A STUDY OF TIME DEPENDENT NATURAL CONVECTION FLOW IN A VERTICAL POROUS CHANNEL

By

TAFIDA, Mohammed Kabir

B.Sc. (UDUS, 2005)

MSC/SCIE/2341/2008-2009

DEPARTMENT OF MATHEMATICS

AHMADU BELLO UNIVERSITY, ZARIA,

NIGERIA

DECEMBER, 2014

A STUDY OF TIME DEPENDENT NATURAL CONVECTION FLOW IN A VERTICAL POROUS CHANNEL

By

TAFIDA, Mohammed Kabir

B.Sc. (UDUS, 2005)

MSC/SCIE/2341/2008-2009

A THESIS SUBMITTED TO THE

POSTGRADUATE SCHOOL,

AHMADU BELLO UNIVERSITY, ZARIA,

NIGERIA

IN PARTIAL FULFILMENT FOR THE AWARD

OF MASTER OF SCIENCE IN

MATHEMATICS

DEPARTMENT OF MATHEMATICS

AHMADU BELLO UNIVERSITY, ZARIA,

NIGERIA

DECEMBER, 2014

Declaration

I declare that the work in this thesis entitled ”A STUDY OF TIME DEPENDENT NATURAL CONVECTION FLOW IN A VERTICAL POROUS CHANNEL” has been performed by me in the Department of Mathematics under the supervision of Dr. A. O. Ajibade and Prof. B. K. Jha. The information derived from the literature has been duly acknowledged in the text and a list of references provided. No part of this thesis was previously presented for another degree or diploma at any university.

TAFIDA, Mohammed Kabir

Name of student	Signature	Date

Certification

This thesis entitled "A STUDY OF TIME DEPENDENT NATURAL CONVECTION FLOW IN A VERTICAL POROUS CHANNEL" by TAFIDA, Mohammed Kabir, meets the regulations governing the award of the degree of Master of Science of Ahmadu Bello University, Zaria, and is approved for its contribution to knowledge and literary presentation.

__________________________ ____________
Dr. A. O. Ajibade Date

Chairman Supervisory Committee

__________________________ ____________
Prof. B. K. Jha Date

Member Supervisory Committee

__________________________ ____________
Dr. B. Sani Date

Head of Department

__________________________ ____________
Prof. I. J. Uwanta Date

External Examiner

__________________________ ____________
Prof. A. H. ZOAKA Date

Dean, School of Postgraduate Studies

Dedication

This thesis is dedicated to Almighty ALLAH (S.W.T). And to my lovely Parents: Late Alhaji Muhammad Tafida Isah and Hajiya Hassana Halliru Amfani who the root of my being in the academic line with all tireless effort and help they have given to me morally and financially. May ALLAH (S.W.T) bestow on them His everlasting blessing in this world and the best of all in the here after. May He also forgive them their shortcomings before, now and in advance. Ameen.

Acknowledgement

In the name of Allah, most merciful and most compassionate. First of all I would like to acknowledge the contribution of my supervisor Dr. A. O. Ajibade, whose ideas, guidance, patience and support have been essential at all stages. I would like to thank him for giving me an insight into his perception of Mathematics, for hours of mathematical and non-mathematical conversations and for careful reading of the draft of this thesis. My sincere thanks are due to my second supervisor Prof. Basant. K. Jha, for his many advices and encouragement.

Next, I would like to thank Professor D. Singh, Dr. B. Sani, Professor A. A. Tijjani, Professor J. Singh, Professor S. B. Junaidu, Dr. A. Yahaya, Dr. A. Mohammed, Dr. A. M. Ibrahim, Mal. A. I. Fulatan, Mal. Y. M. Baraya, Dr. H. M. Jibril, Dr. H. G. Dikko, Mal. A. Alkali, Dr. A. A. Obiniyi, Dr. A. Abdulraheem, Mrs. M. I. Yakubu, Mrs. A. Umar, who gave me the mathematical training needed to survive in the world of Mathematics.

I also like to thank Mr. Joseph, Bakut Sylvester for the most enjoyable time I have had working with him and for providing me with a number of relevant materials. I sincerely hope that this is not the end of our cooperation but merely a stage of it.

I would like to thank my friends and colleagues who have made positive effects in my mathematical and private life. I would especially like to mention Mu'azu Mohammed Tafida, Ayuba Umar Muhammad, Bello Dauda, Dr. Ibrahim Muhammad Sani Gumi, Musa Balarabe, Imam Abdussamad Tanko, Mal. S. Ado, Mal. Mustapha Shettima, Mal. Ja'afar Aliyu, Mal. Sani Sa'idu, Mal. A. T. Rabi'u, Mal. Umar, I. Ohimege, Mal. Alhassan Nalado (DAC A.B.U), Lawal Ahmad (Lawwali Maths), Mal. Umar Ibrahim Haruna, Mal. Ibrahim Abubakar, Mal.

Bashir Balarabe, Mal Haruna Muhammad, Mr. Luka M, Mal A. Zakariyya, Mal. Iliyasu Adamu, Mal. Usman Umar Aliyu.

I also wish to express my obligation to mention the people that I have lived with over the years: My uncle Prof. A. H. Amfani, my brothers and Sisters, Ibrahim, Yusuf, Mahrazu, Bashir, Rilwan, Hamza, Shehu, Ilyasu and Umar, Rukayya, Zuwaira, Harira, Rabi'at, Zulaihat, and my entire family Safiyyat, Maryam, Aishat, Fatima, Abdullahi, Sumayya and Muhammad Auwal Albany (Junior). I have had an extraordinary fortune that these are also the people that I have loved and love most. I am grateful to them for standing by me even in the most difficult moments in my life, and for bearing with me when I am into mathematics. I wish I could give them something they could enjoy reading better than this thesis.

Abstract

The transient motion of a viscous incompressible fluid in vertical porous channel is first analysed considering the effect of transpiration and heat generation in the channel. In the second part of the problem, natural convection flow of an incompressible viscous fluid in a vertical channel is analysed when the porous plates are subjected to isothermal and adiabatic conditions. The governing equations are solved using the Laplace transform method. Due to the difficulties that are associated with the analytical inversion of the obtained results from the Laplace to the time domain, the Riemann-Sum approach is employed to do the inversion, thus obtaining the temperature, velocity, skin-friction and rate of heat transfer. To validate the Riemann-sum approach used, the steady state solution for velocity and temperature are obtained analytically. An excellent agreement is found between the steady state solutions and the transient solutions at large values of time. From the course of investigation, it is revealed that heat generation leads to an increase in heat transfer on the cold plate while it decreases it on the heated plate.

Contents

LIST OF FIGURES

Figures 3.1 and 3.2 show the geometry of the problems.

Figures 4.1 to 4.30 show the velocity field, temperature field, Skin-friction and Rate of heat transfer as functions of the physical parameters.

LIST OF TABLES

LIST OF APPENDICES

NOMENCLATURE AND GREEK LETTERS

Nomenclature

g - acceleration due to gravity

h - width of the channel

H - dimensionless heat sink parameter

k - thermal conductivity of fluid

Nu_0 - Rate of heat transfer at y=0

Nu_1 - Rate of heat transfer at y=1

Pr - Prandtl number

$t^{'}$ - dimensional time

t - dimensionless time

$T^{'}$ - dimensional fluid temperature

$T_w^{'}$ - channel wall temperature $(t^{'} > 0)$

$T_0^{'}$ - initial temperature of fluid at$(t^{'} \leq 0)$

T - dimensionless fluid temperature

$u^{'}$ - dimensional velocity

u - dimensionless velocity

$y^{'}$ - co-ordinate perpendicular to the plate

y - dimensionless co-ordinate perpendicular to the plate

GREEK LETTERS

τ_0 - Skin-friction at y=0

τ_1 - Skin-friction at y=1

β - coefficient of thermal expansion

μ - coefficient of viscosity

ν - kinematic viscosity

CHAPTER 1

GENERAL INTRODUCTION

1.1 Introduction

Fluid flows are of vital interest in almost every sphere of life. This is completely attributed to its wide applications in various fields. These applications are found in the field of Engineering, Geophysical, environmental dynamics, architectural designs and Biological Sciences for better understanding of researches and applications. Fluid through vertical channel is of great importance in many technological flows. Examples of these are found in the design of nuclear reactors, cooling of electronic equipment, heating of building through Trombe walls,ventilating systems, cooling and distillation towers, solar energy systems and many chemical processes. Others are found in the area of solid matrix heat exchanger, nuclear waste disposal and thermal insulations.

In other instances, the natural convection flow driven by internal heat generation/absorption has been of great interest. This interest stems from various possible applications of this type of flow in many process. These include cooling of nuclear reactors and convection in the earth mantle-Mckenzie *et al.* [1974]. Furthermore, perhaps the most wide spread application is in the field of nuclear energy such as in nuclear-reactor core and in post-accident heat removal-

Baker *et al* [1976] and Smith and Hammitt [1996]. Natural convection driven by internal heat generation/absorption, plays an important role in overall heat transfer. Natural convection with internal heat generation also applies to fire and combustion modelling, Delichtsios [1988], the development of metal waste from spent nuclear fuel Westphal *et al.* [1994], and for the storage of spent nuclear fuel.

1.2 Motivation

Due to its wide applications in various disciplines, it has become very imperative to understand the behaviour of a transient natural convection in a vertical channel and natural convection flow of heat generating fluid in a vertical channel with isothermal and adiabatic conditions. This thesis extends the work of Jha and Ajibade [2010] where the effects of suction/injection as well as internal heat generation are absent. However, several fluids of interest exist with heat generation of which results are not applicable. Therefore, we make our channel plates porous so that we have suction/injection effects on the fluid flow.

1.3 Aim and Objectives of the study

The aim of this study is to investigate the effect of transpiration and heat generation on transient natural convection flow in a vertical channel.

To achieve the aim, we set the following objectives

i. Investigate the effect of transpiration and heat generation fluid in an asymmetrical vertical channel

ii. Investigate the effect of transpiration and heat generation of fluid with isothermal and adiabatic boundary conditions.

1.4 Methodology

To achieve the above objectives, the mathematical problems in both cases were formulated and solved analytically, while for steady natural convection flow, the mathematical problem is also solved analytically. Due to the difficulties that are associated with the analytical inversion of the obtained results from the Laplace to the time domain, the Riemann-Sum approach is employed to do the inversion. With the aid of MATLAB programs the numerical values of velocity field, temperature field, skin-friction and rate of heat transfer are obtained and presented graphically in order to see the effects of controlling physical parameters.

1.5 Definition of Basic terms

(1) **Boussinesq approximation:** Is the assumption that the fluid flow is considered under little variations of temperature and density.

(2) **Free or Natural convection:** When fluid flow is induced by different fluid densities which are due to temperature change, the flow is called free or natural convection.

(3) **Prandtl number** (Pr)**:** Is the ratio of momentum to thermal diffusivities and is a function only of fluid properties. Mathematically, Prandtl number is given by the expression $Pr = \frac{\mu C_p}{k}$

(4) **Nusselt number (Nu):** Is the dimensionless rate of heat transfer at the 'wall-fluid' boundary.

(5) **Skin friction (τ):** Is the friction that occurs between the fluid and the solid surface.

(6) **Suction/injection:** Is the increase/decrease of fluid flowing through the wall.

(7) **Heat sources/sinks:** Is the increase/decrease of heat of the fluid in the channel as generated or absorbed by the fluid.

(8) **Convection:** Is the process in which heat is transferred by movement of a heated fluid.

(9) **Radiation:** Is the transfer of energy from a source in the form of rays or waves.

(10) **Porous:** Is the permission of the movement of fluid through a solid surface by way of pores.

(11) **Dimensionless quantity:** Is a quantity without an associated physical dimension.

1.6 Basic Equations

(1) **Continuity equation**

$\frac{\partial \rho}{\partial t} + (\nabla.\rho\overline{U}) = 0$

Since ρ is constant

$(\nabla.\overline{U}) = 0$

(2) **Equation of motion**

$\rho\frac{D\overline{U}}{Dt} = -\nabla p - (\nabla.\tau) + \rho g$

(3) **Equation of energy**

$\rho\widehat{C}_{\rho}\frac{DT}{Dt} = k\nabla^{2}T + \mu\Phi_{v}$

1.7 Organisation of the thesis

In chapter one, the general introduction of the thesis was taken into consideration. The second chapter is the literature review. Mathematical analysis and the solution methods are discussed in chapter three. Chapter four is the discussion of the results of all graphs and chapter five is the summary, conclusions and direction of research. References and appendices follow after.

CHAPTER 2

LITERATURE REVIEW

2.1 Heat Generation/Absorption

A large number of physical phenomena involve natural convection driven by heat generation. The study of heat generation in moving fluids is important in several physical problems dealing with chemical reactions and those concerned with dissociating fluids. Possible heat generation effects may alter the temperature distribution and therefore, the particle deposition rate. In addition, understanding of the effects of internal heat generation is also significant in numerous applications that include reactor safety analysis, metal waste, spent nuclear fuel, fire and combustion studies and strength of radioactive materials (Postelnicu and Pop [1999]). Foraboschi and Federico [1964] investigated steady- state temperature profiles for linear parabolic and piston flow in circular tubes. Vajravelu and Hadjinicolaou [1993] studied the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a linearly stretching continuous surface with viscous dissipation or frictional heating and internal heat generation. Chamkha and Camille [2000] solved hydro magnetic flow with heat and mass transfer over a flat plate in the presence of heat generation or absorption and thermophoresis. Mendez and Trevino [2000] analyzed the effects of the conjugate conduction natural convection heat transfer along a thin vertical plate with non-uniform heat generation. Continuing the work of Vajravelu and Hadjinicolaou [1993], natural convection with heat generation along a uniformly heat vertical wavy surface have been demonstrated by Molla *et al.* [2004]. Besides that, Mohammadein and Gorla [2001], Rahman et al. [2009] and Magyari and Chamkha [2010] take into account the effect of heat generation to investigate the characteristics of heat and mass transfer in a micropolar fluid flow. Then, Ferdousi and Alim [2010] considered the effect of heat generation on natural convection flow from a porous vertical plate. Veena et al. [2006] worked on heat transfer characteristics in the laminar boundary layer flow of a viscoelastic fluid over a linearly stretching continuous surface with variable wall temperature subjected to suction or blowing. Molla *et al.* [2009] examined the natural convection flow of a viscous incompressible fluid past an isothermal horizontal circular cylinder considering the temperature dependent internal heat generation. Mahdy [2010] considered the effects of chemical reaction and heat generation on double-diffusive natural convection heat and mass transfer near a vertical truncated cone in porous media. Afterwards, Siddiqa *et al.* [2010] studied natural convection flow of a viscous incompressible fluid over a semi-infinite flat plate with the effects of exponentially varying temperature dependent viscosity and the internal heat generation.

2.2 Suction/Injection

The significance of suction/injection on the boundary layer control in the field of aerodynamics and space science is well recognized by Singh [1984]. In an article, Ishak *et al.* [2008] showed that suction/injection of a fluid through the bounding surface as for example in mass transfer cooling, can significantly change the flow field and, as a consequence, affect the heat transfer rate from the plate. In another article, Al-sanea [2004] pointed out that in general, suction tends to increase the skin-friction and heat transfer coefficient whereas injection acts in the opposite manner. This is well explored by Shojaeford *et al.* [2005], where suction/injection were used to control fluid flow on the surface of subsonic aircraft. Jha and Ajibade [2010] investigated transient natural convection flow between vertical parallel plates: one plate isothermally heated and other thermally insulated. Recently, Jha and Ajibade [2009] considered the case of a free convective flow of heat generation/absorbing fluid between vertical parallel porous plates due to periodic heating on the porous plates. An analysis of the transient motion of a viscous incompressible fluid between two infinite vertical parallel plates due to natural convection currents occurring as a result of application of isothermal and adiabatic conditions on the plates was investigated by Jha and Ajibade [2010]. Hossain *et al.* [2000] investigated the natural convection flow past a permeable wedge for the fluid having temperature dependent viscosity and thermal conductivity. Kabir *et al.* [2002] have studied the problem of natural convection of fluid with temperature dependent viscosity along a heated vertical wavy surface.

CHAPTER 3

THE PROBLEMS UNDER CONSIDERATION

3.1 The effect of transpiration and heat generation on transient natural convection in a vertical channel

In this problem, the transient natural convection of an incompressible viscous fluid in a vertical channel of width h is considered as shown in figure 3.1.

3.1.1 Mathematical analysis

Consider the effect of transpiration and heat generation of transient natural convection of an incompressible viscous fluid in a vertical channel of width h. The $x^{'}$ -axis is taken along one of the porous channel walls while the $y^{'}$ -axis is normal to it. The fluid was initially static and at the same temperature $T_0^{'}$ with the walls of the channel. At time $t^{'} > 0$, the temperature of the wall $y^{'} = 0$ becomes $T = T_w^{'}$ while the wall $y^{'} = h$ was kept at the temperature $T = T_0^{'}$. Natural convection sets in due to the temperature gradient. Because of the viscosity of the fluid consideration, the velocity of the fluid at both walls of the channel remains $u^{'} = 0$. Figure 3.1 presents the geometry of the problem.

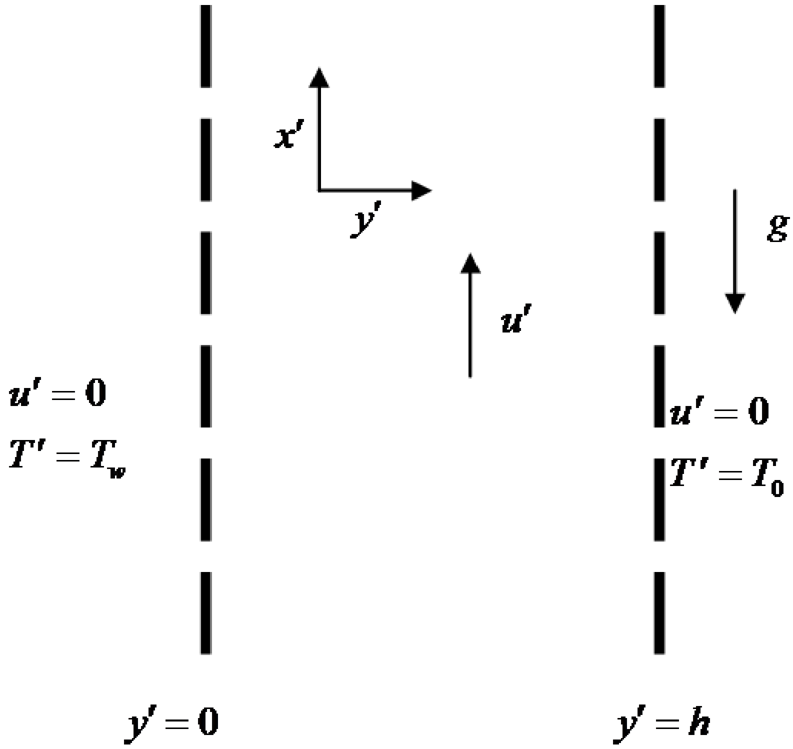

Figure 3.1: Schematic diagram

By the use of Boussinesq's approximation, the governing equations for the flow problem in dimensionless form are:

$$\frac{\partial u}{\partial t} - q\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + T \tag{3.1}$$

$$\frac{\partial T}{\partial t} - q\frac{\partial T}{\partial y} = \frac{1}{Pr}\frac{\partial^2 T}{\partial y^2} - \frac{H}{Pr}T \tag{3.2}$$

The non-dimensional quantities introduced in the above equations are defined as:

$$t = \frac{t'\nu}{h^2}, y = \frac{y'}{h}, T = \frac{T' - T_0'}{T_w' - T_0'}, Pr = \frac{\mu C_p}{k}$$

$$u = \frac{u'\mu}{g\beta h^2(T_w' - T_0')}, H = \frac{Q_0 h^2}{k}, \tag{3.3}$$

The physical quantities used in the above equations are defined in the nomenclature. The initial conditions for velocity and temperature field in dimensionless form are:

$$u = (0, y) = T(0, y) = 0 \tag{3.4}$$

while the boundary conditions in dimensionless form is given as

$$t > 0 : \begin{cases} u = 0, T = 1 \quad at \quad y = 0 \\ u = 0, T = 0 \quad at \quad y = 1 \end{cases} \tag{3.5}$$

Introducing the variables

$\overline{u}(y, s) = \int_0^\infty u(y, t)exp(-st)dt$

$\overline{T}(y, s) = \int_0^\infty T(y, t)exp(-st)dt$ where $s > 0$

Equations (3.1) and (3.2) are transformed into the Laplace domain using the initial condition (3.4) to

$$\frac{d^2\overline{u}}{dy^2} + q\frac{d\overline{u}}{dy} - s\overline{u} = -\overline{T} \tag{3.6}$$

$$\frac{d^2\overline{T}}{dy^2} + \delta\frac{d\overline{T}}{dy} - P\overline{T} = 0 \tag{3.7}$$

where, $\delta = qPr$ and $P = (sPr + H)$

The boundary conditions (3.5) are also transformed into

$$\overline{u} = 0, \overline{T} = \frac{1}{s} \quad at \quad y = 0$$

$$\overline{u} = 0, \overline{T} = 0 \quad at \quad y = 1 \tag{3.8}$$

3.1.2 Solution of problem 3.1 in Laplace domain

Using the method of undetermined coefficients, the solution of equations (3.6) and (3.7) under the boundary condition (3.8) can be written as

$$\overline{T} = c_1 exp\left[y\left(\lambda - \frac{qPr}{2}\right)\right] + c_2 exp\left[-y\left(\lambda + \frac{qPr}{2}\right)\right] \tag{3.9}$$

$$\overline{u} = d_1 exp\left[y\left(\gamma - \frac{q}{2}\right)\right] + d_2 exp\left[-y\left(\gamma + \frac{q}{2}\right)\right] + d_3 exp\left[y\left(\lambda - \frac{qPr}{2}\right)\right] + d_4 exp\left[-y\left(\lambda + \frac{qPr}{2}\right)\right] \tag{3.10}$$

where

$$c_1 = \frac{exp(-2\lambda)}{s[1-exp(-2\lambda)]}$$

$$c_2 = \frac{1}{s[1-exp(-2\lambda)]}$$

$$d_1 = -(d_2 + d_3 + d_4)$$

$$d_2 = \frac{(d_3+d_4)exp\left(\gamma-\frac{q}{2}\right)-d_3exp\left(\lambda-\frac{qPr}{2}\right)-d_4exp\left[-\left(\lambda+\frac{qPr}{2}\right)\right]}{exp\left[-\left(\gamma+\frac{q}{2}\right)\right]-exp\left(\gamma-\frac{q}{2}\right)}$$

$$d_3 = \frac{-c_1}{\left(\lambda-\frac{qPr}{2}\right)^2+q\left(\lambda-\frac{qPr}{2}\right)-s}$$

$$d_4 = \frac{-c_2}{\left(\lambda+\frac{qPr}{2}\right)^2-q\left(\lambda+\frac{qPr}{2}\right)-s}$$

$$\lambda = \sqrt{\frac{\delta^2}{4} + P}$$

$$\gamma = \sqrt{\frac{q^2}{4} + s}$$

By the use of equations (3.9) and (3.10), the skin-friction and the rate of heat transfer on the heated wall ($y = 0$) are respectively

$$\tau_0 = \frac{d\overline{u}}{dy}|_{y=0} = d_1\left(\gamma - \frac{q}{2}\right) - d_2\left(\gamma + \frac{q}{2}\right) + d_3\left(\lambda - \frac{qPr}{2}\right) - d_4\left(\lambda + \frac{qPr}{2}\right) \quad (3.11)$$

$$Nu_0 = \frac{d\overline{T}}{dy}|_{y=0} = c_1\left(\lambda - \frac{qPr}{2}\right) - c_2\left(\lambda + \frac{qPr}{2}\right) \quad (3.12)$$

while on the wall $y = 1$, the skin-friction and the rate of heat transfer are respectively

$$\tau_1 = \frac{d\overline{u}}{dy}|_{y=1} = d_1\left(\gamma - \frac{q}{2}\right) exp\left(\gamma - \frac{q}{2}\right) - d_2\left(\gamma + \frac{q}{2}\right) exp\left[-\left(\gamma + \frac{q}{2}\right)\right]$$

$$+d_3\left(\lambda - \frac{qPr}{2}\right) exp\left(\lambda - \frac{qPr}{2}\right) - d_4\left(\lambda + \frac{qPr}{2}\right) exp\left[-\left(\lambda + \frac{qPr}{2}\right)\right] \quad (3.13)$$

$$Nu_1 = \frac{d\overline{T}}{dy}|_{y=1} = c_1\left(\lambda - \frac{qPr}{2}\right) exp\left(\lambda - \frac{qPr}{2}\right) - c_2\left(\lambda + \frac{qPr}{2}\right) exp\left[-\left(\lambda + \frac{qPr}{2}\right)\right] \quad (3.14)$$

