\frac{SPr}{2}\right)}$$

$$F_5 = \frac{C_2}{RF_1}exp\left(\frac{SPr}{2}\right)$$

$$F_6 = \frac{C_2}{RF_1}exp\left(\frac{-SPr}{2}\right)$$

Thus:

$$F_2 = -\theta_0 Da + F_7 \; where \; F_7 = F_4 Da - F_5$$

$$F_3 = -\theta_0 Da + F_8 \;\; where \; F_8 = F_4 Da - F_6$$

$$F_9 = \frac{Da\ cosh\left(\frac{S}{4R}\right)}{cosh\left(\frac{\delta}{2}\right)}$$

$$F_{10} = -\frac{\left[F_8 exp\left(\frac{S}{4R}\right) + F_7 exp\left(\frac{-S}{4R}\right)\right]}{2cosh\left(\frac{\delta}{2}\right)}$$

$$F_{11} = \frac{Da\ sinh\left(\frac{S}{4R}\right)}{sinh\left(\frac{\delta}{2}\right)}$$

$$F_{12} = \frac{\left[F_7 exp\left(\frac{-S}{4R}\right) - F_8 exp\left(\frac{S}{4R}\right)\right]}{2sinh\left(\frac{\delta}{2}\right)}$$

Using equation (3.28), the critical value of $\theta_0$is

$$\frac{du(y)}{dy} = 0$$

$$\frac{du(y)}{dy} = C_3\left[\delta\ sinh(\delta y) - \frac{S}{2R}cosh(\delta y)\right] exp\left(-\frac{Sy}{2R}\right)$$

$$+ C_4\left[\delta\ cosh(\delta y) - \frac{S}{2R}\ sinh(\delta y)\right] exp\left(-\frac{Sy}{2R}\right)$$

at $y = -\frac{1}{2}\ and\ y = +\frac{1}{2}$ are respectively:

$$0 = -C_3\left[\delta\ sinh\left(\frac{\delta}{2}\right) + \frac{S}{2R}\ cosh\left(\frac{\delta}{2}\right)\right] exp\left(\frac{S}{4R}\right) + C_4\left[\delta\ cosh\left(\frac{\delta}{2}\right) + \frac{S}{2R}\ sinh\left(\frac{\delta}{2}\right)\right] exp\left(\frac{S}{4R}\right)$$

$$0 = -F_{13}[\theta_0 F_9 + F_{10}] + F_{14}[\theta_0 F_{11} + F_{12}]$$

Therefore at $= -\frac{1}{2}$;

$$\theta_0 = \frac{F_{10}F_{13} - F_{14}F_{12}}{F_{14}F_{11} - F_{13}F_9}\ (3.29)$$

And at $= +\frac{1}{2}$;

$$\theta_0 = -\frac{F_{15}F_{10} + F_{12}F_{16}}{F_9F_{15} + F_{11}F_{16}}\ (3.30)$$

Where:

$$F_{13} = \left[\delta\ sinh\left(\frac{\delta}{2}\right) + \frac{S}{2R}cosh\left(\frac{\delta}{2}\right)\right] exp\left(\frac{S}{4R}\right)$$

$$F_{14} = \left[\delta\ cosh\left(\frac{\delta}{2}\right) + \frac{S}{2R}sinh\left(\frac{\delta}{2}\right)\right] exp\left(\frac{S}{4R}\right)$$

$$F_{15} = \left[\delta\ sinh\left(\frac{\delta}{2}\right) - \frac{S}{2R}cosh\left(\frac{\delta}{2}\right)\right] exp\left(\frac{-S}{4R}\right)$$

$$F_{16} = \left[\delta\ cosh\left(\frac{\delta}{2}\right) - \frac{S}{2R}sinh\left(\frac{\delta}{2}\right)\right] exp\left(\frac{-S}{4R}\right)$$

Mass flux

The mass flux Q as defined in the nomenclature is given by;

$$Q = \int_{-\frac{1}{2}}^{+\frac{1}{2}} u(y)dy$$

$$= \int_{-\frac{1}{2}}^{+\frac{1}{2}} \left[cosh(\delta y) exp\left(-\frac{Sy}{2R}\right)\right] dy$$

$$= \frac{1}{2}\int_{-\frac{1}{2}}^{+\frac{1}{2}} \left[[exp(\delta y) + exp(-\delta y)] exp\left(-\frac{Sy}{2R}\right)\right] dy$$

$$= \frac{1}{2}\left[\frac{exp\left(y\left(\delta - \frac{S}{2R}\right)\right)}{\left(\delta - \frac{S}{2R}\right)} - \frac{exp\left(-y\left(\delta + \frac{S}{2R}\right)\right)}{\left(\delta + \frac{S}{2R}\right)}\right]_{-\frac{1}{2}}^{+\frac{1}{2}}$$

$$= \left[\frac{\sinh\left(\frac{1}{2}\left(\delta - \frac{S}{2R}\right)\right)}{\left(\delta - \frac{S}{2R}\right)} + \frac{\sinh\left(\frac{1}{2}\left(\delta + \frac{S}{2R}\right)\right)}{\left(\delta + \frac{S}{2R}\right)}\right]$$

And;

$$= \int_{-\frac{1}{2}}^{+\frac{1}{2}} \left[[exp(\delta y) - exp(-\delta y)] exp\left(-\frac{Sy}{2R}\right)\right] dy$$

$$= \frac{1}{2}\left[\frac{exp\left(y\left(\delta - \frac{S}{2R}\right)\right)}{\left(\delta - \frac{S}{2R}\right)} + \frac{exp\left(-y\left(\delta + \frac{S}{2R}\right)\right)}{\left(\delta + \frac{S}{2R}\right)}\right]_{-\frac{1}{2}}^{+\frac{1}{2}}$$

$$= \left[\frac{\sinh\left(\frac{1}{2}\left(\delta - \frac{S}{2R}\right)\right)}{\left(\delta - \frac{S}{2R}\right)} - \frac{\sinh\left(\frac{1}{2}\left(\delta + \frac{S}{2R}\right)\right)}{\left(\delta + \frac{S}{2R}\right)}\right]$$

Therefore,

$$Q = \int_{-\frac{1}{2}}^{+\frac{1}{2}} u(y)dy = C_3F_{17} + C_4F_{18} + C_1Da + \frac{C_2}{F_1R}\frac{\left[exp\left(-\frac{SPr}{2}\right) - exp\left(\frac{SPr}{2}\right)\right]}{SPr}$$

$$= \theta_0[F_9F_{17} + F_{11}F_{18} + Da] + [F_{10}F_{17} + F_{12}F_{18} + F_4Da - F_{19}](3.31)$$

Where,

$F_{17} = \left[\frac{\sinh\left(\frac{1}{2}\left(\delta - \frac{S}{2R}\right)\right)}{\left(\delta - \frac{S}{2R}\right)} + \frac{\sinh\left(\frac{1}{2}\left(\delta + \frac{S}{2R}\right)\right)}{\left(\delta + \frac{S}{2R}\right)}\right]$ and

$$F_{18} = \left[\frac{\sinh\left(\frac{1}{2}\left(\delta - \frac{S}{2R}\right)\right)}{\left(\delta - \frac{S}{2R}\right)} - \frac{\sinh\left(\frac{1}{2}\left(\delta + \frac{S}{2R}\right)\right)}{\left(\delta + \frac{S}{2R}\right)}\right]$$

3.8Solution of problem 3.2

The solution of equations $(3.18)\ and\ (3.20)$ subject to the boundary condition (3.21) are:

$$\theta_f = y - \theta_0(3.32)$$

$$\theta_p = y - \theta_0(3.33)$$

By substituting equations $(4.6)\ and\ (4.7)$ into equations$(3.17)\ and\ (3.19)$respectively, and solving them using the boundary condition (3.21) gives

$$u_f = \left[\theta_0\frac{y^2}{2} - \frac{y^3}{6} + K_1\,y + K_2\right](3.34)$$

$$u_p = C_5\,cosh(Gy) + C_6\,sinh(Gy) - \theta_0Da + yDa(3.35)$$

Where;

$$C_5 = \frac{[u_i\,sinh(G) + \theta_0A_4 - DaA_5]}{-A_1}$$

$$C_6 = \frac{[u_i\ cosh(G) + \theta_0 A_2 - DaA_3]}{A_1}$$

$$A_1 = sinh(Gd)\ cosh(G) - \ sinh(G)cosh(Gd)$$

$$A_2 = Da\ cosh(G) - Dacosh(Gd)$$

$$A_3 = d\ cosh(G) - cosh(Gd)$$

$$A_4 = Da\ sinh(G) - Da\ sinh(Gd)$$

$$A_5 = d\ sinh(G) - sinh(Gd)$$

Now, for the interfacial velocity, equations (3.12) and (3.15) are substituted in the last part of equation (3.17) and solving at y = d gives

$$\gamma \frac{du_p}{dy} - \frac{du_f}{dy} = \frac{\alpha}{\sqrt{Da}} u_p, \qquad at\ y = d$$

Then ,

$$[\gamma G\{C_5\ sinh(Gd) + \ C_6\ cosh(Gd)\} + \gamma Da] - \theta_0 d + \frac{d^2}{2} - K_1$$

$$= \frac{\alpha}{\sqrt{Da}} [C_5\ cosh(Gd) + C_6 sinh(Gd) - \theta_0 Da + dDa]$$

Thus

$u_i = \frac{[\theta_0 A_{12} + A_{13}]}{A_{11}}$ (3.36)

Where;

$$A_6 = \gamma\ G\ sinh(Gd) - \frac{\alpha}{\sqrt{Da}}\ cosh(Gd)$$

$$A_7 = \gamma\ G\ cosh(Gd) - \frac{\alpha}{\sqrt{Da}}\ sinh(Gd)$$

$$A_8 = -\gamma\ Da - \frac{d^2}{2} + \alpha d\sqrt{Da} + \frac{d^2}{6}$$

$$A_9 = -\frac{d}{2} + d - \alpha\sqrt{Da}$$

$$A_{10} = -\frac{[DaA_5A_6 - DaA_3A_7]}{A_1}$$

$$A_{11} = \left[\frac{cosh(G)A_7 - sinh(G)A_6}{A_1} - \frac{1}{d}\right]$$

$$A_{12} = A_9 - \frac{[A_2A_7 - A_4A_6]}{A_1}$$

$$A_{13} = A_8 + A_{10}$$

CRITICAL VALUE OF $\boldsymbol{\theta_0}$

The critical value of θ_0 is given by $\frac{du}{dy}\Big|_{y=0}$; which gives $\theta_0 = \frac{[d^3A_{11}+6A_{13}]}{3[d^2A_{11}-2A_{12}]}$.

3.9Solution of problem 3.3

The solution of equations (3.23) and (3.25) subject to the boundary condition (3.26) is

$$\theta_f = \theta_p = \frac{ln(r)}{ln(\lambda)} - \theta_0 (3.37)$$

By substituting equation (3.37) into equations (3.22) and (3.24) and solving them using the boundary condition (3.26) gives

$$u_p = C_3 I_0\left(\frac{\alpha}{\sqrt{\gamma Da}}\right) + C_4 k_0\left(\frac{\alpha}{\sqrt{\gamma Da}}\right) + Da\frac{ln(r)}{ln(\lambda)} - \theta_0\, Da (3.38)$$

$$u_f = \frac{r^2\theta_0}{4} - \frac{1}{ln(\lambda)}\left[\frac{r^2}{4}ln(r) - \frac{r^2}{4}\right] + C_1\ ln(r) + \ C_2 (3.39)$$

where $u = \ u_p + \ u_f$

The critical value of θ_0 of the clear fluid is given by $\left.\frac{du}{dr}\right|_{r=1} = 0$; which gives

$$\theta_0 = -\frac{\left[\frac{F_1}{ln(d)} + \frac{F_{13}}{F_{11}ln(d)} + \frac{ln(\lambda)}{4}\right]}{0.5\ \left[1 - \frac{(d^2-1)}{2ln(d)} + \frac{2\ F_{12}}{F_{11}ln(d)}\right]} (3.40)$$

While for the porous region is given by $\left.\frac{du}{dr}\right|_{r=\lambda} = 0$; which gives

$$\theta_0 = \frac{[(F_{17} - F_{16})F_{11} - \ (F_{13}F_{15})]}{[F_{12}F_{15} - \ F_{14}F_{11}]} (3.41)$$

The mass flux (Q) is given by;

$$Q = \int_1^{\lambda} r\ u(r)\ dr$$

$$Q = \int_1^{d} r\ u_f(r)\ dr + \int_d^{\lambda} r\ u_p(r)\ dr$$

$$\int_1^{d} r\ u_f(r)\ dr = \ X_{26}$$

And $\int_d^{\lambda} r\ u_p(r)\ dr = \ X_{27}$

Therefore, $Q = X_{26} + X_{27} (3.42)$

Where,

$$X_1 = \frac{[ln(d) - 1]}{ln(d)}\frac{d^2}{4} + \frac{1}{4\,ln(d)}X_2 = K_0\left(\frac{\lambda}{\sqrt{\gamma Da}}\right)I_0\left(\frac{d}{\sqrt{\gamma Da}}\right) - K_0\left(\frac{d}{\sqrt{\gamma Da}}\right)I_o\left(\frac{\lambda}{\sqrt{\gamma Da}}\right)$$

$$X_3 = \frac{Da\left[I_0\left(\frac{d}{\sqrt{\gamma Da}}\right) - I_0\left(\frac{\lambda}{\sqrt{\gamma Da}}\right)\right]}{X_2}X_{3a} = \frac{\left[I_0\left(\frac{d}{\sqrt{\gamma Da}}\right) - \frac{ln(d)}{ln(\lambda)}I_0\left(\frac{\lambda}{\sqrt{\gamma Da}}\right)\right]}{X_2}$$

$$X_4 = \frac{I_0\left(\frac{\lambda}{\sqrt{\gamma Da}}\right)}{X_2}X_5 = \frac{Da\left[K_0\left(\frac{d}{\sqrt{\gamma Da}}\right) - K_0\left(\frac{\lambda}{\sqrt{\gamma Da}}\right)\right]}{X_2}$$

$$X_{5a} = \frac{Da\left[K_0\left(\frac{d}{\sqrt{\gamma Da}}\right) - \frac{ln(d)}{ln(\lambda)}K_0\left(\frac{\lambda}{\sqrt{\gamma Da}}\right)\right]}{X_2}X_6 = \frac{K_0\left(\frac{\lambda}{\sqrt{\gamma Da}}\right)}{X_2}$$

$$X_7 = \sqrt{\frac{\gamma}{Da}}I_0\left(\frac{d}{\sqrt{\gamma Da}}\right) - \frac{\alpha}{\sqrt{Da}}I_0\left(\frac{d}{\sqrt{\gamma Da}}\right)X_8 = \sqrt{\frac{\gamma}{Da}}K_1\left(\frac{d}{\sqrt{\gamma Da}}\right) + \frac{\alpha}{\sqrt{Da}}K_0\left(\frac{d}{\sqrt{\gamma Da}}\right)$$

$$X_9 = \frac{\alpha\sqrt{Da}\,ln(d)}{ln(\lambda)} - \frac{\gamma\,Da}{d\,ln(\lambda)} - \frac{1}{ln(\lambda)}\left\{\frac{dln(d)}{2} - \frac{d}{4}\right\} + \frac{X_1}{dln(d)}$$

$$X_{10} = -\alpha\sqrt{Da} + \frac{d}{2} - \frac{1}{ln(d)}\left\{\frac{d^2}{4} - \frac{1}{4}\right\}X_{11} = \left[X_6X_7 + X_4X_8 - \frac{1}{dln(d)}\right]$$

$$X_{12} = [X_{10} + X_5X_7 + X_3X_8]X_{13} = [X_9 - X_{5a}X_7 - X_{3a}X_8]$$

$$X_{14} = X_5I_1\left(\frac{\lambda}{\sqrt{\gamma Da}}\right) + X_3K_1\left(\frac{\lambda}{\sqrt{\gamma Da}}\right)X_{15} = X_6I_1\left(\frac{\lambda}{\sqrt{\gamma Da}}\right) + X_4K_1\left(\frac{\lambda}{\sqrt{\gamma Da}}\right)$$

$$X_{16} = X_{5a}I_1\left(\frac{\lambda}{\sqrt{\gamma Da}}\right) + X_{3a}K_1\left(\frac{\lambda}{\sqrt{\gamma Da}}\right)X_{17} = \frac{-Da\sqrt{\gamma Da}}{\lambda ln(\lambda)}$$

$$X_{18} = \frac{\theta_0}{16}(d^2-1) X_{19} = \frac{C_1}{4}[d^2(2ln(d)-1)+1]$$

$$X_{20} = \frac{1}{4ln(\lambda)}\left[d^4\frac{ln(d)}{4} - \frac{5}{16}(d^4-1)\right] X_{21} = \frac{C_2}{2}(d^2-1)$$

$$X_{22} = C_3\left[\lambda I_1\left(\frac{\lambda}{\sqrt{\gamma Da}}\right) - dI_1\left(\frac{d}{\sqrt{\gamma Da}}\right)\right]\sqrt{\gamma Da}$$

$$X_{23} = C_4\left[\lambda K_1\left(\frac{\lambda}{\sqrt{\gamma Da}}\right) - dK_1\left(\frac{d}{\sqrt{\gamma Da}}\right)\right]\sqrt{\gamma Da}$$

$$X_{24} = \frac{Da}{ln(\lambda)}\left[\frac{\left(\lambda^2 ln(\lambda) - d^2 ln(d)\right)}{2} - \frac{1}{4}(\lambda^2 - d^2)\right]$$

$$X_{25} = \frac{\theta_0\, Da}{2}(\lambda^2 - d^2)$$

$$X_{26} = X_{18} + X_{19} - X_{20} + X_{21} X_{27} = X_{22} - X_{23} + X_{24} - X_{25}$$

$$C_1 = \frac{[u_i + F_1]}{ln(d)} - \frac{\left[\theta_0\left(\frac{d^2}{4} - \frac{1}{4}\right)\right]}{ln(d)} C_2 = -\left[\frac{\theta_0}{4} + \frac{1}{4ln(\lambda)}\right]$$

$$C_3 = -\theta_0 X_5 + X_6 u_i + X_{5a} C_4 = -u_i X_4 + \theta_0 X_3 - X_{3a}$$

$$u_i = \frac{(X_{12}\theta_0 + X_{13})}{X_{11}}$$

CHAPTER FOUR

DISCUSSION OF THE RESULTS

4.1INTRODUCTION

This chapter discusses the results obtained in chapter four which are presented graphically.

