TITLE: BUILDING RESPONSE TO DYNAMIC LOADS DUE TO QUARRY BLASTING.

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BUILDING RESPONSE

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D Y N A M I C L O A D S

D U E T O

QUARRY BLASTING

BY

EMMANUEL IKUME YUWA

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and

MISS MEMBER YUWA.

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A B S T R A C T :

The Gxistencc of quarries near residential areas is now becoming inevitable. In order to protect building Structures against quarry operations hazards, there is need to quantify response of these structures to hazards. One of such hazards is the dynonic loads emitted and transmitted through the soil and air media due to blasting of the rock for its excavation. In this research, effort has been made to model the dynamic loads transmitted from the blasting point to the structure. Two path-ways are identified - air and soil - for the transmission of the loads onto a building structure. An assessment of the effect of weather conditions at the time of blasting on the relative contribution of the loads travelling through the air and soil media onto a building structure in the vicinity was made. Modal analysis was utilised effectively for obtaining response of a building structure modelled into an elastic, three degree of

freedom system.

The research results indicate that structural damage is essentially due to ground vibrations striking the foundations of the building. This damage is quantitatively greater than those corresponding to air blast hitting the super structure. Overall, the weather conditions is found to affect the response in all cases.

CONTENTS

LIST OF SYMBOLS

The following symbols are used in the following pages as heredefined, unless defined otherwise.

SYMBOL		DESCRIPTION
C	–	Damping Coefficient Matrix.
d_e	–	Generalised Displacements.
$E I$	–	Flexural Rigidity.
E.R.	–	Energy Ratio.
F_e	–	Flexibility Matrix of Constrained Structure.
G	–	Impulsive Loads.
I	–	Unit Matrix.
K	–	Stiffness Coefficient Matrix.
K_c	–	Constrained Stiffness Matrix.
M	–	Mass Matrix.
M_c	–	Constrained Mass Matrix.
P	–	Dynamic Applied Forces.
ρ_s	–	Density of Soil.
ρ_a	–	Density of Air.
P_e	–	Generalised Forces.
P_o	–	Atmospheric Pressure.
P_r	–	Reflected Pressure.
P_{so}	–	Peak Over-pressure.
q	–	Amplitudes of Displacement.

q_o - Rigid Body Modes.

q_c - Unconstrained Amplitudes of Displacement Corresponding to M_c.

q_e - Elastic Eigenmodes.

ϕ_e - Time Dependent Multiplier.

Q_g - Input - Loads - Equivalent due to Base Acceleration.

R - Range.

σ - Stress.

δ - Time Lag.

t - Time.

t_d - Duration of Over-pressure.

U - Displacement of Masses.

$\ddot{U}$ - Acceleration.

U_s - Velocity of Sound.

v - Air Particle Velocity.

w - Circular Frequency.

X_g - Base Displacement.

$\ddot{X}_g$ - Base Acceleration.

Y - Yield (Explosive Size).

λ - Eigenvalue.

CHAPTER ONE

INTRODUCTION

Nigeria as a developing country has been faced with the problem of construction to expand her cities. This has given rise to the opening of quarries almost everywhere and the proximity of these quarries to residential areas has become inevitable. There is therefore the need for a study of the effect of these quarries on residential areas in their vicinities. These quarries are responsible for the aggregates obtained for construction and in certain cases, rocks obtained from them are used for the production of cement.

The process of obtaining rock material from its main body requires the use of explosives to blast the main body. The blasting has its effect not only on the rock body but also on the surrounding structures. The interest here is to find out the behaviour of a civil engineering structure in the vicinity of a quarry.

A structure located in the near vicinity of a quarry responds to sets of loading conditions which are due to both static and dynamic loads. The static loads are due to the self weight of the structure and some other sets of statically imposed loads, while the dynamic loads could be due to machinery within the structure or due to the blasting of rock at a nearby quarry. A dynamic load is principally defined as that load whose time of rise from zero to a maximum is less than double the natural period of vibration of the structure on which it acts (63). The response of such a structure to both the static and the dynamic loads is of importance to the structural engineer. Herein, the concern is with the response of a building structure, located in the near vicinity of a quarry, to the dynamic loads due to quarry blasting.

E.H. Rockwell made studies in 1927 and he stressed the need for measuring vibrations from quarry blasting (102). Tseng Huang, John A. Blume and others have through the years tried to study the response of structures to aseismic loadings (98,8). While others are working on the response of structures to ground motion - mostly earthquakes (12,57,94), others have concentrated on the response of structures to air blast forces (24).

In quarry blasting these two forms of external forces exist - air blast and ground vibration. It is the intention in this thesis to determine the response of a structure loaded simultaneously by these two forms of dynamic loads - air blast forces and ground vibration.

A three storey building is treated in this work. It is examined using the small deflection theory which limits the analysis to the elastic region of the stress-strain curve. The analysis is done using matrix algebra which has proven very effective. A computer program has been written to use the matrix equations developed. Graphs are plotted to show the variation of the response of the structure with time and its relationship with the period of vibration of the structure for the different forms of input loads. The percentage contribution, on the total response, of the input loads on the structure is considered. It will also be shown that weather conditions at the time of blast influence the maximum response. Suggestions are made on how the knowledge of the properties shown could be used to predict the response of a structure in the near vicinity of a quarry.

CHAPTER TWO

LITERATURE REVIEW

In 1905, Stephen Timoshenko published, in the Bulletin of the Polytechnical Institute of St. Petersbury a paper titled "O Yavleniyakh Rezonansa v Valakh (on Resonanse Phenomena in Shafts)" (96). It showed how the effect of the distributed mass of a shaft on period of free torsional vibration could be accounted for approximately using Rayleighs method on sound theory. This marked the begining of vibration problems in Engineering.

The birth of explosives and their development for mining purposes brought in problems of what effects the air and ground vibrations resulting from blasting have on structures of various types. This problem was observed to be more acute for surface operations (20) such as open-pit mines, quarries and construction projects that are in the proximity of residential areas.

The first investigation carried out in the field was done in 1927 by Rockwell (102). From his instrumented studies, he concluded that quarry blasting would not produce damage to residential structures that were more than 60 to 100 m from the quarry. He also pointed out the need for measuring the vibrations from quarry blasting in order to establish the level of vibration as a function of charge size and distance.

During the period from 1935 to 1942, the Bureau of mines in USA conducted on extensive investigation on the problem of seismic effects of quarry blasting (104,105). This work was summarized in Bulletin 442 (104) and aimed at establishing damage criteria for residential structures which when exceeded results in some degree of failure within the structure. This established a criteria based on acceleration, and labelled them as - vibration levels below 0.1g: no damage, between 0.1 and 1g: caution and above 1g: damage. Between 1949 and 1960 interest in the problem of seismic vibration from blasting and their effects upon structures continued to increase and many publications (16,17) were released. These investigators tried to measure and/or analyse the ground seismic waves resulting from blasting.

It has been observed that a lot of work has been done on the problem of vibrations of structures in the near vicinity of quarry blasting. Quarry blasting results in the emission of two types of loads that travel through two different media to nearby structures - air blast pressure through air medium and ground vibration through the soil medium. The problem of analysing these two forms of dynamic forces on structures has been treated by many authors and in each case - considered the response of a structure to a single form of the loading condition.

In 1956, Schenker (116) presented in his paper some aspects of the analytical procedures for computing the response of structures to dynamic loads. He treated the case of lateral loads, both the close coupled and the far coupled, for response. His emphasis was on the problem of joint rotation, deformation into the inelastic range and damping. In 1961, Burkhartt (15) presented the vibration analysis for structural floor systems. This also considered lateral loads as dynamic input functions. Tseng Hung in 1964 (98) studied on a more specific basis and presented a study on the response of structures to triangular pulse loading. This, of course, was treated on the understanding that the loading was lateral. Chiu, Henderson, Hart, Morrone (24,31,35,67) in their papers have presented the response of structures to lateral loads such as air blast pressure.

The work on response of structures has not been limited to that of its response to lateral loads such as air blast pressure alone. Work has been done extensively on the response of structures to ground vibrations. In 1949, Crandell (39) published his study on "Ground vibrations due to blasting and its effect upon structures". This paper was aimed at helping the contractor establish the amount of explosive to use so that no damage will be caused to nearby residential areas during blasting. He established the energy ratio concept which he defined as:

$$E.R. = \frac{a^2}{n^2} \qquad ..... (2.1)$$

where E.R. - Energy ratio

a - acceleration of ground wave

n - frequency.

This was related to distance and weight of explosive as

$$E.R. = \frac{(50)^2}{(D)^2} C^2K \qquad ..... (2.2)$$

Where D - distance in feet

C - weight of explosive in pounds

K - proportionality factor depending on the terrain which transmits vibrations

Most of the work on ground vibration has been aimed at trying to define analytically the motion of ground vibration reaching a structure. Frank Neuman, Kisaraj, Bycroft, Oguntala (30,55,10,95) have presented in their papers different forms of solution for ground vibration records and the effect of soil media modeling on the obtained optimum result. In recent times the use of response spectra has been suggested (9) as a reasonable form of representing ground vibrations as input loads for the analysis of response of structures to ground vibration.

A real structure exposed to dynamic forces from quarry blasting will not respond to either air blast pressure alone or ground vibration alone. The maximum response, which is important to the structural engineer, will occur as a result of the combined action of these input forces. Work on this aspect has been done only at the instrumentation level. The Bureau of mines in USA has organised many investigations and presented reports on the maximum response of residential structures subjected to quarry blast loads. In the results of these investigations by the Bureau of mines in USA many factors that contribute to the maximum response of structures due to quarry blast were not carefully considered. In some of its reports attention was mainly paid to ground vibration alone.

It is the intention here to use the already established modal analysis theory to determine the combined effect of ground vibration and air blast pressure from quarry blasting on Civil Engineering structures.

CHAPTER THREE

LOADS

In order to determine the response of any structure, the loads acting on the structure, referred to as the input functions, must be identified and determined. For any structure, there are two types of loads that act - dead loads and imposed loads. The dead loads include the self weight of the structure while the imposed loads are the live loads on the structure. For a structure in the near vicinity of a mining area with quarry operations, the live loads include dynamic loads due to the quarry blasting.

Dynamic loads can be divided into two types - periodic and non-periodic. The loads due to quarry blasting are classified as a non-periodic type of dynamic loads. One useful defination of dynamic loads is that the time in which the load rises from zero to its maximum value is less than double the natural period of vibration of the structure on which it acts (63). The dynamic loads discussed in this chapter are those due to quarry blasting. The loads are transient in nature and are referred to as inpulsive loads.

3.1 THE QUARRY

Consider a quarry site as shown in fig. 3.1. A structure in the near vicinity, as shown, of the blast site is at a distance d from the blast. (d is between 60 to 100m).