3.1.3 Steady state problem 3.1

To validate the Riemann-sum approximation used, the steady-state solutions for velocity and temperature fields are derived by taking $\frac{\partial()}{\partial t} = 0$ in the equations (3.1) and (3.2) reduce to

$$\frac{d^2u}{dy^2} + q\frac{du}{dy} = -T \quad (3.15)$$

$$\frac{d^2T}{dy^2} + \delta\frac{dT}{dy} - HT = 0 \quad (3.16)$$

3.1.4 Solution of Steady state of problem 3.1

The steady state equations (3.15) and (3.16) are solved analytically under the boundary conditions (3.5) to obtain the following steady-state solutions.

$$\widehat{T} = F_1exp(my) + F_2exp(ny) \quad (3.17)$$

$$\widehat{u} = \frac{-F_1exp(my)}{m(m+q)} - \frac{F_2exp(ny)}{n(n+q)} + \frac{F_3}{q} + F_4exp(-qy) \quad (3.18)$$

$$\widehat{\tau}_0 = \frac{d\widehat{u}}{dy}|_{y=0} = \frac{-mF_1}{m(m+q)} - \frac{nF_2}{n(n+q)} - qF_4 \quad (3.19)$$

$$\widehat{N}u_0 = \frac{d\overline{T}}{dy}|_{y=0} = mF_1 + nF_2 \quad (3.20)$$

$$\widehat{\tau}_1 = \frac{d\widehat{u}}{dy}|_{y=1} = \frac{-mF_1exp(m)}{m(m+q)} - \frac{nF_2exp(n)}{n(n+q)} - qF_4exp(-q) \quad (3.21)$$

$$\widehat{N}u_1 = \frac{d\widehat{T}}{dy}|_{y=1} = mF_1exp(m) + nF_2exp(n) \quad (3.22)$$

where

$$F_1 = \frac{exp(n)}{exp(m) - exp(n)}$$

$$F_2 = \frac{-exp(m)}{exp(m)-exp(n)}$$

$$F_3 = \frac{F_1 q exp(m)}{m(m+q)} + \frac{F_2 q exp(n)}{n(n+q)} - F_4 q exp(-q)$$

$$F_4 = \frac{F_1(1-exp(m))}{m(m+q)(1-exp(-q))} + \frac{F_2(1-exp(n))}{n(n+q)(1-exp(-q))}$$

$$m = \frac{-Prq+\sqrt{(Prq)^2+4H}}{2}$$

$$n = \frac{-Prq-\sqrt{(Prq)^2+4H}}{2}$$

3.2 Effect of transpiration on natural convection flow of heat generating fluid in a vertical channel with isothermal and adiabatic conditions

The present work considered the effect of transpiration on natural convection flow of heat generating incompressible viscous fluid in a vertical channel formed by two parallel porous plates, having isothermal and adiabatic boundary conditions. The $x^{'}$ -axis is taken along one of the porous channel walls while the $y^{'}$ -axis is normal to it. The fluid was initially static and at the same temperature $T_0^{'}$ with the walls of the channel. At time $t > 0$, the temperature of the wall $y^{'} = h$ becomes $T^{'} = T_w^{'}$ while the wall $y^{'} = 0$ was kept insulated (see fig 3.2). Natural convection sets in due to the temperature gradient. Because of the viscosity of the fluid consideration, the velocity of the fluid at both walls of the channel remains $u^{'} = 0$.

Figure 3.2 shows the geometry of the problem.

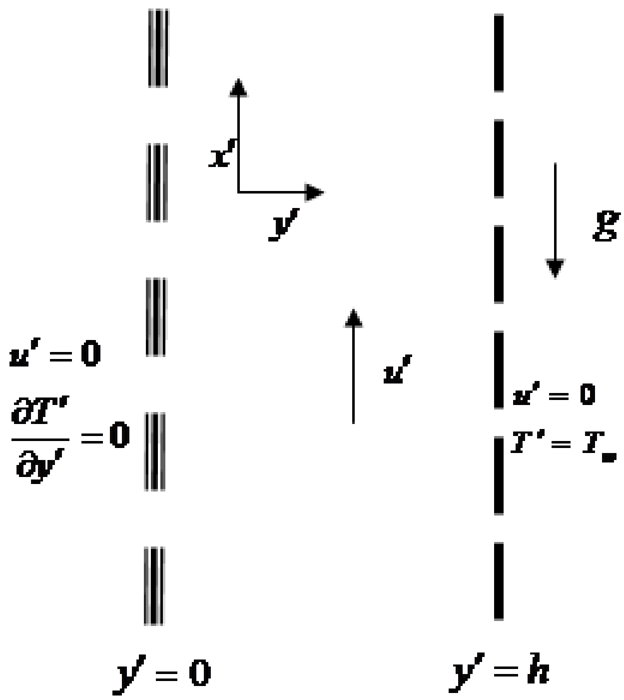

Figure 3.2: Schematic diagram

By the use of Boussinesq's approximation, the governing equations for the flow problem in dimensionless form are:

$$\frac{\partial u}{\partial t} - q\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + T \tag{3.23}$$

$$\frac{\partial T}{\partial t} - q\frac{\partial T}{\partial y} = \frac{1}{Pr}\frac{\partial^2 T}{\partial y^2} - \frac{H}{Pr}T \tag{3.24}$$

The non-dimensional quantities introduced in the above equations are defined as:

$$t = \frac{t'\nu}{h^2}, y = \frac{y'}{h}, T = \frac{T' - T_0'}{T_w' - T_0'}, Pr = \frac{\mu C_p}{k}$$

$$u = \frac{u'\mu}{g\beta h^2(T_w' - T_0')}, H = \frac{Q_0 h^2}{k}, \tag{3.25}$$

The physical quantities used in the above equations are defined in the nomenclature.

The initial conditions for velocity and temperature field in dimensionless form are:

$$u = (0, y) = T(0, y) = 0, \quad 0 \leq y \leq 1 \tag{3.26}$$

while the boundary conditions in dimensionless form is given as

$$t > 0 : \begin{cases} u = 0, \frac{\partial T}{\partial y} = 0 \quad at \quad y = 0 \\ u = 0, T = 1 \quad at \quad y = 1 \end{cases} \tag{3.27}$$

Introducing the variables

$\overline{u}(y, s) = \int_0^\infty u(y, t) exp(-st)dt$

$\overline{T}(y, s) = \int_0^\infty T(y, t) exp(-st)dt$ where $s > 0$

Equations (3.23) and (3.24) are transformed into the Laplace domain using the initial condition (3.26) to

$$\frac{d^2\overline{u}}{dy^2} + q\frac{d\overline{u}}{dy} - s\overline{u} = -\overline{T} \tag{3.28}$$

$$\frac{d^2\overline{T}}{dy^2} + \delta\frac{d\overline{T}}{dy} - P\overline{T} = 0 \tag{3.29}$$

where, $\delta = qPr$ and $P = (sPr + H)$

The boundary conditions (3.27) becomes

$$\overline{u} = 0, \frac{d\overline{T}}{dy} = 0 \quad at \quad y = 0$$

$$\overline{u} = 0, \overline{T} = \frac{1}{s} \quad at \quad y = 1 \tag{3.30}$$

3.2.1 Solution of problem 3.2 in Laplace domain

Using the method of undetermined coefficients, the solution of equations (3.25) and (3.29) under the boundary condition (3.30) can be written as

$$\overline{T} = c_1 exp\left[y\left(\lambda - \frac{qPr}{2}\right)\right] + c_2 exp\left[-y\left(\lambda + \frac{qPr}{2}\right)\right] \tag{3.31}$$

$$\overline{u} = c_3 exp\left[y\left(\gamma - \frac{q}{2}\right)\right] + c_4 exp\left[-y\left(\gamma + \frac{q}{2}\right)\right] + c_5 exp\left[y\left(\lambda - \frac{qPr}{2}\right)\right] + c_6 exp\left[-y\left(\lambda + \frac{qPr}{2}\right)\right] \quad (3.32)$$

where

$$c_1 = \frac{\left(\lambda+\frac{qPr}{2}\right)}{s\left[\left(\lambda+\frac{qPr}{2}\right)exp\left(\lambda-\frac{qPr}{2}\right)+\left(\lambda-\frac{qPr}{2}\right)exp\left[-\left(\lambda+\frac{qPr}{2}\right)\right]\right]}$$

$$c_2 = \frac{c_1\left(\lambda-\frac{qPr}{2}\right)}{\left(\lambda+\frac{qPr}{2}\right)}$$

$$c_3 = -(c_4 + c_5 + c_6)$$

$$c_4 = \frac{(c_5+c_6)exp\left(\gamma-\frac{q}{2}\right)-c_5 exp\left(\lambda-\frac{qPr}{2}\right)-c_6 exp\left[-\left(\lambda+\frac{qPr}{2}\right)\right]}{exp\left[-\left(\gamma+\frac{q}{2}\right)\right]-exp\left(\gamma-\frac{q}{2}\right)}$$

$$c_5 = \frac{-c_1}{\left(\lambda-\frac{qPr}{2}\right)^2+q\left(\lambda-\frac{qPr}{2}\right)-s}$$

$$c_6 = \frac{-c_2}{\left(\lambda+\frac{qPr}{2}\right)^2-q\left(\lambda+\frac{qPr}{2}\right)-s}$$

$$\lambda = \sqrt{\frac{\delta^2}{4} + P}$$

$$\gamma = \sqrt{\frac{q^2}{4} + s}$$

By the use of equations (3.31) and (3.32), the skin-friction at the adiabatic wall can be written as

$$\tau_0 = \frac{d\overline{u}}{dy}|_{y=0} = c_3\left(\gamma - \frac{q}{2}\right) - c_4\left(\gamma + \frac{q}{2}\right) + c_5\left(\lambda - \frac{qPr}{2}\right) - c_6\left(\lambda + \frac{qPr}{2}\right) \quad (3.33)$$

while at the isothermal wall, the skin-friction and the rate of heat transfer are respectively

$$\tau_1 = \frac{d\overline{u}}{dy}|_{y=1} = c_3\left(\gamma - \frac{q}{2}\right) exp\left(\gamma - \frac{q}{2}\right) - c_4\left(\gamma + \frac{q}{2}\right) exp\left[-\left(\gamma + \frac{q}{2}\right)\right]$$
$$+c_5\left(\lambda - \frac{qPr}{2}\right) exp\left(\lambda - \frac{qPr}{2}\right) - c_6\left(\lambda + \frac{qPr}{2}\right) exp\left[-\left(\lambda + \frac{qPr}{2}\right)\right] \quad (3.34)$$
$$Nu_1 = \frac{d\overline{T}}{dy}|_{y=1} = c_1\left(\lambda - \frac{qPr}{2}\right) exp\left(\lambda - \frac{qPr}{2}\right) - c_2\left(\lambda + \frac{qPr}{2}\right) exp\left[-\left(\lambda + \frac{qPr}{2}\right)\right] \quad (3.35)$$

3.2.2 Steady state of problem 3.2

To validate the Riemann-sum approximation used, the steady-state solutions for velocity and temperature fields are derived by taking $\frac{\partial()}{\partial t} = 0$ in the equations (3.23) and (3.24) which reduces to

$$\frac{d^2u}{dy^2} + q\frac{du}{dy} = -T \quad (3.36)$$
$$\frac{d^2T}{dy^2} + \delta\frac{dT}{dy} - HT = 0 \quad (3.37)$$

3.2.3 Steady state solution of problem 3.2

These are solved analytically under the boundary conditions (3.27) to obtain the following steady-state solutions.

$$\widehat{T} = f_1 exp(my) + f_2 exp(ny) \quad (3.38)$$
$$\widehat{u} = \frac{-f_1 exp(my)}{m(m+q)} - \frac{f_2 exp(ny)}{n(n+q)} + \frac{f_3}{q} + f_4 exp(-qy) \quad (3.39)$$
$$\widehat{\tau_0} = \frac{d\widehat{u}}{dy}|_{y=0} = \frac{mf_1}{m(m+q)} - \frac{nf_2}{n(n+q)} - qf_4 \quad (3.40)$$
$$\widehat{\tau_1} = \frac{d\widehat{u}}{dy}|_{y=1} = \frac{mf_1 exp(m)}{m(m+q)} - \frac{nf_2 exp(n)}{n(n+q)} - qf_4 exp(-q) \quad (3.41)$$
$$\widehat{N}u_1 = \frac{d\widehat{T}}{dy}|_{y=1} = mf_1 exp(m) + nf_2 exp(n) \quad (3.42)$$

where

$$f_1 = \frac{n}{nexp(m) - mexp(n)}$$

$$f_2 = \frac{-m}{nexp(m) - mexp(n)}$$

$$f_3 = \frac{f_1 qexp(m)}{m(m+q)} + \frac{f_2 qexp(n)}{n(n+q)} - f_4 qexp(-q)$$

$$f_4 = \frac{f_1(1 - exp(m))}{m(m+q)(1 - exp(-q))} + \frac{f_2(1 - exp(n))}{n(n+q)(1 - exp(-q))}$$

$$m = \frac{-Prq + \sqrt{(Prq)^2 + 4H}}{2}$$

$$n = \frac{-Prq - \sqrt{(Prq)^2 + 4H}}{2}$$

3.3 Riemann-sum inversion of Laplace transform functions

To obtain the temperature, velocity, Nusselt number and skin-friction in the time domain, it is necessary to invert the obtained results (3.9) to (3.14) and (3.31) to (3.35) by any of the known method. However, due to the bottlenecks involved in the analytical inversion, we use the Riemann-sum approximations, a method based on a numerical technique for the inversion as required.

$$u(y,t) = \frac{\exp(\varepsilon t)}{t}\left[\frac{1}{2}\overline{u}(y,\varepsilon) + Re\sum_{n=1}^{n}\overline{U}\left(y,\varepsilon+\frac{in\pi}{t}\right)(-1)^n\right] \tag{3.43}$$

$$T(y,t) = \frac{\exp(\varepsilon t)}{t}\left[\frac{1}{2}\overline{T}(y,\varepsilon) + Re\sum_{n=1}^{n}\overline{T}\left(y,\varepsilon+\frac{in\pi}{t}\right)(-1)^n\right] \tag{3.44}$$

$$\tau(y,t) = \frac{\exp(\varepsilon t)}{t}\left[\frac{1}{2}\overline{\tau}(y,\varepsilon) + Re\sum_{n=1}^{n}\overline{\tau}\left(y,\varepsilon+\frac{in\pi}{t}\right)(-1)^n\right] \tag{3.45}$$

$$Nu(y,t) = \frac{\exp(\varepsilon t)}{t}\left[\frac{1}{2}\overline{N}u(y,\varepsilon) + Re\sum_{n=1}^{n}\overline{N}u\left(y,\varepsilon+\frac{in\pi}{t}\right)(-1)^n\right] \tag{3.46}$$

where

$\overline{u}(y,\varepsilon) = \overline{u}(y,s), \overline{T}(y,\varepsilon) = \overline{T}(y,s), \overline{\tau}(y,\varepsilon) = \overline{\tau}(y,s)$ and $\overline{N}u(y,\varepsilon) = \overline{N}u(y,s)$

Re refers to the 'real part of', $i = \sqrt{-1}$ is the imaginary number, N is the number of terms used in the Riemann-sum approximation and ε is the real part of the Bromwich contour that is used in inverting Laplace transforms. The Riemann-sum approximation for the Laplace inversion involves a single summation for the numerical process. Its accuracy depends on the value of ε and the truncation error dictated by N. The value of ε must be selected so that the Bromwich contour encloses all the branch points, Tzou [1997]. For faster convergence, the quantity $\varepsilon\ t$=4.7 gives the most satisfactory results since other tested values of $\varepsilon\ t$ selected need longer computational time, Khadrawi and Al-Nimr [2007].

CHAPTER 4

DISCUSSION OF THE RESULTS

This chapter gives the discussion of the results obtained from chapter three. The analytical solutions for temperature, velocity, skin-friction and Nusselt number are obtained from the Laplace domain to the time domain using the Riemann-sum approximation. The results of the numerical simulations are presented graphically for some carefully selected values of the governing parameters.

4.1 Discussion of problem 3.1

The present work analyses the effects of transpiration and heat generation on transient natural convection in vertical channel using the Riemann-Sum approach. The velocity field, temperature field, Skin-friction and rate of heat transfer are presented graphically in figures , for various values of Prandtl number (Pr), heat source/sink (H), suction/injection (q), and time (t), for the purpose of discussion.

Figures 4.1 and 4.2, present the temperature and velocity profiles for different values of time for fixed values of $Pr = 0.71, H = -2.0, q = -2.0$. It is clearly observed in these figures that fluid temperature increases with time and attains a steady-state for large values of time. This strengthens the convection current within the channel and it results to an increase in the fluid velocity.

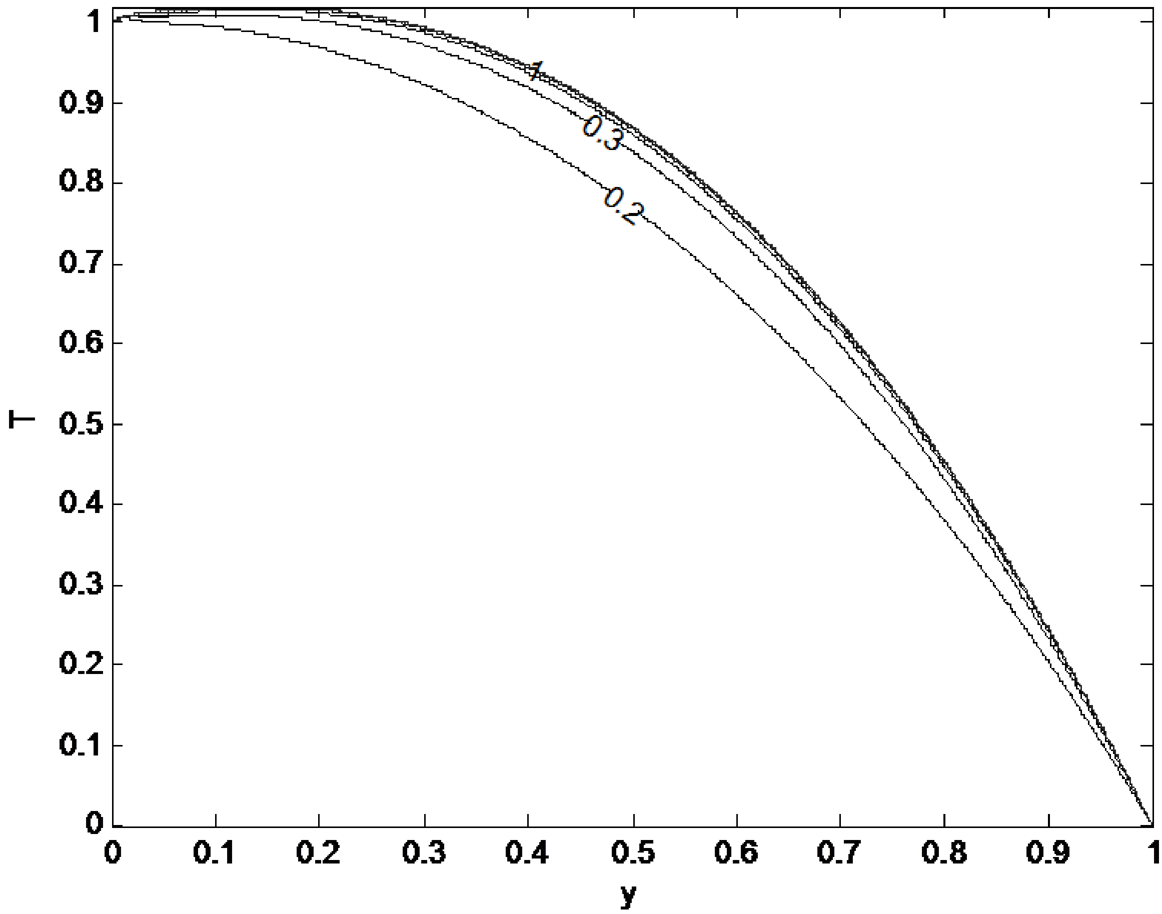

Figure 4.1: Temperature profile for different values of time ($Pr = 0.71, H = -2.0, q = -2.0$)