4.2Discussing the results of problem 3.1

In order to see the effect of the suction/injection(S), Prandtl number(Pr) and Darcy number(Da) on the velocity $\big(u(y)\big)$ and temperature$\big(\theta(y)\big)$, graphs are plotted for various values of the governing parameters.

Figures4.1a and 4.2b present the influence of suction/injection on the temperature, an increase in suction/injection parameter(S) leads to an increase in temperature for both $Pr = 7.0$ (water) and$Pr = 0.71$ (air). It is, however, observed that injection through the heated plate act in support of heat flux to boost temperature in the channel. On the other hand, suction through the heated plate lead to temperature difference within the channel.

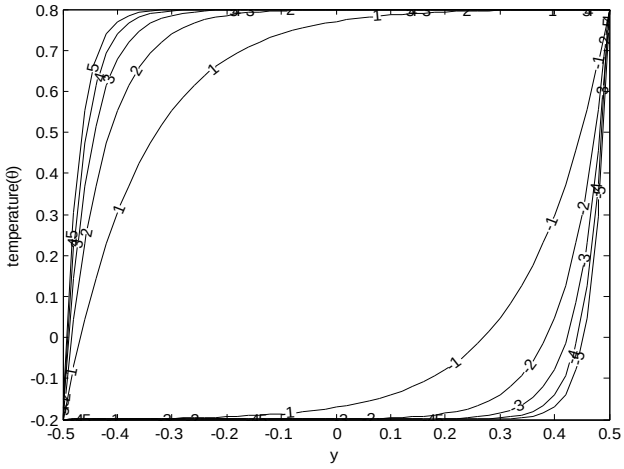


Figure4.1a. Temperature Profile $\theta(y)$ for Different S: $(Pr = 7.0, \theta_0 = 0.2)$

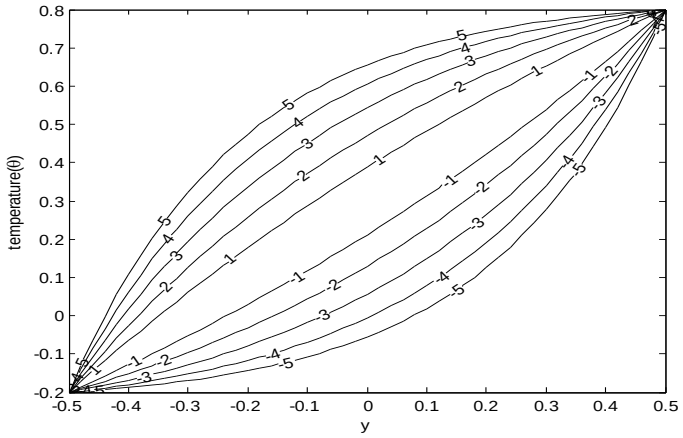


Figure4.1b. Temperature Profile $\theta(y)$ for Different S: $(Pr = 0.71, \theta_0 = 0.2)$

Figures 4.2a and 4.2b reveals the temperature profile for different values of θ_0 when suction is considered $(S < 0)$. It is observed that as θ_0 increases, temperature decreases. While in Figures 4.3a and 4.3b when injection is considered $(S > 0)$, it is also observed that as θ_0 increases, temperature decreases. However, as observed in Figures 4.2a and 4.3a, it reveals that temperature is higher near to the plate $\left(y = +\frac{1}{2}\right)$ when fluid is being injected in to the channel than to the plate $\left(y = -\frac{1}{2}\right)$ when suction of fluid is taking place.

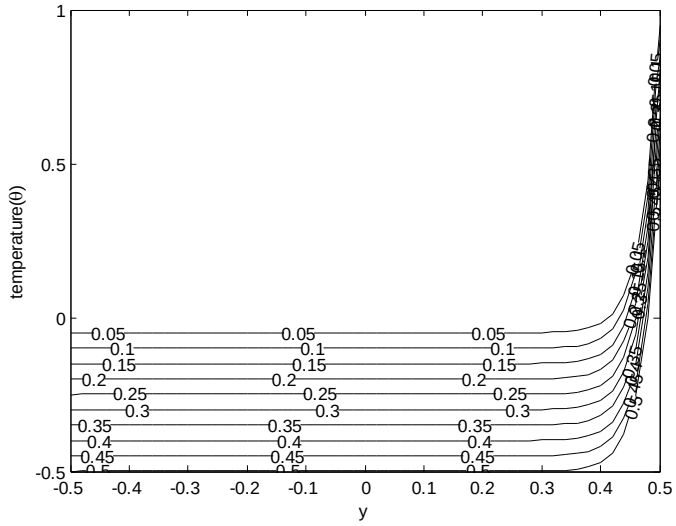


Figure4.2a. Temperature Profile $\theta(y)$ for Different θ_0: $(Pr = 7.0, S = -5.0)$

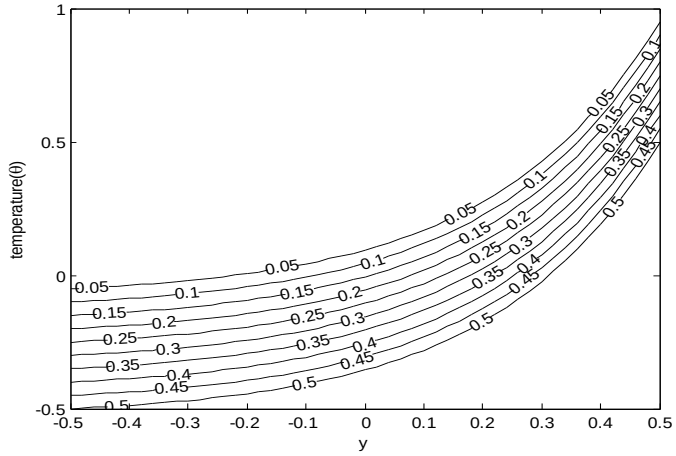


Figure4.2b. Temperature Profile $\theta(y)$ for Different θ_0: $(Pr = 0.71, S = -5.0)$

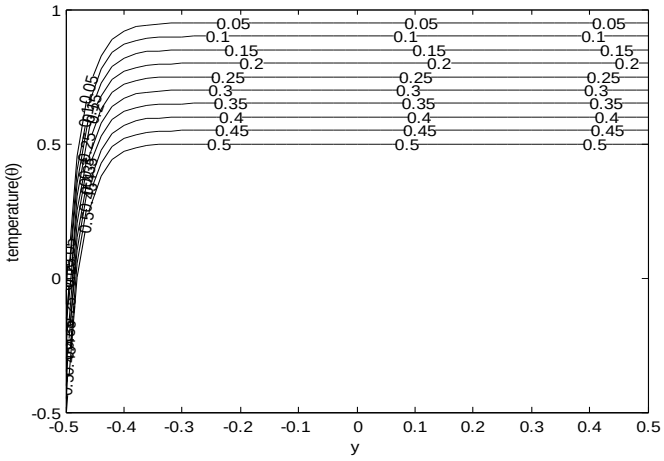


Figure4.3a. Temperature Profile $\theta(y)$ for Different θ_0: $(Pr = 7.0, S = 5.0)$

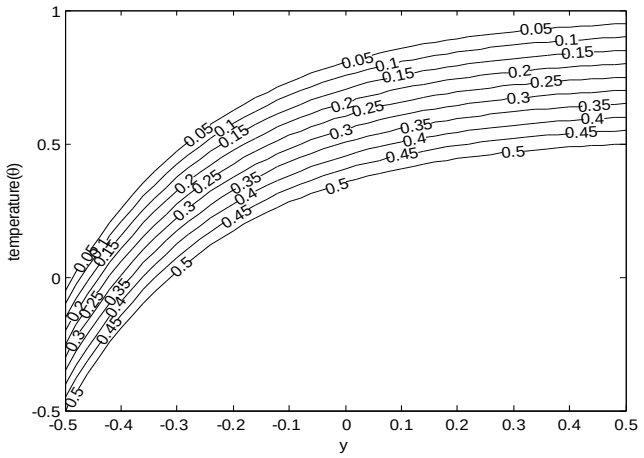


Figure4.3b. Temperature Profile $\theta(y)$ for Different θ_0: $(Pr = 0.71, S = 5.0)$

Figures 4.4a and 4.4b show the influence of suction/injection on the velocity profile, an increase in suction/injection parameter(S) leads to an increase in velocity for both $Pr = 7.0$ (water) and$Pr = 0.71$ (air). However, the effect of suction/injection on velocity of fluid varies from plate to plate as it can be observed in Figures 4.4a and 4.4b that fluid velocity is higher near the plate with injection.

Figures4.5a, 4.5b, 4.6a and 4.6b depict the effect of θ_0 on the velocity profile. It is observed in figures 4.5a and 4.5b that, velocity decreases with increase in θ_0 when $(S < 0)$ for both $Pr = 0.71$ and$Pr = 7.0$. While the velocity also decreases as θ_0 increases when $(S > 0)$ for both $Pr = 0.71$ and$Pr = 7.0$. This shows that an increase in θ_0 will reduce the fluid velocity throughout the channel.

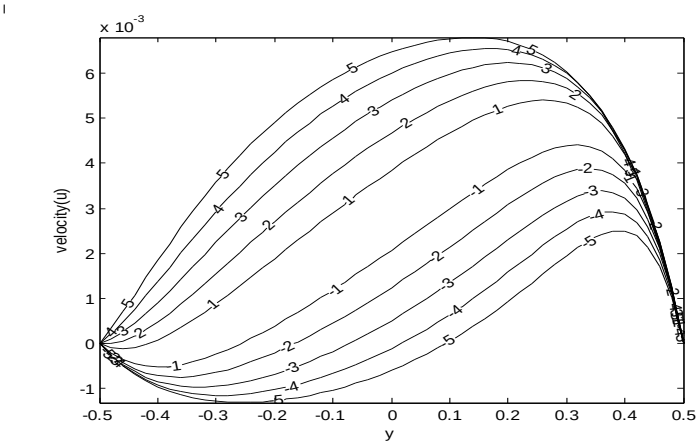


Figure4.4a. Velocity Profile $u(y)$ for Different S: $(Pr = 0.71, Da = 0.01, R = 1.0, \theta_0 = 0.2$

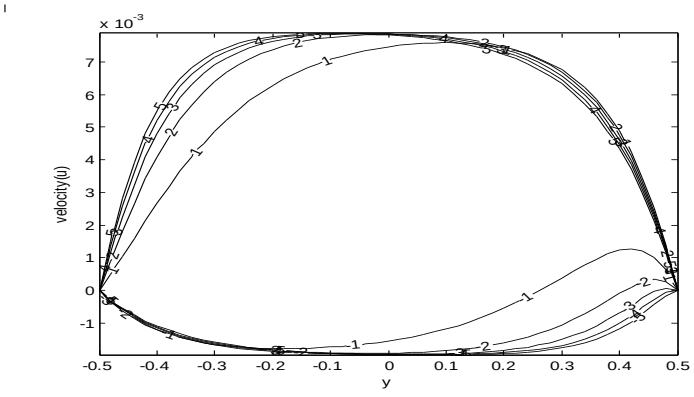


Figure4.4b. Velocity Profile $u(y)$ for Different S: $(Pr = 7.0, Da = 0.01, R = 1.0, \theta_0 = 0.2)$

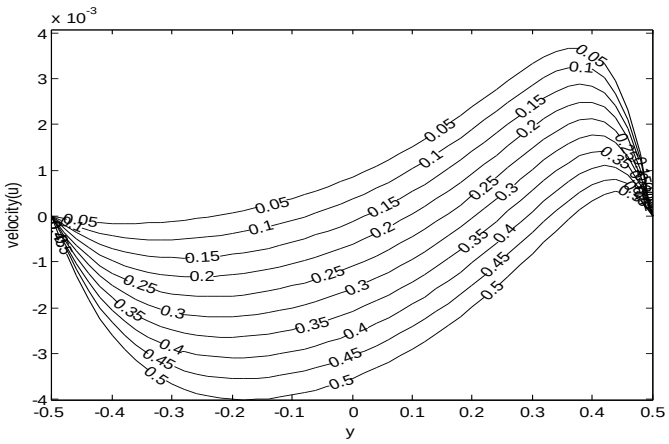


Figure4.5a. Velocity Profile $u(y)$ for Different θ_0:($Pr = 0.71, Da = 0.01, R = 1.0, S = -5.0$

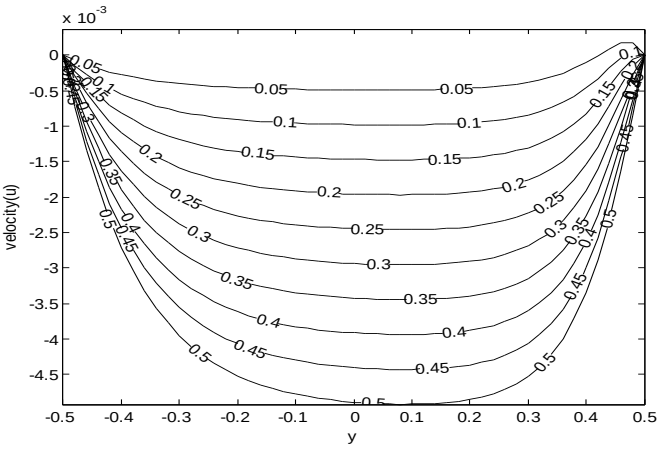


Figure4.5b.Velocity Profile $u(y)$ for Different θ_0:($Pr = 7.0, Da = 0.01, R = 1.0, S = -5.0$

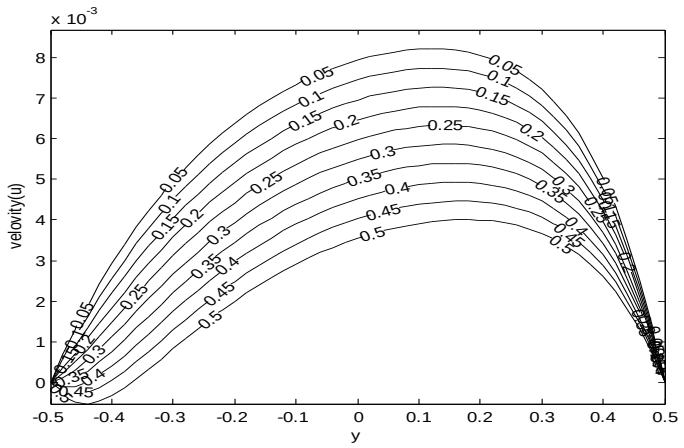


Figure4.6a.Velocity Profile $u(y)$ for Different θ_0:($Pr = 0.71, Da = 0.01, R = 1.0, S =$ $+5.0$

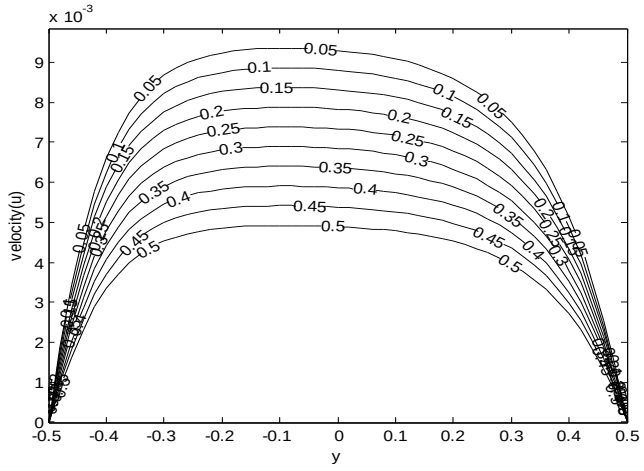


Figure4.6b.Velocity Profile $u(y)$ for Different θ_0:($Pr = 7.0, Da = 0.01, R = 1.0, S =$ $+5.0$

Figures 4.7a and 4.7b shows the combined effect of the suction/injection and Pr on the critical value of θ_0. From Figure4.7a, it is observed that near the plate$\left(y = -\frac{1}{2}\right)$ where fluid is being sucked off the channel, the critical value of θ_0 increases asPr is increasing. This is normal because of increased thermal diffusivity at low Pr. Figure.4.8a reveals the combined effect of the

suction/injection parameter (S) and the Darcy number(Da) on the critical value of θ_0. It can be observed that near the plate $\left(y = -\frac{1}{2}\right)$, the critical value of θ_0 decreases as Da increases at some certain values of (S) whereas the reverse trend occurs at some other range of (S). Figure4.8b also depicts the combined effect of the suction/injection parameter$(S)$and the Darcy number(Da) near the plate $\left(y = +\frac{1}{2}\right)$. It can be observed that asDa decreases the critical value of θ_0 decreases. Figures 4.9a and 4.9b shows the combined influence of Da and S. In Figure4.9a near the plate $\left(y = +\frac{1}{2}\right)$, the critical value of θ_0 decreases as Da increases at some values of (S), and it increases with an increase in Da at some other values of (S). In figure4.9b, it is observed that near the plate $\left(y = -\frac{1}{2}\right)$ the critical value of θ_0 is decreasing as Da is increasing. However, it is interesting to note in figures4.8a and 4.9a that there is a value of (S) at which the effect of Da on critical θ_0 is nullified.