When the rock bed is blasted, two forms of media are used to transmit dynamic loads to this nearby structure - the air medium and the soil medium. The dynamic loads through the air medium are due to air blast at the time of blasting. The intensity and travel time of this dynamic load, which is in the form of a pressure wave, depend considerably on the atmospheric conditions existing around the site at the time of blast (112). In 1960, Perkins, Lorrain, and Townsend published a report (113) which showed that local atmospheric conditions were responsible for the focusing of air blast and that these focal points were centres of some damage.

The dynamic loads on the structure through the soil media are measured in the form of displacement, velocity and acceleration of the soil mass at the foundation of the structure. Due to soil-structure interaction (80) the structure's foundation vibrates at the same rate as the soil mass on which it is placed. The nature in which this structure vibrates depends on the nature (magnitude, direction and arrival time) of the input dynamic loads.

The magnitude of the two input loads (forcing dynamic functions) is discussed below.

3.2. AIR BLAST PRESSURE

When an explosion occurs, a circular shock front is propagated away from the point of burst. At any instant in time the distribution of over-pressure (the excess above atmospheric pressure) along a radial line is as shown in fig. 3.2 (9).

The shock front travels with a velocity u and has a peak pressure P_{so} which decays behind the front as indicated in fig. 3.2 (a). When the shock front strikes an object such as a building, there is a 'diffraction' effect producing forces which result from the higher pressures due to reflection of the wave on the front face of the object and also from the time lag before the over pressure acts on the face. At the same time the air behind the shock front produces drag forces on any object encountered. Thus the total loading consists of three parts:

(a) The initial diffraction effect

(b) The effects of the general over pressure P_s.

(c) The drag loading.

At a fixed point on the ground the variation of overpressure and dynamic pressure with time is as indicated in fig. 3.2 (b). The dynamic pressure P_d is merely given as:

$$P_d = 0.5\, \rho_a\, v^2 \qquad .....(3.1)$$

where ρ_a - air density

v - air particle velocity.

The duration of the positive phase t_d of over-pressure and dynamic pressure can be found from (9):-

$$\frac{t_{d1}}{t_{d2}} = \left\{\frac{Y_1}{Y_2}\right\}^{\frac{1}{3}} \qquad .....(3.2).$$

where t_d - duration of over-pressure

Y - Yield (explosive size)

The distance at chich this occurs (referred to as the range) is found from (9):

$$\frac{R_1}{R_2} = \left\{\frac{Y_1}{Y_2}\right\}^{\frac{1}{3}} \qquad .....(3.3).$$

where R - the range.

The velocity of shock front depends on the peak over-pressure P_{so} and is given by (9):

$$U = U_s \left\{1 + \frac{6P_{so}}{7P_o}\right\}^{\frac{1}{2}} \qquad .....(3.4).$$

where U_s - velocity of sound (m/s)

P_o - atmospheric pressure (N/m^2)

The rate of decay of the pressure at a point on the ground depends on the intensity of the peak pressure and the positive phase duration.

When the shock front strikes a solid surface placed normal to the direction of shock travel, there is an instantaneous increase in pressure above that of the shock front itself. The total pressure, which is normally referred to as the reflected pressure, is given by (9):

$$P_r = 2P_{so} \left(\frac{7P_o + 4P_{so}}{7P_o + P_{so}} \right) \quad .....(3.5)$$

where P_{so} is the maximum side-on pressure, and P_o the ambient atmospheric pressure.

The reflected overpressure on the front wall decays rapidly to a value corresponding to the side-on overpressure plus some proportion of the dynamic pressure, in a time called the "clearing time" (time to clear face of reflection effects). This is shown in fig. 3.3 as t_c. The clearing time at a point on the front wall is a function of the shortest distance from the point on the structure to the edge where pressures all spill around the structure. The clearing time t_c is given as (9);

$$t_c = \frac{3a}{U} \quad .....(3.6)$$

Where a - the shortest distance described above

u - the velocity of the shock front.

After the clearing time, the pressure on the front wall decays to zero at a rate determined by the decay of the side-on overpressure and dynamic pressure. During the decay period, the pressure is the sum of the side-on pressure and dynamic pressure.

For purposes of analysis, the significant loading parameters for a structure subjected to blast loading are:-

1. The peak applied pressure P_r

2. The effective duration t_d of the load pulse, considered as a triangular load-time curve.

Extensive work has been done to establish air blast pressure generated from detonation of explosive in quarry operations. Charts such as the ones given in fig. 3.4 have been developed for determining the air blast pressure from such detonations of explosive in quarry operations (112). The charts given in fig. 3.4 can be used to determine the peak pressure P_r and used as input loads for structural analysis.

3.3. GROUND VIBRATION

Blasting at quarries results in ground vibrations which are transmitted to nearby structures. In the near vicinity of a blast, ground motions of sufficient magnitude to cause significant structural damage may be encountered (33). Ground motion resulting from blasts are transient motions often containing a large number of frequency components.

During the blast, compressive pressure waves are sent through the soil medium. The velocity of travel of such waves is given as (9)

$$V_p^2 = \frac{\lambda + 2G}{\rho_s} \quad .....(3.7)$$

where ρ_s - density of soil media

G - shear modulus

λ - Lames constant - responsible for lateral expansion. A constant of cubical expansion.

The soil particle velocity associated with this wave is given as (9).

$$V_x = \frac{\sigma_x \; V_p}{G} \quad .....(3.8)$$

where σ_x - the stress intensity in the

x - direction of that point.

It will be noticed that particle velocity is dependent on the intensity of stress but the wave propagation is only a function of the material properties.

The most useful type of measurement of this type of transient ground motion caused by an explosion is an acceleration-time record obtained from an accelerograph of relatively high frequency response. An acceleration measurement is prefered to a velocity or displacement measurement because:

1. Accelerations at the bases of structures are needed to determine structural responses and such accelerations cannot be determined with the required accuracy from a velocity or displacement record.
2. Velocities or displacements can be determined if desired by integrations of acceleration-time record. In general, a reverse process of differentiation cannot be performed with a satisfactory accuracy.
3. Accelerometers are inherently rugged, stable devices which can be calibrated easily.

The ground motions due to blasting, reaching a structure, can then be determined if the relationship between ground motion, amount of explosive charge and the distance from the explosion are known. The chart given in fig. 3.5 gives this relationship. The relatioship between charge weight W in tons, the distance d in feet, and the peak acceleration a, in units of gravity (33) is given as:

$$a = \frac{9500 \, W^{\frac{3}{4}}}{d^2} \qquad .....(3.9)$$

Acceleration-time records for blasts have been obtained and are typical of that shown in fig. 3.6. The shape of this record can be approximated into a triangular shape for an acceleration-time curve as shown by the dotted line in fig. 3.6. The modeled triangular shape of zero time rise is used as an input function. Since all acceleration-time curves of blast conditions have roughly the same shape, a knowledge of the peak acceleration as well as the rate of decay (or decceleration/retardation) should be necessary to adequately describe the acceleration-time curve (33).

3.4. WEATHER CONDITIONS

The velocity of travel and the intensity of air blast along a radial line from the point of blast is affected by the weather conditions at the time of blast. In 1960, Perkins, Lorrain and Townsend showed in a report (113) that local atmospheric conditions were responsible for the focussing of air blasts and that these focal points were centres of most damage.

The intensity of air blast at a point is proportional to the intensity of noise (7), weather conditions affect the intensity of noise from a blast as it is heard at a distance. These conditions determine the velocity of the sound in different directions and at different altitudes. The variables that influence sound propagation are wind, velocity and temparature as a function of altitude.

Normally, air temperature decreases as the altitude from the earth's surface increases, but in most cases, different conditions from the above exist and are discussed below.

3.4.1 ISOTHERMAL AIR

There are certain periods when the temperature of air is constant over a range of altitudes, then an isotherm is said to exist. This type of isothermal air exists with zero wind velocity and if blasting occurs in such an atmospheric condition, straight sound wave rays are produced with a spherical wave front as shown in fig. 3.7.

3.4.2 SOUND SPEED INVERSION

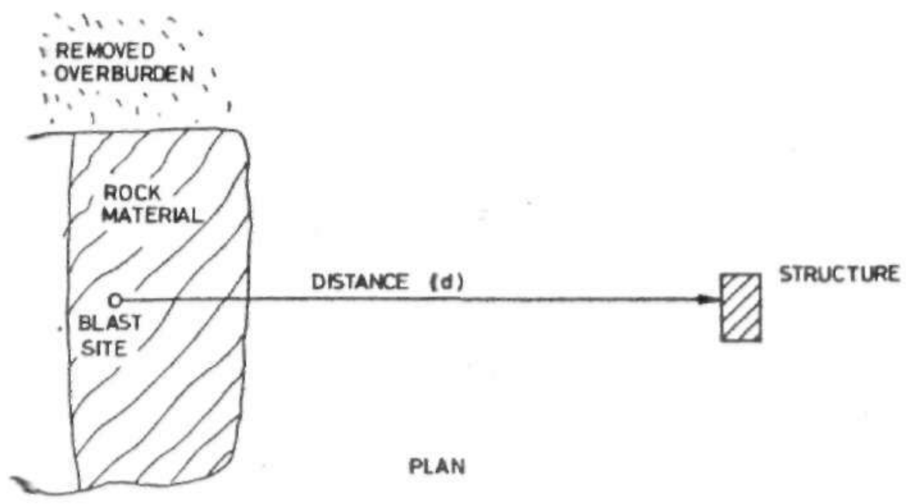
Weather conditions exist at certain periods such that a greater sound velocity in one direction exist above the earth's surface. Sound speed inversion can be said to exist then and in this case. the sound wave in the direction of velocity increase may cause bending of rays and therefore louder noise in certain areas. The direction of wind flow has a significant effect on the focussing of sound rays and the focal points are the points of higher sound intensity (7).

3.4.3 ARRIVAL TIME

The above discussion indicates that the position of a structure in relation to the direction of wind at the time of blast is significant when considering the arrival time of air blast on the structure. If the structure is away from the direction of wind, the air blast reaches it at a slower rate and the intensity is smaller than when it is in the direction of wind flow. Since wind velocity is not constant all year round, the arrival time of air blast on a structure situated in the vicinity of blast will vary in direct proportion to the variation of wind velocity. This leads to a consideration of this wind velocity effect on the maximum response in relation to the arrival time of air blast on the structure.

3.5 TIME LAG

It has been explained above that at any instant of quarry blast, a structure in its vicinity experiences forcing dynamic loads that arrive on it through two forms of media - air and soil media. Since the soil conditions remain approximately the same with respect to velocity of wave through it, it can be said that the arrival time of ground vibration on a structure in the vicinity remains approximately constant with all blastings in the area. In the case of air blast travel, the velocity of travel of air blast is affected by wind velocity and direction, therefore the arrival time of air blast on such a structure changes with atmospheric conditions. There will result a time-lag between the arrival of the ground vibration and the arrival of the air blast pressure on the structure. The maximum response will be affected by this time-lag phenomenon.