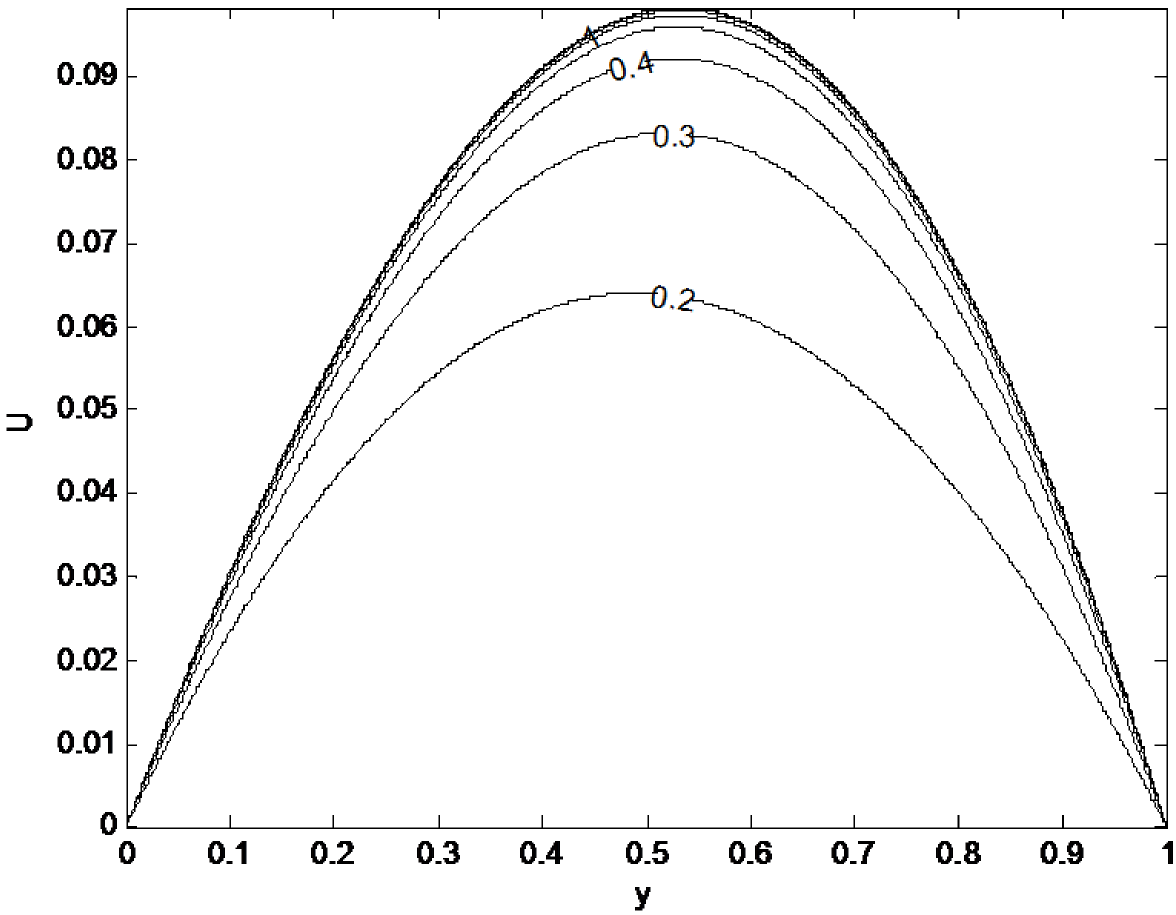

Figure 4.2: Velocity profile for different values of time $(Pr = 0.71, H = -2.0, q = -2.0)$

Figures 4.3 and 4.4. depict the effect of suction/injection on the fluid temperature and velocity when $Pr = 0.71, H = 2.0, t = 0.3$. It should be noted that $q > 0$ signifies suction on the heated plate with a corresponding injection on the cold plate. In this situation, it is observed that as suction increases on the heated plate, temperature within the channel responded with a decrease as shown in figure 4.3. This is physically expected since horizontal fluid motion is in the direction opposing the heat flux from the heated plate. The decrease in temperature weakens the convection current and as such, fluid velocity decreases as suction increases through the heated plate, when fluid is injected through the heated plate, horizontal fluid motion and heat flux to boost the fluid temperature in the channel. Hence the fluid temperature as well as velocity grow with increasing injection. This agrees well with the results of Jha and Ajibade [2010]. In addition, it is observed that the effect of transpiration on fluid velocity is negligible at fluid sections close to the heated wall.

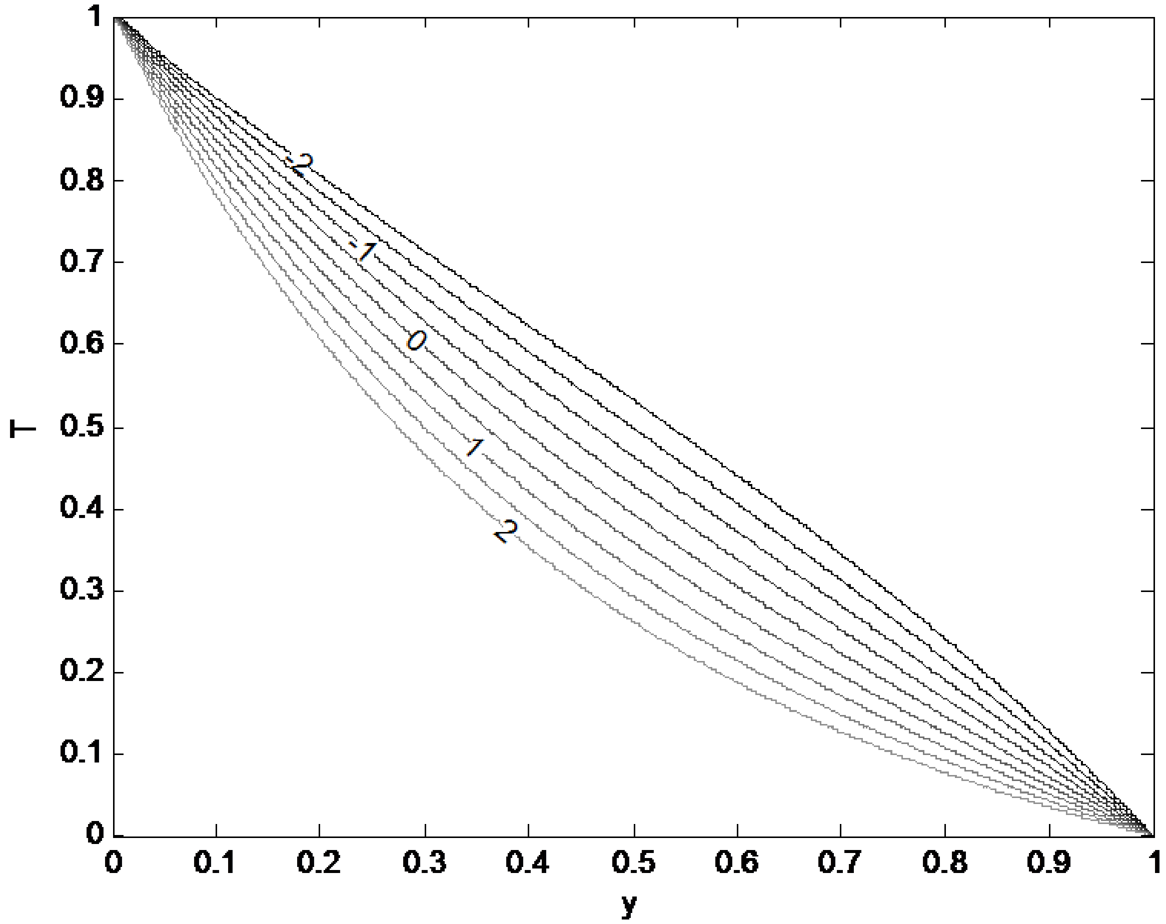

Figure 4.3: Temperature profile for different values of q ($Pr = 0.71, t = 0.3, H = 2.0$)

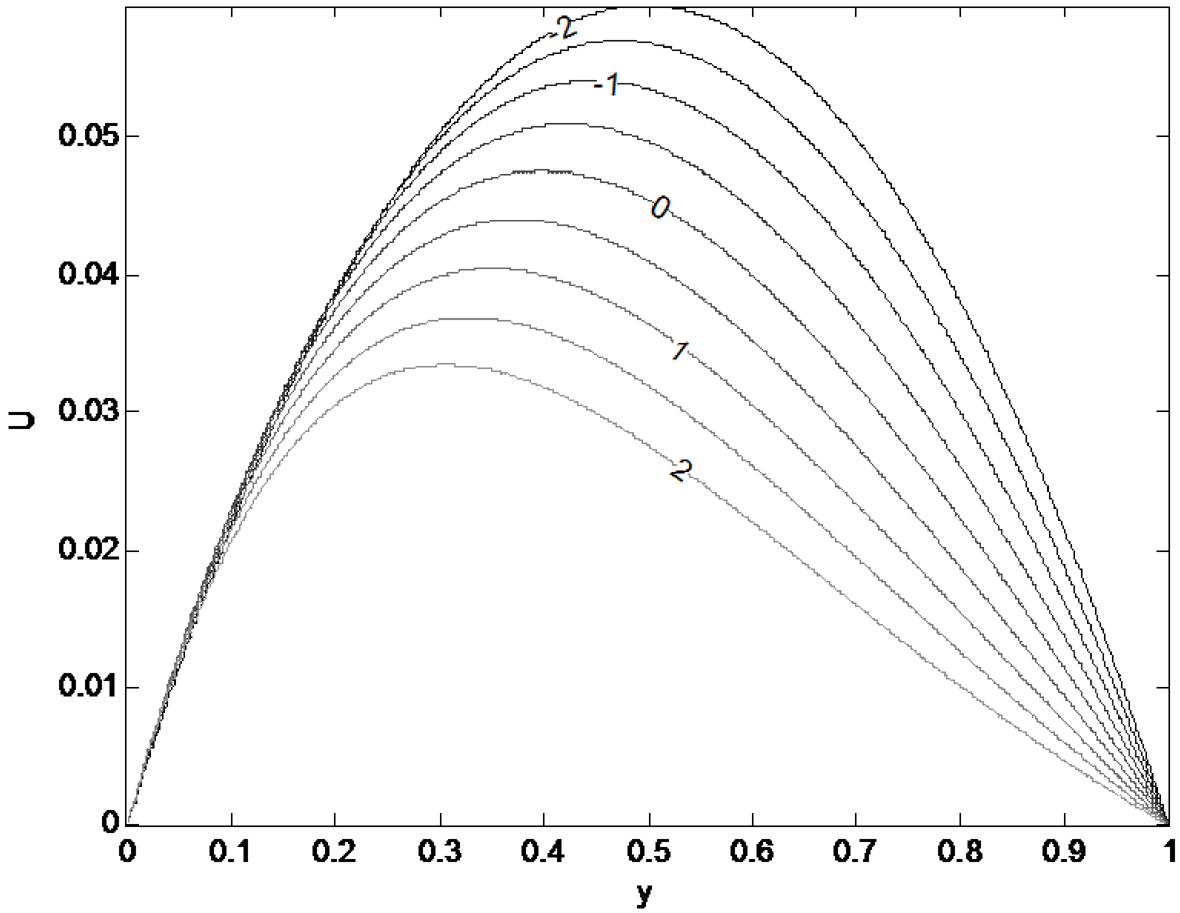

Figure 4.4: Velocity profile for different values of q ($Pr = 0.71, t = 0.3, H = 2.0$)

Figures 4.5 and 4.6 illustrate the effect of heat source/sink on the fluid temperature and velocity for fixed values of Pr, q and t. It is seen from the figures that as the heat source ($H < 0$) increases, fluid temperature and velocity increase while it decreases with increase in heat sink ($H > 0$). Increasing the heat source parameter causes the fluid temperature to increase and it strengthens the convection current within the channel. In addition, increasing the heat sink parameter causes a drop in fluid temperature and the thermal boundary layer becomes thinner thereby reduces the velocity distribution of the fluid as shown in figure 4.6.

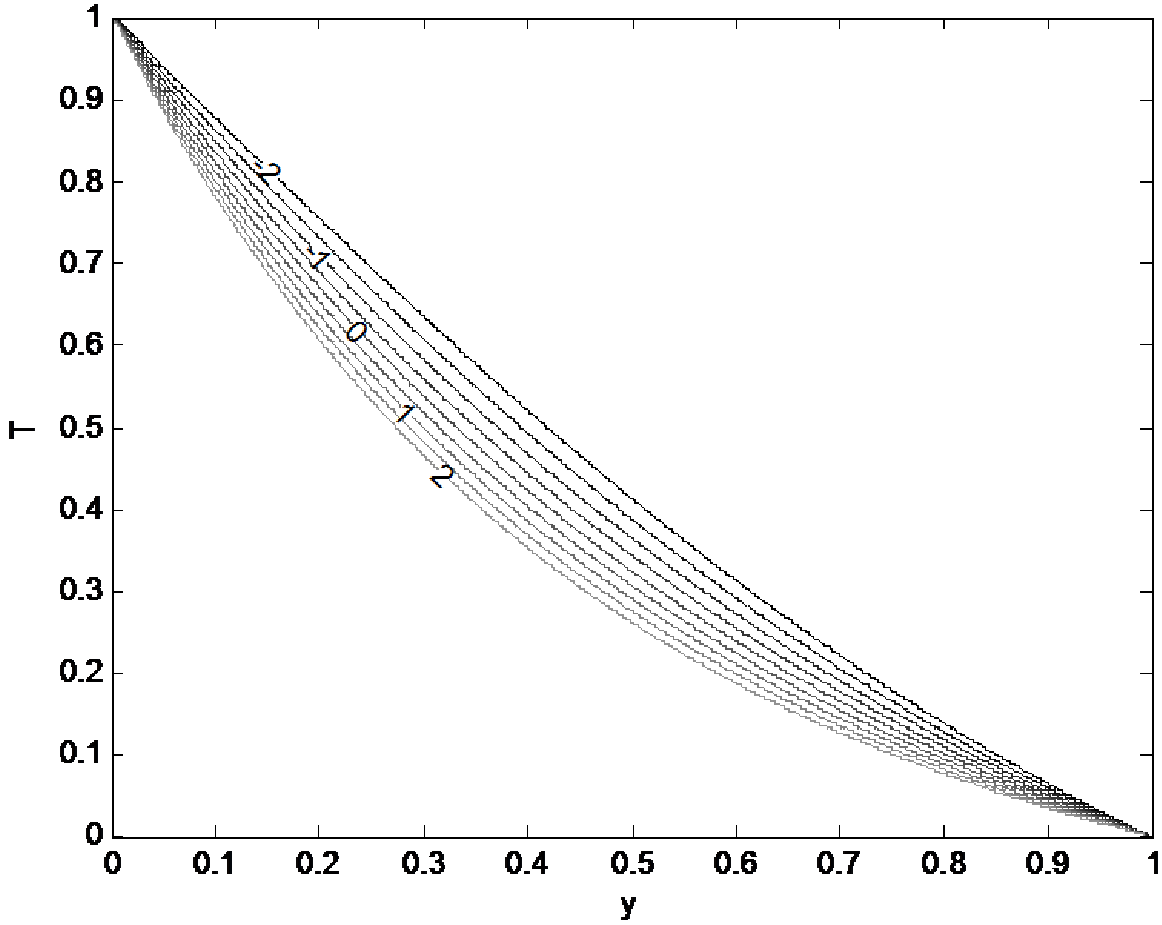

Figure 4.5: Temperature profile for different values of H $(Pr = 0.71, t = 0.3, q = 2.0)$

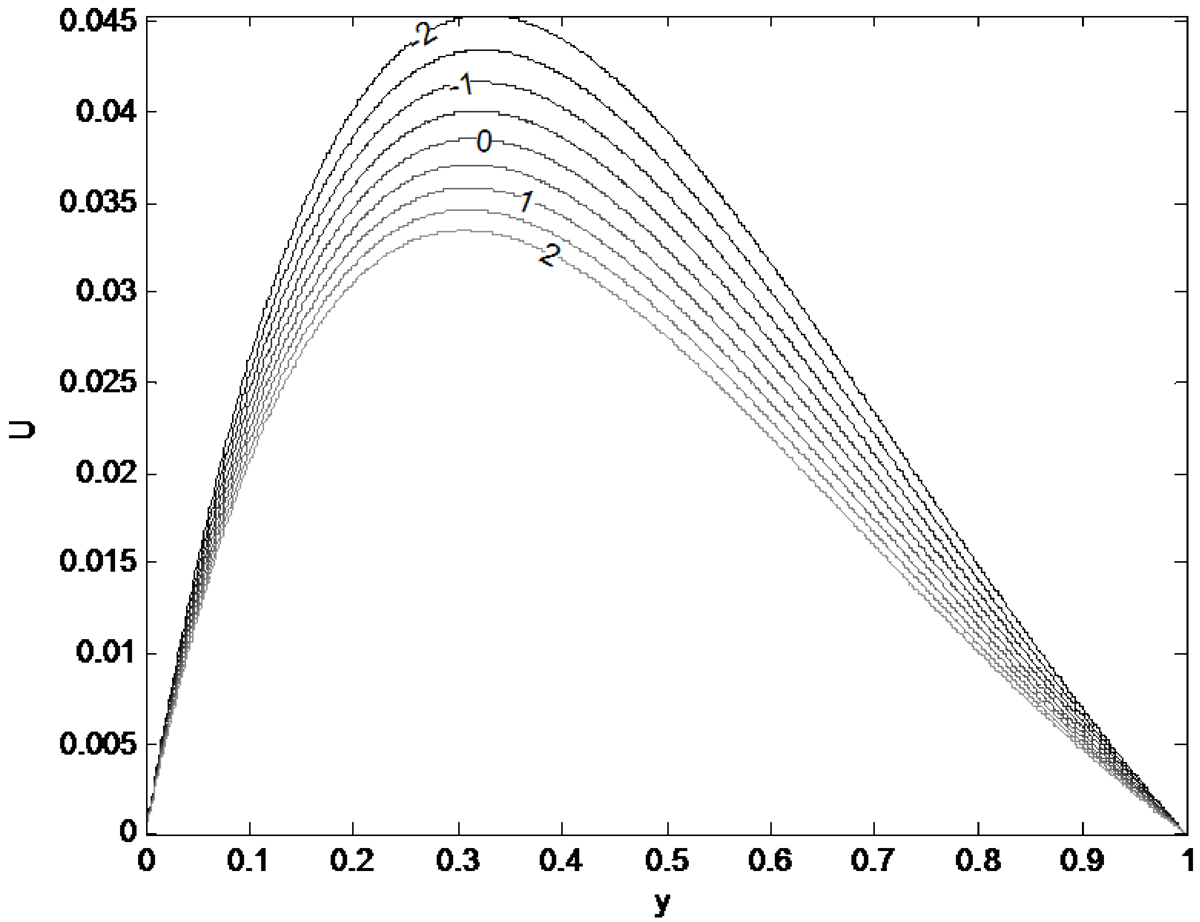

Figure 4.6: Velocity profile for different values of H $(Pr = 0.71, t = 0.3, q = 2.0)$

Figures 4.7 and 4.8 show the temperature and velocity profiles for different values of Pr when $t = 0.3, H = 2.0, q = 2.0$. An obvious fact in figures 4.7 and 4.8 is that the fluid temperature and velocity decreases with the increase of Pr. This is physically expected because an increase in Pr decreases the thermal diffusivity of the working fluid. Consequently the temperature of the fluid decrease and as such, convection current is weakened leading to a decrease in velocity as shown in figure 4.8.

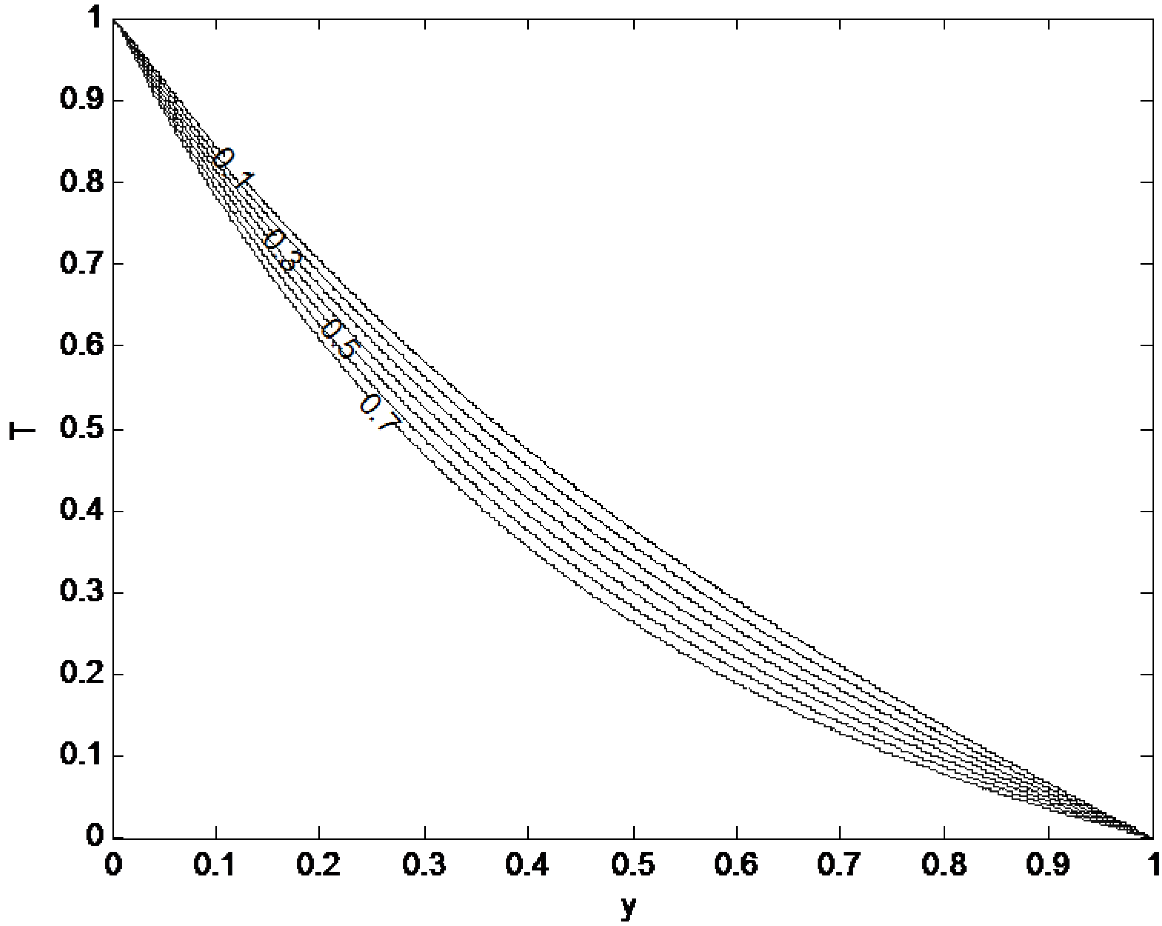

Figure 4.7: Temperature profile for different values of Pr $(t = 0.3, H = 2.0, q = 2.0)$