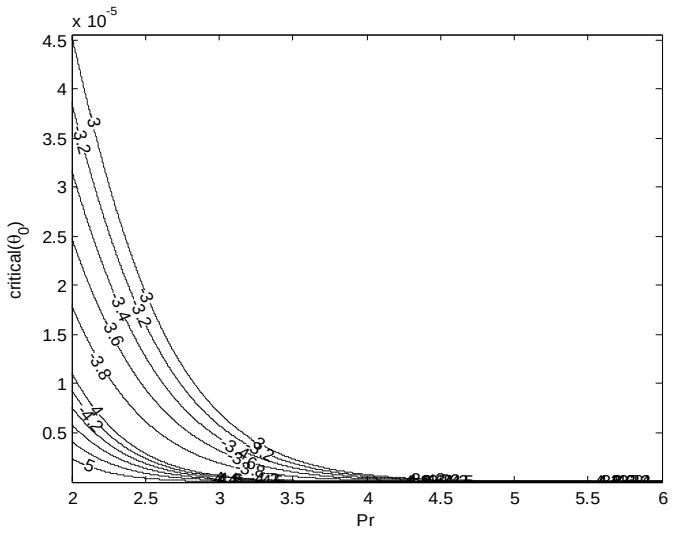


Figure4.7a.Critical Value of θ_0 at $y = -\frac{1}{2}$ for Different Pr *and* S:$(Da = 0.001, R = 1.0)$

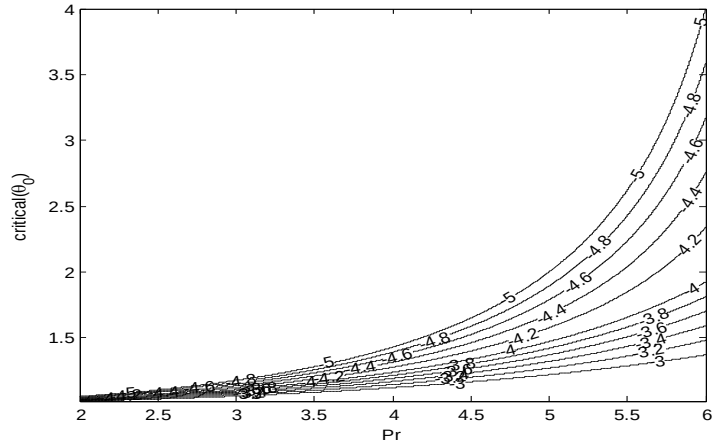


Figure4.7b.Critical Value of θ_0 at $y = +\frac{1}{2}$ for Different Pr *and* S:$(Da = 0.001, R = 1.0)$

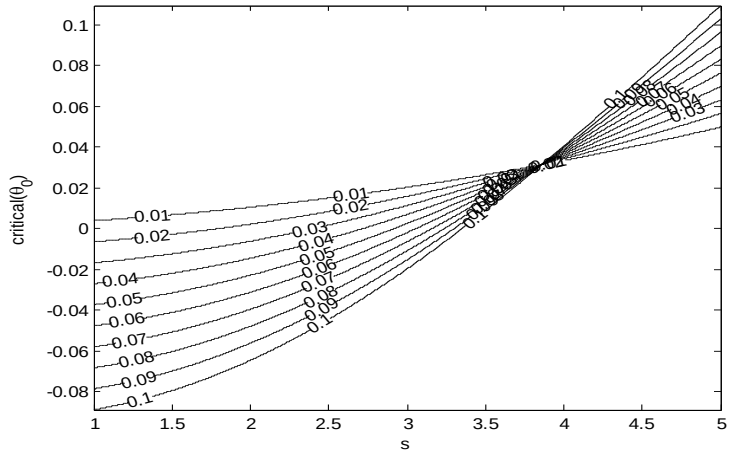


Figure4.8a.Critical Value of θ_0 at $y = -\frac{1}{2}$ for Different*Da and S*:$(Pr = 0.71, R = 1.0, S =$

1.0 to 5.0

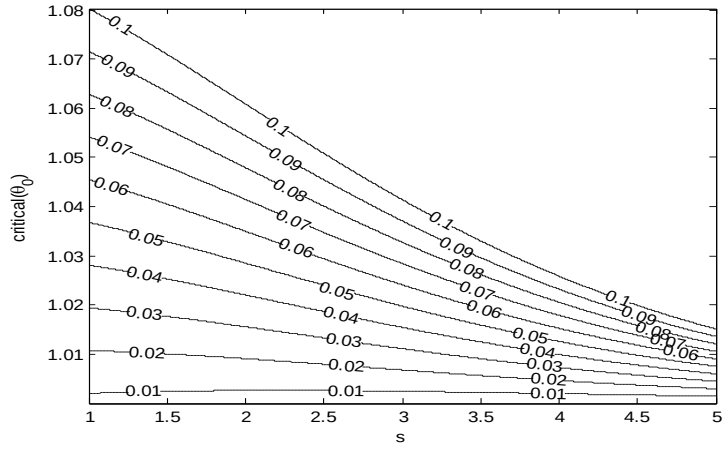


Figure4.8b.Critical Value of θ_0 at $y = +\frac{1}{2}$ for DifferentDa and S:($Pr = 0.71, R = 1.0, S =$ 1.0 to 5.0

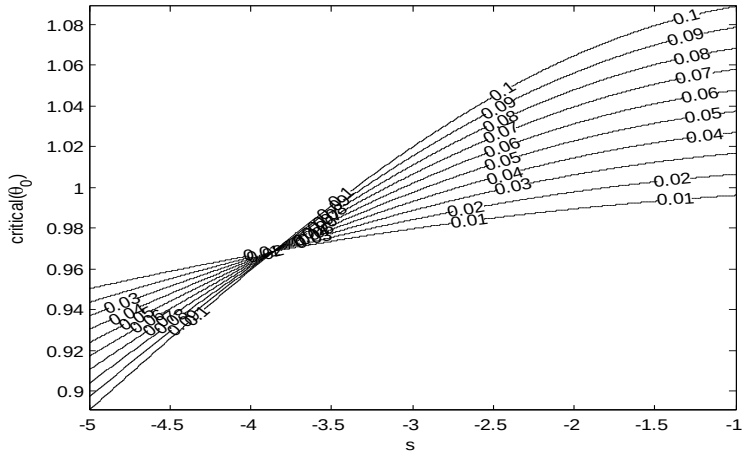


Figure4.9a.Critical Value of θ_0 at $y = +\frac{1}{2}$ for DifferentDa and S:($Pr = 0.71, R = 1.0, S =$ -1.0 to -5.0

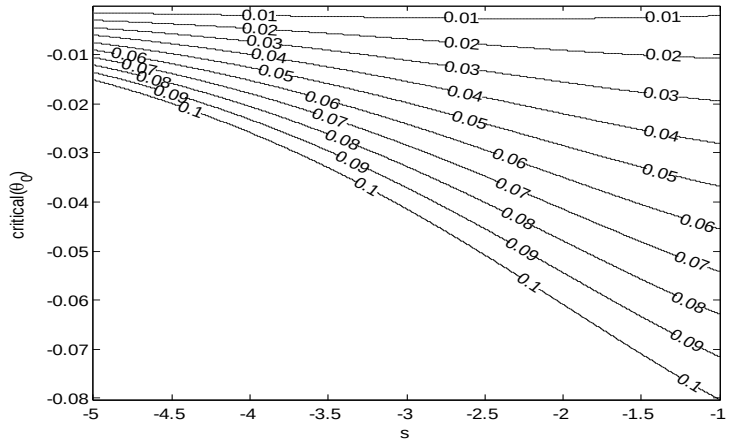


Figure4.9b.Critical Value of θ_0 at $y = -\frac{1}{2}$ for DifferentDa $and S$:($Pr = 0.71, R = 1.0, S =$ -1.0 to -5.0

Figure4.10a shows the combined effect of Da and S on the mass flux. It can be seen that mass flux increases with increase in Da when injection is considered through the heated plate $(S > 0)$. While in Figure 4.10b it is observed that for small value of S, mass flux is an increasing function of Da. Furthermore, when S becomes large Q decreases as Da increases. Which shows that the response of mass flux variations in the porosity is dependent on the suction/injection parameter. In figure4.11a, the combined effect of $(Pr > 1)$ and S on mass flux is depicted. It is observed that, as$S$ decreases mass flux decreases. In figure4.11b, it is seen that as Pr increases mass flux decreases. Figure4.12a also shows the combined effect of S and $(Pr < 1)$ on mass flux. It can be observed that as S increases mass flux increases at all values of Pr. Figure4.12b also shows the combined effect of S and$(Pr > 1)$. It can be seen that as S increases mass flux increases at all values of Pr.

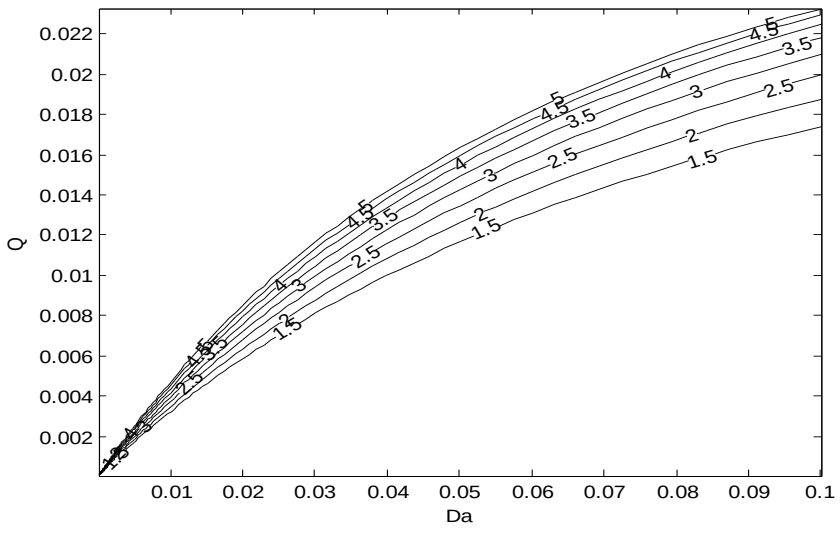


Figure4.10 a. Mass Flux for Different Da and $(+S)$: $(Pr = 0.71, \theta_0 = 0.2, R = 1.0)$

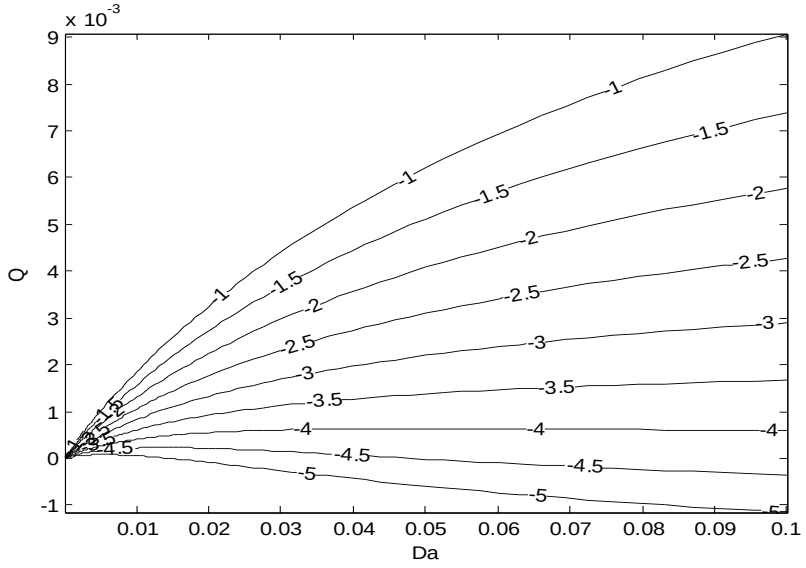


Figure4.10 b. Mass Flux for Different Da and $(-S)$:$(Pr = 0.71, \theta_0 = 0.2, R = 1.0)$

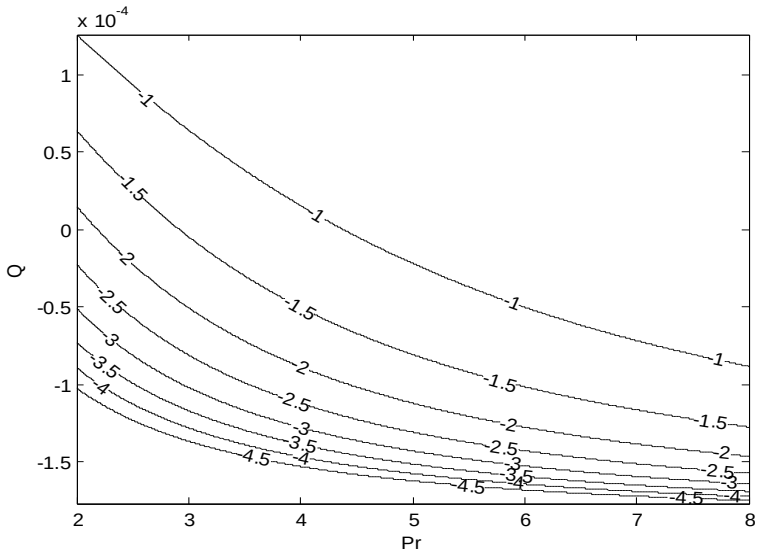


Figure4.11a. Mass Flux for Different $\mathbf{Pr} > 1$ $\boldsymbol{and}$ $(-\boldsymbol{S})$:$(\boldsymbol{Da} = \mathbf{0.001}, \boldsymbol{\theta_0} = \mathbf{0.2}, \boldsymbol{R} = \mathbf{1.0})$

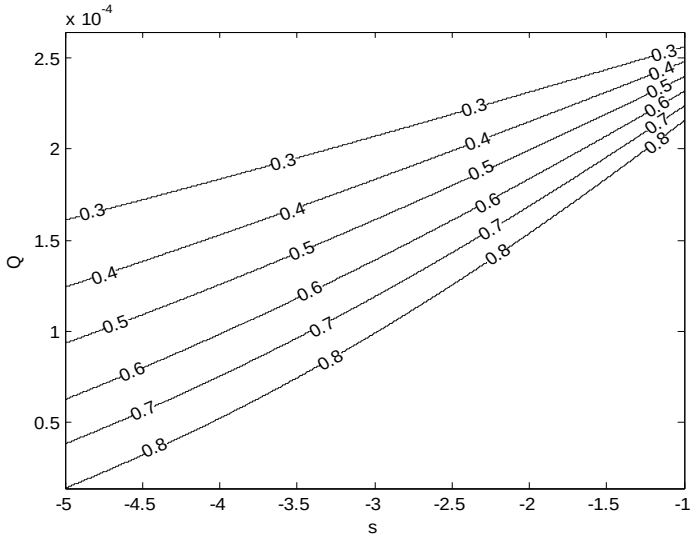


Figure4.11 b. Mass Flux for Different $\mathbf{Pr} < 1$ $\boldsymbol{and}$ $(-\boldsymbol{S})$:$(\boldsymbol{Da} = \mathbf{0.001}, \boldsymbol{\theta_0} = \mathbf{0.2}, \boldsymbol{R} = \mathbf{1.0})$

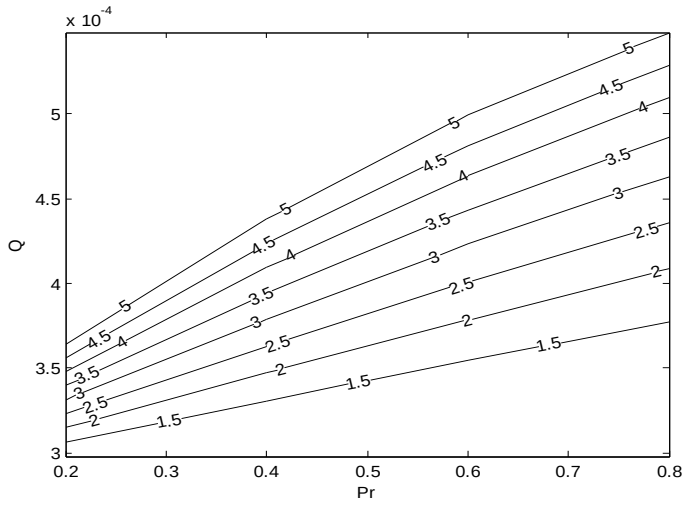


Figure4.12 a. Mass Flux for Different $\mathbf{Pr} < 1$ $\boldsymbol{and}$ $(+\boldsymbol{S})$:$(\boldsymbol{Da} = \mathbf{0.001}, \boldsymbol{\theta_0} = \mathbf{0.2}, \boldsymbol{R} = \mathbf{1.0})$

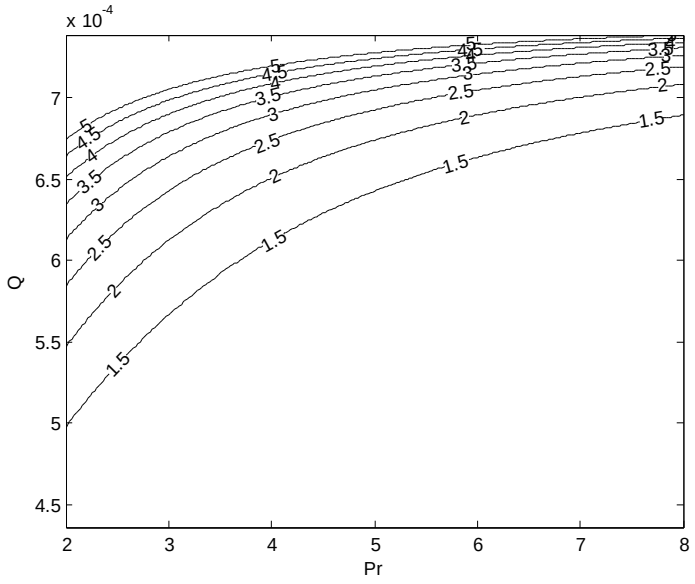


Figure4.12 b. Mass Flux for Different $\mathbf{Pr} > 1$ $\boldsymbol{and}$ $(+\boldsymbol{S})$:$(\boldsymbol{Da} = \mathbf{0.001}, \boldsymbol{\theta_0} = \mathbf{0.2}, \boldsymbol{R} = \mathbf{1.0})$

4.3Discussing the results of problem 3.2

To discuss the effect of transport parameters on the fluid velocity, interface velocity and the critical value of θ_0, numerical values are evaluated for some typical values of the governing parameters.