(a) SITTE OF QUARRY AND NEARBY RESIDENTIAL STRUCTURE

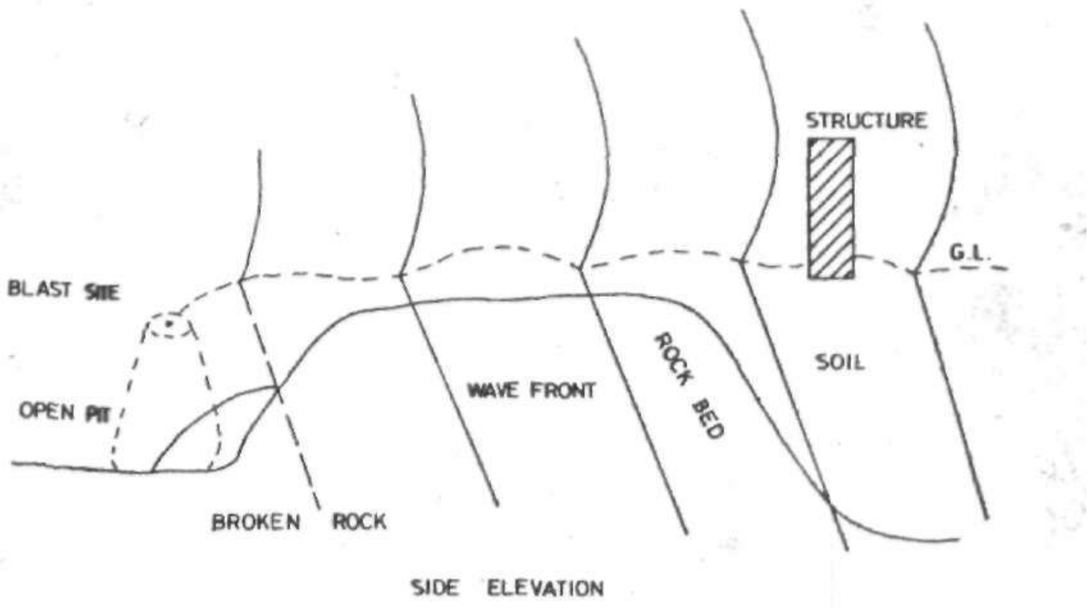
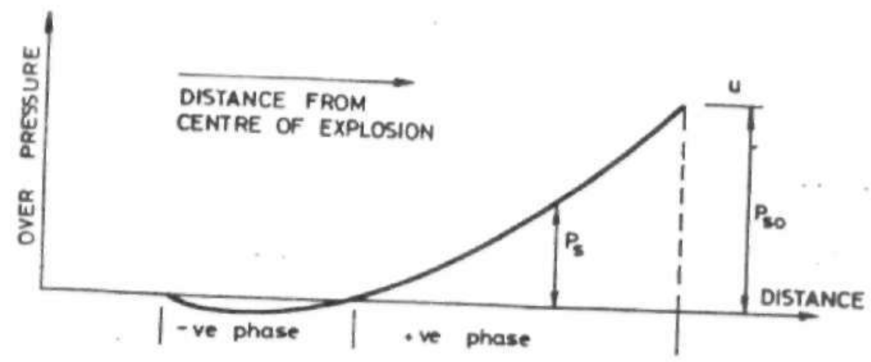
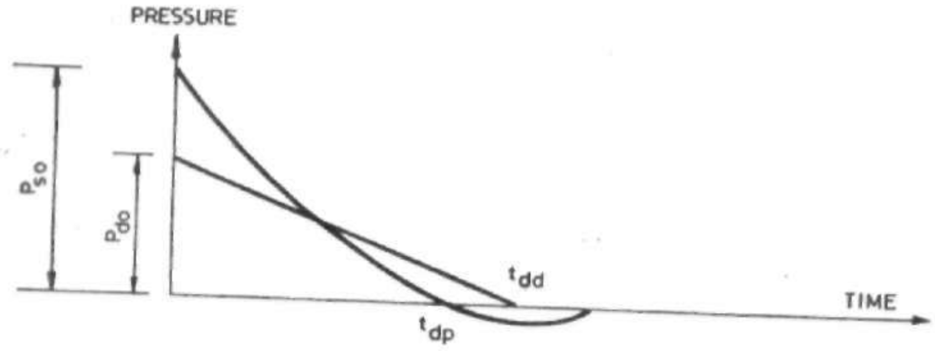


FIG. 3.1



(a) OVERPRESSURE VERSUS DISTANCE FROM CENTRE OF EXPLOSION.



(b) PRESSURE VERSUS TIME AFTER DETONATION

FIG. 3.2

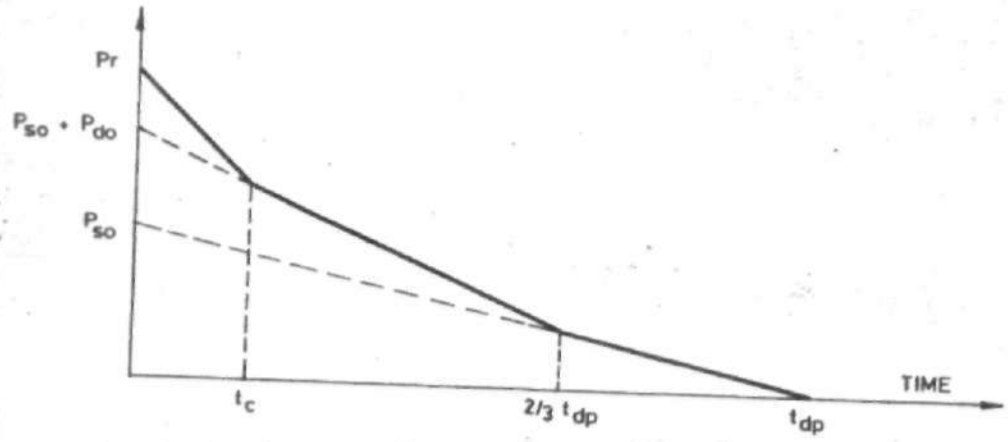


FIG. 3.3 PRESSURE VERSUS TIME.

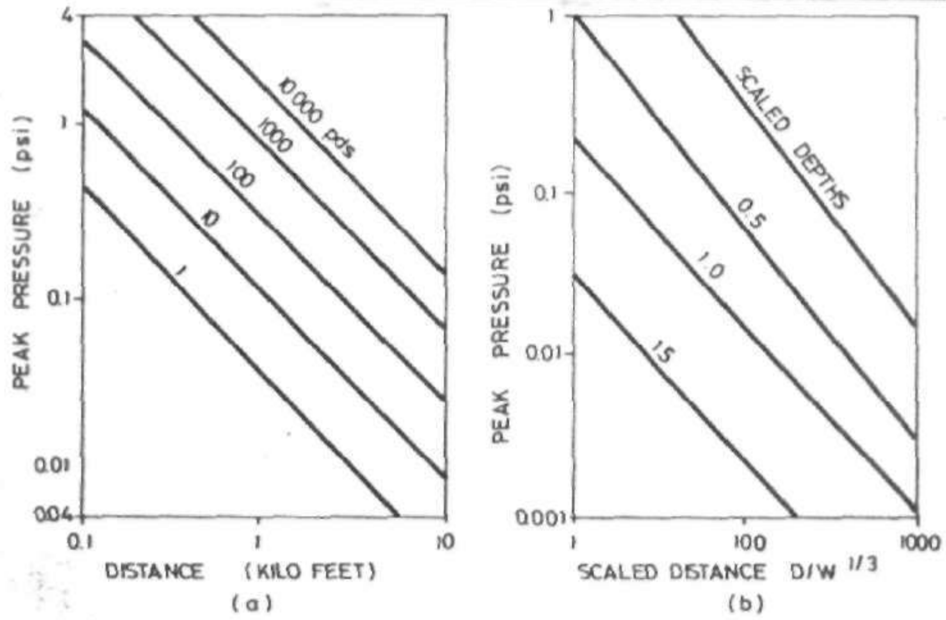


FIG. 3.4 AIR BLAST PRESSURE VERSUS DISTANCE AND DEPTH OF DRILL HOLE FROM QUARRY BLAST. (112)

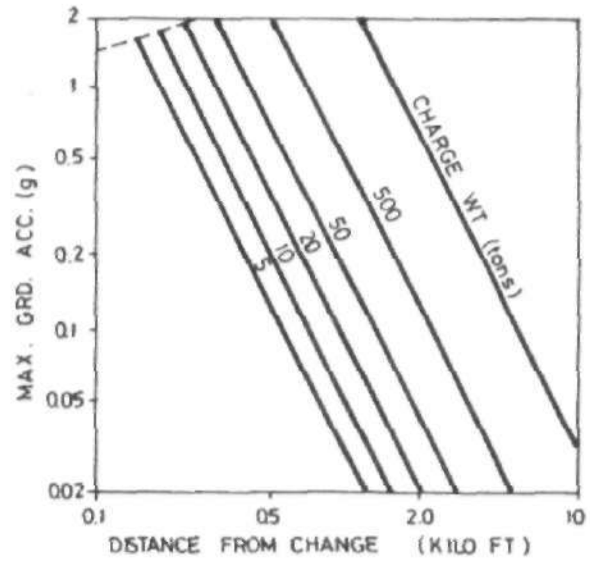


FIG. 3.5 MAXIMUM GROUND ACCELERATION AS A FUNCTION OF TOTAL CHARGE WEIGHT AND DISTANCE FROM DETONATION FOR LARGE BLAST.

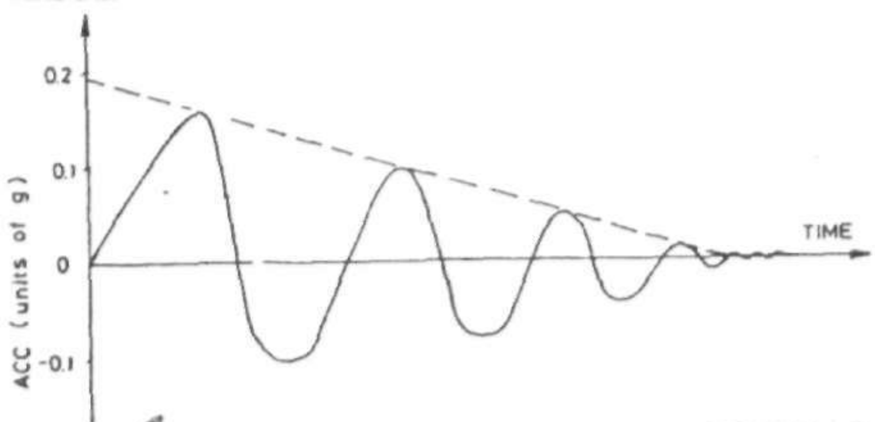


FIG. 3.6 ACCELERATION TIME RECORD.