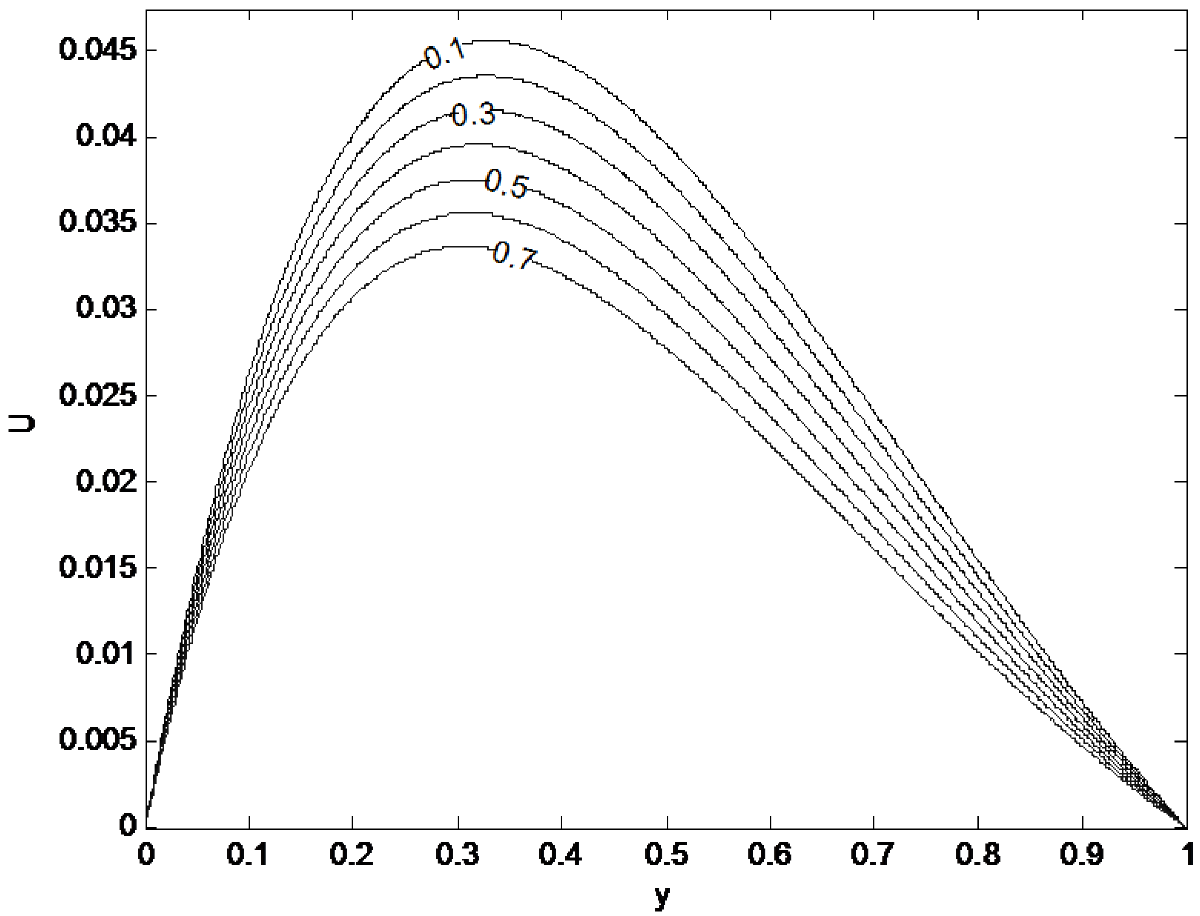

Figure 4.8: Velocity profile for different values of Pr $(t = 0.3, H = 2.0, q = 2.0)$

A clear view from figure 4.9 and 4.10 show that as Pr increases, rate of heat transfer increases on the heated plate, while it decreases on the cold plate. This is due to the fact that temperature decreases with increase in Pr (see figure 4.7), thus leading to an increase in the temperature gradient on the heated plate, hence an increase in the rate of heat transfer. In addition, as suction (q) increases on the heated plate, rate of heat transfer is seen to linearly increase. The reverse is observed in case of heated wall $(y = 1)$ as shown in figure 4.10. Comparisons between both plates show that heat transfer is higher on the cold plate in comparison to the heated plate.

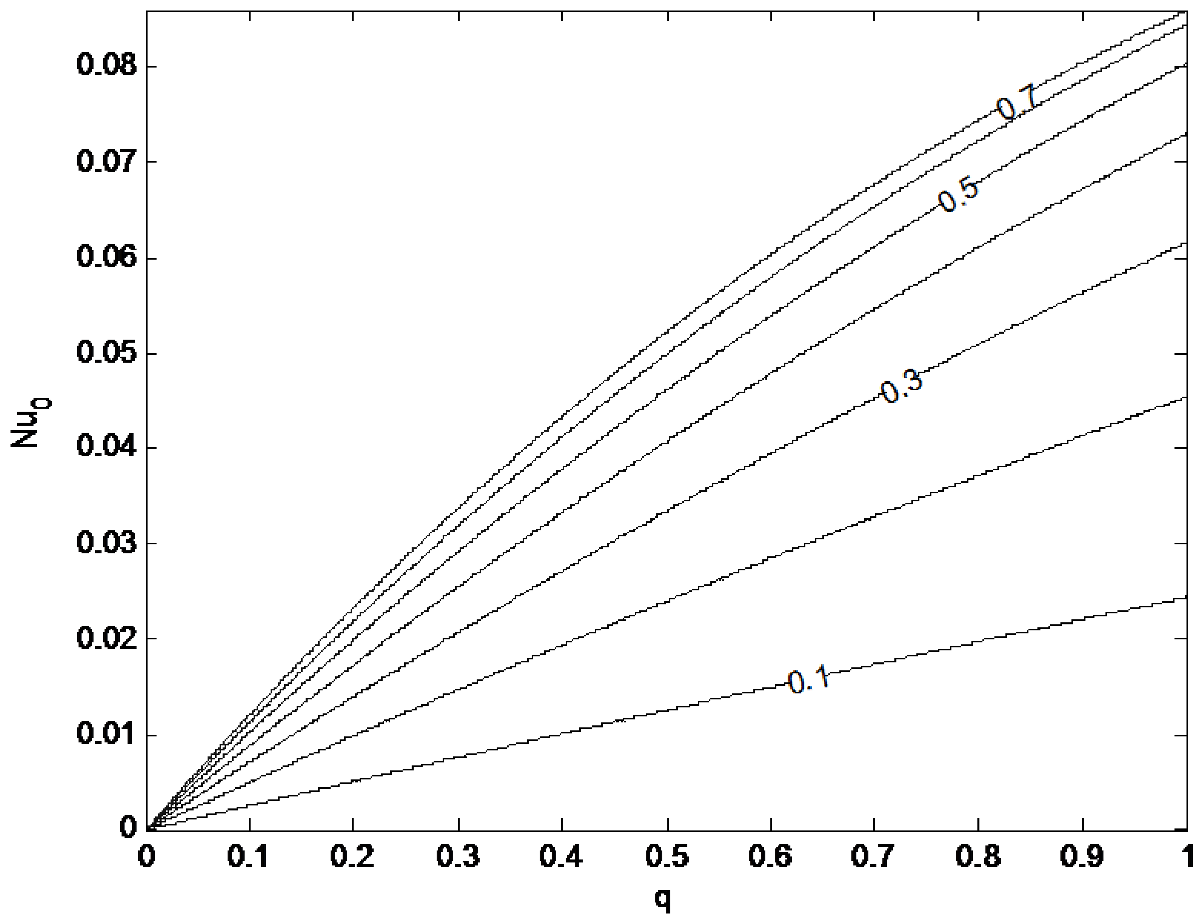

Figure 4.9: Rate of heat transfer on the heated plate for Pr $(t = 0.3, H = 2.0)$ at $y = 0$

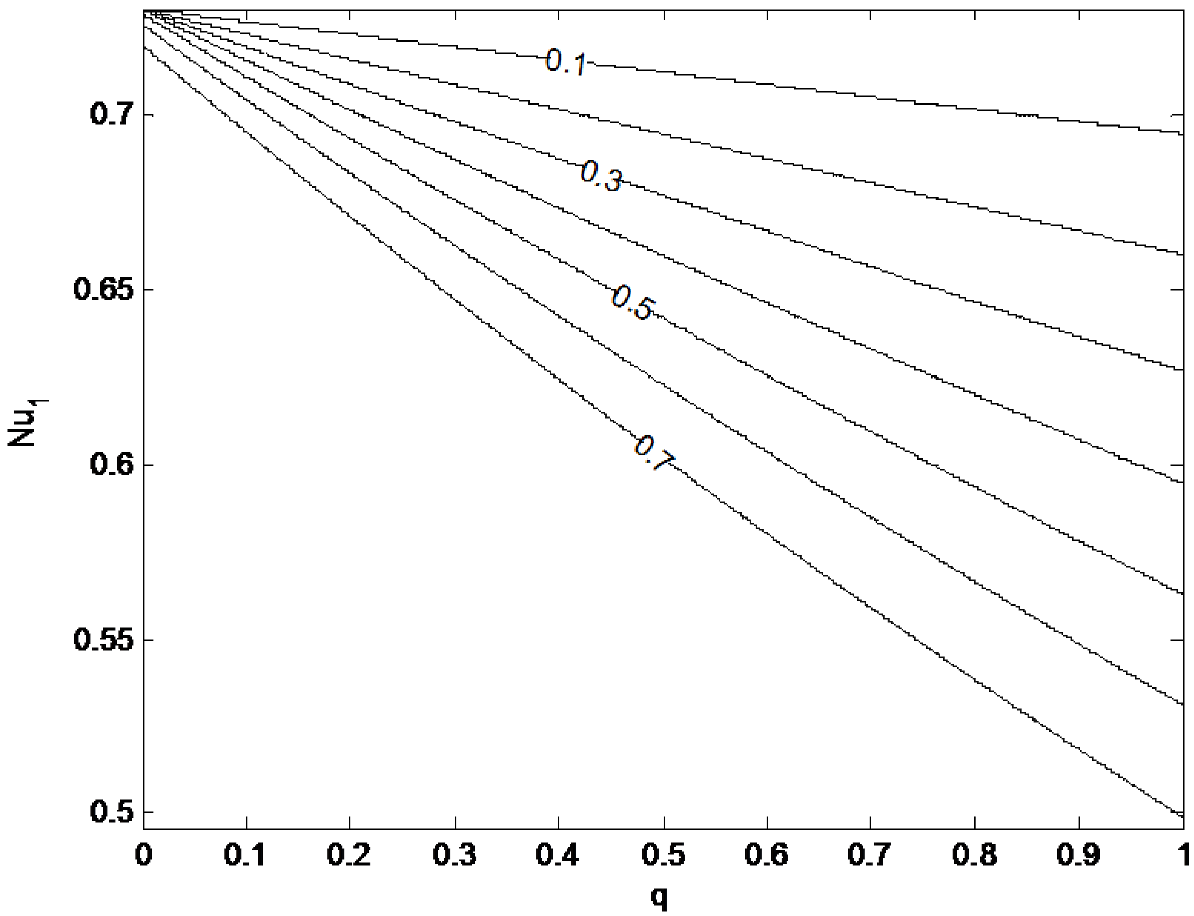

Figure 4.10: Rate of heat transfer on the cold plate for Pr $(t = 0.3, H = 2.0)$ at $y = 1$

Figures 4.11 and 4.12 depict the effect of heat source/sink on the rate of heat transfer at both walls $(y = 0, y = 1)$ in this situation, the heat source $(H < 0)$ decreases the rate of heat transfer on the heated wall, while it increases same on the cold wall $(y = 1)$. The reverse phenomenon is observed in case of heat sink $(H > 0)$. This is physically expected since fluid temperature increases with heat source and this decreases the temperature gradient on the heated plate while heat transfer to the cold plate increases.

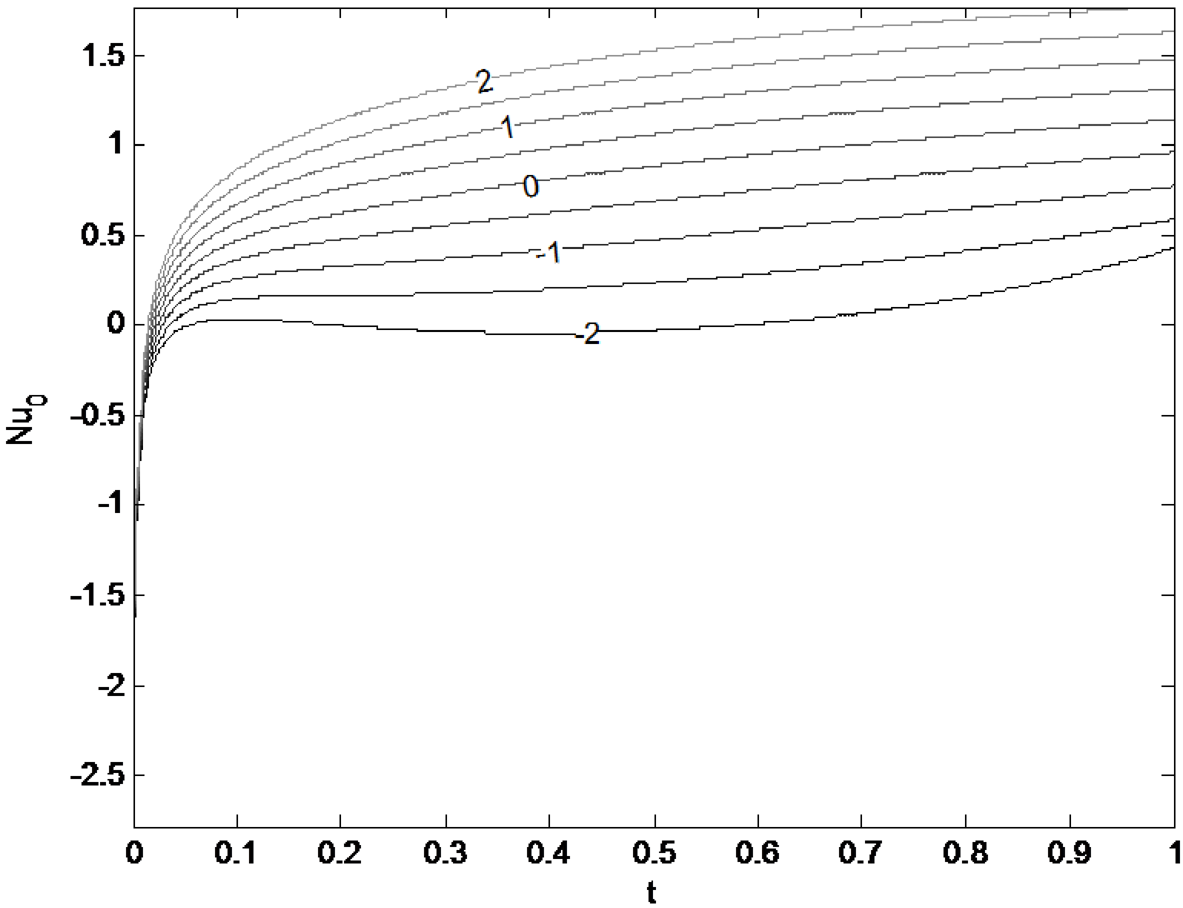

Figure 4.11: Rate of heat transfer on the heated plate for H $(Pr = 0.7, q = 2.0)$ at $y = 0$

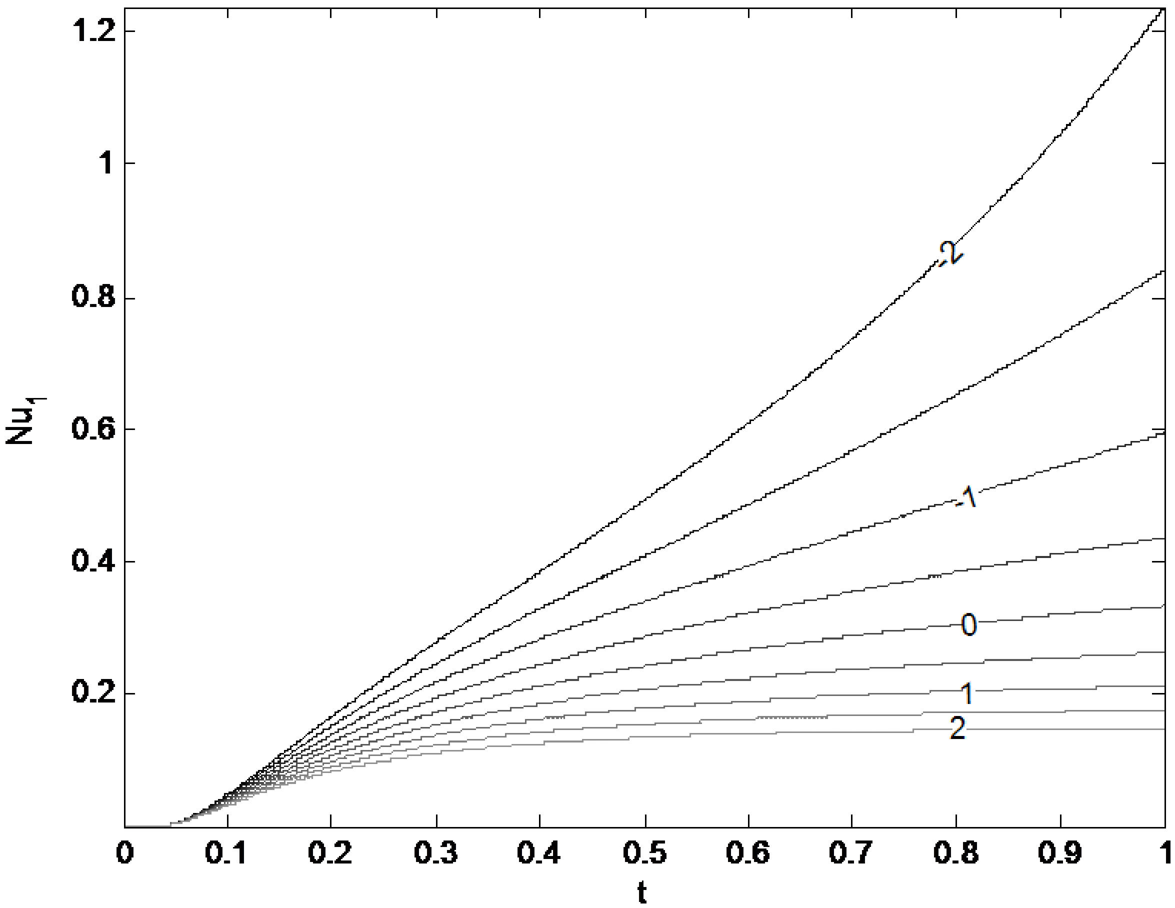

Figure 4.12: Rate of heat transfer on the cold plate for H $(Pr = 0.7, q = 2.0)$ at $y = 1$

The skin-friction on the heated plate is shown in figure 4.13 for different values of heat source/sink (H). An observation from this figure indicates that the skin-friction increases with increase in heat source while it decreases with increase in heat sink.

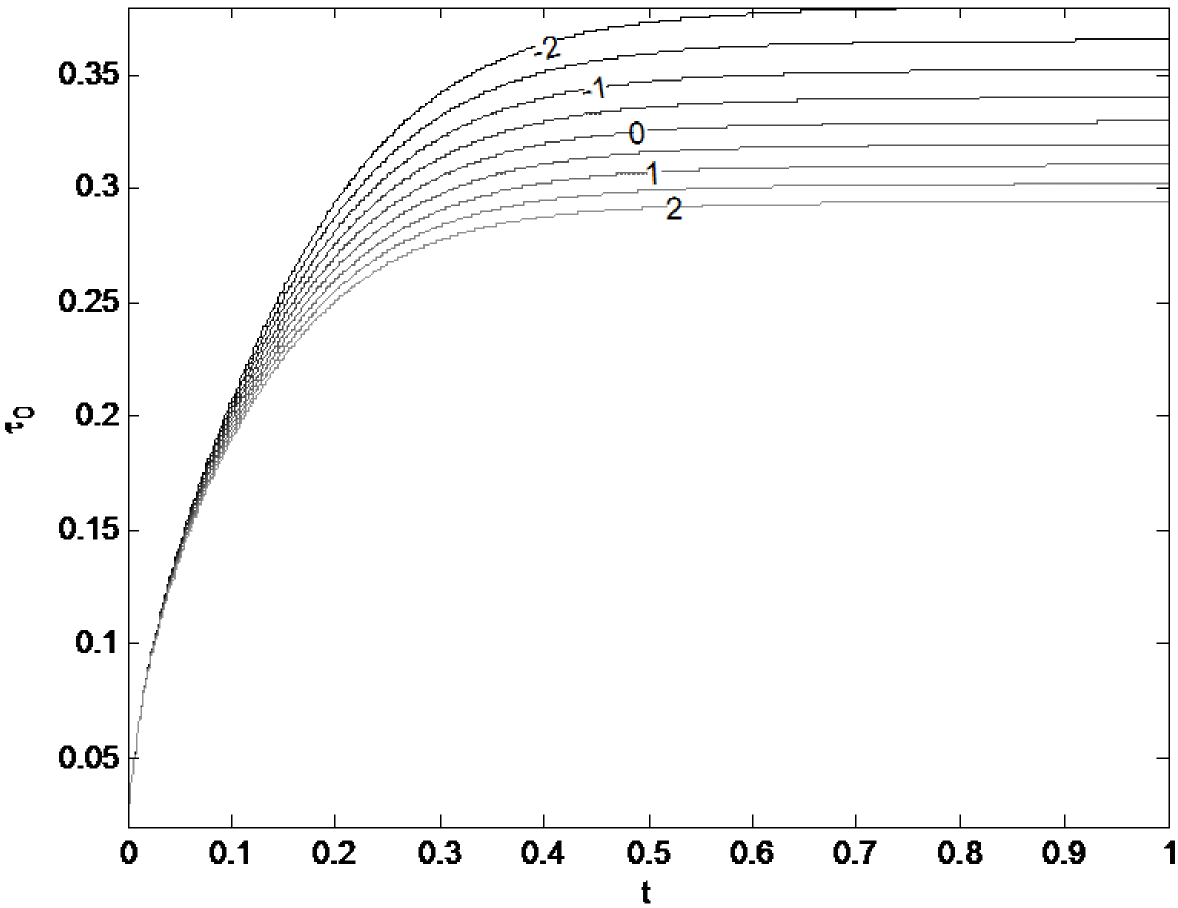

Figure 4.13: Skin friction at the cold wall for different H $(Pr = 0.71, q = 2.0)$ at $y = 0$

Figure 4.14 shows that the skin-friction decreases as Pr increases. Physically, this is expected because an increase in Pr decreases the velocity as shown in fig 4.8.