Figure4.13 shows the velocity profile at different values of θ_0 with $Da = 0.1$ and $d = 0.5$. It can be seen that as θ_0 increases, fluid velocity is decreasing in the clear fluid region and the same behavior is also observed in the porous region. Figure4.14 also shows velocity profile for different values of θ_0 with $Da = 0.01$. By comparing Figures4.13 and 4.14, it is clear that impact of θ_0 on flow formation is weaker for small value of Darcy number inside the composite channel. Also, Figure4.15 shows that for small Darcy number the effect of θ_0 on velocity is negligible inside the porous domain.

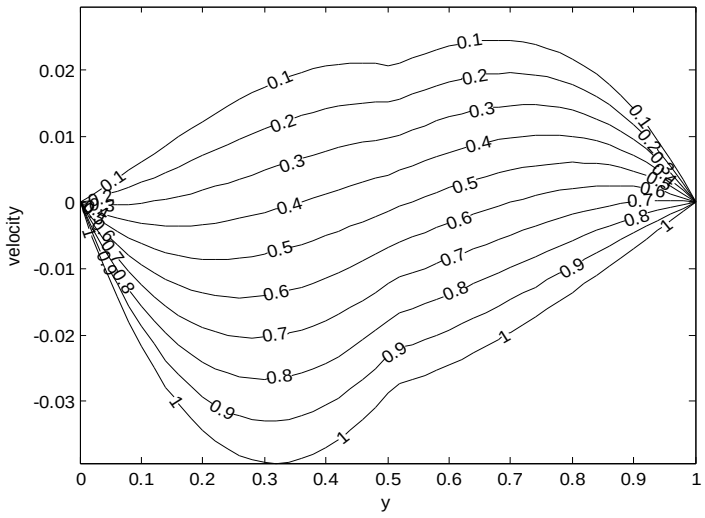


Figure4.13 Velocity Profile at Different values of θ_0 with $Da = 0.1$

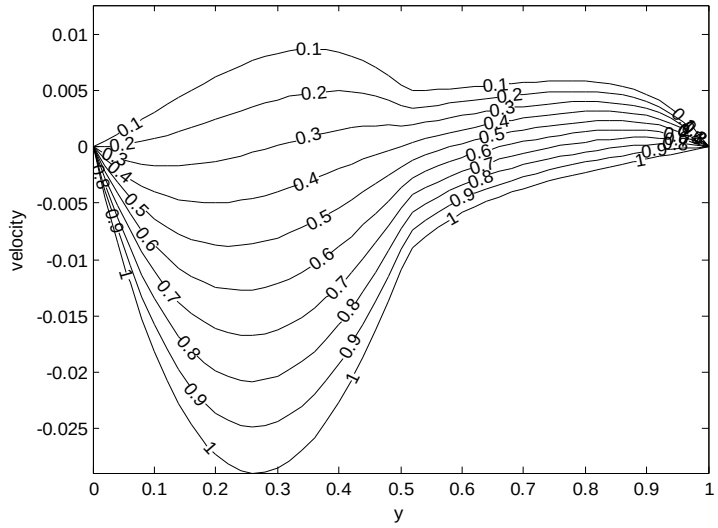


Figure4.14 velocity Profile at Different Value of θ_0 with $Da = 0.01$

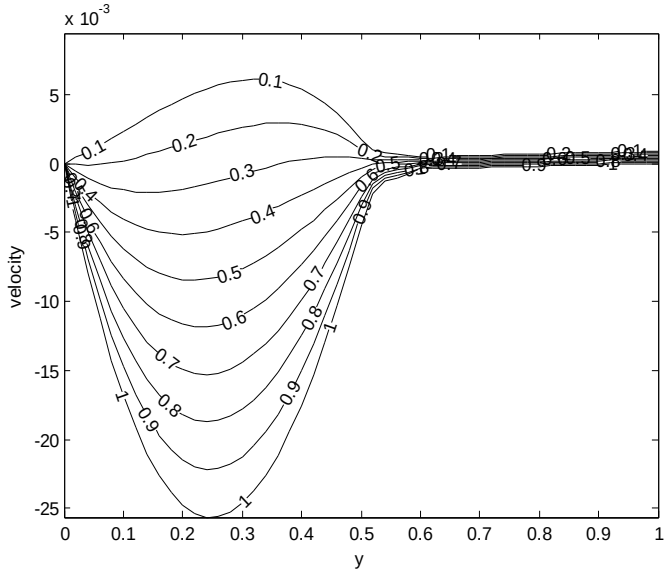


Figure4.15 Velocity Profile at Different Value of θ_0 with $Da = 0.001$

Figure4.16 shows that by reducing the porous layer thickness inside the channel, the probability of reverse flow at the cold plate decreases while the increase in Darcy number reduces the

numerical value of critical θ_0. It is also interesting to note that the variation of critical value of θ_0 is responsible for reverse flow formation at the left plate.

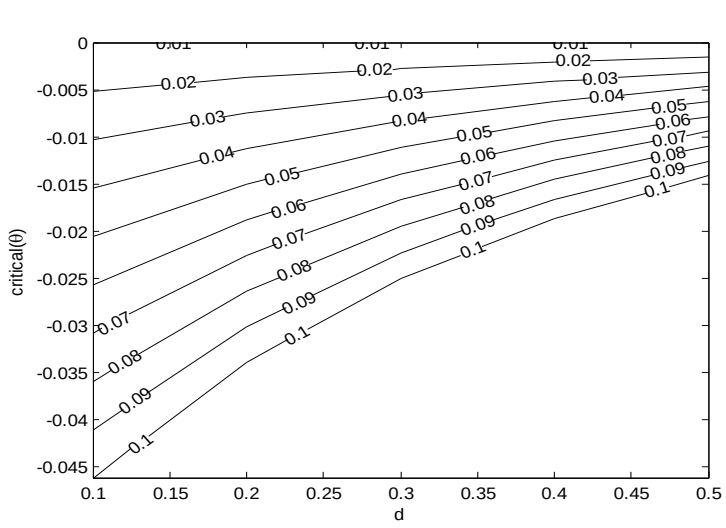


Figure4.16 Criticalθ_0 at Different Values of Da

Figure4.17 reveals the variation of interface velocity with θ_0 and Darcy number. It is clear that role of Darcy number on interface velocity is strongly dependent of θ_0. From this figure it is clear that there exist a value of $\theta_0(\theta_0 = 0.5)$ below which interface velocity increases with increase of Darcy number while it decreases with increase of Darcy number for $\theta_0 > 0.5$.

It is noticed from figure 4.18 that interface velocity increases with decrease in the porous layer thickness inside the composite channel for fixed value of Darcy number. While for fixed porous layer thickness, interface velocity increases with increase in Darcy number. Figure 4.19 demonstrate the interface velocity variation with porous layer thickness and adjustable co-efficient in the stress jump condition. This indicates that for negative value of adjustable co-efficient in the stress jump condition, interface velocity is strongly dependent on porous layer thickness inside the composite channel while the impact of porous layer thickness on interface

velocity is moderate for positive value of α. It is also clear that the role of porous layer thickness on interface velocity is strongly dependent of α.

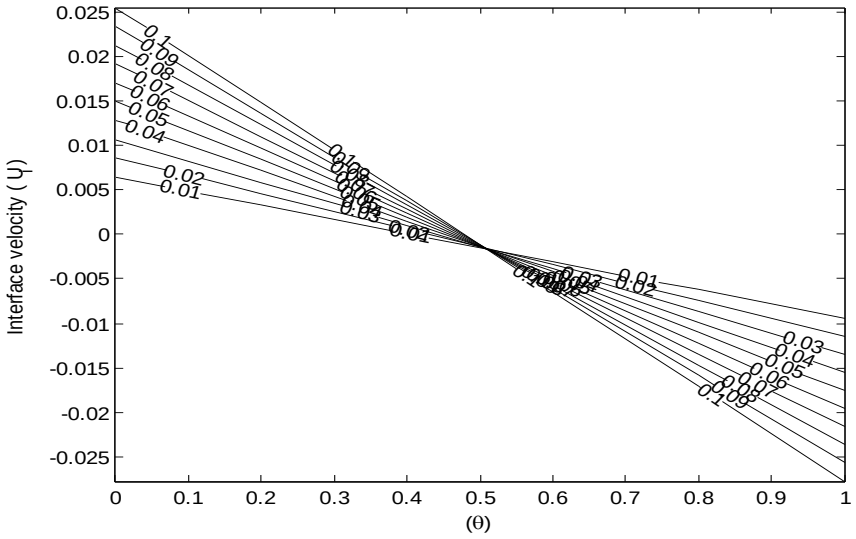


Figure4.17 Interface Velocity at Different Valuesθ_0 and Da

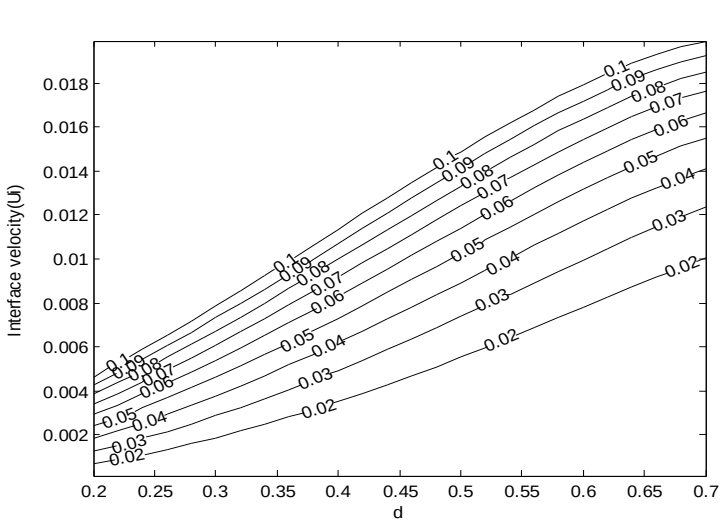


Figure4.18 Interface Velocity with Different Values of Da and d ($\theta_0 = 0.2$)

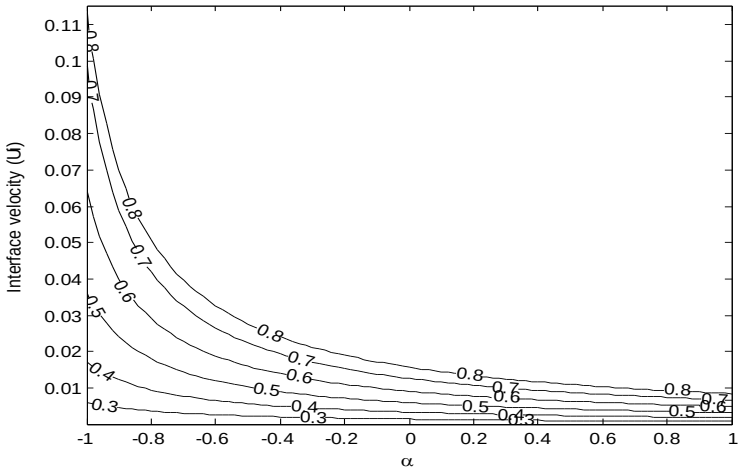


Figure4.19. Interface Velocity with Different Values of α and d ($Da = 0.01, \theta_0 = 0.2$)

It is noticed from Figure4.20 that the mass flux inside the channel increases by reducing the porous layer thickness while it decreases with decrease of Darcy number.

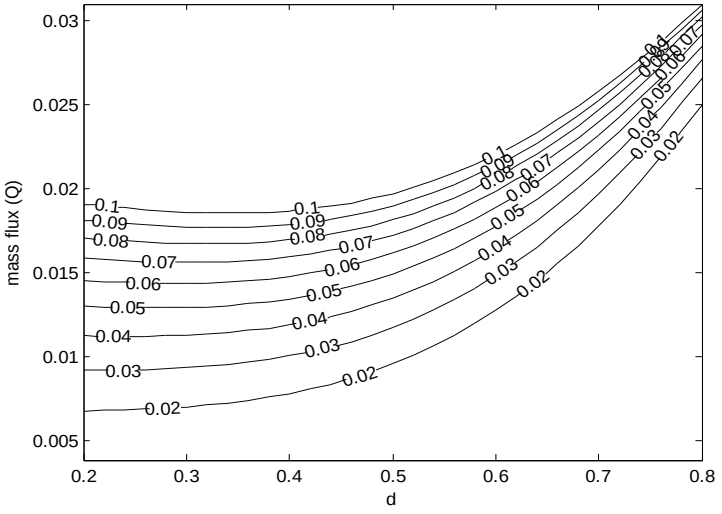


Figure4.20 Mass Flux with Different Values of d and Da ($\theta_0 = 0.02,\ \alpha = 1.0$)

4.4Discussing the results of problem 3.3

To discuss the effect of transport parameters on the fluid velocity, interface velocity and the critical value of θ_0, numerical values are evaluated for some typical values of the governing parameters.

Figure4.21 shows the velocity (u) plotted against r at different values of θ_0 when $Da = 0.2$ and $d = 0.5$. It can be seen that as θ_0 is increasing velocity is decreasing in the annulus gap, while in Figure4.22 velocity (u) is also plotted against r at different values of θ_0 when $Da = 0.01$. It shows that impact of θ_0 is more pronounce in the clear fluid region in the annulus. Furthermore, Figure4.23 shows the velocity profile at different values of θ_0 with $Da = 0.001$. It can be seen that for small Darcy number the effect of θ_0 on velocity is negligible inside the porous domain.