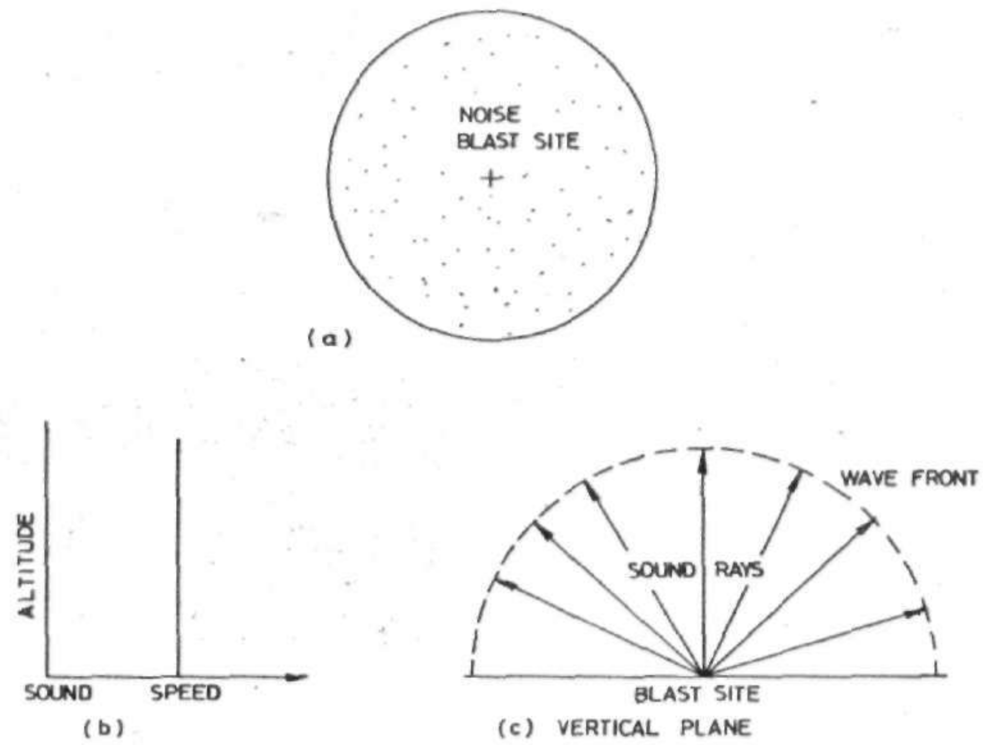


FIG. 3.7 SOUND PROPAGATION FROM AN ISOTHERMAL CONDITION.

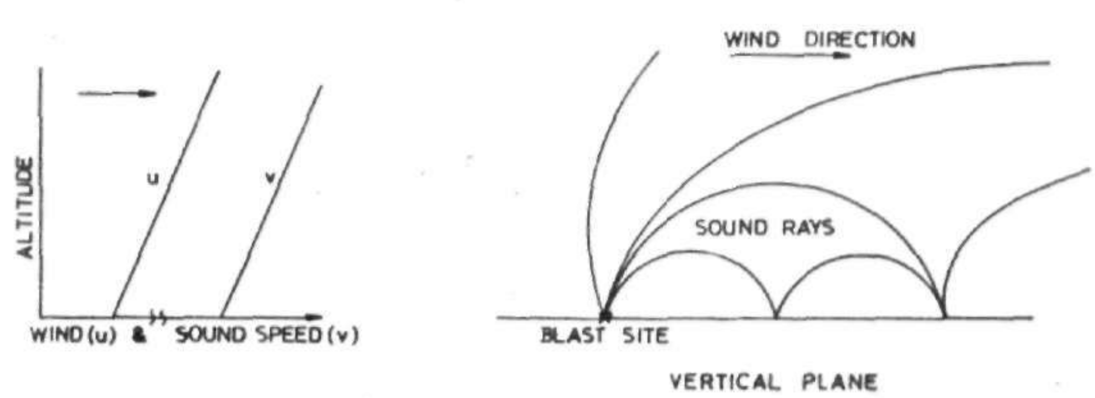


FIG. 3.8 SOUND PROPAGATION FROM + VE WIND GRADIENT.

CHAPTER FOUR

MATHEMATICAL MODEL

The analysis of any real structure requires the model representation of the structure. The model bears significantly on the form of solution obtained.

The real structure considered is a three storey building. This height of building has been found in many cases (96) to be a useful case in dynamics analysis study. Its characteristics are typical of multidegree systems and therefore is used for this study.

4.1 IDEALIZATION

The three storey building idealised is that given in fig. 4.1. The structure is assumed to have the following properties:-

a. The single bay representation is repeated along the length of the building at equal spacing.

b. The structural properties are uniform along the length of the building.

c. The loading is considered uniform along the length of the building.

d. The distributed dynamic pressure applied to the walls is assumed concentrated at the floor levels.

From the assumptions above, the analysis of an interior frame will yield the response of the entire building. In a real structure the mass is continously distributed over the entire structure. Consequently the dynamic analysis of such a structure leads to a solution of an infinite number of degrees of freedom (5). However it is acceptable to replace the real structure by an idealised one consisting of discrete number of lumped masses. These lumped masses are assumed to be connected to one another through elastic massless elements which generally retain the actual behaviour of the original structure. The simplest lumping procedure is to lump the masses at floor levels. The lumped masses are then vertically connected by weightless elastic joints of a moment of inertia equal to the sum of the moments of inertia of the actual vertical members in each storey (2). For further idealization the following assumptions are also made:-

1. The structure deflects in a linearly elastic manner. This is permissible for most civil engineering structures. Minor structural failure (78), such as cracks in walls or columns, which occur very often

appear even when the structure is still within the elastic range of its design.

2. The structure is stable under its system of static loads.

3. Only horizontal motions are considered and these are assumed to be independent of vertical motions. This is permissible because vertical motions due to changes in column length or flexure of girders have relatively small amplitude and hence little effect on the horizontal response.

4. The response is measured in the direction of the applied dynamic loads.

5. Damping will be neglected since most Civil Engineering structures reach maximum response values from impulsive loading before damping effects come into play.

Under the assumptions stated above the real structure in Fig. 4.1 (a) can be represented by a close coupled system where the lumped masses at floor levels are connected by massless springs of stiffnesses k_1, k_2, and k_3.

unconstrained elastic system. Since the free oscillations are harmonic, the displacements U can be written as:

$$U = q\, e^{iwt} \qquad .....(4.4)$$

where q - a column matrix of the amplitudes of the displacements U

w - the circular frequency of oscillations

t - the time.

Using Eq. 4.4 in Eq. 4.3 and then cancelling the common factor e^{iwt}, we obtain:

$$(-w^2 M + K)\, q = 0 \qquad .....(4.5)$$

Eq. 4.5 can be regarded as the equation of motion for an undamped freely oscillating system. Eq. 4.5 has a non-zero solution for q provided:

$$\det(-w^2 M + K) = 0 \qquad .....(4.6)$$

Eq. 4.6 is the characteristic equation for which the natural frequencies of free oscillations can be calculated. The determinant in Eq. 4.6, when expanded, yields a polynomial of n^{th} degree in w^2, the roots of which give the natural frequencies. Only for these natural frequencies will there be a non-zero solution obtained for q in Eq. 4.5. The number of frequencies thus obtained are equal to the number of non-zero mass coefficients on the principal diagonal of the equivalent mass matrix M. This number also includes zero frequencies for the rigid body degrees of freedom. Eq. 4.5 represents a set of homogeneous linear equations; therefore, only the relative values or ratios of q can be obtained.

In order to employ certain numerical methods for solution, it is sometimes necessary to transform Eq. 4.5 by first premultiplying the equation by M^{-1}, so that, after changing signs, we have:-

$$(w^2 I - M^{-1} K) q = 0 \qquad .....(4.7)$$

which is now the standard form for eigenvalue calculations.

It will not be possible at this time to make the transformation with $\frac{K^{-1}}{w^2}$ because the stiffness matrix K for an unconstrained structure is singular.

4.2.2 CONSTRAINED STRUCTURE

Most civil engineering structures have constrained characteristics i.e. they are always supported in one form or the other. Most of them are simply supported or fixed at certain ends. The important thing is that motion in certain directions is restricted. For such a system, the corresponding stiffness matrix will be non-singular.

Denoting the constrained stiffness matrix by K_c and the corresponding mass matrix by M_c, we can write Eq. 4.6

as follows:-

$$(-w^2 M_c + K_c) q_c = 0 \qquad .....(4.8)$$

where q_c - refers to the unconstrained degrees of freedom corresponding to M_c.

4.3 FLEXIBILITY METHOD

4.3.1 CONSTRAINED STRUCTURE

For certain Civil Engineering systems (statically determinate) it is sometimes more convenient to work with displacement equations of motion. This involves the writing of the equations of motion in displacement coordinates using flexibility coefficients

The equation of motion for an undamped constrained structure can be written as:-

$$F_c M_c \ddot{U} + U = F_c P \qquad .....(4.9)$$

where F_c - is the flexibility matrix of the constrained structure

Considering the structure in free oscillations i.e. $P = 0$. The Eq. 4.9 becomes

$$F_c M_c \ddot{U} + U = 0 \qquad .....(4.10)$$

Since free oscillations are harmonic, we substitute Eq. 4.4 in Eq. 4.10 and replace q by q_c so that we have:

$$(-w^2 F_c M + I) q_c = 0 \qquad .....(4.11)$$

where I - is an identity matrix. Premultiplying Eq. 4.11

by F_c^{-1} gives

$$(-w^2 M_c + F_c^{-1})\, q_c = 0 \qquad .....(4.12)$$

Eq. 4.12 is similar to Eq. 4.8 with

$$F_c^{-1} = K_c \qquad .....(4.12a)$$

This relationship is useful when, in trying to obtain a solution using flexibility coefficients, it is found that stiffness coefficients are more easily obtainable. The transformation can then be used after obtaining the stiffness coefficients.

If we divide Eq. 4.11 by w^2 and change signs, we get

$$A_c\, q_c = \lambda q_c \qquad .....(4.13)$$

where

$$A_c = F_c M_c \qquad .....(4.13a)$$

$$\lambda = \frac{1}{w^2} \qquad .....(4.13b)$$

Eq. 4.13 is the standard eigenvalue equation.

4.4 EIGENMODES RELATIONSHIPS

The mode q obtained from Eq. 4.5 can be seperated into the rigid body modes q_o and normal modes of vibration (elastic eigenmodes) q_e so that

$$q = q_o + q_e \qquad .....(4.14a)$$

where

$$q_o = q_1 \quad q_2 \quad \text{-----} \quad q_z \qquad \ldots\ldots(4.14)$$

$$q_e = q_{z+1} \quad q_{z+2} \quad \text{-----} \quad q_{z+n} \qquad \ldots\ldots(4.15)$$

where z represents the number of unconstrained rigid-body degrees of freedom and n represents the number of eigenmodes.