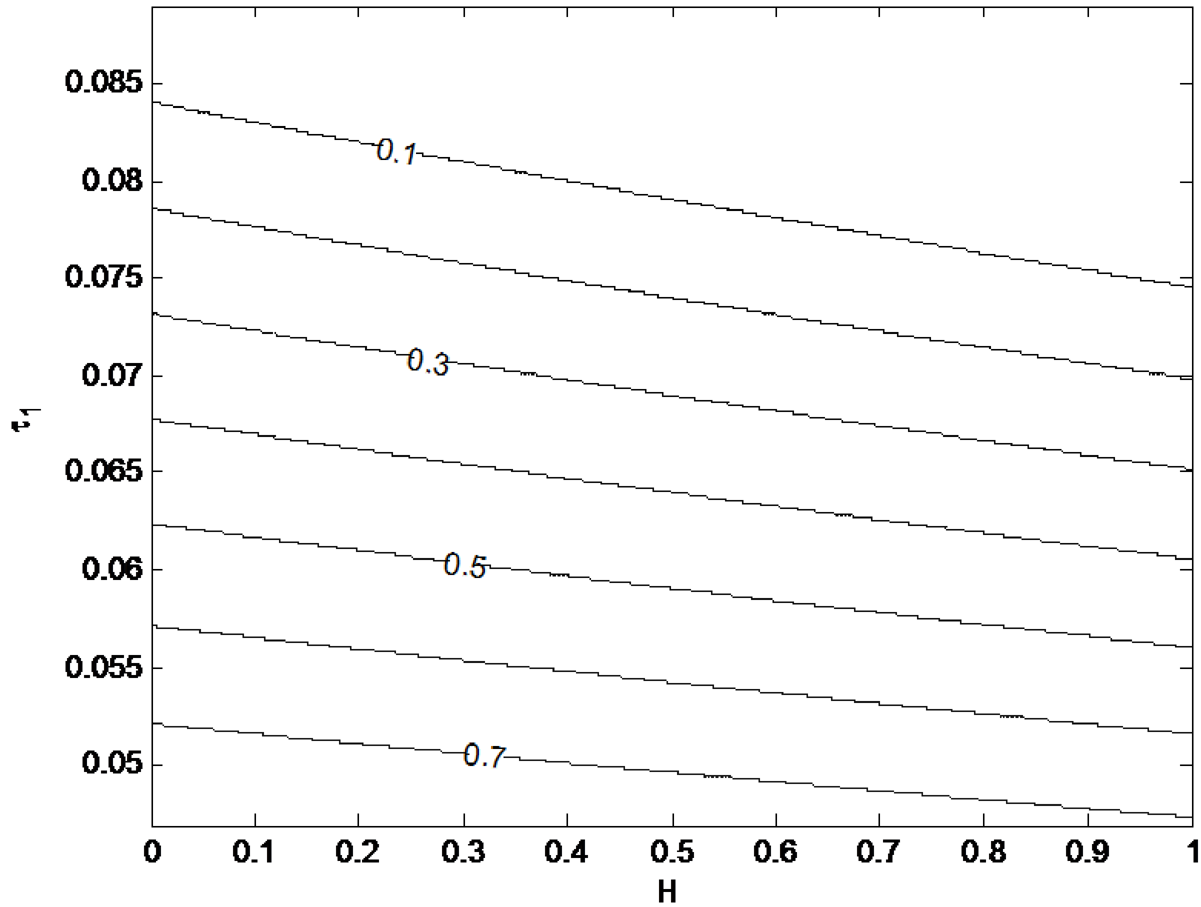

Figure 4.14: Skin friction at the heat wall for different Pr $(t = 0.3, q = 2.0)$ at $y = 1$

Figures 4.15 and 4.16 present the skin-friction at both walls for different values of Prandtl number (Pr). A clear view from these figures is that on both walls as Pr increases skin-friction decreases. Physically, this is expected because an increase in Pr decreases the velocity as shown in fig 4.8. However, skin-friction grows with increasing suction on the cold plate while it decreases on the hot plate as suction is increased on the cold plate $q > 0$.

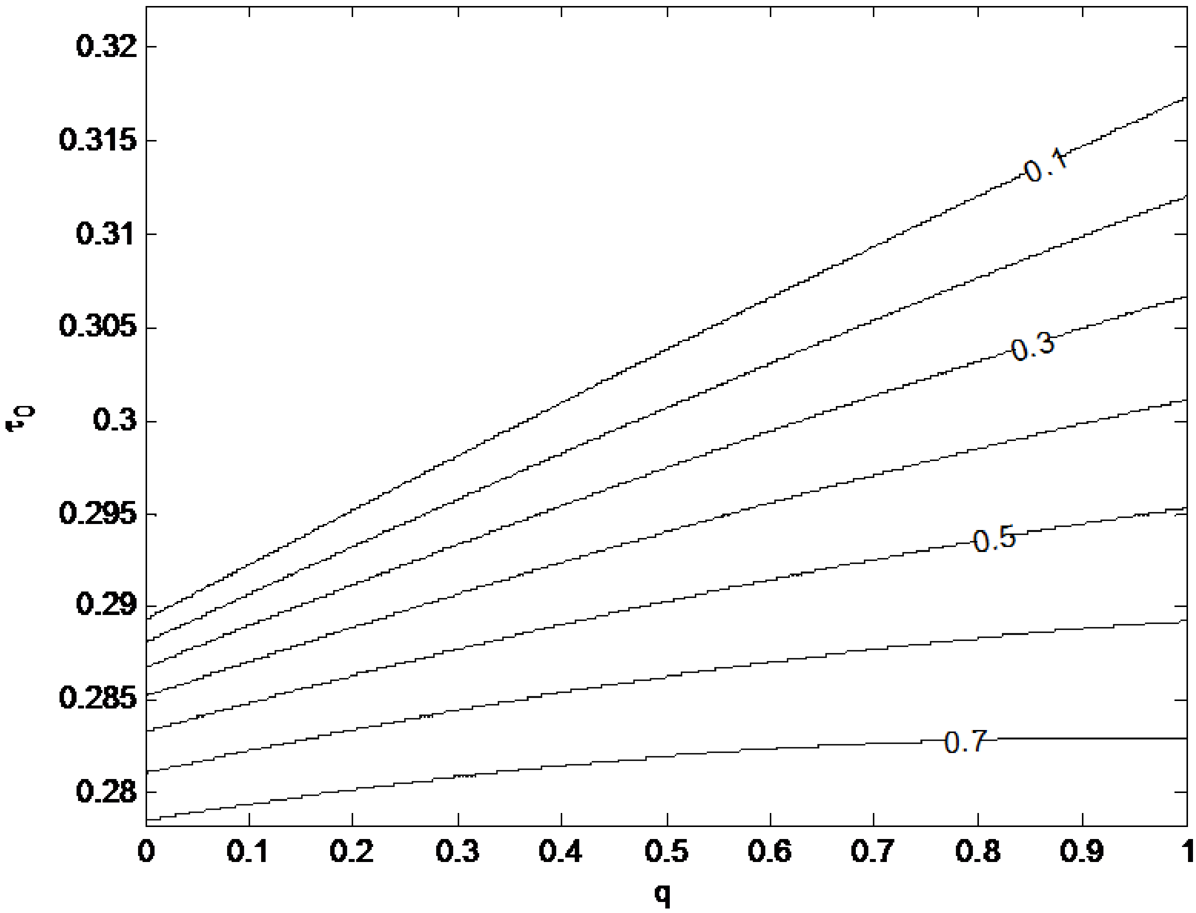

Figure 4.15: Skin friction at the heated wall for different Pr $(t = 0.3, H = 2.0)$ at $y = 0$

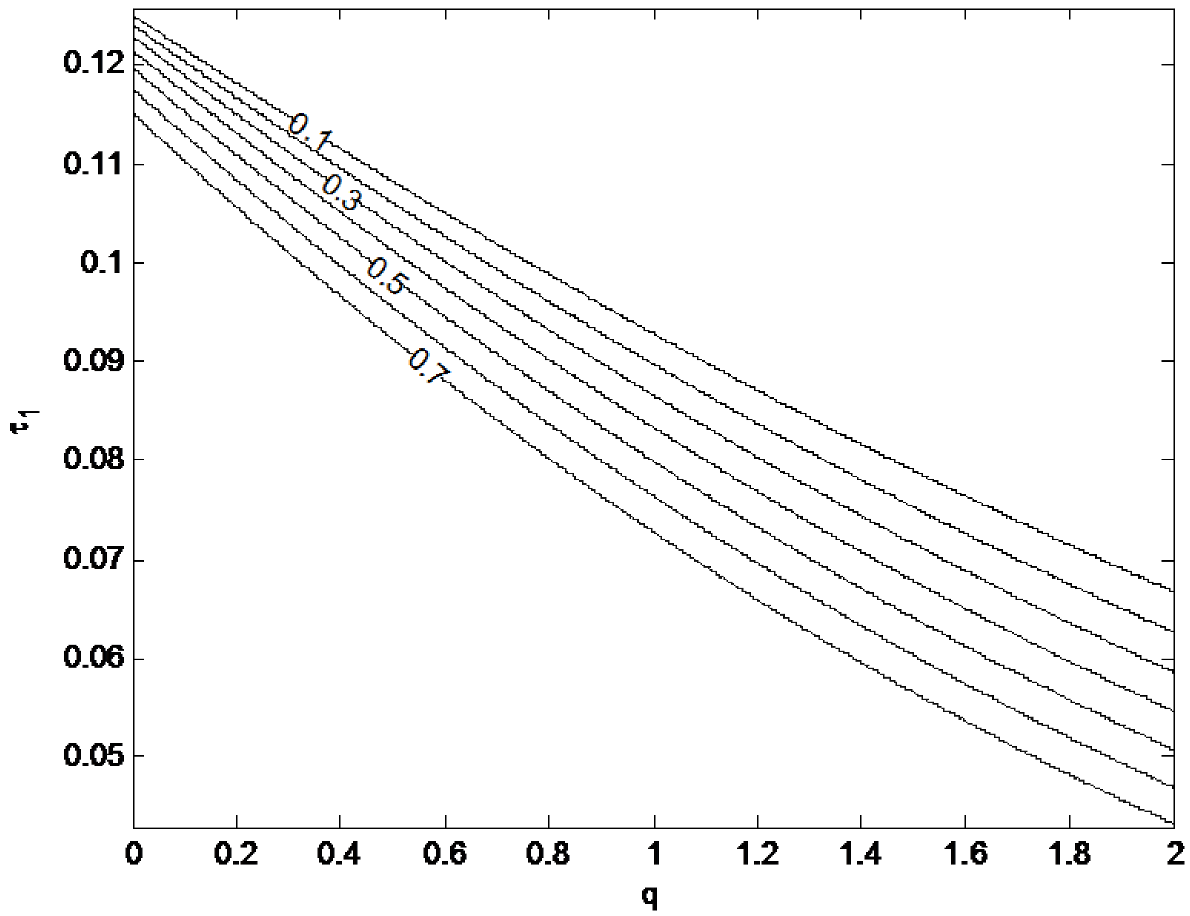

Figure 4.16: Skin friction at the heated wall for different Pr ($t = 0.3, H = 2.0$) at $y = 1$

Table 4.1 shows that transient temperature and velocity increase with increasing time and finally attains a steady state for large values of time, which coincides with the steady state temperature and velocity.

Table 4.1: Validation of problem 3.1

Transient (Using Riemann-Sum)	Temperature	Velocity
t	$H = -2.0, q = -2.0, Pr = 0.71$	
0.1	0.5420	0.0293
0.3	0.8399	0.0829
0.5	0.8681	0.0951
0.7	0.8707	0.0971
0.8	0.8709	0.0973
Steady state	0.8709	0.0973

4.2 Discussion of problem 3.2

The analysis of the effects of transpiration on natural convection flow of heat generating fluid in a vertical channel is presented here under the isothermal/adiabatic thermal conditions. The fluid velocity, fluid temperature, skin-friction and rate of heat transfer at the isothermal/adiabatic plates are presented graphically in figures 4.17 to 4.30 for carefully selected values of Pr (Prandtl number), H (heat source/sink), q (suction/injection) and t (time). For the purpose of discussion, the value of Pr chosen is $Pr = 0.71$ corresponding to air, heat generation parameter $H = 1.0$, suction/injection parameter $q = 2.0$.

Figure 4.17 illustrates the variations for the temperature profiles for different values of time. A clear view from this figure is that fluid temperature increases with time and finally attains a steady-state for large values of time. This strengthens the convection current within the channel, which results to an increase in the fluid velocity with growing time as shown in figure 4.18.

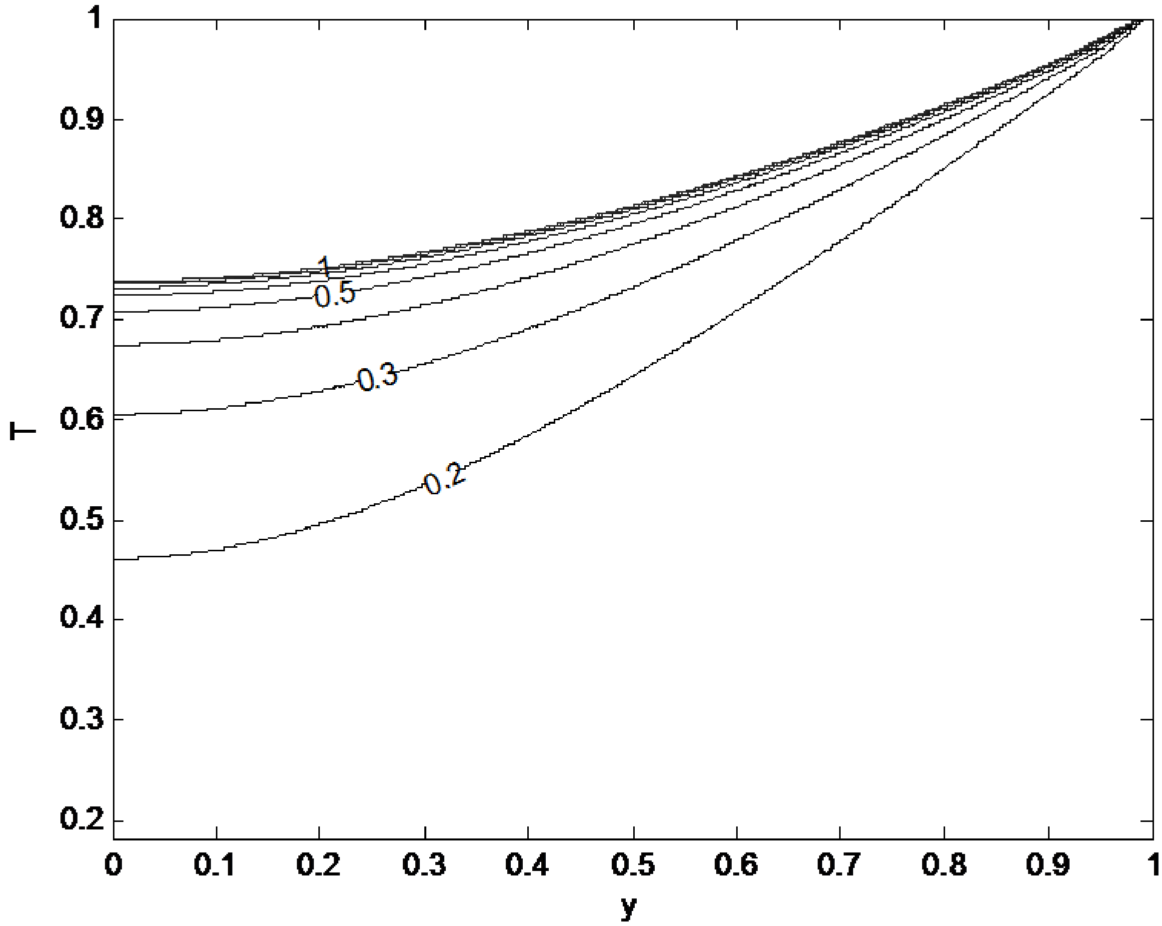

Figure 4.17: Temperature profile for different values of time $(Pr = 0.71, H = 1.0, q = 2.0)$

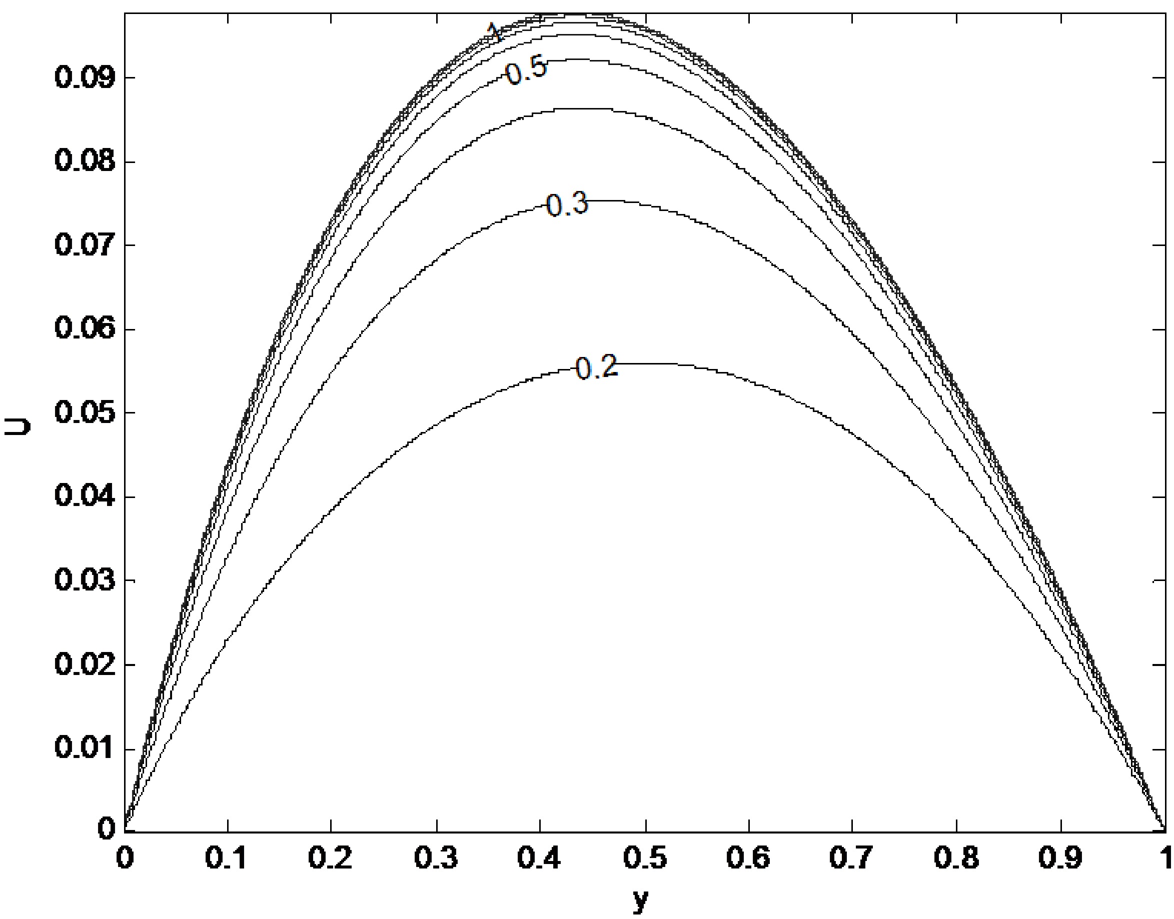

Figure 4.18: Velocity profile for different values of time ($Pr = 0.71, H = 1.0, q = 2.0$)

Figures 4.19 and 4.20 describe the effect of suction/injection on the fluid temperature and velocity. It should be noted that $q > 0$ signifies suction on the adiabatic plate with a corresponding injection on the isothermal plate, while $q < 0$ indicates suction on the isothermal plate with a corresponding injection on the adiabatic plate. Observation from these figures show that both temperature and velocity increase with increase in injection on the isothermal plate ($q > 0$) while the reverse phenomena is observed for $q < 0$. This is attributed to that fact when $q > 0$, direction of the injection is same as that of heat flux through the isothermal plate. Therefore, injection acts in support of heat penetration to boost fluid temperature in the channel. On the other hand, when $q < 0$, the direction of horizontal fluid flow opposes that of the heat flux on the isothermal plate thereby decreasing the heat penetration and fluid temperature. As the fluid temperature increases or decreases convection current increases or decreases thus increasing or decreasing the velocity respectively.