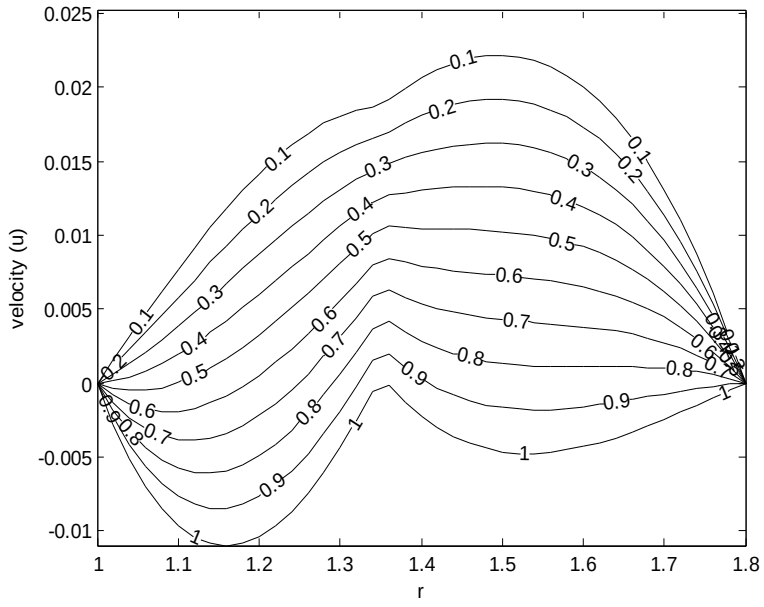


Figure 4.21 Velocity Profile for Different Values of $\theta_0(Da = 0.2)$

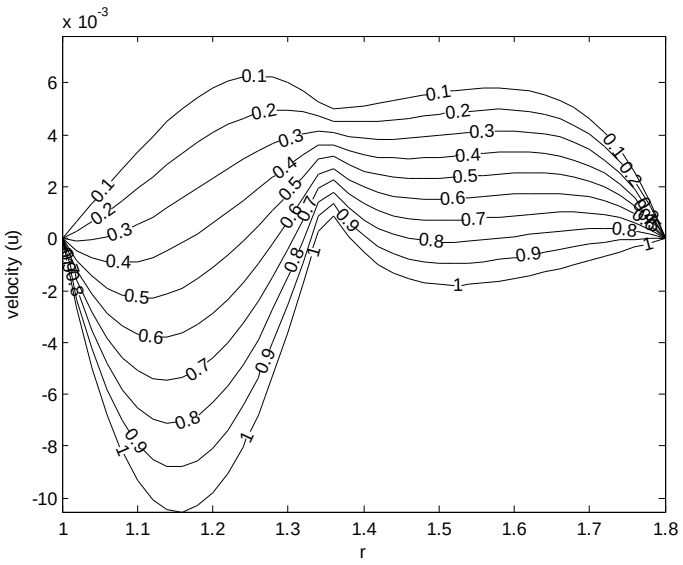


Figure 4.22 Velocity Profile for Different Values of $\theta_0 (Da = 0.01)$

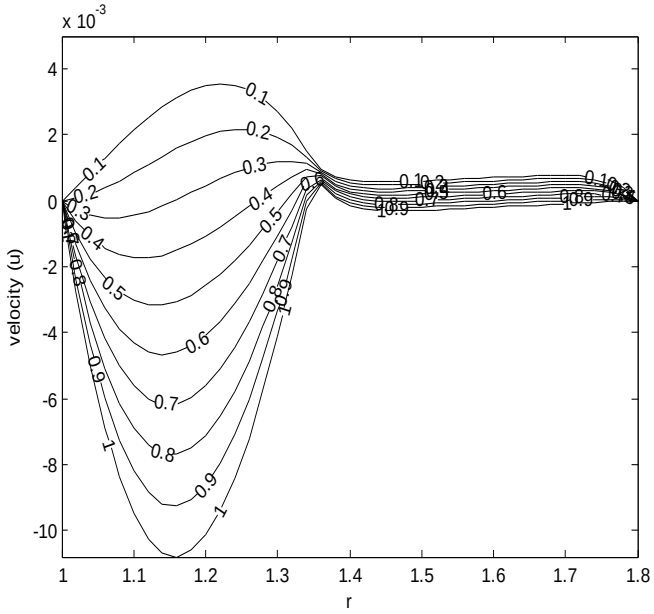


Figure 4.23 Velocity Profile for Different Values of $\theta_0 (Da = 0.001)$

Figure 4.24 reveals the variation of interface velocity with θ_0 and Da. It is clear that the role of Darcy number on interface velocity is strongly dependent of θ_0. From this figure it is clear that

there exist a value of $\theta_0$at which the influence of Da on interface velocity is negligible. In addition, the interface velocity increases with increase of Da, while it decreases with decrease of Da for $\theta_0 > 0.8$. Figure 4.25 shows that the interface velocity increases with decrease in the porous layer thickness inside the annulus for fixed value of Da. While for fixed porous layer thickness, interface velocity increases with increase of Da. While Figure 4.26 show the variation of interface velocity with porous layer thickness and adjustable coefficient in the stress jump condition. This indicates that for negative values of adjustable coefficient in the stress jump condition, interface velocity is strongly dependent on porous layer thickness inside the annulus. While the impact of porous layer thickness on interface velocity is moderate for positive values of α.

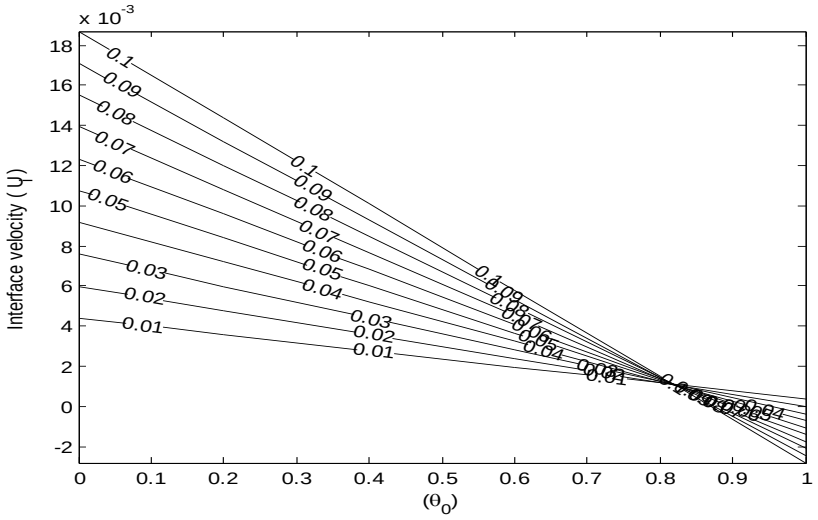


Figure 4.24 Interface Velocity Profile for Different values of θ_0 and Da

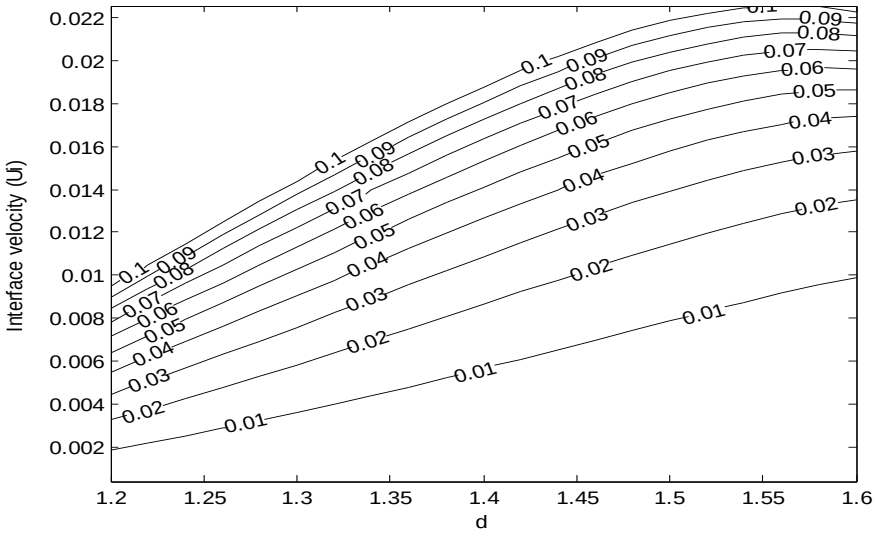


Figure 4.25 Interface Velocity Profile for Different values of d and Da

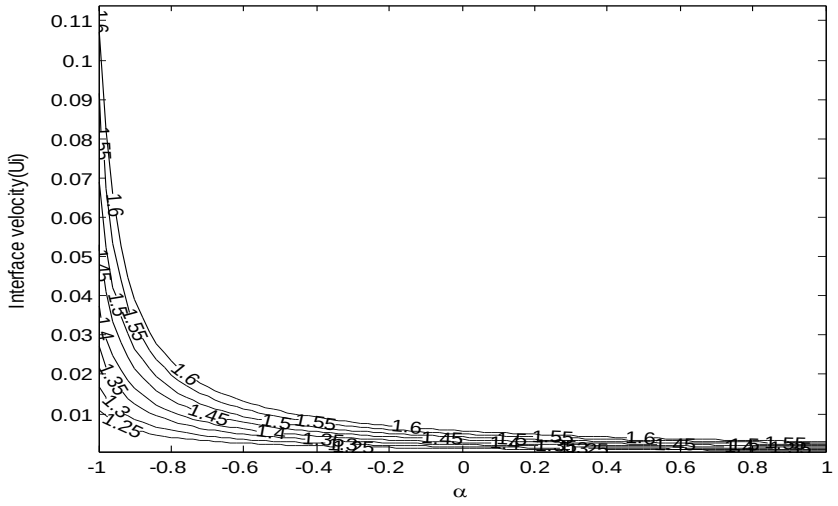


Figure 4.26 Interface Velocity Profile for Different Values of

α and d($Da = 0.001$ and $\theta_0 = 0.2$)

From Figure 4.27 it can be observed that by increasing Darcy number, the chance of reverse flow at the clear fluid region decreases while the decrease in porous layer thickness reduces the numerical value of critical θ_0. It is also interesting to note that the variation of critical θ_0 is

responsible for reverse flow formation at the clear fluid region. Figure 4.28 also reveals the variation of critical θ_0 with Darcy number and porous layer thickness. It can be seen clearly that the role of Darcy number on critical θ_0 is strongly dependent on porous layer thickness. From this figure it is clear that there exist a value of $d(d = 1.5)$ below which critical θ_0 decreases with increase of Darcy number, while it increases with increase of Darcy number for $d > 1.5$.

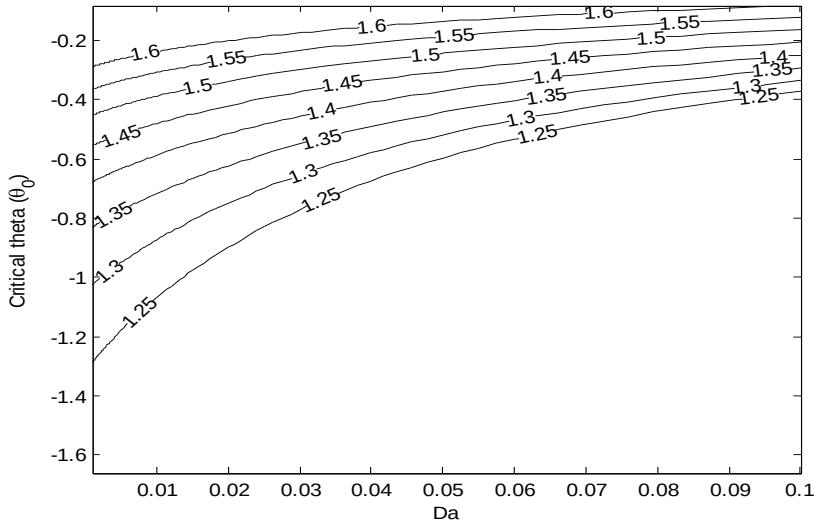


Figure 4.27 Critical θ_0 $at\ r = 1$ for Different Values of Da and d

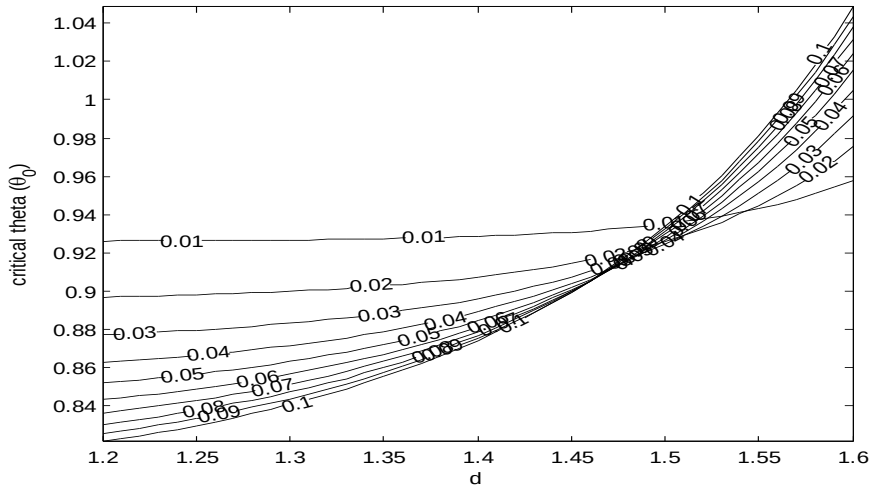


Figure 4.28 Critical θ_0 $at\ r = \lambda$ for Different Values of d and Da

It is noticed from Figure 4.29 that mass flux inside the annulus increases by reducing the porous layer thickness while it decreases with decrease of Darcy number.

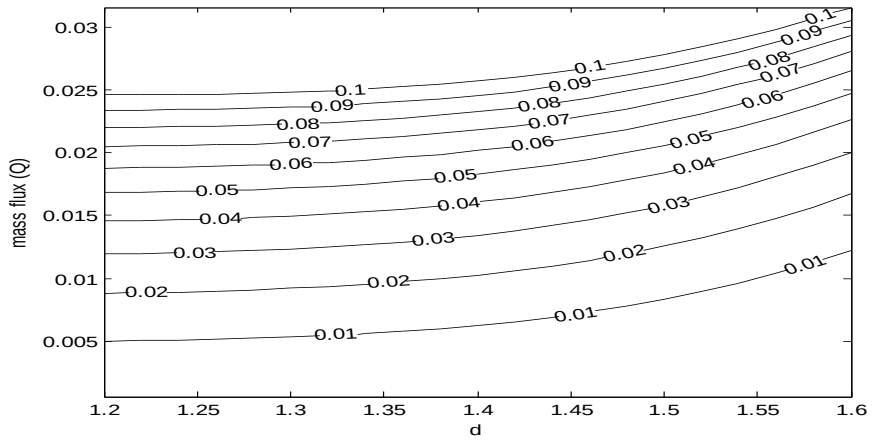


Figure 4.29 Mass Flux at Different Values of d and Da

CHAPTER FIVE

SUMMARY AND CONCLUSIONS

5.0 SUMMARY

The first problem reports the analytical study of the impact of suction/injection on mixed convection flow between vertical porous plates filled with porous material. The second problemis the fully developed mixed convection flow between vertical parallel plates partially filled with porous material in which the interface velocity, the velocity and the temperature are obtained analytically. The third problem studies the effect of Darcy number and the stress jump condition at the interface of fluid/porous layer for the steady fully developed non-Darcian mixed convection flow in an annulus partially filled with porous material. In the first problem, the plates are heated such that one plate is maintained at a temperature T_1 while the other plate is maintained at a temperature T_2. And in the second and third problems, the wall of the clear fluid is heated while that of the porous medium is cooled.The dimensionless form of the governing equations are solved analytically using the method of undetermined coefficients and the analytical solutions obtained were presented graphically and discussed.

5.1 CONCLUSIONS

In order to analyze the impact of the suction/injection parameter(S) on mixed convection flow between vertical porous plates filled with porous material, a mathematical model has been developed. The governing equations have been solved analytically and the numerical results obtained are presented graphically. Results show the reference temperature(θ_0) decreases the temperature as it increases. And it is observed that by increasing injection $(S > 0)$through the hot plate, temperature increases while by increasing suction through the cold plate it decreases temperature. It is observed also that the fluid velocity is strongly affected by the suction/injection parameter(S) and the reference temperature(θ_0) . Also, Darcy number Da increases the fluid velocity and it can be used to suppress the onset of flow reversal.For $S \to 0$, the solution is recovered to the solution by Weidman and Medina(2008).

A fully developed mixed convection flow between vertical parallel plates partially filled with clear fluid and partially filled with porous material is considered. The effects of $\theta_0, Da, d \ and \ \alpha$ are investigated. It is found that by increasing the porous layer thickness the probability of reverse flow is less.The velocity is significantly affected throughout the porous region for higher Darcy number. For all values of Darcy number, the interface velocity is influenced by the adjustable coefficient in the stress jump condition. It can be concluded that fluid flow can be suppressed by imposing porous matrix in the channel.

5.2 RECOMMENDATIONS

This research work investigated mixed convection flow in different geometries in which the fluid considered is non-Darcian where the walls of the channels are maintained at different temperature. To have a wider understanding and application of this phenomenon of mixed convection, the effect of the governing parameters can be investigated on the boundary conditions. Darcian fluids can also be considered in the presence of magneto-hydrodynamic with uniform temperature on the walls of the plates.

REFERENCES

Al-Nimr, M.A., and Darabsch, T.T (1995) Analytical solutions for transient laminar fully Developed free convection in open-ended vertical concentric porous annul.*ASME j. Heat Transfer*, vol. 117, pp. 762-764

Aung, W. and Woyku, G. (1986) Theory of fully developed combined convection including Flow reversal.*ASME Journal of Heat Transfer* vol.108, pp.485-488

Barletta, A.(1998) Laminar mixed convection with viscous dissipation in a vertical channel. *International Journal of Heat Mass Transfer*, vol.41 pp.3501-3513.

Bejan, A., Dincer, I., Lorente, S, Miguel, A.F, Reis, A.H.(2004) Porous and Complex Flow Structures in Modern Technologies. *Springer, New York*.

Beavers, G.S, Joseph, D.D, (1967) Boundary Conditions at a Natural Permeable Wall *j. Fluid Mechanics j. Fluid Mechanics* vol. 30 pp. 197-207

Bua, H.H, Torrence, K.E, (1981) Onset of Convection in a permeable medium between vertical coaxial cylinders. *Phys. Fluids* vol. 25 pp. 1755-1766

Buhler, K., (2003) Special solutions of the Boussinesq-equations for free convection between vertical walls. *Heat Mass Transfer* vol.39 pp.631-638

Chaudhary, M.A., Merkin, J.H., (1993) The effect of blowing and suction on free-convection boundary layers on vertical surfaces with prescribed heat flux. *Jr. Engng. Math.* Vol. 27 pp. 265-292.