A multi-storey building structure has its foundation fixed in the soil on which it is placed, in which case rigid body motions are restricted. This means that q_o in Eq. 4.14a is equal to zero. If $w^2 \neq 0$; Eq. 4.5 can be re-written as:

$$(w^{-2} K - M) q = 0 \qquad \ldots\ldots(4.16)$$

Substituting the elastic modes of Eq. 4.15 in Eq. 4.16 yields:

$$\left.\begin{array}{l} w_{z+1}^{-2} K q_{z+1} - M q_{z+1} = 0 \\ w_{z+2}^{-2} K q_{z+2} - M q_{z+2} = 0 \\ \text{-----------------} \\ w_{z+n}^{-2} K q_{z+n} - M q_{z+n} = 0 \end{array}\right\} \qquad \ldots\ldots(4.17)$$

which can be combined into a single matrix equation as:

$$K q_e W^{-2} - M q_e = 0 \qquad \ldots\ldots(4.18)$$

where

$$W^{-2} = w_{z+1}^{-2} \quad w_{z+2}^{-2} \quad \ldots\ldots \quad w_{z+n}^{-2} \qquad \ldots\ldots(4.19)$$

Pre-multiplying Eq. 4.18 by q_e^T, we obtain

$$q_e^T K q_e W^{-2} - q_e^T M q_e = 0 \quad .....(4.20)$$

or $$K_e W^{-2} = M_e \quad .....(4.21)$$

where

$$K_e = q_e^T K q_e \quad .....(4.22)$$

which represents the generalised or principal stiffness matrix, and

$$M_e = q_e^T M q_e \quad .....(4.23)$$

represents the generalised or principal mass matrix. Eqs. 4.22 and 4.23 represent the diagonalisation of matrices K and M.

In order to take advantage of the diagonalisation process, let us reconsider the actions equation of motion for free vibration of undamped multi-degree of freedom system. Eq. 4.2 states:

$$M \ddot{U} + K U = 0 \quad .....(4.24)$$

Pre-multiplication of Eq. 4.24 by q_e^T and inserting

$$I = q_e q_e^{-1}$$

before $\ddot{U}$ and U produces

$$q_e^T M q_e q_e^{-1} \ddot{U} + q_e^T K q_e q_e^{-1} U = 0 \quad ...(4.25)$$

which can be restated as

$$M_e \ddot{d}_e + K_e d_e = 0 \quad .....(4.26)$$

where

$$\ddot{d}_e = q_e^{-1} \ddot{U} \quad .....(4.27)$$

and

$$d_e = q_e^{-1} U \quad .....(4.28)$$

The generalised displacements d_e given by Eq. 4.28 are also referred to as the principal coordinates. The generalised displacements d_e then operate as scale factors upon the modal columns q_e. The transformation in Eq. 4.28 is very useful and is used in the computer program.

Since the modal vectors or elastic modes may be scaled arbitrarily, the principal coordinates are not unique. To remove the uncertainties, we scale it using a factor that transforms the mass matrix M_e into an identity matrix.

Consider the i^{th} column, then

$$q_{eni}^T M q_{eni} = M_{ei} = I \quad .....(4.29)$$

in which

$$q_{eni} = \frac{q_{ei}}{C_i} \quad .....(4.30)$$

The eigenvector q_{eni} is said to be normalised with respect to the mass matrix. The scalar C_i in Eq. 4.30 can be computed as in Eq. 4.31.

$$C_i = \pm \sqrt{q_{ei}^T M q_{ei}} \quad .....(4.31)$$

When all the vectors in the modal matrix are normalised in this manner, the principal mass matrix M_e becomes unity

i.e. $q_{en}^T \, M \, q_{en} = M_e = I$ .....(4.32)

This set of principal coordinates is known as normal coordinates.

If the principal modes or elastic modes of vibration for a multi-degree system are used as generalised coordinates, the equations of undamped motion become uncoupled. In these coordinates, each equation may be solved as if it pertained to a system with only one degree of freedom.

4.5 DYNAMIC RESPONSE OF CONSTRAINED SYSTEMS

Most civil engineering structures are constrained in one form or the other. That is motion in certain directions are restricted. Rigid body modes are eliminated and analysis is on the elastic modes of vibration.

For an elastic undamped system subjected to a time varying forcing function p(t), the equation of motion is given by

$M \ddot{U} + K U = p(t)$ .....(4.33)

Since we are interested only in the elastic modes, we can replace the displacement U with q_e and some time dependent multiplier ϕ_e such that

$U = q_e \, \phi_e$ .....(4.34)

where

$$\phi_e = \phi_1 \quad \phi_2 \quad \text{-----} \quad \phi_n \qquad .....(4.35)$$

Eq. 4.35 is a column matrix of unknown functions of time. Substituting Eq. 4.34 into Eq. 4.33 we get

$$M\, q_e\, \ddot{\phi}_e + K\, q_e\, \phi_e = p(t) \qquad .....(4.36)$$

Pre-multiplication of Eq. 4.36 by q_e^T gives:

$$q_e^T\, M\, q_e\, \ddot{\phi}_e + q_e^T\, K\, q_e\, \phi_e = q_e^T\, p(t) \quad ...(4.37)$$

Using Eqs. 4.22 and 4.23 in Eq. 4.37 we obtain

$$M_e\, \ddot{\phi}_e + K_e\, \phi_e = q_e^T\, p(t) \qquad .....(4.38)$$

Premultiply Eq. 4.38 by M_e^{-1} and get

$$\ddot{\phi}_e + M_e^{-1}\, K_e\, \phi_e = M_e^{-1}\, P_e(t) \qquad .....(4.39)$$

where

$$P_e(t) = q_e^T\, p(t) \qquad .....(4.40)$$

$P_e(t)$ is a column matrix of the generalised forces for the elastic modes q_e.

But from Eq. 4.21 we note that

$$W^2 = M_e^{-1}\, K_e \qquad .....(4.41)$$

Then

$$\ddot{\phi}_e + W^2\, \phi_e = M_e^{-1}\, P_e(t) \qquad .....(4.42)$$

Since W^2 is a diagonal matrix, Eq. 4.42 constitutes a set of uncoupled second order differential equations, a typical set of which is of the form:

$$\ddot{\phi}_{ei} + W^2 \phi_{ei} = M_{ei} P_{ei}(t) \quad .....(4.43)$$

where i represents the row number in the column matrix $P_e(t)$.

The solution to Eq. 4.43 can be obtained using the Duhamel's Integral such that

$$\phi_{ei} = \dot{\phi}_{ei}(0)\, w_i^{-1} \operatorname{Sin}(w_i t) + \phi_{ei}(0) \operatorname{Cos}(w_i t) + w_i^{-1} M_{ei}^{-1} \int_0^t \operatorname{Sin}(w_i(t-t_1))\, P_{ei}(t_1)\, dt_1 \quad ..(4.44)$$

For i = 1 to n we can combine Eq. 4.44 into matrix form to be

$$\phi_e = W^{-1} \operatorname{Sin}(wt)\, \dot{\phi}_e(0) + \operatorname{Cos}(wt)\, \phi_e(0) + W^{-1} M_e^{-1} \int_0^t \operatorname{Sin}(w(t-t_1))\, P_e(t_1)\, dt_1 \quad ...(4.45)$$

where

$$\operatorname{Sin}(wt) = \operatorname{Sin}(w_1 t) \;\; \operatorname{Sin}(w_2 t) \;\; --- \;\; \operatorname{Sin}(w_n t) ..(4.46)$$

$$\operatorname{Cos}(wt) = \operatorname{Cos}(w_1 t) \;\; \operatorname{Cos}(w_2 t) \;\; --- \;\; \operatorname{Cos}(w_n t) ..(4.47)$$

Eq. 4.45 gives the general solution of an elastic system responding to a dynamic forcing function.

4.6 RESPONSE DUE TO IMPULSIVE LOADS

Dynamic loads that have a relatively short-time history are usually referred to as impulsive loads. This class of loads are typical of loads on a structure as a result of rock blasting.

Suppose a system is subjected to an impulsive force at time $t = 0$. Assuming that the system does not experience an initial velocity at $t = 0$; if the impulsive load duration is t_d, the displacement function for the elastic modes of the system can then be written, using Eq. 4.45 as:

$$\phi_e = W^{-1} M_e^{-1} \int_0^t \mathrm{Sin}(w(t-t_1))\, G\, dt_1 \quad \ldots(4.49)$$

for $t \leqslant t_d$

where

G is the column matrix of the impulsive loads.

At $t > t_d$ $\qquad G = 0$

then

$$\phi_e = W^{-1} \mathrm{Sin}(wt_2)\, \dot{\phi}_e(t_d) + \mathrm{Cos}(wt_2)\, \phi_e(t_d) \quad \ldots\ldots(4.50)$$

where $\qquad t_2 = t-t_d$

The solutions in Eqs. 4.49 and 4.50 are the elastic displacements response of the system subjected to impulsive forces of duration t_d.

4.7 RESPONSE DUE TO BASE MOTION

Response of structures is sometimes due to base excitation. The ground motion induces base motion of equal magnitude to the motion of the soil particles next to the base of the structure (95). This motion is transmitted to the various masses of the structure.

The method of estimation depends on the record of the ground motion available. In most cases, the records available are the ground acceleration records. This analysis will therefore be based on the use of ground acceleration - time record.

If the base of the structure translates in the X - direction in accordance with the function in Eq. 4.51

$$X_g = f_g(t) \quad .....(4.51)$$

The relative displacement will be written in vector form as

$$X_1 = U - I x_g \quad .....(4.52)$$

where I - denotes a vector that is filled with values of unity.

The relative acceleration from Eq. 4.52 is then

$$\ddot{X}_1 = \ddot{U} - I \ddot{x}_g \quad .....(4.53)$$

where $\ddot{x}_g$ is the base acceleration.

Substituting Eqs. 4.52 and 4.53 in Eq. 4.3 we get

$$M \ddot{X}_1 + K X_1 = Q_g \quad .....(4.54)$$

where

$$Q_g = - M I \ddot{x}_g \quad .....(4.55)$$

Q_g is the equivalent input loads due to the base displacement and can be used in place of p(t) in Eq. 4.40. The displacements obtained using Eq. 4.54 are relative displacements. In the computer programme, these are found and then transformed to the actual displacements using Eq. 4.52.