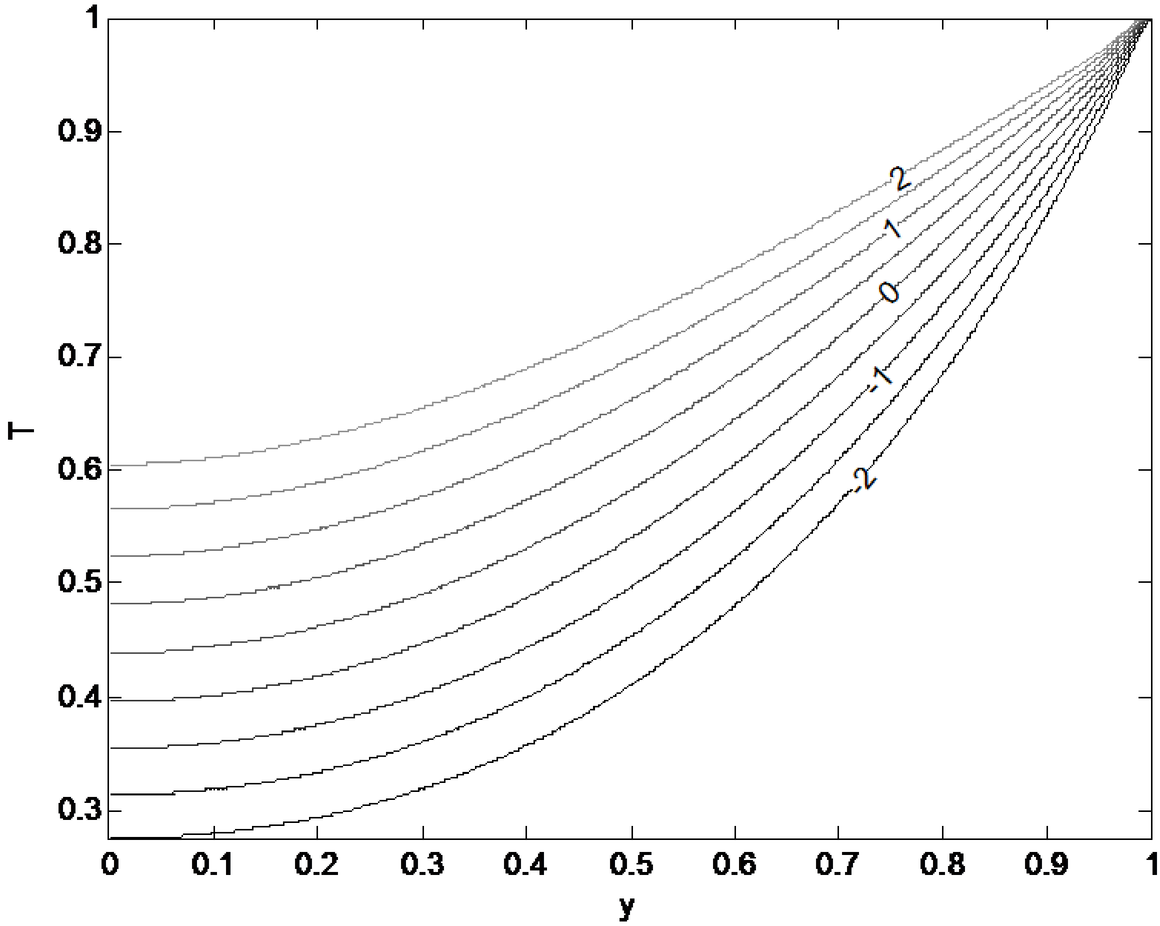

Figure 4.19: Temperature profile for different values of q ($Pr = 0.71, t = 0.3, H = 1.0$)

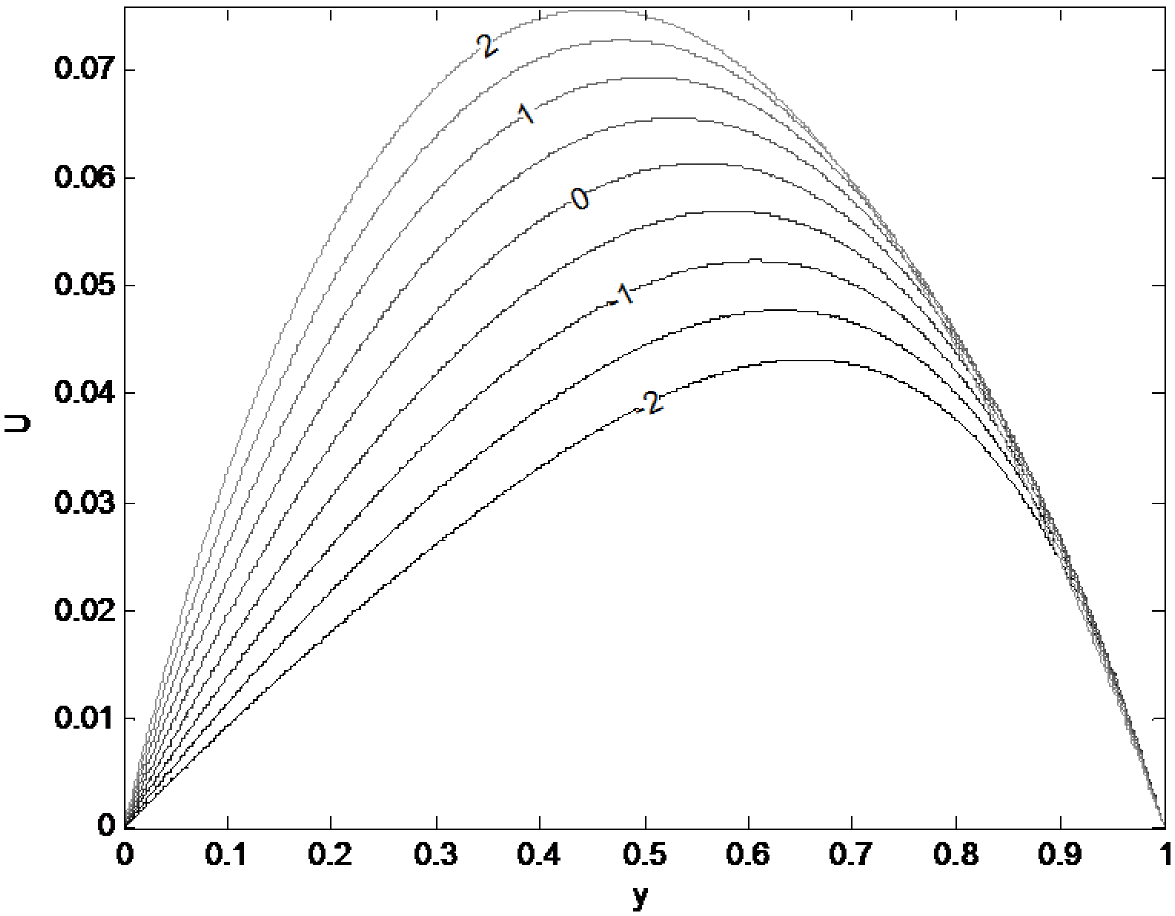

Figure 4.20: Velocity profile for different values of q $(Pr = 0.71, t = 0.3H = 1.0)$

Figures 4.21 and 4.22 depict the effect of heat source/sink on the fluid temperature and velocity. It is clear from the figures that as the heat source $(H < 0)$ increases, fluid temperature and velocity increase while they decrease with increase in heat sink. Increasing the heat sink parameter $(H > 0)$ causes the fluid to cool and the thermal boundary layer becomes thinner thereby reducing thermal buoyancy effect and hence reduces the velocity distribution of the fluid as shown in figure 4.22.

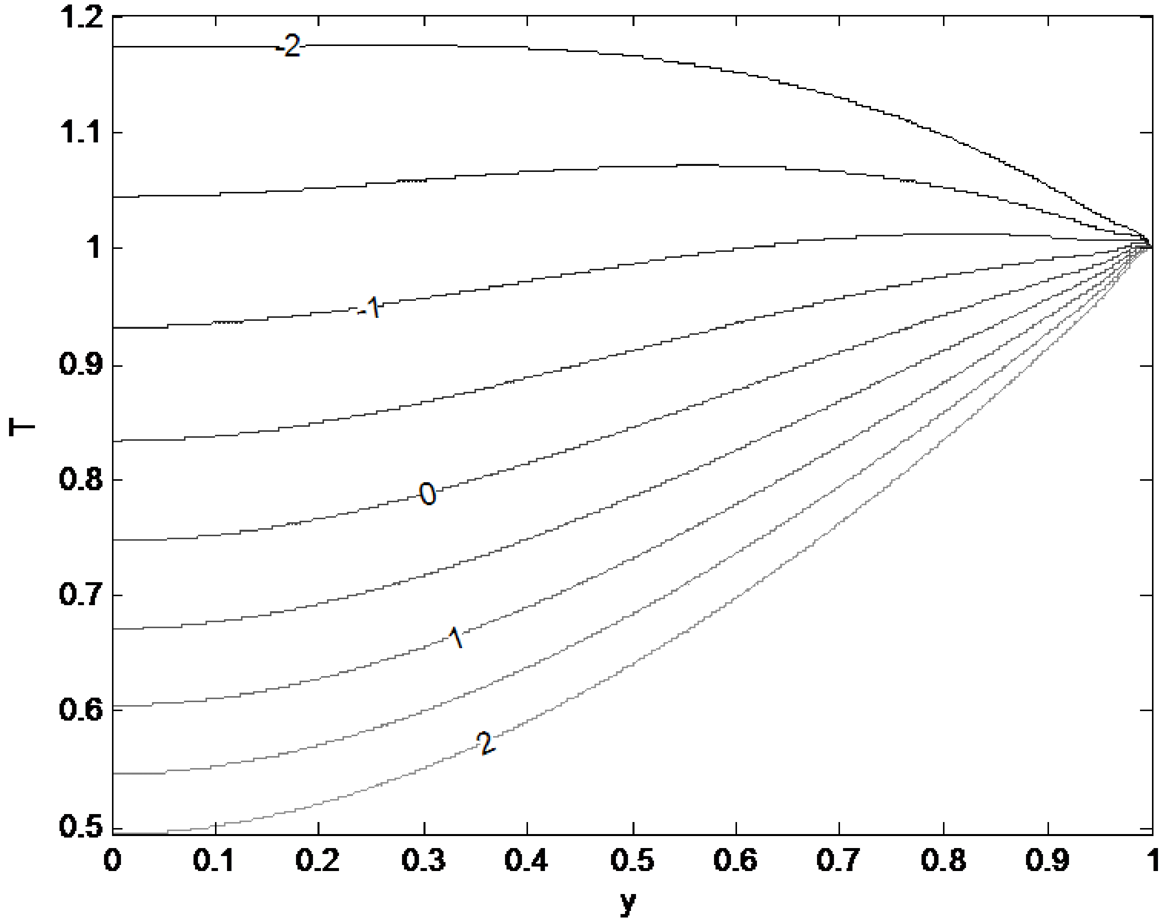

Figure 4.21: Temperature profile for different values of H $(Pr = 0.71, t = 0.3, q = 2.0)$

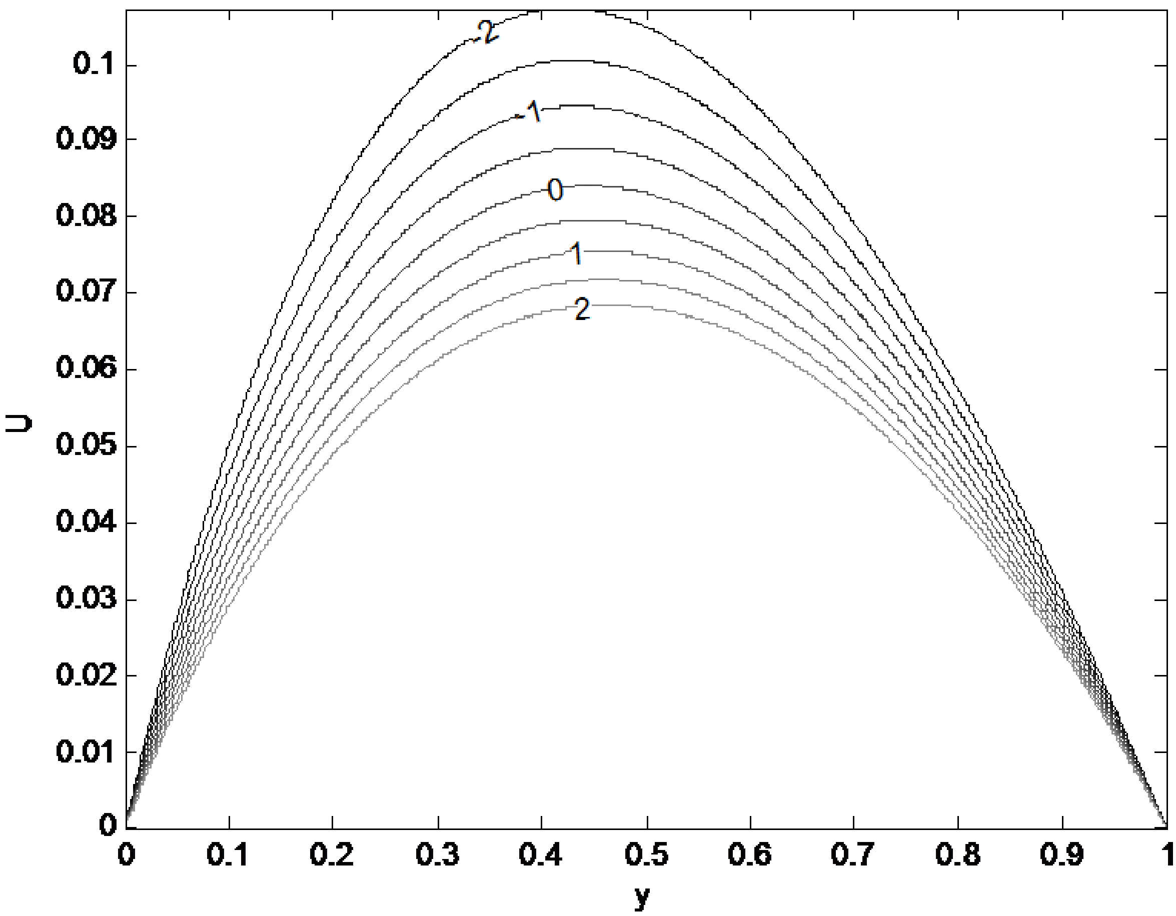

Figure 4.22: Velocity profile for different values of H $(Pr = 0.71, t = 0.3, q = 2.0)$

Figures 4.23 and 4.24 present the temperature and velocity profiles for different value of Pr. In this situation fluid temperature and velocity decreases with an increase in Pr. This is physically true because an increase in Pr decreases the thermal diffusivity of the working fluid, therefore, heat penetration from the isothermal plate decreases which leads to a thinning of the thermal boundary layer and hence a decrease in the temperature. Moreover, in figure 4.24, the velocity is clearly seen to decrease with an increase in Prandtl number Pr which is as a result of weak convection current due to decrease in temperature as observed in figure 4.23.

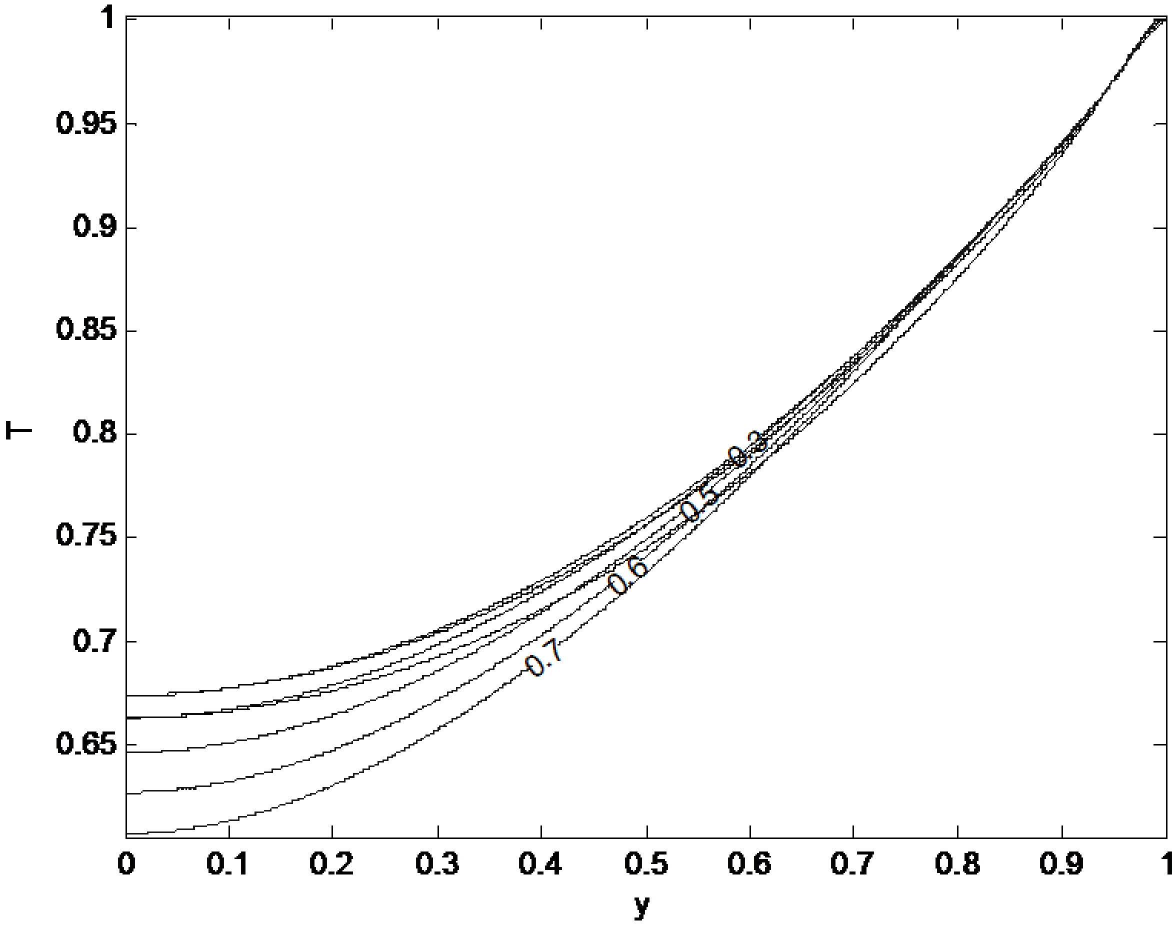

Figure 4.23: Temperature profile for different values of Pr $(t = 0.3, H = 1.0, q = 2.0)$

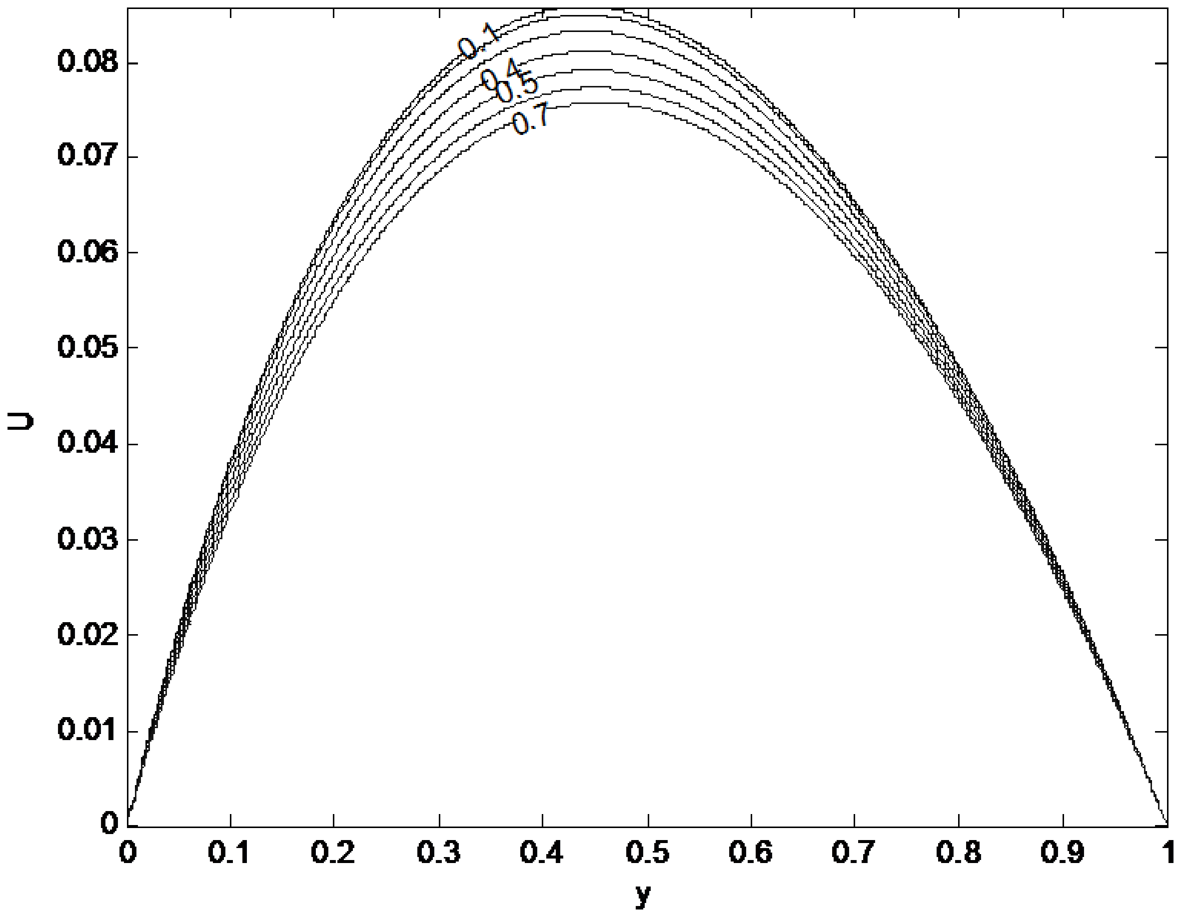

Figure 4.24: Velocity profile for different values of Pr $(t = 0.3, H = 1.0q = 2.0)$

Figure 4.25 describes the rate of heat transfer at the isothermal plate due to variations in Pr. Observation from the figure shows that as Pr increases, rate of heat transfer decreases. This is due to the fact that temperature decreases with an increase in Pr as shown in figure 4.23, thus leading to a decrease in the temperature gradient hence a decrease in the rate of heat transfer. Furthermore, the heat transfer rate is observed to decrease with growing injection on the isothermal plate while it increases with growing suction. This is due to the fact that fluid temperature increases with increase in injection $q > 0$ on the isothermal plate, thus the temperature gradient decreases leading to a decrease in Nusselt number.

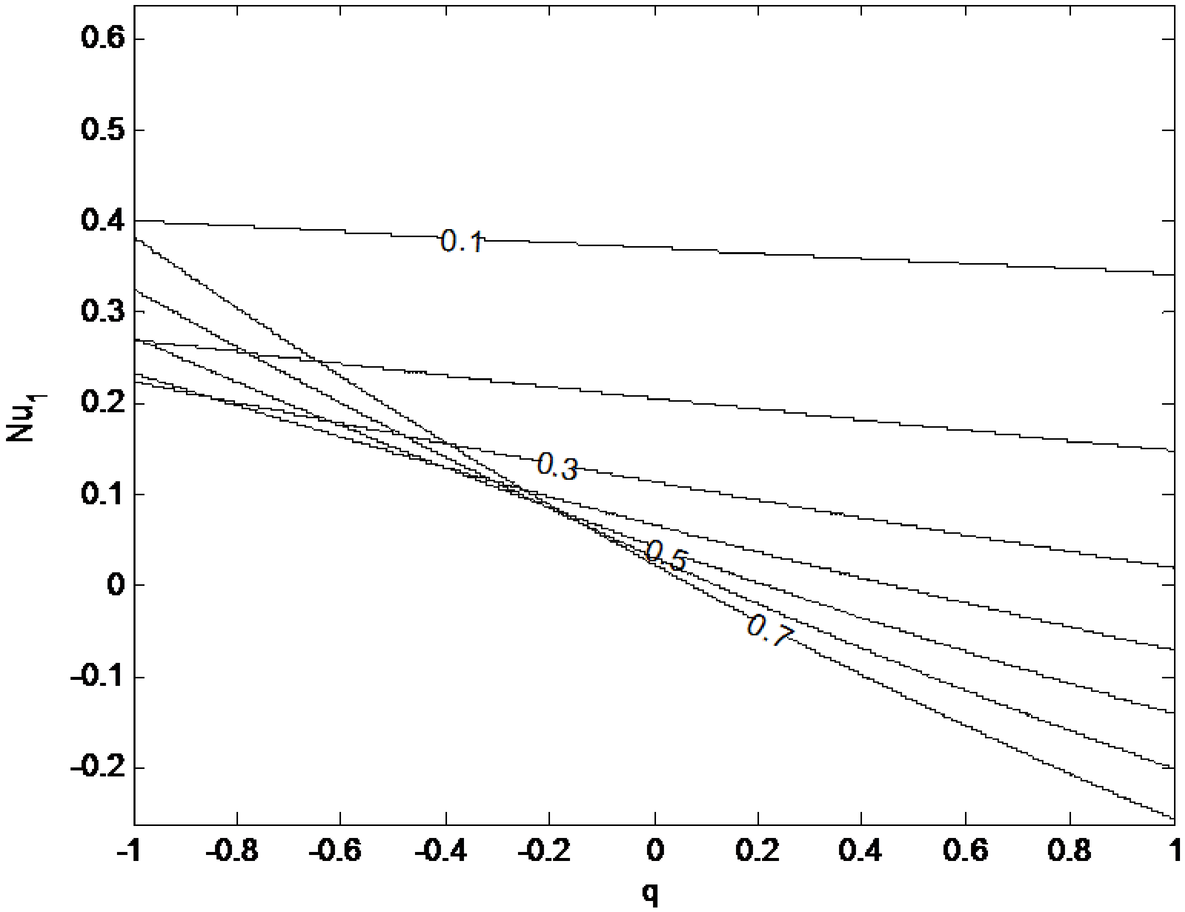

Figure 4.25: Rate of heat transfer for Pr $(t = 0.3, H = 1.0)$ at the isothermal plate

Figure 4.26. Shows the effect of heat source/sink on the rate of heat transfer at the isothermal plate. A clear view from this figure shows that as heat source $(H < 0)$ increases the rate of heat transfer increase, while it decreases with increase in heat sink $(H > 0)$. This is physically expected because, fluid temperature increases and it decreases with increase in heat sink as shown in figure 4.21.