Cheng, C.H, Huag, H.S, Huang, W.H, (1990) Flow reversal and heat transfer of fully developed Mixed convection in vertical channels. *Journal of Thermophysics Heat Transfer*,vol.4 pp.375-383.

De Lemos, M.J.S., (2006) Turbulence in porous media: Modeling and Applications, *Elsevier,* Oxford.

Grosan, T. and Pop, I., (2007) Thermal radiation effect on fully developed mixed convection Flow in a vertical channel. *Tech. Mech,* vol.1 pp.37-47.

Hamadah, T.T and Wirtz, R.A., (1991) Analysis of laminar fully developed mixed convection in A vertical channel with opposing buoyancy. *ASME Journal of Heat Transfer,* vol. 113 pp. 507-510.

Havstad, M.A., Burns, P.J., (1982) Convective Heat Transfer in vertical cylindrical annuli filled With a porous medium. *Int. J. Heat Mass Transfer* vol. 25 pp.1755-1766.

Ingham, D.B., Pop, I.,(Eds) (2005) Transport Phenomena in porous media, *Elsevier,* Oxford.

Jha, B.K., Ajibade, A.O., (2009) Free convective flow of heat generating/absorbing fluid Between vertical porous plates with periodic heat input. *Int. Commun. Heat Mass Trans.* Vol. 36 pp. 624-631.

Jha, B.K., Ajibade, A.O., (2010)Free convective flow between vertical porous plates with Periodic heat input. *Z. Angew. Math. Mech.* Vol. 90 (3) pp. 185-193.

Jha, B.K., Odengle, J.O., Kaurangini, M.L., (2011) Effect of transpiration on free-convective Couette flow in a composite channel. *J. Porous Media* vol. 14(7) pp. 627-635.

Joshi, H.M., (1987) Fully developed natural convection in an isothermal vertical annular duct. *International communication of Heat Transfer,* vol.14 pp.657-667.

Kaurangini, M.L., Jha, B.K., (2010) Steady and Transient investigation of generalized couette Flow in a compositechannel with suction/injection. *J. Porous Media* vol. 13(10) pp.931-943.

Kaurangini, M.L., Jha, B.K., (2011) Analysis of fluid flow in the interface region between a

Uniform porous medium and a clear fluid in parallel-plate with suction/injection. *Int. J. Management Sci. Eng. Management*

Kaviany, M., (1991) Principles of heat transfer in porous media.*New York:Springer-Verlag.*

Khaefinejad, B. and Aghanajafi, C., (2009) Laminar flow with simultaneous effect of mixed Convection and thermal radiation in a channel. *Journal of Fusion Energy*, vol. 28 pp. 83-90.

Kuznetsov, A.K., (1998) Analytical investigation of Couette flow in a composite channels partially filled with a clear fluid. *Int. J. Heat Mass Transfer* vol. 41 pp. 2556-2560.

Kuznetsov, A.K., (1996a) Analysis of a non-thermal equilibrium fluid flow in a concentric tube annulus filled with a porous medium. *Int. Com. Heat Mass Transfer*. Vol. 23 pp. 926-938.

Kuznetsov, A.K.,(1996b) Analytical investigation of the fluid flow in the interface region between a porous medium and a clear fluid in channels partially filled with a porous medium. *Applied Scientific Research*. Vol.56 pp.653-662.

Minkowyck, W.J. Cheng, P.,(1976) Free Convection about a vertical cylinder embedded in a porous medium. *Int. J Heat Mass Transfer*. Vol.19 pp. 805-813.

Muralidhar, K.,(1989) Mixed convection flow in a saturated porous annulus. *Int. J. Heat Mass Transfer*. Vol.32 pp.881-888.

Murthy, PVSN. Mukherjee, S. Srinivascharya, D. Krishna, PVSSR., (2004) Combined radiation And mixed convection from a vertical wall with suction/injection in a non-Darcy porous Medium .*ACTA Mechanica*, vol.168 pp.145-156.

Nield, D.A., Bejan, A.,(1992) Convection in porous media. *New York:Springer-Verlag.*

Nield, D.A., Bejan, A.,(2006) Convection in porous media, third ed. *Springer, New York.*

Ochoa-Tapia, J.A., Whitaker, S.,(1995a) Momentum transfer at the boundary between a porous medium and a homogeneous fluid-I.Theoretical Development.*Int. J. Heat Mass Transfer*. Vol. 38 pp.2635-2646.

Ochoa-Tapia, J.A., Whitaker, S.,(1995b) Momentum transfer at the boundary between a porous medium and a homogeneous fluid-II. Comparison with experiment. *Int. J. Heat Mass Transfer*. Vol. 38 pp.2647-2655.

Ostrich, S. Combined natural and mixed convection in laminar flow and heat transfer of fluids with and without linearly varying wall temperatures. NASA, Report No. NACA-TN-3141.

Pantokratoras, A.(2007)Fully developed couette flow of three fluids with variable thermophysical properties flowing through a porous medium channel heated asymmetrically with large temperature difference. *Journal of Heat Transfer,*vol.129 pp.1742-1747.

Paul,T., Singh, A.K.,(1998) Natural convection between coaxial cylinders partially filled with a porous material. *Foeschung im Ingenieurwesen*. Vol.64 pp. 157-162.

Paul,T., Singh, A.K.,(2002) Effects of interfacial stress jump condition on free convection flow between vertical walls partially filled with porous medium. *Modeling, Simulation & Control.*

Paulikakos, D., Kazmierczak, M., (1987) Forced convection in a duct partially filled with a porous material. *ASME J. Heat Transfer.* Vol.109 pp. 653-662.

Pop, I. Ingham, D.B., (2001) convective Heat Transfer: Mathematical and Computational Modeling of Viscous Fluids and Porous media. *Pergamon.Oxford*.

Pop, I., Grosan, T., Cornelia, R. (2009) Effect of heat generated by an exothermic reaction on the fully developed mixed convection flow in a vertical channel.*Communication in Non-Linear Simulation.*

Ramanaiah, G. and Malarvizhi, G. (1990) Unified treatment of free and mixed convection on a permeable vertical cylinder in a saturated porous media. *Indian Journal of Technology,* vol.28 pp.604-608.

Rossi di Sichio, E. Celli, M. Pop, I. (2011) Buoyant flow in a vertical fluid saturated porous annulus: The Brinkman model. *International Journal of Heat Mass Transfer,* vol.54 pp.1665-1670.

Singh, A.K and Gorla, R.S.R. (2008) Heat transfer between two vertical parallel walls partially filled with a porous medium:Use of Brinkman-extended Darcy model. *Journal of Porous Media,* vol.11 pp.457-466.

Umavathi, J.C. and Malashetty, M.S. (2005) Magneto hydrodynamicmixed convection in a vertical channel. *International Journal of Non-Linear Mechanic.* Vol.40 pp.90-101.

Vadesz, P. (Ed.) Emerging topics in heat and mass transfer in porous media. *Springer, New York,* 2008.

Vafai, K. and Thiyagaraja, R. (1987) Analysis of flow and heat transfer at the interface region of a porous medium. *International Journal of Heat Mass Transfer,* vol.30 pp. 1391-1405.

Vafai, K and Kim, S.J. (1990)Fluid mechanics of the interface between a porous medium and a fluid layer-An exact solution. *International Journal of Heat and FluidFlow,* vol.11 pp.254-256.

Verma P.D. and Chauhan, D.S., (1986) Unsteady flow of a viscous imcomparable fluid through a circular naturally permeable tube surrounded by a porous material. *Indian Journal Tech.,* vol.24 pp. 63-65.

Weidman, P.D. , Medina, A. (2008) Porous media convection between vertical walls: continuum of solutions from capped to open ends. *Acta Mechnica* Vol.199 pp 209-216.

APPENDICES

APPENDIX I

Relationship between the parameter θ_0 and the parameter $\frac{Gr}{Re}$.

The difference between natural convection and mixed convection is due to the driving forces. Natural convection arises only if the fluid flow is due to buoyancy force, while mixed convection flow arises if the buoyancy effect acts in the presence of other external mechanical devices such as compressors, pumps, fans etc. In both cases, the pressure gradient does not generally vanish.

However, the overall force driving the flow in the local momentum balance equation is

$$\rho_0 g\beta(T - T_0) = -\frac{d\bar{p}}{dz} + \rho_0 g\beta(T - T_1) \qquad \text{(I)}$$

where $\rho_0, g, \beta, T, T_0, T_1 \; and \; T_2$ are all defined in the nomenclature. Moreover, the choice of the reference temperature does not only affects the buoyancy force term but also the hydrodynamic pressure gradient $\frac{d\bar{p}}{dz}$. In natural convection, a reference temperature is usually chosen such that;

$$-\frac{d\bar{p}}{dz} = (T_1 - T_0)\rho_0 g\beta \text{(II)}$$

so that $\frac{d\bar{p}}{dz} = 0$.

Of course, it is possible to perform this choice also in the case of combined forced and free convection, as proposed in Rossi di Schio et al. (2011).

By comparing the non- dimensionalization introduced in this paper with the more traditional non-dimensionalization based on the parameter $\frac{Gr}{Re}$. In this work, the reference temperature T_1 is

chosen such that the hydrodynamic pressure drop vanishes, so that the overall force can be written as;

$\rho_0 g\beta(T_0-T_1)$ (III)

However, if a different reference temperature T_2 is introduced, the sum of the hydrodynamic pressure gradient and the buoyancy force is given by

$-\frac{d\bar{p}}{dz}+\rho_0 g\beta(T-T_2)$ (IV)

By comparing (III) and (IV) it shows that the choices are equivalent provided that

$\frac{d\bar{p}}{dz}=\rho_0 g\beta(T_2-T_1)\theta_0$. (V)

By defining the following dimensionless parameters;

$U_0=-\frac{d\bar{p}}{dz}\frac{d^2}{\bar{\mu}}, Re=\frac{U_0 d}{\nu}, Gr=\frac{g\beta(T_2-T_1)d^3}{\nu^2}, Da=\frac{k}{d^2}, \gamma=\frac{\bar{\mu}}{\mu}$. (VI)

Equation (V) becomes

$\frac{Gr}{Re}=-\frac{\gamma}{\theta_0}$. (VII)

Finally, it shows that for a fixed γ there exist a one to one correspondence between the parameter θ_0 and the parameter $\frac{Gr}{Re}$.

APPENDIX II

Constants used in problem one are as follows:

$$f_1 = S^2Pr^2 - \frac{S^2Pr}{R} - \frac{1}{R\,Da} f_2 = c_1 Da - \frac{C_2}{Rf_1} exp\left(\frac{SPr}{2}\right)$$

$$f_3 = c_1 Da - \frac{C_2}{Rf_1} exp\left(\frac{-SPr}{2}\right) f_4 = \frac{exp\left(\frac{SPr}{2}\right)}{2sinh\left(\frac{SPr}{2}\right)} f_5 = \frac{C_2}{Rf_1} exp\left(\frac{SPr}{2}\right)$$

$$f_6 = \frac{C_2}{RF_1} exp\left(\frac{-SPr}{2}\right)$$

$$f_2 = -\theta_0 Da + f_7 \; where \; f_7 = f_4 Da - f_5$$

$$f_3 = -\theta_0 Da + f_8 \;\; where \; f_8 = f_4 Da - f_6$$

$$f_9 = \frac{Da\, cosh\left(\frac{S}{4R}\right)}{cosh\left(\frac{\delta}{2}\right)} f_{10} = -\frac{\left[f_8 exp\left(\frac{S}{4R}\right) + f_7 exp\left(\frac{-S}{4R}\right)\right]}{2cosh\left(\frac{\delta}{2}\right)} F_{11} = \frac{Da\, sinh\left(\frac{S}{4R}\right)}{sinh\left(\frac{\delta}{2}\right)}$$

$$f_{12} = \frac{\left[F_7 exp\left(\frac{-S}{4R}\right) - f_8 exp\left(\frac{S}{4R}\right)\right]}{2sinh\left(\frac{\delta}{2}\right)}$$

$$f_{13} = \left[\delta\, sinh\left(\frac{\delta}{2}\right) + \frac{S}{2R} cosh\left(\frac{\delta}{2}\right)\right] exp\left(\frac{S}{4R}\right) f_{14} = \left[\delta\, cosh\left(\frac{\delta}{2}\right) + \frac{S}{2R} sinh\left(\frac{\delta}{2}\right)\right] exp\left(\frac{S}{4R}\right)$$

$$f_{15} = \left[\delta\, sinh\left(\frac{\delta}{2}\right) - \frac{S}{2R} cosh\left(\frac{\delta}{2}\right)\right] exp\left(\frac{-S}{4R}\right) f_{16} = \left[\delta\, cosh\left(\frac{\delta}{2}\right) - \frac{S}{2R} sinh\left(\frac{\delta}{2}\right)\right] exp\left(\frac{-S}{4R}\right)$$

$$f_{17} = \left[\frac{\sinh\left(\frac{1}{2}\left(\delta - \frac{S}{2R}\right)\right)}{\left(\delta - \frac{S}{2R}\right)} + \frac{\sinh\left(\frac{1}{2}\left(\delta + \frac{S}{2R}\right)\right)}{\left(\delta + \frac{S}{2R}\right)}\right] f_{18} = \left[\frac{\sinh\left(\frac{1}{2}\left(\delta - \frac{S}{2R}\right)\right)}{\left(\delta - \frac{S}{2R}\right)} - \frac{\sinh\left(\frac{1}{2}\left(\delta + \frac{S}{2R}\right)\right)}{\left(\delta + \frac{S}{2R}\right)}\right]$$

$$f_{19} = \frac{2sinh\left(\frac{SPr}{2}\right) C_2}{Rf_1 SPr}$$

$$c_1 = -\theta_0 + f_4 c_2 = -\frac{1}{2sinh\left(\frac{SPr}{2}\right)}$$

$$c_3 = \theta_0 f_9 + f_{10} c_4 = \theta_0 f_{11} + f_{12}$$

Constants used in problem two are:

$$F_1 = sinh(Gd)\ cosh(G) - sinh(G)\ cosh(Gd) F_2 = Da\ cosh(G) - Da\ cosh(Gd)$$

$$F_3 = d\ cosh(G)\ cosh(Gd) F_4 = Da\ sinh(G) - Da\ sinh(Gd)$$

$$F_5 = d\ sinh(G) - sinh(Gd) F_6 = \gamma G\ sinh(Gd) - \frac{\alpha}{\sqrt{Da}} cosh(Gd)$$

$$F_7 = \gamma G\ cosh(Gd) - \frac{\alpha}{\sqrt{Da}} sinh(Gd) F_8 = -\gamma Da - \frac{d^2}{2} + \alpha d\sqrt{Da} + \frac{d^2}{6}$$

$$F_9 = -\frac{d}{2} + d - \alpha\sqrt{Da} F_{10} = -\frac{(DaF_5F_6 - DaF_3F_7)}{F_1}$$

$$F_{11} = \left[\frac{F_7\ cosh(G) - F_6\ sinh(G)}{F_1} - \frac{1}{d}\right] F_{12} = F_9 - \frac{(F_2F_7 - F_4F_6)}{F_1}$$

$$F_{13} = F_8 + F_{10}$$

$$C_1 = \frac{[u_i\ sinh(G) + \theta_0 F_4 - DaF_5]}{-F_1} C_2 = \frac{[u_i\ cosh(G) + \theta_0 F_2 - DaF_3]}{F_1}$$

$$u_i = C_1\ cosh(Gd) + C_2\ sinh(Gd) - \theta_0 Da + dDa$$

$$u_i = \frac{[\theta_0 F_{12} + F_{13}]}{F_{11}}$$

Constants used in problem three are:

$$F_1 = \frac{[ln(d) - 1]}{ln(d)}\frac{d^2}{4} + \frac{1}{4\,ln(d)}F_2 = K_0\left(\frac{\lambda}{\sqrt{\gamma Da}}\right)I_0\left(\frac{d}{\sqrt{\gamma Da}}\right) - K_0\left(\frac{d}{\sqrt{\gamma Da}}\right)I_o\left(\frac{\lambda}{\sqrt{\gamma Da}}\right)$$

$$F_3 = \frac{Da\left[I_0\left(\frac{d}{\sqrt{\gamma Da}}\right) - I_0\left(\frac{\lambda}{\sqrt{\gamma Da}}\right)\right]}{F_2}F_{3a} = \frac{\left[I_0\left(\frac{d}{\sqrt{\gamma Da}}\right) - \frac{ln(d)}{ln(\lambda)}I_0\left(\frac{\lambda}{\sqrt{\gamma Da}}\right)\right]}{F_2}$$

$$F_4 = \frac{I_0\left(\frac{\lambda}{\sqrt{\gamma Da}}\right)}{F_2}F_5 = \frac{Da\left[K_0\left(\frac{d}{\sqrt{\gamma Da}}\right) - K_0\left(\frac{\lambda}{\sqrt{\gamma Da}}\right)\right]}{F_2}$$

$$F_{5a} = \frac{Da\left[K_0\left(\frac{d}{\sqrt{\gamma Da}}\right) - \frac{ln(d)}{ln(\lambda)}K_0\left(\frac{\lambda}{\sqrt{\gamma Da}}\right)\right]}{F_2}F_6 = \frac{K_0\left(\frac{\lambda}{\sqrt{\gamma Da}}\right)}{F_2}$$