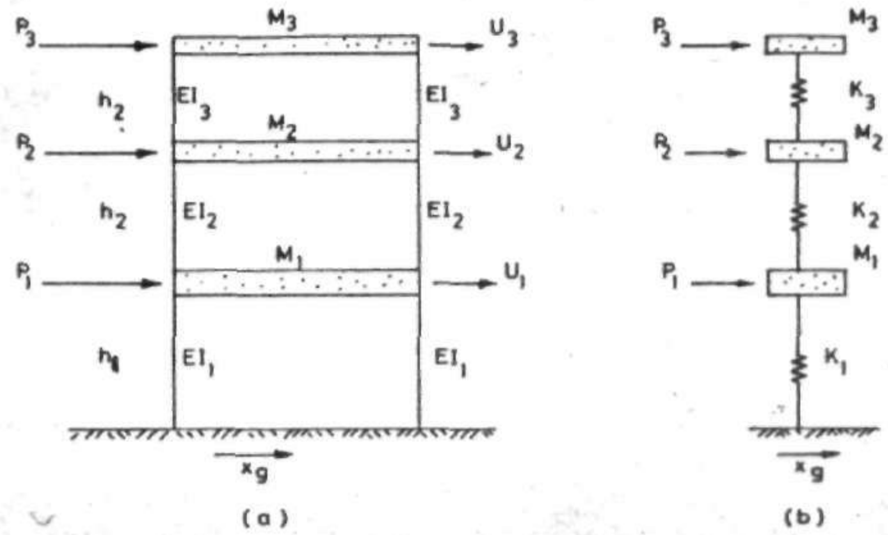


FIG. 4.1 SINGLE BAY THREE STOREY FRAMES.

CHAPTER FIVE

RESULTS

In chapter four, the equations for the dynamic analysis were presented. The theory formed the basis of the computer programme developed by the author to obtain the results presented in this chapter. The results are those obtained in analysing the structure shown in Fig. 4.1. The selection of this structure was described in section 4.1 and the model shown in Figs. 4.1 a & b.

Since the results are meant for comparative work, the dimensions of the structure are chosen as unity whenever it is deemed that the relative contributions of the different sections will not be altered. In view of this the dimensions of the structure selected for the computer programme were selected as follows:-
(Refer to Fig. 4.1)

$h_1 = 1.0$

$h_2 = 1.0$

$h_3 = 1.0$

where h is the height between floors and the units given in meters:

$M_1 = 1.0$

$M_2 = 1.0$

$M_3 = 1.0$

where M is mass of lumped masses with the units in kg:

$EI_1 = 3.0$

$EI_2 = 2.0$

$EI_3 = 1.0$

where EI is the flexural rigidity of the vertical walls connecting the floors. Units: E in N/m^2, I in m^4.
It should be noted that in the input record, as much as possible, the parameters were considered on the relative weights. This is for the purposes of this study.

5.1 DYNAMIC FORCING FUNCTIONS

On the basis of the theory presented in chapter three the dynamic forcing functions were estimated. The quantity of explosive used to determine the magnitude of air blast loading and base acceleration magnitude were based on the operational explosive weights obtained from Nkalagu Cement Company and Benue Cement Company. At these two sites the blasting method involves relays and the weight of explosive per drill hole is 10 lbs. (4.53 kg.). The relays have the effect of delaying explosion per drill hole. Thus it had been considered that since the study is for comparative effect, the effect emitted per drill hole would give a good indication of the response pattern. On this basis the air blast pressure is estimated from Fig. 5.2 using an explosive charge weight of 10 lbs. (4.53 kg.) (see chart Fig. 5.2).

The distribution of air blast pressure on the structure was scaled using the scaling factors given by Biggs (9). Since the structure studied is only a 3-floor building, it is felt that the distribution factors in the above reference would serve the purpose. The distribution of the air blast pressure on the lumped masses of the structure were weighted as follows:-

Mass	Distribution factor
M_1	1.0
M_2	0.8
M_3	0.5

The ground vibration input load on the base of structure was measured in terms of peak value of acceleration of the soil particles at the structure's base. This was estimated from Eq. 3.9 which is:-

$$a = \frac{9500\ W^{\frac{3}{4}}}{d^2} \qquad .....(3.9)$$

where W - weight of Explosive in tons

d - distance from blast source in feet

a - acceleration in units of gravity.

When Eq. 3.9 is multiplied by a gravity value, the actual value of acceleration can be obtained.

With the peak values of air blast pressure and base acceleration obtained it became a problem of obtaining the required dynamic forcing function's duration t_d. The shape of this forcing function had earlier been explained in sections 3.2 and 3.3 to be that of a triangular pulse load with zero-time rise. The duration of this pulse was then chosen as 0.5 sec. This is less than 0.25 and greater than 0.10 of the fundamental period of vibration of the assumed structure (48). Other typical dynamic forcing functions durations are given by Hudson D.E.; Andrews A.B. and Biggs J.M. (33, 1, 9).

5.2 CHARACTERISTIC SHAPES

The three floor structure whose dimensions were described in section 5.1 was solved to obtain the following natural characteristics given in table 5.1.

The characteristics shapes for the different modes of vibration are shown in Fig. 5.3.

5.3 DYNAMIC RESPONSE

In order to obtain the dynamic response (as used here response means displacement), use is made of Eq. 4.45. The results obtained are tabulated in Table 5.2 to Table 5.10 and plotted in Fig. 5.4 to 5.14.

Results in Table 5.2 are plotted in Fig. 5.4. The plot shows the response of the lumped masses when subjected to the air blast pressure. The response is analysed for the duration of impulse. As expected of the response pattern during this time, the response of the masses has not yet shown typical dynamic behaviour (31, 98). The impulsive load is the decaying type and in many instances the load may be past-history when the structure finally responds properly. The response record from the analysis takes the form of the dynamic load's static equivalent as input. This behaviour does not continue and the dynamic response of the different masses, due to the impulse load, becomes prominent as we see in Fig. 5.5 (Table 5.3). In Fig. 5.5 it can be seen that the top floor mass M_3 displaces largest followed by the second floor mass M_2 and the least is the first floor mass M_1. The maximum displacement of the masses occurs after the duration of impulse which is the free vibration zone.

A structure subjected to dynamic loads responds in different modes of vibration. For n-degrees of freedom there are n-modes of vibration. The response of the top floor mass M_3 in the first three modes of vibration are shown in Fig. 5.6 (Table 5.4). The response in the different modes is seen to be harmonic. The displacements are highest for the first mode, followed by the second mode and then the third mode. The response shown in Fig. 5.6 can then be said to be dominated by the first (fundamental) mode of vibration.

The blast sends waves through the soil media on which the structure's foundation is placed. The waves reaching the foundation could be measured in terms of acceleration as shown in section 3.3. Response of the different masses to this base acceleration, as a dynamic input function, is measured relative to base displacement as shown in Fig. 5.7 (Table 5.5) and the actual displacement given in Fig. 5.8 (Table 5.6). Here again the displacement of top floor mass M_3 is highest and occurs during the free vibration phase.

Fig. 5.9 (Table 5.7) shows the total response due to combined action of air blast pressure and base displacement. Fig. 5.10 (Table 5.8) then indicates that the dynamic response of the structure (top floor mass M_3 used as case study) is dominated by ground vibrations disturbing the base of the structure.

The time interval between the instant the air blast pressure and ground vibration reach the structure (time-lag) has its effect on the time of maximum response. The case when air blast pressure leads ground vibration is shown in Fig. 5.11 (Table 5.9). It shows that the maximum response is initially dominated by ground vibration effect until the lag is greater than the pressure duration of the air blast pressure. The critical point is at $\delta = t_d$ when the initial peak response becomes instantly dominated by the effect of the air blast pressure, Fig. 5.13. In the case when ground vibration leads the air blast pressure, the time of maximum response remains constant, Fig. 5.12, since it is completely dominated by ground vibration effect. The maximum response values increase with increasing time-lag.

The implication of these results to structural behaviour is discussed in chapter six.

TABLE 5.1

MODE	EIGENVALUE $\lambda = \frac{1}{w^2}$	NATURAL FREQUENCY w^2	PERIOD T(Sec)	EIGENVECTORS (CHARACTERISTICS SHPES)		
				FIRST FLOOR M_1	SECOND FLOOR M_2	THIRD FLOOR M_3
1^{st}	0.20043	4.9894	2.8133	0.25499	0.5842	1.0000
2^{nd}	0.03632	27.5290	1.1977	0.7388	1.0000	-0.6450
3^{rd}	0.01324	75.4750	0.7233	1.0000	-0.7725	0.1219

TABLE 5.2 = RESPONSE OF MASSES TO AIR BLAST PRESSURE WITHIN THE IMPULSE DURATION (t_d).

TIME (SEC)	DISPLACEMENT (in) = (x 2.54 cm)		
	M_1	M_2	M_3
0	0	0	0
0.05	0.55095×10^{-3}	0.44482×10^{-3}	0.27836×10^{-3}
0.10	0.20727×10^{-2}	0.17190×10^{-2}	0.10800×10^{-2}
0.15	0.43012×10^{-2}	0.37306×10^{-2}	0.23615×10^{-2}
0.20	0.69116×10^{-2}	0.63776×10^{-2}	0.40871×10^{-2}
0.25	0.95597×10^{-2}	0.95373×10^{-2}	0.62267×10^{-2}
0.30	0.11926×10^{-1}	0.13056×10^{-1}	0.87531×10^{-1}
0.35	0.13754×10^{-1}	0.16743×10^{-1}	0.11638×10^{-1}
0.40	0.14877×10^{-1}	0.20371×10^{-1}	0.14848×10^{-1}
0.45	0.15229×10^{-1}	0.23687×10^{-1}	0.18337×10^{-1}
0.50	0.14838×10^{-1}	0.26429×10^{-1}	0.22042×10^{-1}

TABLE 5.3

DISPLACEMENT OF MASSES

DUE TO AIR BLAST PRESSURE

TIME (SEC)	DISPLACEMENT (in) = (x 2.54 cm)		
	M_1	M_2	M_3
0	0	0	0
0.25	0.95597×10^{-2}	0.95373×10^{-2}	0.62267×10^{-2}
0.50	0.14838×10^{-1}	0.26429×10^{-1}	0.22042×10^{-1}
0.75	0.84561×10^{-2}	0.26570×10^{-1}	0.40961×10^{-1}
1.00	0.63108×10^{-2}	0.13293×10^{-1}	0.48825×10^{-1}
1.25	0.30321×10^{-2}	0.11597×10^{-1}	0.32530×10^{-1}
1.50	0.43980×10^{-2}	0.12003×10^{-1}	0.16480×10^{-2}
1.75	0.49254×10^{-2}	0.29637×10^{-2}	0.23012×10^{-1}
1.95	-0.76724×10^{-2}	-0.18606×10^{-1}	-0.32575×10^{-1}

TABLE 5.4

DISPLACEMENT OF MASS M_3 IN THE FIRST THREE MODES DUE TO AIR BLAST PRESSURE

TIME (SEC)	DISPLACEMENT (in) = (x 2.54 cm)		
	FIRST MODE	SECOND MODE	THIRD MODE
0	0	0	0
0.25	0.101×10^{-1}	-0.425×10^{-2}	0.355×10^{-3}
0.50	0.293×10^{-1}	-0.734×10^{-2}	0.412×10^{-4}
0.75	0.417×10^{-1}	-0.423×10^{-3}	0.314×10^{-3}
1.00	0.414×10^{-1}	0.713×10^{-2}	0.314×10^{-3}
1.25	0.285×10^{-1}	0.407×10^{-2}	-0.412×10^{-4}
1.50	0.696×10^{-2}	-0.504×10^{-2}	-0.268×10^{-3}
1.75	-0.167×10^{-1}	-0.665×10^{-2}	0.344×10^{-3}
1.95	-0.323×10^{-1}	-0.321×10^{-3}	0.304×10^{-4}

TABLE 5.5

RELATIVE DISPLACEMENT OF MASSES DUE TO BASE ACCELERATION.