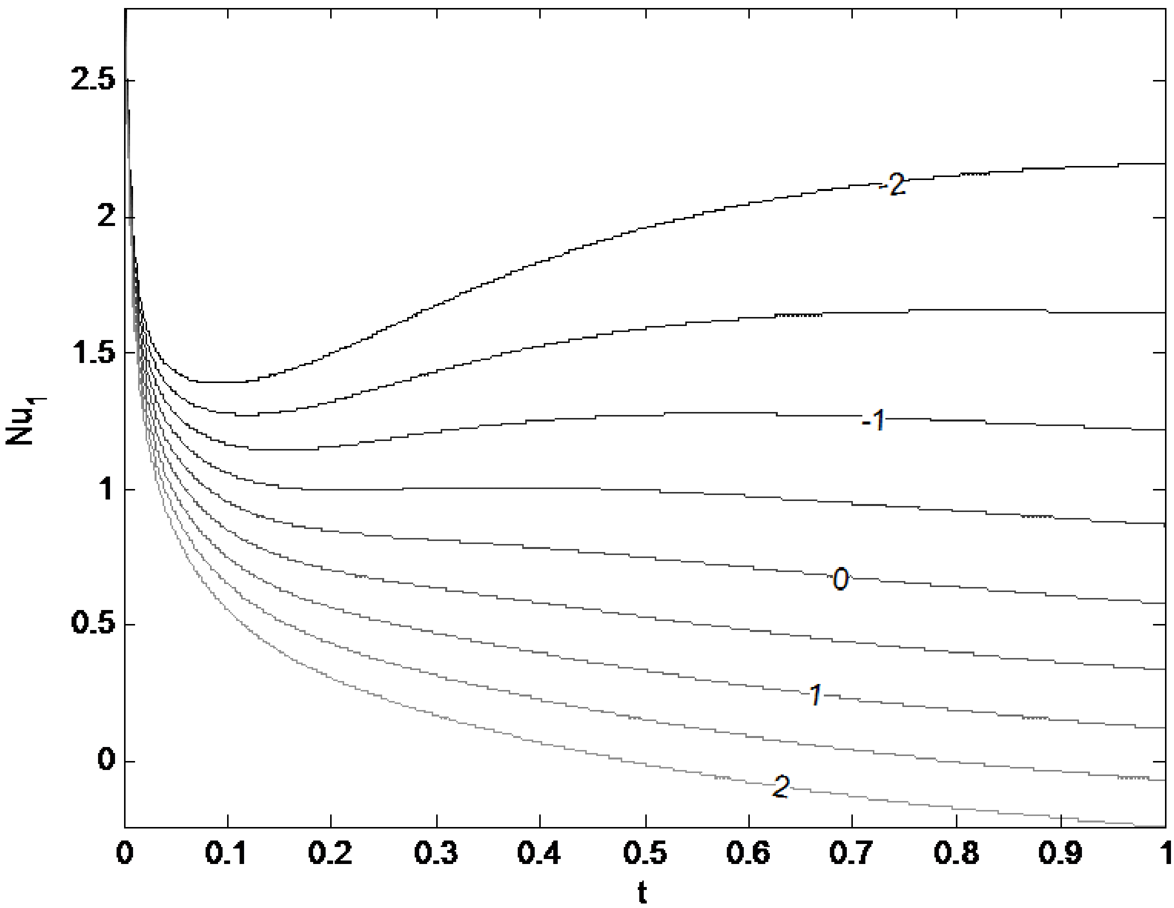

Figure 4.26: Rate of heat transfer for H $(Pr = 0.71, q = 2.0)$ at the isothermal plate

Figures 4.27 and 4.28 present the skin-friction at adiabatic and isothermal wall for different values of Prandtl number (Pr). Observations from these figures is that as Pr increases skin-friction decreases. Physically, this is expected because an increase in Pr decreases the velocity as shown in figure 4.24. Comparing these figures show that skin-friction is less at the adiabatic plate in comparison to the isothermal plate. This is so because velocity gradient is higher on the isothermal plate in comparison to the adiabatic plate. In addition, it is observed that the skin friction on both plates grows with increase in suction on the plate.

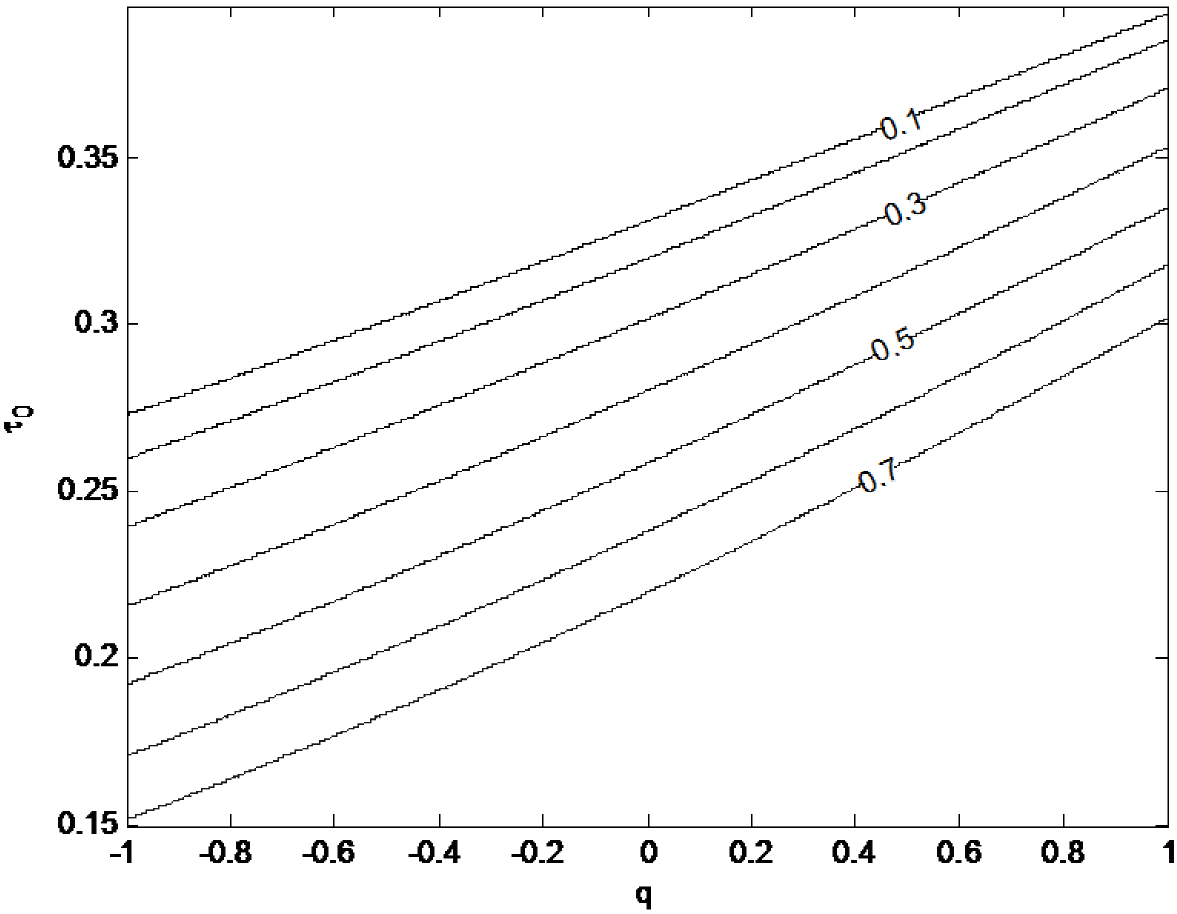

Figure 4.27: Skin friction at the adiabatic wall for different Pr $(t = 0.3, H = 1.0)$ at y=0

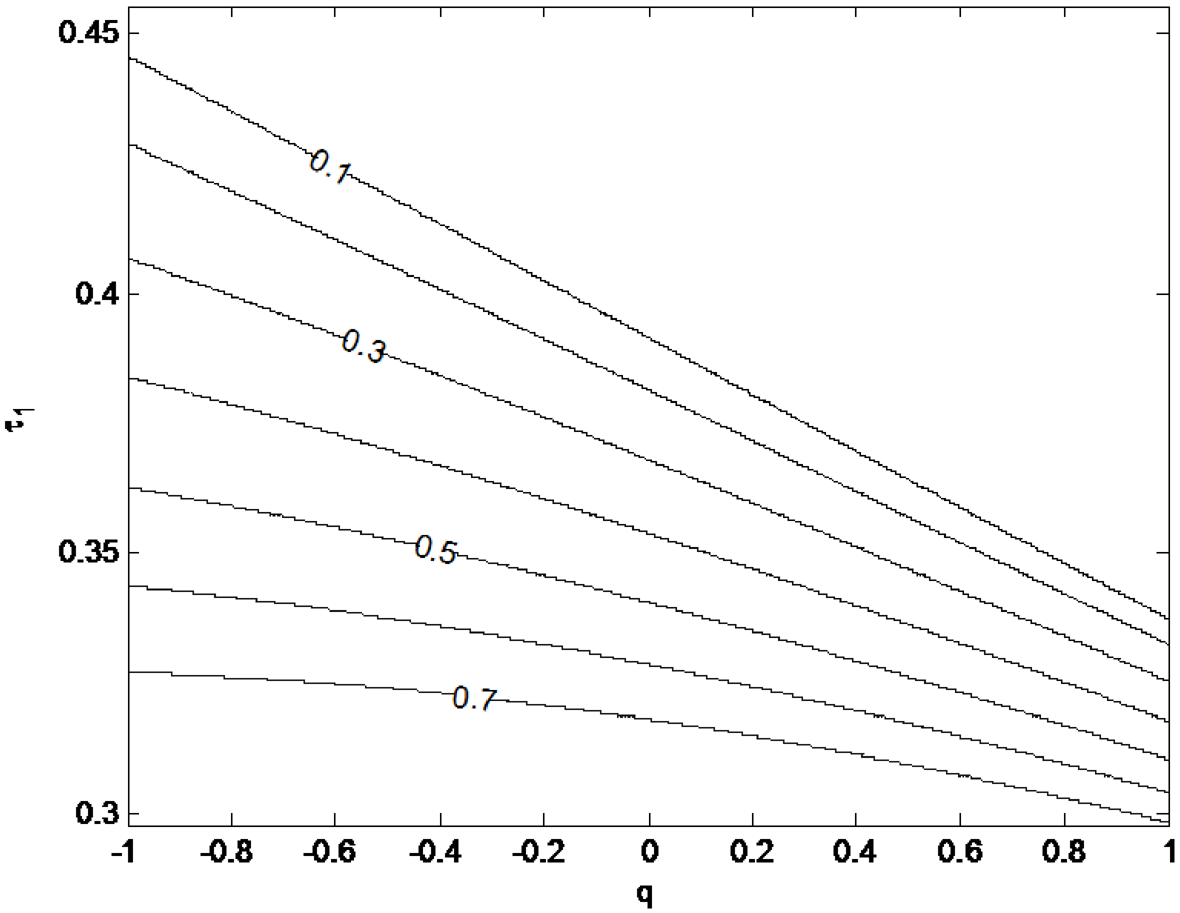

Figure 4.28: Skin friction at the isothermal wall for different Pr $(t = 0.3, q = 2.0)$ at y=1

Figure 4.29 Illustrates the behaviour of skin-friction at the adiabatic boundary for different values of heat source/sink (H). An observation from this figure indicates that the skin-friction increases with increase in heat source while it decreases with increase in heat sinks. This is physically expected since an increase in heat sink $(H > 0)$ decrease the velocity of the fluid (see figure 4.22).

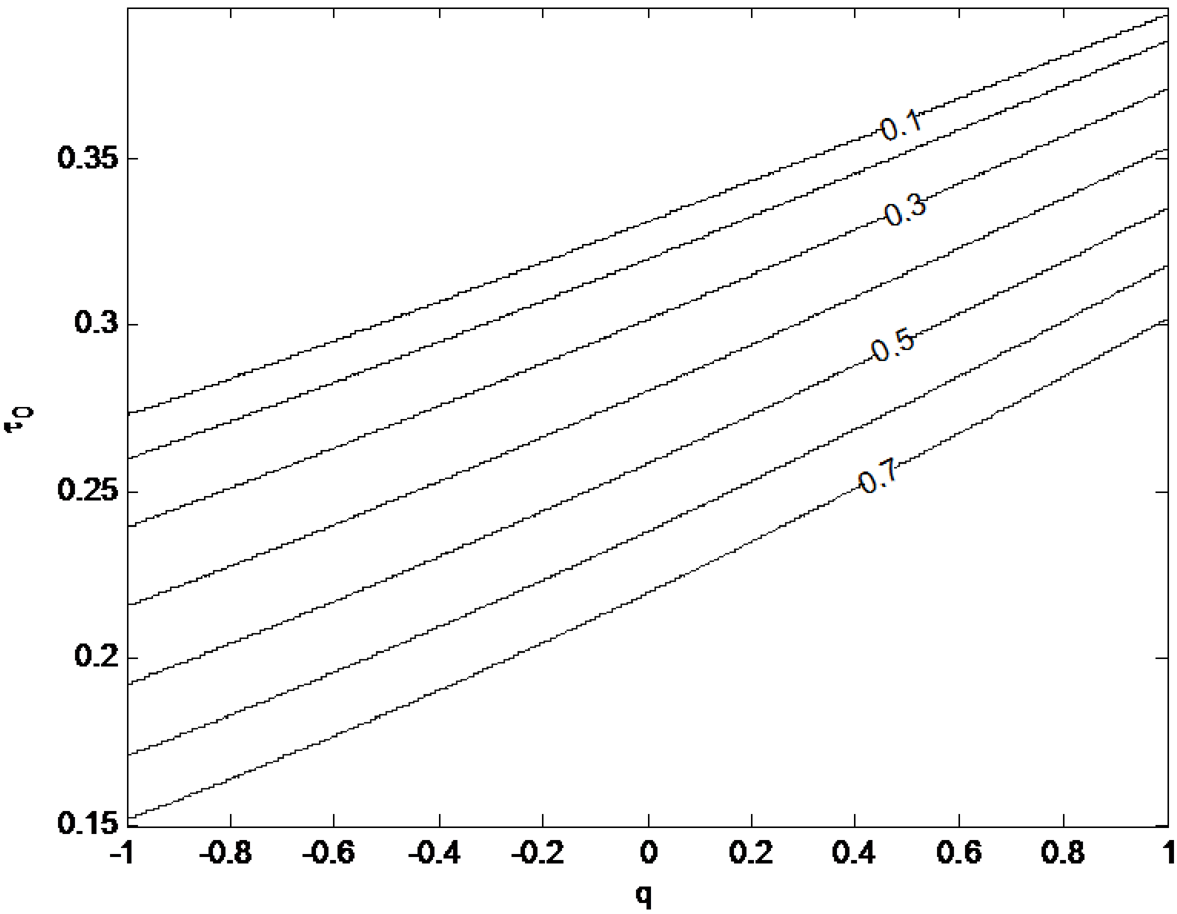

Figure 4.29: Skin friction at the adiabatic wall for different H $(Pr = 0.71, q = 2.0)$ at y=0

Figure 4.30 depicts the effect of Prandtl number on the skin-friction at isothermal plate ($y = 0$). This figure shows that as Pr increases, skin-friction decreases.

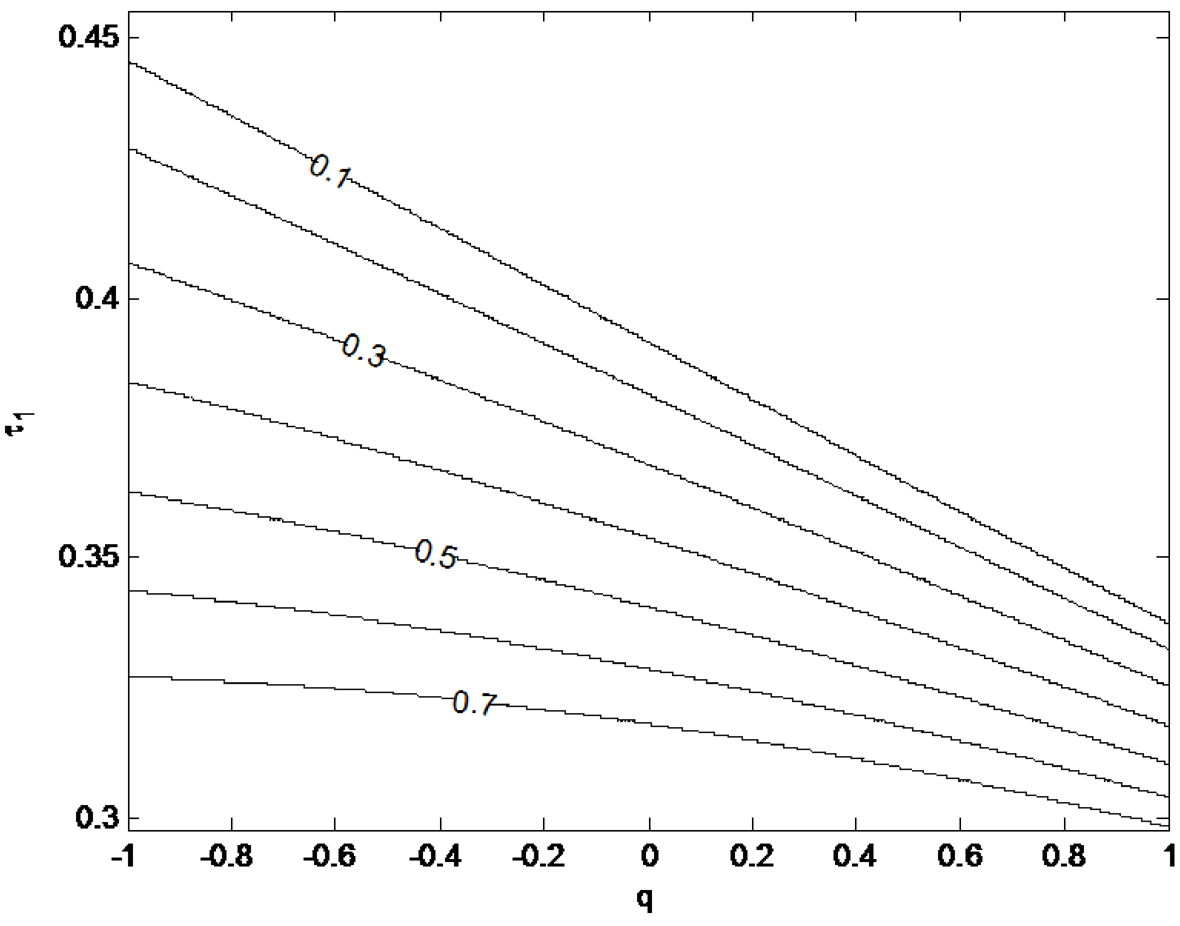

Figure 4.30: Skin friction at the isothermal wall for different Pr $(t = 0.3, q = 2.0)$ at y=1

Table 4.2 shows that transient temperature and velocity increase with increasing time and finally attains a steady state for large values of time, which coincides with the steady state temperature and velocity.

Table 4.2: Validation of problem 3.2

Transient (Using Riemann-Sum)	Temperature	Velocity
t	$H = 1.0, q = 2.0, Pr = 0.71$	
0.2	0.6433	0.0559
0.5	0.7953	0.0921
0.8	0.8122	0.0972
1.1	0.8140	0.0976
1.3	0.8142	0.0978
Steady state	0.8142	0.0978

4.3 Validation

To validate the second problem, we compare our steady-state results for temperature as well as velocity with that of Jha and Ajibade [2010].

Setting $q = 0$ in the present problem we have $m = -n = \sqrt{H}$. Also $f_1 = f_2 = f_3 = -f_4 = \frac{1}{2}$. So that equation (3.38) becomes $\widehat{T} = \cosh\, y\sqrt{H}$. However, as $H \to 0$, the steady-state temperature become $\widehat{T} = 1$, which coincides with equation (29) in Jha and Ajibade [2010]. Similarly, the steady-state velocity (3.39) becomes $\widehat{u} = \frac{1}{2}(y - y^2)$.

In the work of Jha and Ajibade [2010], the effect of suction/injection as well as internal heat generation is absent. However, several fluids of interest exist with heat generation of which results is not applicable.

CHAPTER 5

SUMMARY AND CONCLUSION

5.1 Summary

An analysis of the transient free convective flow of a viscous incompressible fluid in a vertical channel formed by two parallel porous plates in the presence of heat generation is carried out. The Laplace transform method is used to solve the dimensionless governing linear partial differential equation. The conversion from the Laplace domain to the time domain is achieved by the Riemann-sum approximation method. The effects of governing parameters are studied. Two problems, each with the transpiration and heat generation are discussed. The first problem is the effect of transpiration and heat generation on transient natural convection in a vertical channel. In this problem, the effect of Prandtl number (Pr), heat source/sink (H), suction/injection (q), and time t on the flow behaviour is investigated. While the second problem derives the solutions for the natural convection flow of heat generating fluid in a vertical channel with isothermal and adiabatic conditions. The controlling parameters for this problem are Prandtl number (Pr), heat source/sink (H), suction/injection (q), and time t. The effect of these parameters is ivestigated on the flow behaviour.

5.2 Conclusion

In the first problem, it is discovered that fluid temperature and velocity increase with time and attain steady-states for large values of time. In addition, fluid temperature and velocity increase on the heated plate, temperature within the channel responded with a decrease. Moreover, as the heat source increases, fluid temperature and velocity increase while it decreases with the increase in heat sink. The skin-friction increases with increase in heat source while it decreases with increase in heat sink. The rate of heat transfer decreases with increase in Prandtl number Pr. In addition, heat source decreases the rate of heat transfer on the heated wall, while it increases same on the cold wall.