$$F_7 = \sqrt{\frac{\gamma}{Da}}I_0\left(\frac{d}{\sqrt{\gamma Da}}\right) - \frac{\alpha}{\sqrt{Da}}I_0\left(\frac{d}{\sqrt{\gamma Da}}\right)F_8 = \sqrt{\frac{\gamma}{Da}}K_1\left(\frac{d}{\sqrt{\gamma Da}}\right) + \frac{\alpha}{\sqrt{Da}}K_0\left(\frac{d}{\sqrt{\gamma Da}}\right)$$

$$F_9 = \frac{\alpha\sqrt{Da}\,ln(d)}{ln(\lambda)} - \frac{\gamma\,Da}{d\,ln(\lambda)} - \frac{1}{ln(\lambda)}\left\{\frac{dln(d)}{2} - \frac{d}{4}\right\} + \frac{F_1}{dln(d)}$$

$$F_{10} = -\alpha\sqrt{Da} + \frac{d}{2} - \frac{1}{ln(d)}\left\{\frac{d^2}{4} - \frac{1}{4}\right\}F_{11} = \left[F_6F_7 + F_4F_8 - \frac{1}{dln(d)}\right]$$

$$F_{12} = [F_{10} + F_5F_7 + F_3F_8]F_{13} = [F_9 - F_{5a}F_7 - F_{3a}F_8]$$

$$F_{14} = F_5I_1\left(\frac{\lambda}{\sqrt{\gamma Da}}\right) + F_3K_1\left(\frac{\lambda}{\sqrt{\gamma Da}}\right)F_{15} = F_6I_1\left(\frac{\lambda}{\sqrt{\gamma Da}}\right) + F_4K_1\left(\frac{\lambda}{\sqrt{\gamma Da}}\right)$$

$$F_{16} = F_{5a}I_1\left(\frac{\lambda}{\sqrt{\gamma Da}}\right) + F_{3a}K_1\left(\frac{\lambda}{\sqrt{\gamma Da}}\right)F_{17} = \frac{-Da\sqrt{\gamma Da}}{\lambda ln(\lambda)}$$

$$F_{18} = \frac{\theta_0}{16}(d^2 - 1)F_{19} = \frac{C_1}{4}[d^2(2ln(d) - 1) + 1]$$

$$F_{20} = \frac{1}{4ln(\lambda)}\left[d^4\frac{ln(d)}{4} - \frac{5}{16}(d^4-1)\right] F_{21} = \frac{C_2}{2}(d^2-1)$$

$$F_{22} = C_3\left[\lambda I_1\left(\frac{\lambda}{\sqrt{\gamma Da}}\right) - dI_1\left(\frac{d}{\sqrt{\gamma Da}}\right)\right]\sqrt{\gamma Da}$$

$$F_{23} = C_4\left[\lambda K_1\left(\frac{\lambda}{\sqrt{\gamma Da}}\right) - dK_1\left(\frac{d}{\sqrt{\gamma Da}}\right)\right]\sqrt{\gamma Da}$$

$$F_{24} = \frac{Da}{ln(\lambda)}\left[\frac{\left(\lambda^2 ln(\lambda) - d^2 ln(d)\right)}{2} - \frac{1}{4}(\lambda^2 - d^2)\right]$$

$$F_{25} = \frac{\theta_0\, Da}{2}(\lambda^2 - d^2)$$

$$A = F_{18} + F_{19} - F_{20} + F_{21} B = F_{22} - F_{23} + F_{24} - F_{25}$$

$$C_1 = \frac{[u_i + F_1]}{ln(d)} - \frac{\left[\theta_0\left(\frac{d^2}{4} - \frac{1}{4}\right)\right]}{ln(d)} C_2 = -\left[\frac{\theta_0}{4} + \frac{1}{4ln(\lambda)}\right]$$

$$C_3 = -\theta_0 F_5 + F_6 u_i + F_{5a} C_4 = -u_i F_4 + \theta_0 F_3 - F_{3a}$$

$$u_i = \frac{(F_{12}\theta_0 + F_{13})}{F_{11}}$$

APPENDIX III

Computer programming for 'Role of suction/injection on mixed convection flow in a vertical porous channel filled with porous material

```
%   WIEDMAN WITH SUCTION TEMPERATURE
    [y,s]=meshgrid(-0.5:0.02:0.5,-5.0:1.0:5.0);
%[y,tz]=meshgrid(-0.5:0.02:0.5,0.0:0.1:0.5);
%w=(y-y+1).*(tz-tz+1);

     w=(y-y+1).*(s-s+1);
%pr=0.71.*w;
     pr=7.0.*w;

     tz=0.2.*w;
%s=-5.0.*w;
     x1=0.5.*s.*pr;
     x2=-s.*pr.*y;
     x3=sinh(x1);
     x4=(exp(x1)-exp(x2))./(2.*x3);
     th=x4-tz
     [C,h]=contour(y,th,s);
   set(h,'showText','on','textStep',get(h,'levelStep')*1);
   colormap gray
```

```
%   WIEDMAN WITH SUCTION VELOCITY
    [y,s]=meshgrid(-0.5:0.02:0.5,-5.0:1.0:5.0);
%[y,tz]=meshgrid(-0.5:0.02:0.5,0.0:0.1:0.5);
%w=(y-y+1).*(tz-tz+1);
     w=(y-y+1).*(s-s+1);
     pr=0.71.*w;
%pr=7.0.*w;
     tz=0.2.*w;
%s=-5.0.*w;
```

```
      r=1.0.*w;
      d=0.01.*w;
      x1=(s./(2.*r)).^2;
      x2=1./(r.*d);
      de=sqrt(x1+x2);
      f1=((s.*pr).^2)-((s.*s.*pr)./r)-x2;
      x3=0.5.*s.*pr;
      x4=exp(x3);
      x5=2.*sinh(x3);
      c1=-tz+(x4./x5);
      c2=-1./x5;
      f2=c1.*d-(c2./(f1.*r)).*x4;
      f3=c1.*d-(c2./(f1.*r)).*exp(-x3);
      x6=s./(4.*r);
      c3=-(f3.*exp(x6)+f2.*exp(-x6))./(2.*cosh(0.5.*de));
      c4=(f2.*exp(-x6)-f3.*exp(x6))./(2.*sinh(0.5.*de));
      x7=exp(-(0.5.*s.*y)./r);
      x8=c3.*cosh(de.*y)+c4.*sinh(de.*y);
      x9=-(c2.*exp(-s.*pr.*y))./(r.*f1);
      u=x7.*x8+c1.*d+x9
      [C,h]=contour(y,u,s);
   set(h,'showText','on','textStep',get(h,'levelStep')*1);
   colormap gray

%   WIEDMAN WITH SUCTION CRITICAL THEATA
%[pr,s]=meshgrid(2.0:2.0:6.0,3.0:1.0:5.0);
    [pr,s]=meshgrid(2.0:2.0:6.0,-3.0:-1.0:-5.0);

    w=(pr-pr+1).*(s-s+1);
     r=1.0.*w;
     da=0.001.*w;
     x1=(s./(2.*r)).^2;
     x2=1./(r.*da);
     de=sqrt(x1+x2);
     f1=((s.*pr).^2)-((s.*s.*pr)./r)-x2;
     x3=0.5.*s.*pr;
     x4=exp(x3);
     x5=2.*sinh(x3);
     c2=-1./x5;
     f4=x4./x5;
     f5=(c2.*exp(x3))./(f1.*r);
     f6=(c2.*exp(-x3))./(f1.*r);
     f7=f4.*da-f5;
     f8=f4.*da-f6;
     x6=s./(4.*r);
     x7=0.5.*de;
     x8=(0.5.*s)./r;
     f9=(da.*cosh(x6))./cosh(x7);
     f10=-(f8.*exp(x6)+f7.*exp(-x6))./(2.*cosh(x7));
     f11=(da.*sinh(x6))./sinh(x7);
     f12=(f7.*exp(-x6)-f8.*exp(x6))./(2.*sinh(x7));
     f13=(de.*sinh(x7)+(x8.*cosh(x7))).*exp(x6);
     f14=(de.*cosh(x7)+(x8.*sinh(x7))).*exp(x6);
     x9=(f10.*f13)-(f14.*f12);
     x10=(f14.*f11)-(f13.*f9);
     th1=x9./x10
```

```
      f15=(de.*sinh(x7)-(x8.*cosh(x7))).*exp(-x6);
      f16=(de.*cosh(x7)-(x8.*sinh(x7))).*exp(-x6);
      x11=(f15.*f10)+(f12.*f16);
      x12=(f9.*f15)+(f11.*f16);
      th2=-(x11./x12)
      [C,h]=contour(pr,th1,s);
    set(h,'showText','on','textStep',get(h,'levelStep')*1);
    colormap gray
```

```
%   WIEDMAN WITH SUCTION mass flux
% [pr,s]=meshgrid(2.0:2.0:8.0,-1.0:-1.0:-5.0);
%[pr,s]=meshgrid(0.2:0.2:0.8,-1.0:-1.0:-5.0);
% [pr,s]=meshgrid(0.2:0.2:0.8,1.0:1.0:5.0);
%[pr,s]=meshgrid(2.0:2.0:8.0,1.0:1.0:5.0);
% [da1,s]=meshgrid(1.0:1.0:4.0,1.0:1.0:5.0);
    [da1,s]=meshgrid(1.0:1.0:4.0,-1.0:-1.0:-5.0);
%w=(pr-pr+1).*(s-s+1);

     w=(da1-da1+1).*(s-s+1);
     da=10.0.^-da1;
     pr=0.71.*w;
%da=0.001.*w;
     tz=0.2.*w;
%s=-5.0.*w;
     r=1.0.*w;
     x1=(s./(2.*r)).^2;
     x2=1./(r.*da);
     de=sqrt(x1+x2);
     f1=((s.*pr).^2)-((s.*s.*pr)./r)-x2;
     x3=0.5.*s.*pr;
     x4=exp(x3);
     x5=2.*sinh(x3);
     c2=-1./x5;
     f4=x4./x5;
     f5=(c2.*exp(x3))./(f1.*r);
     f6=(c2.*exp(-x3))./(f1.*r);
     f7=f4.*da-f5;
     f8=f4.*da-f6;
     x6=s./(4.*r);
     x7=0.5.*de;
     x8=(0.5.*s)./r;
     f9=(da.*cosh(x6))./cosh(x7);
     f10=-(f8.*exp(x6)+f7.*exp(-x6))./(2.*cosh(x7));
     f11=(da.*sinh(x6))./sinh(x7);
     f12=(f7.*exp(-x6)-f8.*exp(x6))./(2.*sinh(x7));
     f13=(de.*sinh(x7)+(x8.*cosh(x7))).*exp(x6);
     f14=(de.*cosh(x7)+(x8.*sinh(x7))).*exp(x6);
```

```
      x9=(f10.*f13)-(f14.*f12);
      x10=(f14.*f11)-(f13.*f9);
      f15=(de.*sinh(x7)-(x8.*cosh(x7))).*exp(-x6);
      f16=(de.*cosh(x7)-(x8.*sinh(x7))).*exp(-x6);
      x11=de-(s./(2.*r));
      x12=de+(s./(2.*r));
      x13=(sinh(0.5.*x11))./x11;
      x14=(sinh(0.5.*x12))./x12;
      f17=x13+x14;
      f18=x13-x14;
      f19=(2.*c2.*sinh(0.5.*s.*pr))./(r.*f1.*s.*pr);
      x15=((f9.*f17)+(f11.*f18)-da).*tz;
      x16=(f10.*f17)+(f12.*f18)+(f4.*da)-f19;
      q=x15+x16;
      [C,h]=contour(da,q,s);
   set(h,'showText','on','textStep',get(h,'levelStep')*1);
   colormap gray
```

Computer programming for 'Fully developed mixed convection flow between vertical parallel plates partially filled with porous material

```
%   velocity in composite channel
%[y,th]=meshgrid(0.0:0.03:1.0,0.0:0.1:1.0);
    [y,th]=meshgrid(0.0:0.02:1.0,0.0:0.2:1.0);
      w=(y-y+1).*(th-th+1);
      d=0.5.*w;
      da=0.1.*w;
      ga=1.0.*w;
      al=1.0.*w;
      g=1./(sqrt(ga.*da));
      f1=((sinh(g.*d)).*cosh(g))-((sinh(g)).*cosh(g.*d));
      f2=da.*(cosh(g)-cosh(g.*d));
      f3=d.*cosh(g)-cosh(g.*d);
      f4=da.*(sinh(g)-sinh(g.*d));
      f5=d.*sinh(g)-sinh(g.*d);
      x1=sqrt(da);
      x2=al./x1;
      f6=(ga.*g.*sinh(d.*g))-(x2.*cosh(g.*d));
      f7=(ga.*g.*cosh(d.*g))-(x2.*sinh(g.*d));
      f8=-(ga.*da)-(0.5.*d.*d)+(al.*d.*x1)+((d.*d)./6.);
      f9=(0.5.*d)-al.*x1;
      f10=da.*((f3.*f7)-(f5.*f6))./f1;
      f11=(((cosh(g).*f7)-(sinh(g).*f6))./f1)-(1./d);
      f12=((f4.*f6-f2.*f7)./f1)+f9;
      f13=f8+f10;
      ui=((th.*f12)+f13)./f11;
      c1=-((ui.*sinh(g))+(th.*f4)-(da.*f5))./f1;
      c2=((cosh(g).*ui)+(th.*f2)-(da.*f3))./f1;
      ak1=(ui./d)-(0.5.*th.*d)+((d.*d)./6);
      u11=((0.5.*y.*y.*th)-((y.^3)./6)+(y.*ak1));
      u1=u11.*heaviside(d-y);
      u12=c1.*cosh(g.*y)+c2.*sinh(g.*y)+(da.*(y-th));
```

```
      u2=u12.*heaviside(y-d);
      u=u1+u2
      th1=(d.*d.*d.*f11)+(6.*f13);
      th2=3.*((d.*d.*f11)-2.*f12);
      t0=th1./th2
     [C,h]=contour(y,u,th);
   set(h,'showText','on','textStep',get(h,'levelStep')*1);
   colormap gray



%   convection in composite channel
%   ui(iterface velocity)
%[y,th]=meshgrid(0.0:0.03:1.0,0.0:0.1:1.0);
     [da1,th]=meshgrid(1.0:1.0:3.0,0.0:0.2:1.0);
      w=(da1-da1+1).*(th-th+1);
      d=0.5.*w;
      da=(10.0).^-da1.*w;
      ga=1.0.*w;
      al=1.0.*w;
      g=1./(sqrt(ga.*da));
      f1=((sinh(g.*d)).*cosh(g))-((sinh(g)).*cosh(g.*d));
      f2=da.*(cosh(g)-cosh(g.*d));
      f3=d.*cosh(g)-cosh(g.*d);
      f4=da.*(sinh(g)-sinh(g.*d));
      f5=d.*sinh(g)-sinh(g.*d);
      x1=sqrt(da);
      x2=al./x1;
      f6=(ga.*g.*sinh(d.*g))-(x2.*cosh(g.*d));
      f7=(ga.*g.*cosh(d.*g))-(x2.*sinh(g.*d));
      f8=-(ga.*da)-(0.5.*d.*d)+(al.*d.*x1)+((d.*d)./6.);
      f9=(0.5.*d)-al.*x1;
      f10=da.*((f3.*f7)-(f5.*f6))./f1;
      f11=(((cosh(g).*f7)-(sinh(g).*f6))./f1)-(1./d);
      f12=((f4.*f6-f2.*f7)./f1)+f9;
      f13=f8+f10;
      ui=((th.*f12)+f13)./f11
% c1=-((ui.*sinh(g))+(th.*f4)-(da.*f5))./f1;
%c2=((cosh(g).*ui)+(th.*f2)-(da.*f3))./f1;
%ak1=(ui./d)-(0.5.*th.*d)+((d.*d)./6);
%u11=((0.5.*y.*y.*th)-((y.^3)./6)+(y.*ak1));
%u1=u11.*heaviside(d-y);
%u12=c1.*cosh(g.*y)+c2.*sinh(g.*y)+(da.*(y-th));
%u2=u12.*heaviside(y-d);
%u=u1+u2
%th1=(d.*d.*d.*f11)+(6.*f13);
%th2=3.*((d.*d.*f11)-2.*f12);
%t0=th1./th2
     [C,h]=contour(th,ui,da);
   set(h,'showText','on','textStep',get(h,'levelStep')*1);
   colormap gray
    xlabel('(\theta)')
ylabel('Interface velocity ( U_i )')
```

```
%   convection in composite channel
%   critical value of tha0
     [da1,d]=meshgrid(1.0:1.0:3.0,0.1:0.1:0.5);
      w=(da1-da1+1).*(d-d+1);
%d=0.5.*w;
      da=(10.0).^-da1.*w;
      ga=1.0.*w;
      al=1.0.*w;
      g=1./(sqrt(ga.*da));
      f1=((sinh(g.*d)).*cosh(g))-((sinh(g)).*cosh(g.*d));
      f2=da.*(cosh(g)-cosh(g.*d));
      f3=d.*cosh(g)-cosh(g.*d);
      f4=da.*(sinh(g)-sinh(g.*d));
      f5=d.*sinh(g)-sinh(g.*d);
      x1=sqrt(da);
      x2=al./x1;
      f6=(ga.*g.*sinh(d.*g))-(x2.*cosh(g.*d));
      f7=(ga.*g.*cosh(d.*g))-(x2.*sinh(g.*d));
      f8=-(ga.*da)-(0.5.*d.*d)+(al.*d.*x1)+((d.*d)./6.);
      f9=(0.5.*d)-al.*x1;
      f10=da.*((f3.*f7)-(f5.*f6))./f1;
      f11=(((cosh(g).*f7)-(sinh(g).*f6))./f1)-(1./d);
      f12=((f4.*f6-f2.*f7)./f1)+f9;
      f13=f8+f10;
%ui=((th.*f12)+f13)./f11
%c1=-((ui.*sinh(g))+(th.*f4)-(da.*f5))./f1;
%c2=((cosh(g).*ui)+(th.*f2)-(da.*f3))./f1;
%ak1=(ui./d)-(0.5.*th.*d)+((d.*d)./6);
%u11=((0.5.*y.*y.*th)-((y.^3)./6)+(y.*ak1));
%u1=u11.*heaviside(d-y);
%u12=c1.*cosh(g.*y)+c2.*sinh(g.*y)+(da.*(y-th));
%u2=u12.*heaviside(y-d);
%u=u1+u2
      th1=(d.*d.*d.*f11)+(6.*f13);
      th2=3.*((d.*d.*f11)-2.*f12);
      t0=th1./th2
     [C,h]=contour(da,t0,d);
   set(h,'showText','on','textStep',get(h,'levelStep')*1);
   colormap gray



%   convection in composite channel
%   mass flux
     [da1,d]=meshgrid(1.0:1.0:2.0,0.2:0.1:0.8);
      w=(da1-da1+1).*(d-d+1);
      th=0.02.*w;
      da=(10.0).^-da1.*w;
      ga=1.0.*w;
      al=1.0.*w;
      g=1./(sqrt(ga.*da));
      f1=((sinh(g.*d)).*cosh(g))-((sinh(g)).*cosh(g.*d));
```

```
      f2=da.*(cosh(g)-cosh(g.*d));
      f3=d.*cosh(g)-cosh(g.*d);
      f4=da.*(sinh(g)-sinh(g.*d));
      f5=d.*sinh(g)-sinh(g.*d);
      x1=sqrt(da);
      x2=al./x1;
      f6=(ga.*g.*sinh(d.*g))-(x2.*cosh(g.*d));
      f7=(ga.*g.*cosh(d.*g))-(x2.*sinh(g.*d));
      f8=-(ga.*da)-(0.5.*d.*d)+(al.*d.*x1)+((d.*d)./6.);
      f9=(0.5.*d)-al.*x1;
      f10=da.*((f3.*f7)-(f5.*f6))./f1;
      f11=(((cosh(g).*f7)-(sinh(g).*f6))./f1)-(1./d);
      f12=((f4.*f6-f2.*f7)./f1)+f9;
      f13=f8+f10;
      ui=((th.*f12)+f13)./f11;
      c1=-((ui.*sinh(g))+(th.*f4)-(da.*f5))./f1;
      c2=((cosh(g).*ui)+(th.*f2)-(da.*f3))./f1;
      x3=-((1./12.).*th.*(d.^3))+((1./24.).*(d.^4))+(ui.*d.*0.5);
      x4=(c1./g).*(sinh(g)-sinh(g.*d))+(c2./g).*(cosh(g)-cosh(g.*d));
      x5=-th.*da.*(1.-d)+(0.5.*da.*(1.-(d.*d)));
      q=x3+x4+x5
      [C,h]=contour(d,q,da);
   set(h,'showText','on','textStep',get(h,'levelStep')*1);
   colormap gray
```