TIME (SEC)	RELATIVE DISPLACEMENT (in) = (x 2.54 cm)		
	M_1	M_2	M_3
0	0	0	0
0.25	-0.33697×10^{-1}	-0.40635×10^{-1}	-0.41014×10^{-1}
0.50	-0.59702×10^{-1}	-0.11495	-0.13022
0.75	-0.48199×10^{-1}	-0.13206	-0.21268
1.00	-0.42507×10^{-1}	-0.93114×10^{-1}	-0.23462
1.25	-0.25235×10^{-1}	-0.71305×10^{-1}	-0.15840
1.50	-0.16394×10^{-1}	-0.43905×10^{-1}	-0.20565×10^{-1}
1.75	-0.47823×10^{-2}	-0.30999×10^{-1}	0.10412
1.95	0.40768×10^{-1}	0.96464×10^{-1}	0.16716

TABLE 5.6

ACTUAL DISPLACEMENT OF MASSES DUE TO BASE DISPLACEMENT

TIME (SEC)	DISPLACEMENT (in) = (x 2.54 cm)		
	M_1	M_2	M_3
0	0	0	0
0.50	0.86394×10^{-1}	0.16305×10^{-1}	0.10338×10^{-2}
1.00	0.88751×10^{-1}	0.38145×10^{-1}	-0.10336
1.50	0.11486	0.87353×10^{-1}	0.11069
2.00	0.18506	0.24161	0.30873
2.50	0.17836	0.25917	0.31915
3.00	0.11689	0.10268	0.13682
3.50	0.10548	0.34765×10^{-1}	-0.93395×10^{-1}
3.90	0.58896×10^{-1}	0.21283×10^{-2}	-0.51628×10^{-1}

TABLE 5.7

TOTAL DISPLACEMENT OF MASSES DUE TO

COMBINED AIR BLAST AND BASE DISPLACEMENT

TIME (SEC)	DISPLACEMENT (in) = (x 2.54 cm)		
	M_1	M_2	M_3
0	0	0	0
0.50	0.866394×10^{-1}	0.42734×10^{-1}	0.23076×10^{-1}
1.00	0.95062×10^{-1}	0.51438×10^{-1}	-0.54539×10^{-1}
1.50	0.11926	0.99356×10^{-1}	0.11234
2.00	0.17351	0.21952	0.27496
2.50	0.16950	0.23229	0.28381
3.00	0.12153	0.11166	0.13327
3.50	0.10651	0.49888	-0.46673×10^{-1}
3.90	0.76249×10^{-1}	0.29449×10^{-1}	-0.18293×10^{-1}

TABLE 5.8

COMPARATIVE EFFECT OF BASE DISPLACEMENT AND AIR BLAST PRESSURE ON TOP FLOOR MASS (M_3) AS CASE STUDY.

TIME (SEC)	DISPLACEMENT (in) = (x 2.54 cm)		
	Due to Air Blast Pressure	Due to Base Displacement	Total Displacement
0	0	0	0
0.50	0.22042×10^{-1}	0.10338×10^{-2}	0.23076×10^{-1}
1.00	0.48825×10^{-1}	-0.10336	-0.54539×10^{-1}
1.50	0.16480×10^{-2}	0.11069	0.11234
2.00	-0.33775×10^{-1}	0.30873	0.27496
2.50	-0.35344×10^{-2}	0.31915	0.28381
3.00	-0.35544×10^{-2}	0.13682	0.13327
3.50	0.46722×10^{-1}	-0.93395×10^{-1}	-0.46673×10^{-1}
3.90	0.33335×10^{-1}	-0.51628×10^{-1}	-0.18293×10^{-1}

TABLE 5.9

TIME OF MAXIMUM RESPONSE (t_m) AS AGAINST TIME LAG IN ARRIVAL OF AIR BLAST PRESSURE AND GROUND VIBRATION (AIR BLAST LEADING)

δ/t_d	t_m (sec)		DISPLACEMENT (in) = (x 2.54 cm)	
	FIRST	SECOND	FIRST	SECOND
0.2	1.1	-	-0.58105×10^{-1}	-
0.4	1.1	-	-0.65745×10^{-1}	-
0.6	1.3	-	-0.76548×10^{-1}	-
0.8	1.4	-	-0.89066×10^{-1}	-
1.0	1.0	1.5	0.49859×10^{-1}	-0.10172
1.2	1.0	1.6	0.49021×10^{-1}	-0.11317
1.4	1.0	1.7	0.48845×10^{-1}	-0.12257
1.6	1.0	1.8	0.48845×10^{-1}	-0.12959
1.8	1.0	1.9	0.48825×10^{-1}	-0.13432
2.0	1.0	2.0	0.48825×10^{-1}	-0.13714

δ - difference in arrival time on structure of the two input loads (time lag)

td - Duration of Air Blast Pressure

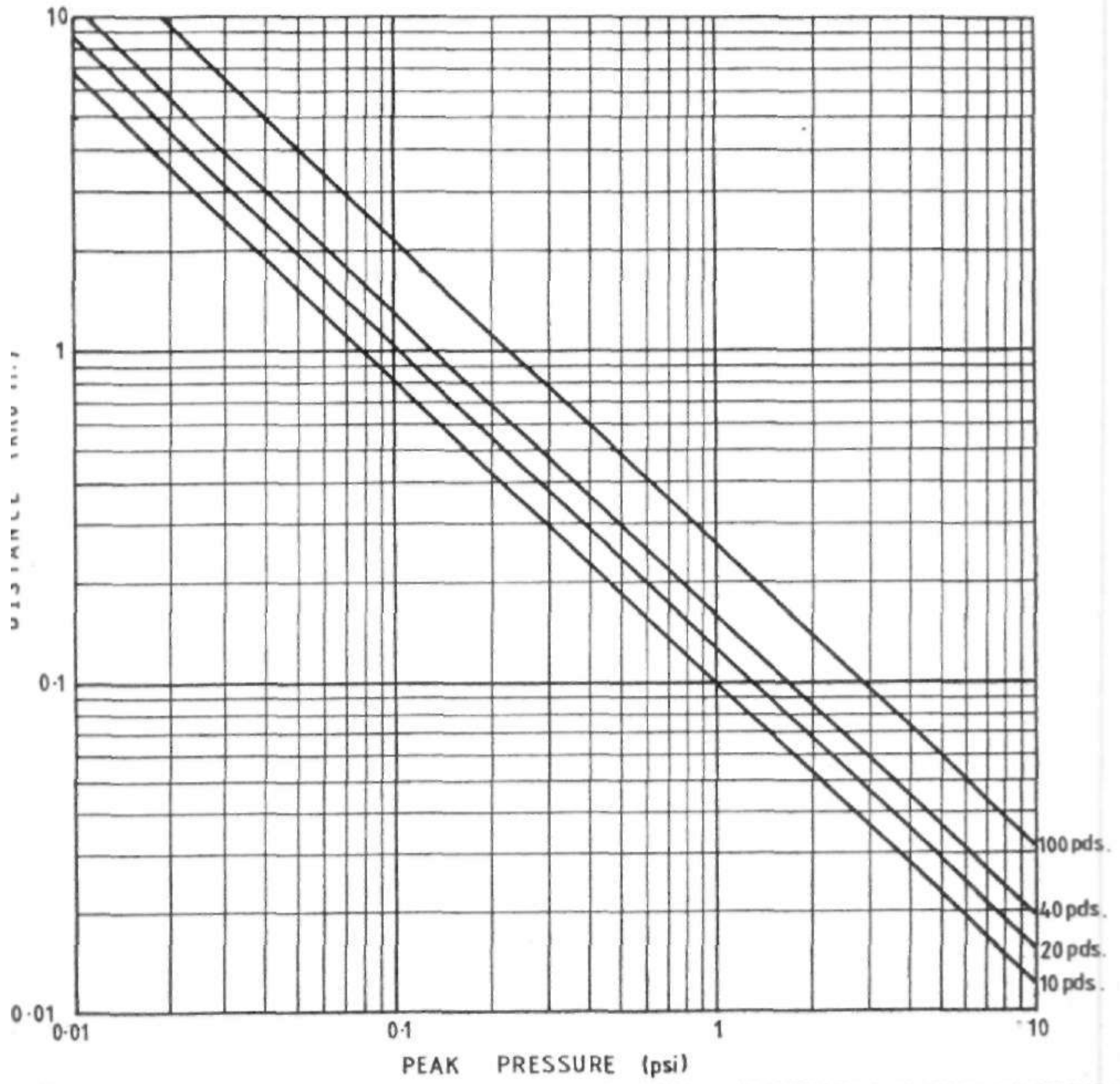
tm - time of maximum response

TABLE 5.10

TIME OF MAXIMUM RESPONSE(t_m) AS AGAINST TIME LAG IN ARRIVAL TIME OF GROUND VIBRATION AND AIR BLAST PRESSURE (GROUND VIBRATION LEADING)

δ/t_d	t_m (sec)	DISPLACEMENT
0.2	1.0	-0.61451×10^{-1}
0.4	1.0	-0.68005×10^{-1}
0.6	1.0	-0.75672×10^{-1}
0.8	1.0	-0.83432×10^{-1}
1.0	1.0	-0.90625×10^{-1}
1.2	1.0	-0.96720×10^{-1}
1.4	1.0	-0.10139
1.6	1.0	-0.10439
1.8	1.0	-0.11081
2.0	1.0	-0.12377

Fig. 5·2 : PEAK PRESSURE vs DISTANCE
for different explosive (charge) weights.



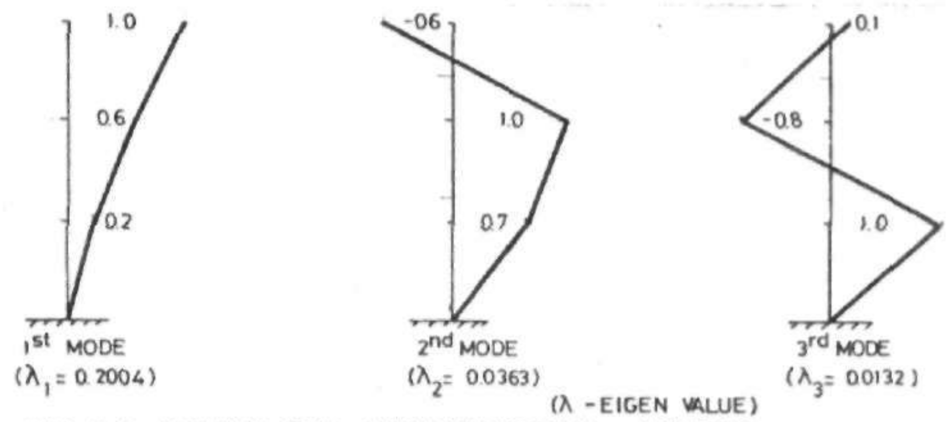


FIG. 5.3 NORMALISED CHARACTERISTIC SHAPES .