Also, in the second problem, it is discovered that the fluid temperature and velocity increase with time and finally attain steady states. Both temperature and velocity increase with increase in injection on the isothermal plate while the reverse phenomenon is observed with injection on the adiabatic plate. In addition, as the heat source increases, fluid temperature and velocity increase and it decreases with increase in heat sink. The skin-friction increases with increase in heat source and it decreases with increase in heat sink. The rate of heat transfer for fluids with higher Pr has high rate of increase as suction/injection grows on the isothermal plate.

REFERENCES

[1] Al-sanea, S. A. (2004), Mixed convection heat transfer along a continuously moving heated vertical plate with suction or injection. *International Journal of Heat and Mass Transfer.* 47: 1445-1465.

[2] Baker, L., Faw, R.E. and Kulacki, F. A., (1976), Post-accident Heat Removal-part I: Heat transfer within an internally heated, Non-boiling liquid Layer, *Nuclear Sciences and Engineering.* 61: 222-230.

[3] Chamkha, A. J., and Camille, I. (2000), Effects of heat generation/absorption and the thermophoresis on hydromagnetic flow with heat and mass transfer over a flat plate. *International Journal of Numerical Methods for Heat and Fluid Flow.* 3: 432-438.

[4] Delichatsios, M. A. (1988), Air experiment into Buoyant jet Flames and pool fires. *The SFPA Handbook or Fire protection Engineering, P. J. Dinenno et al., eds., NFPA publication, Quincy, M. A.* 19: 306-314.

[5] Ferdousi, A., and Alim, M. A. (2010), Natural convection flow from a porous vertical plate in the presence of heat generation. *Daffodil International University Journal of Science and Technology.* 5(1): 73-80.

[6] Foraboschi, F. P. and Federico, I. D. (1964), Heat transfer in a laminar flow of non Newtonian heat generating fluids. *International Journal of Heat and Mass Transfer.* 7: 315.

[7] Hossain, M. A., Munir, M. S. and Rees, D. A. S. (2000), "Flow of viscous incompressible Fluid with Temperature Dependent viscosity and Thermal Conductivity past a permeable wedge with Uniform surface Heat Flux," *International Journal of Thermal Sciences,* 39(6): 635-644. Doi:10.1016/s1290-0729(00)00227-1.

[8] Ishak, A., Merkin, J. H. Nazar., and Pop, I. (2008), Mixed convection boundary layer flow over a permeable vertical surface with prescribed wall heat flux. *Zeitschriftfur Angewandte Mathematik und Physik,* 59: 100-123.

[9] Jha, B. K. and Ajibade, A. O. (2009), Free convection flow of heat generation/absorbing fluid between vertical porous plates with periodic heat input. *International Communication in Heat and Mass Transfer.* 36: 624-631.

[10] Jha, B. K. and Ajibade. A. O., (2010), Transient Natural Convection flow between Vertical parallel plates- one plate Isothermally Heated and other Thermally Insulated, *Journal of Process Mechanical Engineering.* Doi:10.1243/09544089JPME319.

[11] Kabir, S., Hossain, M. A. and Rees, D. A. S., (2002), 'Natural convection of fluid with Temperature Dependent viscosity from Heated vertical wavy surface," *Zeitschriftfur Angewandte Mathematik und Physik,* 53: 48-57 Doi:10.1007/s00033-002-8141-z.

[12] Khadrawi, A. F. and Al-nimr, M. A., (2007), Unsteady natural convection fluid flow in a vertical microchannel under the effect of the dual-phase-lag heat conduction model. *International Journal of Thermophysics,* 28: 1387-1400.

[13] Magyari, E., and Chamkha, A. J. (2010), Combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flows over a uniformly stretched permeable surface: The full analytical solution. *International Journal of Thermal Science* 33(6): 1-8.

[14] Mahdy, A. (2010), Effect of chemical reaction and heat generation or absorption on double-diffusive convection from a vertical truncated cone in porous media with variable viscosity. *International Communication in Heat and Mass Transfer.* 37: 548-554.

[15] Mckenzie, D. P., Roberts, J. M., and Weiss, N. O. (1974), "Convection in Earth's mantle: Toward a Numerical Simulation," *Journal of Fluid Mechanics.* 62: 465-538.

[16] Mendez, F., and Trevino, C. (2000), The conjugate conduction-natural convection heat transfer along a thin vertical plate with non-uniform internal heat generation. *International Journal of Heat and Mass Transfer.* 43: 2739-2748.

[17] Mohammadein, A. A., and Gorla, R. S. (2001), Heat transfer in a micropolar fluid over a stretching sheet with viscous dissipation and internal heat generation. *International Journal of Numerical Methods for Heat and Fluid Flow.* 11(1): 50-58.

[18] Molla, M. M., Paul, S. C., and Hossain, M. A (2009), Natural convection flow from a horizontal circular cylinder with uniform heat flux in presence of heat generation. *Applied Mathematical Modelling.* 33: 3226-3236.

[19] Molla, M. M., Hossain, M. A., and Yao, L. S. (2004), Natural convection flow along a vertical wavy surface with heat generation/absorption. *International Journal of Thermal Science.* 43: 157-163.

[20] Postelnicu, A. and Pop, I. (1999), Similarity solutions of free convection boundary layers over vertical and horizontal surface in porous media World Academy of Science, Engineering and Technology 75 2011 535 with internal heat generation. *International Communication in Heat and Mass Transfer.* 26: 1183-1191.

[21] Rahman, M. M., Eltayeb, I. A., and Rahman, S. M. M. (2009), Thermomicropolar fluid flow along a vertical permeable plate with uniform surface heat flux in the presence of heat generation. *Thermal Science.* 13(1): 23-36.

[22] Smith, W., and Hammitt, F. G. (1996), Natural convection in a rectangular cavity with internal Heat Generation, *Nuclear Sciences and Engineering.* 25: 328-342.

[23] Siddiqa, S., Asghar, S., and Hossain, M. A. (2010), Natural convection flow over an inclined flat plate with internal heat generation and variable viscosity. *Mathematics and Computer Modelling.* 52: 1739-1751.

[24] Singh, A. K. (1984), Stokes problem for a porous vertical plate with sinks by finite difference method. *Astrophysics Space Sciences.* 103: 241-248.

[25] Shojaefard, M. H., Noorpoor, A. R., Avanacians, A., and Chaffapour, M. (2005), Numerical investigation of flow control by section and injection on a subsonic airfoil. *American Journal of Applied Sciences.* 20(10): 1474-1480.

[26] Tzou, D. Y. (1997), *Macro to microscale heat transfer*: The lagging behavior. (Taylor and Francis. Washington DC,) 1: 1-64.

[27] Vajravelu, K., and Hadjinicolaou, A. (1993), Heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation. *International Communication in Heat and Mass Transfer.* 20: 417-430.

[28] Westphal, B. R., Keiser, D. D., Rigg, R. H., and Lang, D. V., (1994), Production of metal waste form spent nuclear fuel treatment, *DOE Spent Nuclear Fuel Conference, Salt Lake City,* 35: 288-294.

APPENDICES

Appendix I

MATLAB PROGRAM FOR TRANSIENT TEMPERATURE

Effect of transpiration and heat generation on transient natural convection in a vertical channel

$[y, t] = meshgrid(0.0 : 0.001 : 1.0, 0.1 : 0.1 : 1.0);$

$[y, q] = meshgrid(0.0 : 0.001 : 1.0, -2.0 : 0.1 : 2.0);$

$[y, H] = meshgrid(0.0 : 0.001 : 1.0, -2.1 : 0.1 : 2.0);$

[y,pr]=meshgrid(0.0:0.001:1.0,0.01:0.1:0.71);

clc

$w = (y - y + 1). * (t - t + 1);$

$w = (y - y + 1). * (q - q + 1);$

$w = (y - y + 1). * (H - H + 1);$

$w = (y - y + 1). * (pr - pr + 1);$

$pr = 0.71. * w;$

$H = 2 * w;$

$t = 0.3 * w;$

$q = 2 * w;$

$ep = 4.7. * w;$

$eps = ep./t;$

$se = 4.7. * w;$

$zx = 0 * w;$

n=1;

while$n < 1000;$

$x1 = (-1).^n;$

$x2 = eps + (i. * n. * pi./t);$

$x3 = q. * pr;$

$x4 = x2. * pr + H;$

$x5 = sqrt((x3.^2)./4 + x4);$

$x6 = x2. * (1 - exp(-2. * x5));$

$c1 = -exp(-(2. * x5))./x6;$

$c2 = (1. * w)./x6;$

$x7 = c1. * exp(y. * (x5 - 0.5. * x3));$

$x8 = c2. * exp(-y. * (x5 + 0.5. * x3));$

$w1 = x7 + x8;$

$zx = zx + w1. * x1;$

$n = n + 1;$

end

$re = real(zx);$

$z1 = eps;$

$zx3 = q. * pr;$

$zx4 = z1. * pr + H;$

$zx5 = sqrt((zx3.^2)./4 + zx4);$

$zx6 = z1. * (1 - exp(-2. * zx5));$

$zc1 = -exp(-(2. * zx5))./zx6;$

$zc2 = (1. * w)./zx6;$

$zx7 = zc1. * exp(y. * (zx5 - 0.5. * zx3));$

$zx8 = zc2. * exp(-y. * (zx5 + 0.5. * zx3));$

$w2 = zx7 + zx8;$

$z2 = exp(se)./t;$

$tt = z2. * (0.5 * w2 + re);$

$[C, h] = contour(y, tt, t);$

$[C, h] = contour(y, tt, q);$

$[C, h] = contour(y, tt, H);$

$[C, h] = contour(y, tt, pr);$

clabel(C,h,'manual')

colormap gray

Appendix II

PROGRAM FOR TRANSIENT TEMPERATURE

MATLAB PROGRAM FOR TRANSIENT VELOCITY

Effect of transpiration and heat generation on transient natural convection in a vertical channel

$[y, t] = meshgrid(0.0 : 0.001 : 1.0, 0.1 : 0.1 : 1.0);$

$[y, q] = meshgrid(0.0 : 0.001 : 1.0, -2.0 : 0.1 : 2.0);$

$[y, H] = meshgrid(0.0 : 0.001 : 1.0, -2.0 : 0.1 : 2.0);$

$[y, pr] = meshgrid(0.0 : 0.001 : 1.0, 0.01 : 0.1 : 0.71);$

clc

$w = (y - y + 1). * (t - t + 1);$

$w = (y - y + 1). * (q - q + 1);$

$w = (y - y + 1). * (H - H + 1);$

$w = (y - y + 1). * (pr - pr + 1);$

$pr = 0.71. * w;$

$H = 2 * w;$

$t = 0.3 * w;$

$q = 2 * w;$

$ep = 4.7. * w;$

$eps = ep./t;$

$se = 4.7. * w;$

$zx = 0 * w;$

n=1;

while$n < 100;$

$x1 = (-1).^n;$

$x2 = eps + (i. * n. * pi./t);$

$x3 = q. * pr;$

$x4 = x2. * pr + H;$

$x5 = sqrt((x3.^2)./4 + x4);$

$x6 = x2. * (1 - exp(-2. * x5));$

$c1 = -exp(-(2. * x5))./x6;$

$c2 = 1./x6;$

$x7 = sqrt((q.^2)./4 + x2);$

$x8 = (x5 - 0.5. * x3).^2;$

$x9 = q. * (x5 - 0.5. * x3) - x2;$

$x10 = -c1./(x8 + x9);$

$x11 = (x5 + 0.5. * x3).^2;$

$x12 = q. * (x5 + 0.5. * x3) + x2;$

$x13 = -c2./(x11 - x12);$

$x14 = (x10 + x13). * exp(x7 - 0.5. * q);$

$x15 = -x10. * exp(x5 - 0.5. * x3) - x13. * exp(-(x5 + 0.5. * x3));$

$d2 = (x14 + x15)./(exp(-(x7 + 0.5. * q)) - exp(x7 - 0.5. * q));$

$d1 = -(d2 + x10 + x13);$

$u1 = d1. * exp(y. * (x7 - 0.5. * q)) + d2. * exp(-(y. * (x7 + 0.5. * q)));$

$u2 = x10. * exp(y. * (x5 - 0.5. * x3)) + x13. * exp(-(y. * (x5 + 0.5. * x3)));$

$uu = u1 + u2;$

$w1 = uu;$

$zx = zx + w1. * x1;$

$n = n + 1;$

end

$re = real(zx);$

$z1 = eps;$

$zx3 = q. * pr;$

$zx4 = z1. * pr + H;$

$zx5 = sqrt((zx3.^2)./4 + zx4);$

$zx6 = z1. * (1 - exp(-2. * zx5));$

$zc1 = -exp(-(2. * zx5))./zx6;$

$zc2 = 1./zx6;$

$zx7 = sqrt((q.^2)./4 + z1);$

$zx8 = (zx5 - 0.5. * zx3).^2;$

$zx9 = q. * (zx5 - 0.5. * zx3) - z1;$

$zx10 = -zc1./(zx8 + zx9);$

$zx11 = (zx5 + 0.5. * zx3).^2;$

$zx12 = q. * (zx5 + 0.5. * zx3) + z1;$

$zx13 = -zc2./(zx11 - zx12);$

$zx14 = (zx10 + zx13). * exp(zx7 - 0.5. * q);$

$zx15 = -zx10. * exp(zx5 - 0.5. * zx3) - zx13. * exp(-(zx5 + 0.5. * zx3));$

$zd2 = (zx14 + zx15)./(exp(-(zx7 + 0.5. * q)) - exp(zx7 - 0.5. * q));$

$zd1 = -(zd2 + zx10 + zx13);$

$zu1 = zd1. * exp(y. * (zx7 - 0.5. * q)) + zd2. * exp(-(y. * (zx7 + 0.5. * q)));$

$zu2 = zx10. * exp(y. * (zx5 - 0.5. * zx3)) + zx13. * exp(-(y. * (zx5 + 0.5. * zx3)));$

$zuu = zu1 + zu2;$

$w2 = zuu;$

$z2 = exp(se)./t;$

$u = z2. * (0.5 * w2 + re);$

$[C, h] = contour(y, u, t);$

$[C, h] = contour(y, u, q);$

$[C, h] = contour(y, u, H);$

$[C, h] = contour(y, u, pr);$

clabel(C,h,'manual')

colormap gray

Appendix III

MATLAB PROGRAM FOR TRANSIENT TEMPERATURE FOR ISOTHERMAL AND ADIABATIC CONDITIONS.

Effect of transpiration on natural convection flow of heat generating fluid in a vertical channel with isothermal and adiabatic conditions.

$[y, t] = meshgrid(0.0 : 0.001 : 1.0, 0.1 : 0.1 : 1.0);$

$[y, q] = meshgrid(0.0 : 0.001 : 1.0, -2.0 : 0.1 : 2.0);$

$[y, H] = meshgrid(0.0 : 0.001 : 1.0, -2.1 : 0.1 : 2.0);$

$[y, pr] = meshgrid(0.0 : 0.001 : 1.0, 0.01 : 0.1 : 0.71);$

clc

$w = (y - y + 1). * (t - t + 1);$

$w = (y - y + 1). * (q - q + 1);$

$w = (y - y + 1). * (H - H + 1);$

$w = (y - y + 1). * (pr - pr + 1);$

$pr = 0.71. * w;$

$H = 1 * w;$

$t = 0.3 * w;$

$q = 2 * w;$

$ep = 4.7. * w;$

$eps = ep./t;$

$se = 4.7. * w;$

$zx = 0 * w;$

n=1;

while$n < 1000$;

$x1 = (-1).^n$;

$x2 = eps + (i. * n. * pi./t)$;

$x3 = q. * pr$;

$x4 = x2. * pr + H$;

$x5 = sqrt((x3.^2)./4 + x4)$;

$x6 = x2. * (1 - exp(-2. * x5))$;

$c1 = (x5 + 0.5. * x3)./(((x5 + 0.5. * x3). * exp(x5 - 0.5. * x3) + (x5 - 0.5. * x3). * exp(-(x5 +$
$0.5. * x3))). * x2)$;

$c2 = c1. * (x5 - 0.5. * x3)./(x5 + 0.5. * x3)$;

$x7 = c1. * exp(y. * (x5 - 0.5. * x3))$;

$x8 = c2. * exp(-y. * (x5 + 0.5. * x3))$;

$w1 = x7 + x8$;

$zx = zx + w1. * x1$;

$n = n + 1$;

end

$re = real(zx)$;

$z1 = eps$;

$zx3 = q. * pr$;

$zx4 = z1. * pr + H$;

$zx5 = sqrt((zx3.^2)./4 + zx4)$;

$zx6 = z1. * (1 - exp(-2. * zx5))$;

$zc1 = (zx5 + 0.5. * zx3)./(((zx5 + 0.5. * zx3). * exp(zx5 - 0.5. * zx3) + (zx5 - 0.5. * zx3). *$
$exp(-(zx5 + 0.5. * zx3))). * z1)$;

$zc2 = zc1. * (zx5 - 0.5. * zx3)./(zx5 + 0.5. * zx3);$

$zx7 = zc1. * exp(y. * (zx5 - 0.5. * zx3));$

$zx8 = zc2. * exp(-y. * (zx5 + 0.5. * zx3));$

$w2 = zx7 + zx8;$

$z2 = exp(se)./t;$

$tt = z2. * (0.5 * w2 + re);$

$[C, h] = contour(y, tt, t);$

$[C, h] = contour(y, tt, q);$

$[C, h] = contour(y, tt, H);$

$[C, h] = contour(y, tt, pr);$

clabel(C,h,'manual')

colormap gray

Appendix IV

MATLAB PROGRAM FOR TRANSIENT VELOCITY FOR ISOTHERMAL AND ADIABATIC CONDITIONS.

Effect of transpiration on natural convection flow of heat generating fluid in a vertical channel with isothermal and adiabatic conditions.

$[y,t] = meshgrid(0.0 : 0.001 : 1.0, 0.1 : 0.1 : 1.0);$

$[y,q] = meshgrid(0.0 : 0.001 : 1.0, -2.0 : 0.1 : 2.0);$

$[y,H] = meshgrid(0.0 : 0.001 : 1.0, -2.0 : 0.1 : 2.0);$

$[y,pr] = meshgrid(0.0 : 0.001 : 1.0, 0.01 : 0.1 : 0.71);$

clc

$w = (y - y + 1). * (t - t + 1);$

$w = (y - y + 1). * (q - q + 1);$

$w = (y - y + 1). * (H - H + 1);$

$w = (y - y + 1). * (pr - pr + 1);$

$pr = 0.71. * w;$

$H = 1 * w;$

$t = 0.3 * w;$

$q = 2 * w;$

$ep = 4.7. * w;$

$eps = ep./t;$

$se = 4.7. * w;$

$zx = 0 * w;$

n=1;

while$n < 100;$

$x1 = (-1).^n;$

$x2 = eps + (i. * n. * pi./t);$

$x3 = q. * pr;$

$x4 = x2. * pr + H;$

$x5 = sqrt((x3.^2)./4 + x4);$

$x6 = x2. * (1 - exp(-2. * x5));$

$c1 = (x5 + 0.5. * x3)./(((x5 + 0.5. * x3). * exp(x5 - 0.5. * x3) + (x5 - 0.5. * x3). * exp(-(x5 + 0.5. * x3))). * x2);$

$c2 = c1. * (x5 - 0.5. * x3)./(x5 + 0.5. * x3);$

$x7 = sqrt((q.^2)./4 + x2);$

$x8 = (x5 - 0.5. * x3).^2;$

$x9 = q. * (x5 - 0.5. * x3) - x2;$

$x10 = -c1./(x8 + x9);$

$x11 = (x5 + 0.5. * x3).^2;$

$x12 = q. * (x5 + 0.5. * x3) + x2;$

$x13 = -c2./(x11 - x12);$

$x14 = (x10 + x13). * exp(x7 - 0.5. * q);$

$x15 = -x10. * exp(x5 - 0.5. * x3) - x13. * exp(-(x5 + 0.5. * x3));$

$c4 = (x14 + x15)./(exp(-(x7 + 0.5. * q)) - exp(x7 - 0.5. * q));$

$c3 = -(c4 + x10 + x13);$

$u1 = c3. * exp(y. * (x7 - 0.5. * q)) + c4. * exp(-(y. * (x7 + 0.5. * q)));$

$u2 = x10. * exp(y. * (x5 - 0.5. * x3)) + x13. * exp(-(y. * (x5 + 0.5. * x3)));$

$uu = u1 + u2;$

$w1 = uu;$

$zx = zx + w1. * x1;$

$n = n + 1;$

end

$re = real(zx);$

$z1 = eps;$

$zx3 = q. * pr;$

$zx4 = z1. * pr + H;$

$zx5 = sqrt((zx3.^2)./4 + zx4);$

$zx6 = z1. * (1 - exp(-2. * zx5));$

$zc1 = (zx5 + 0.5. * zx3)./(((zx5 + 0.5. * zx3). * exp(zx5 - 0.5. * zx3) + (zx5 - 0.5. * zx3). *$

$exp(-(zx5 + 0.5. * zx3))). * z1);$

$zc2 = zc1. * (zx5 - 0.5. * zx3)./(zx5 + 0.5. * zx3);$

$zx7 = sqrt((q.^2)./4 + z1);$

$zx8 = (zx5 - 0.5. * zx3).^2;$

$zx9 = q. * (zx5 - 0.5. * zx3) - z1;$

$zx10 = -zc1./(zx8 + zx9);$

$zx11 = (zx5 + 0.5. * zx3).^2;$

$zx12 = q. * (zx5 + 0.5. * zx3) + z1;$

$zx13 = -zc2./(zx11 - zx12);$

$zx14 = (zx10 + zx13). * exp(zx7 - 0.5. * q);$

$zx15 = -zx10. * exp(zx5 - 0.5. * zx3) - zx13. * exp(-(zx5 + 0.5. * zx3));$

$zc4 = (zx14 + zx15)./(exp(-(zx7 + 0.5. * q)) - exp(zx7 - 0.5. * q));$

$zc3 = -(zc4 + zx10 + zx13);$

$zu1 = zc3. * exp(y. * (zx7 - 0.5. * q)) + zc4. * exp(-(y. * (zx7 + 0.5. * q)));$

$zu2 = zx10. * exp(y. * (zx5 - 0.5. * zx3)) + zx13. * exp(-(y. * (zx5 + 0.5. * zx3)));$

$zuu = zu1 + zu2;$

$w2 = zuu;$

$z2 = exp(se)./t;$

$u = z2. * (0.5 * w2 + re);$

$[C, h] = contour(y, u, t);$

$[C, h] = contour(y, u, q);$

$[C, h] = contour(y, u, H);$

$[C, h] = contour(y, u, pr);$

clabel(C,h,'manual')

colormap gray