Computer programming for 'Steady fully developed non-Darcian mixed convection in an annulus partially filled with porous material and partially filled with clear fluid

```
%   convection in annulus partiallyfilled with porous material
     [r,th]=meshgrid(1.0:0.02:1.8,0.0:0.2:1.0);
      w=(r-r+1).*(th-th+1);
      d=1.3.*w;
      da=0.1.*w;
      ga=1.0.*w;
      al=1.0.*w;
      am=1.8.*w;
      g=1./(sqrt(ga.*da));
      x1=((log(d)-1.).*d.*d)./(4.*log(am));
      x2=1./(4.*log(am));
      f1=x1+x2;
      x3=am.*g;
      x4=d.*g;
      f2=besselk(0,x3).*besseli(0,x4)-besselk(0,x4).*besseli(0,x3);
      f3=(da./f2).*(besseli(0,x4)-besseli(0,x3));
      f4=besseli(0,x3)./f2;
      f5=(da./f2).*(besselk(0,x4)-besselk(0,x3));
      f6=besselk(0,x3)./f2;
      x5=sqrt(ga./da);
      x6=al./sqrt(da);
      f7=x5.*besseli(1,x4)-x6.*besseli(0,x4);
      f8=x5.*besselk(1,x4)+x6.*besselk(0,x4);
```

```
    x7=al.*sqrt(da).*log(d);
    x8=log(am);
    x9=x7./x8;
    x10=-(da.*ga)./(d.*x8);
    x11=0.5.*d.*log(d)-(d./4.);
    x12=-x11./x8;
    x13=f1./(d.*log(d));
    f9=x9+x10+x12+x13;
    f10=-al.*sqrt(da)+(0.5.*d)-(1./(4.*log(d))).*((d.*d)-1.);
    x14=1./(d.*log(d));
    xx1=1./log(d);
    xx2=log(d)./log(am);
    sf5=-(da./f2).*(xx2.*besselk(0,x3)-besselk(0,x4));
    sf3=(da./f2).*(besseli(0,x4)-xx2.*besseli(0,x3));
    f11=f6.*f7+f4.*f8-x14;
    f12=f10+(f7.*f5)+(f3.*f8);
    f13=f9-sf5.*f7-sf3.*f8;
    ui=((th.*f12)+f13)./f11
    c1=((ui+f1)./log(d))-xx1.*th.*((d./2).*(d./2.)-(1./4.));
    x15=th./4.;
    x16=1./(4.*log(am));
    c2=-(x15+x16);
    c3=-th.*f5+(ui.*f6)+sf5;
    c4=-(ui.*f4)+(th.*f3)-sf3;
    x17=(r.*r)./4.;
    x18=x17.*th;
    x19=-(x17.*log(r)-x17)./log(am);
    x20=c1.*log(r)+c2;
    u1=x18+x19+x20
    x21=r./(sqrt(ga.*da));
    x22=(da.*log(r))./log(am);
    x23=th.*da;
    x24=c3.*besseli(0,x21)+c4.*besselk(0,x21);
    u2=x24+x22-x23
    uf=u1.*heaviside(d-r);
    up=u2.*heaviside(r-d);
    u=uf+up;
    x25=f11.*log(d)+(4.*f1.*f11.*log(am))+(4.*f13.*log(am));
    x26=f12+(f11.*log(d).*(1.-(0.5.*d.*d)));
    t0=-x25./x26;
    [C,h]=contour(r,u,th);
set(h,'showText','on','textStep',get(h,'levelStep')*1);
colormap gray



%   convection in annulus partiallyfilled with porous material
     [da1,th]=meshgrid(1.0:1.0:3.0,0.0:0.2:1.0);
     w=(da1-da1+1).*(th-th+1);
     d=1.3.*w;
     da=(10.0).^-da1.*w;
     ga=1.0.*w;
     al=1.0.*w;
     am=1.8.*w;
     g=1./(sqrt(ga.*da));
```

```
      x1=((log(d)-1.).*d.*d)./(4.*log(am));
      x2=1./(4.*log(am));
      f1=x1+x2;
      x3=am.*g;
      x4=d.*g;
      f2=besselk(0,x3).*besseli(0,x4)-besselk(0,x4).*besseli(0,x3);
      f3=(da./f2).*(besseli(0,x4)-besseli(0,x3));
      f4=besseli(0,x3)./f2;
      f5=(da./f2).*(besselk(0,x4)-besselk(0,x3));
      f6=besselk(0,x3)./f2;
      x5=sqrt(ga./da);
      x6=al./sqrt(da);
      f7=x5.*besseli(1,x4)-x6.*besseli(0,x4);
      f8=x5.*besselk(1,x4)+x6.*besselk(0,x4);
      x7=al.*sqrt(da).*log(d);
      x8=log(am);
      x9=x7./x8;
      x10=-(da.*ga)./(d.*x8);
      x11=0.5.*d.*log(d)-(d./4.);
      x12=-x11./x8;
      x13=f1./(d.*log(d));
      f9=x9+x10+x12+x13;
      f10=-al.*sqrt(da)+(0.5.*d)-(1./(4.*log(d))).*((d.*d)-1.);
      x14=1./(d.*log(d));
      xx1=1./log(d);
      xx2=log(d)./log(am);
      sf5=-(da./f2).*(xx2.*besselk(0,x3)-besselk(0,x4));
      sf3=(da./f2).*(besseli(0,x4)-xx2.*besseli(0,x3));
      f11=f6.*f7+f4.*f8-x14;
      f12=f10+(f7.*f5)+(f3.*f8);
      f13=f9-sf5.*f7-sf3.*f8;
      ui=((th.*f12)+f13)./f11
% c1=((ui+f1)./log(d))-xx1.*th.*((d./2).*(d./2.)-(1./4.));
% x15=th./4.;
% x16=1./(4.*log(am));
%c2=-(x15+x16);
%c3=-th.*f5+(ui.*f6)+sf5;
%c4=-(ui.*f4)+(th.*f3)-sf3;
%x17=(r.*r)./4.;
%x18=x17.*th;
%x19=-(x17.*log(r)-x17)./log(am);
%x20=c1.*log(r)+c2;
%u1=x18+x19+x20
%x21=r./(sqrt(ga.*da));
%x22=(da.*log(r))./log(am);
%x23=th.*da;
%x24=c3.*besseli(0,x21)+c4.*besselk(0,x21);
%u2=x24+x22-x23
%uf=u1.*heaviside(d-r);
%up=u2.*heaviside(r-d);
%u=uf+up;
%x25=f11.*log(d)+(4.*f1.*f11.*log(am))+(4.*f13.*log(am));
%x26=f12+(f11.*log(d).*(1.-(0.5.*d.*d)));
%t0=-x25./x26;
      [C,h]=contour(th,ui,da);
   set(h,'showText','on','textStep',get(h,'levelStep')*1);
   colormap gray
```

```
   xlabel('(\theta_0)')
ylabel('Interface velocity ( U_i )')


%   convection in annulus partiallyfilled with porous material
%    critical value of thet at r=1
     [da1,d]=meshgrid(1.0:1.0:3.0,1.2:0.1:1.6);
      w=(da1-da1+1).*(d-d+1);
%d=1.3.*w;
      da=(10.0).^-da1.*w;
      ga=1.0.*w;
      al=1.0.*w;
      am=2.0.*w;
      g=1./(sqrt(ga.*da));
      x1=((log(d)-1.).*d.*d)./(4.*log(am));
      x2=1./(4.*log(am));
      f1=x1+x2;
      x3=am.*g;
      x4=d.*g;
      f2=besselk(0,x3).*besseli(0,x4)-besselk(0,x4).*besseli(0,x3);
      f3=(da./f2).*(besseli(0,x4)-besseli(0,x3));
      f4=besseli(0,x3)./f2;
      f5=(da./f2).*(besselk(0,x4)-besselk(0,x3));
      f6=besselk(0,x3)./f2;
      x5=sqrt(ga./da);
      x6=al./sqrt(da);
      f7=x5.*besseli(1,x4)-x6.*besseli(0,x4);
      f8=x5.*besselk(1,x4)+x6.*besselk(0,x4);
      x7=al.*sqrt(da).*log(d);
      x8=log(am);
      x9=x7./x8;
      x10=-(da.*ga)./(d.*x8);
      x11=0.5.*d.*log(d)-(d./4.);
      x12=-x11./x8;
      x13=f1./(d.*log(d));
      f9=x9+x10+x12+x13;
      f10=-al.*sqrt(da)+(0.5.*d)-(1./(4.*log(d))).*((d.*d)-1.);
      x14=1./(d.*log(d));
      xx1=1./log(d);
      xx2=log(d)./log(am);
      sf5=-(da./f2).*(xx2.*besselk(0,x3)-besselk(0,x4));
      sf3=(da./f2).*(besseli(0,x4)-xx2.*besseli(0,x3));
      f11=f6.*f7+f4.*f8-x14;
      f12=f10+(f7.*f5)+(f3.*f8);
      f13=f9-sf5.*f7-sf3.*f8;
%ui=((th.*f12)+f13)./f11;
%c1=((ui+f1)./log(d))-xx1.*th.*((d./2).*(d./2.)-(1./4.));
%x15=th./4.;
%x16=1./(4.*log(am));
%c2=-(x15+x16);
%c3=-th.*f5+(ui.*f6)+sf5;
%c4=-(ui.*f4)+(th.*f3)-sf3;
      cx7=log(d);
      x17=(f1./cx7)+(f13./(f11.*cx7))+(log(am)./4);
      x18=1.-0.5.*((d.*d)-1.)./(log(d));
      x19=2.*(f12./(cx7.*f11));
```

```
    x20=0.5.*(x18+x19);
    th0=-x17./x20
    [C,h]=contour(da,th0,d);
set(h,'showText','on','textStep',get(h,'levelStep')*1);
colormap gray



%   convection in annulus partiallyfilled with porous material
%    mass flux
    [da1,d]=meshgrid(1.0:1.0:3.0,1.2:0.1:1.6);
     w=(da1-da1+1).*(d-d+1);
     th=0.2.*w;
     da=(10.0).^-da1.*w;
     ga=1.0.*w;
     al=1.0.*w;
     am=2.0.*w;
     g=1./(sqrt(ga.*da));
     x1=((log(d)-1.).*d.*d)./(4.*log(am));
     x2=1./(4.*log(am));
     f1=x1+x2;
     x3=am.*g;
     x4=d.*g;
     f2=besselk(0,x3).*besseli(0,x4)-besselk(0,x4).*besseli(0,x3);
     f3=(da./f2).*(besseli(0,x4)-besseli(0,x3));
     f4=besseli(0,x3)./f2;
     f5=(da./f2).*(besselk(0,x4)-besselk(0,x3));
     f6=besselk(0,x3)./f2;
     x5=sqrt(ga./da);
     x6=al./sqrt(da);
     f7=x5.*besseli(1,x4)-x6.*besseli(0,x4);
     f8=x5.*besselk(1,x4)+x6.*besselk(0,x4);
     x7=al.*sqrt(da).*log(d);
     x8=log(am);
     x9=x7./x8;
     x10=-(da.*ga)./(d.*x8);
     x11=0.5.*d.*log(d)-(d./4.);
     x12=-x11./x8;
     x13=f1./(d.*log(d));
     f9=x9+x10+x12+x13;
     f10=-al.*sqrt(da)+(0.5.*d)-(1./(4.*log(d))).*((d.*d)-1.);
     x14=1./(d.*log(d));
     xx1=1./log(d);
     xx2=log(d)./log(am);
     sf5=-(da./f2).*(xx2.*besselk(0,x3)-besselk(0,x4));
     sf3=(da./f2).*(besseli(0,x4)-xx2.*besseli(0,x3));
     f11=f6.*f7+f4.*f8-x14;
     f12=f10+(f7.*f5)+(f3.*f8);
     f13=f9-sf5.*f7-sf3.*f8;
     ui=((th.*f12)+f13)./f11;
     c1=((ui+f1)./log(d))-xx1.*th.*((d./2).*(d./2.)-(1./4.));
     x15=th./4.;
     x16=1./(4.*log(am));
     c2=-(x15+x16);
     c3=-th.*f5+(ui.*f6)+sf5;
```

```
      c4=-(ui.*f4)+(th.*f3)-sf3;
%x17=(r.*r)./4.;
%x18=x17.*th;
%x19=-(x17.*log(r)-x17)./log(am);
%x20=c1.*log(r)+c2;
      x21=th.*((d.^4)-1.)./16;
      x22=((d.^4).*log(d)./4.)-(5./16).*((d.^4)-1.);
      x23=-x22./(4.*log(am));
      x24=(c1./4.).*((d.*d.*(2.*log(d)-1.))+1);
      x25=0.5.*c2.*((d.*d)-1.);
      qf1=x21+x23+x24+x25;
      x26=sqrt(ga.*da);
      x27=x26.*c3.*(am.*besseli(1,x3)-(d.*besseli(1,x4)));
      x28=-x26.*c4.*(am.*besselk(1,x3)-(d.*besselk(1,x4)));
      x29=da./log(am);
      x30=(am.*am.*(log(am)))-(d.*d.*(log(d)));
      x31=-(1./4.).*((am.*am)-(d.*d));
      x32=x29.*((0.5.*x30)+x31);
      x33=-(0.5.*da.*th).*((am.*am)-(d.*d));
      qf2=x27+x28+x32+x33;
      q=qf1+qf2
      [C,h]=contour(d,q,da);
   set(h,'showText','on','textStep',get(h,'levelStep')*1);
   colormap gray
```