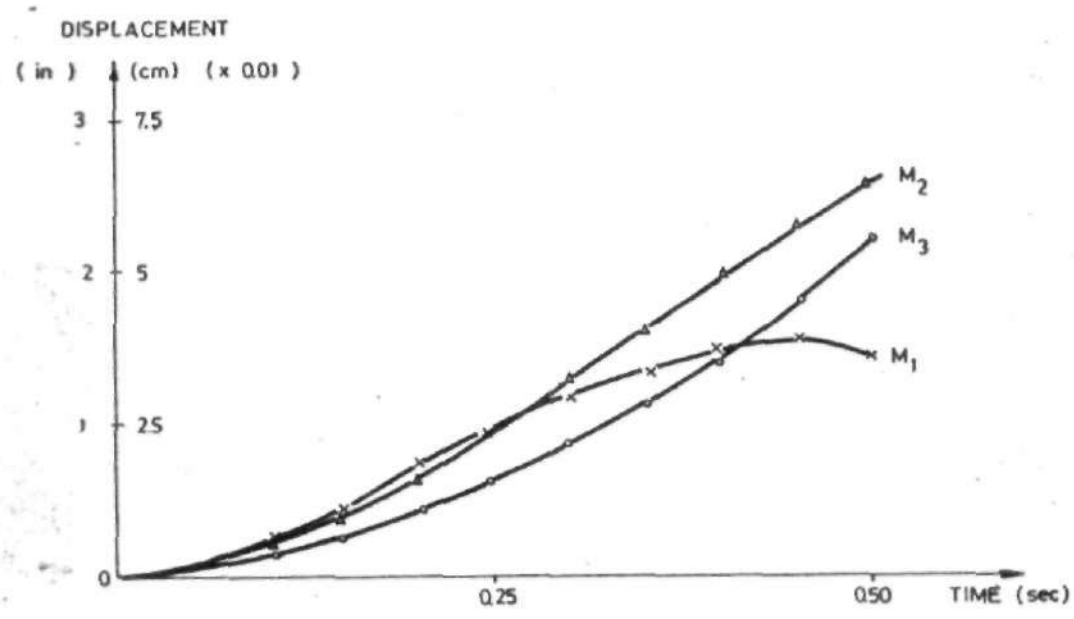


FIG. 5.4 RESPONSE OF THE DIFFERENT MASSES TO AIR BLAST LOADS - TRIANGULAR IMPULSE OF ZERO-TIME-RISE (t_d = 0.5 sec)

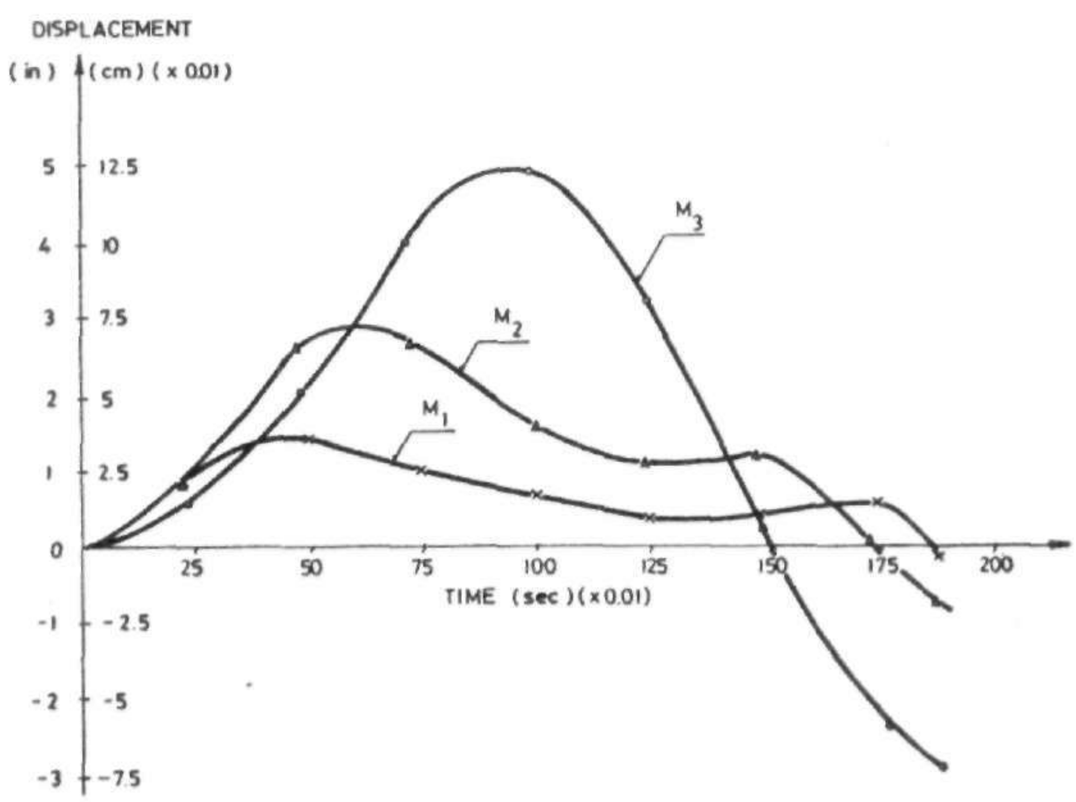


FIG. 5.5 RESPONSE OF THE MASSES TO AIR-BLAST LOADS TRIANGULAR IMPULSE OF ZERO-TIME-RISE. (IMPULSE DURATION 0.5 sec)

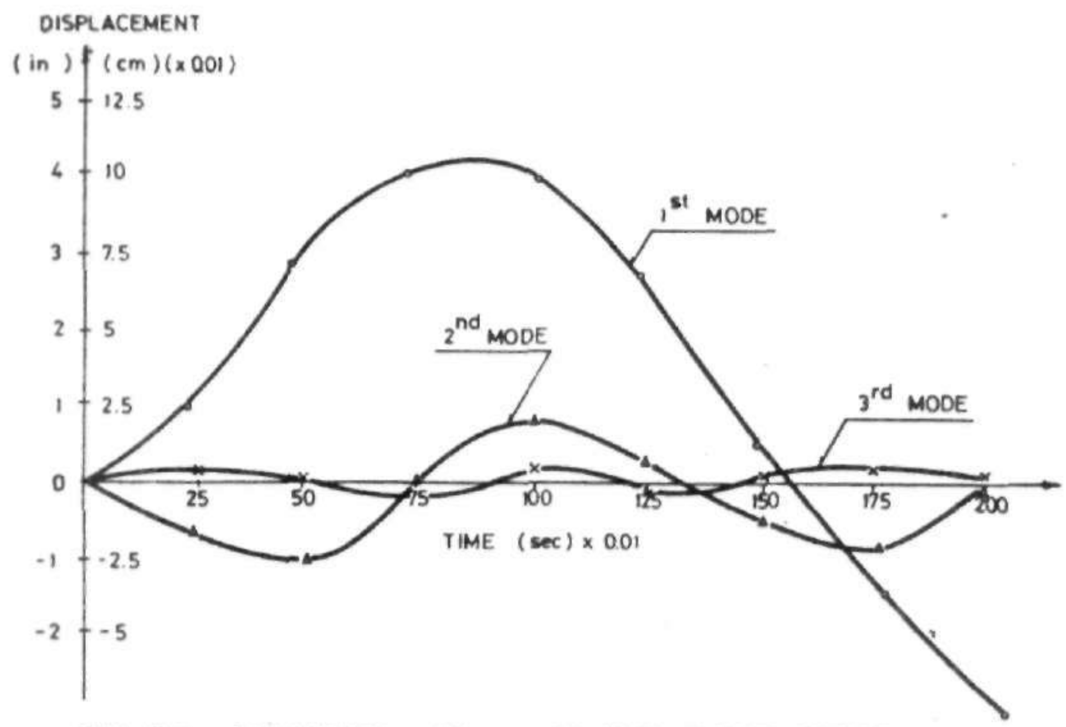


FIG. 5.6 RESPONSE OF M_3 IN THE FIRST THREE MODES
(TRIANGULAR IMPULSE t_d = 0.5 sec)

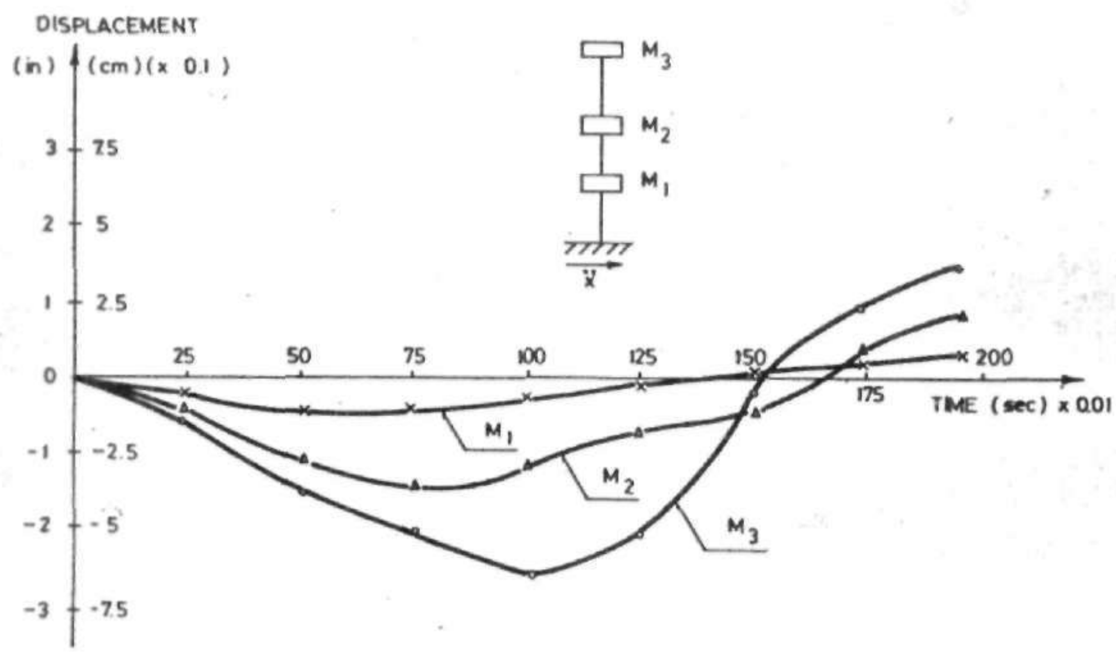


FIG. 5.7 RELATIVE RESPONSE OF LUMPED MASSES
DUE TO BASE ACC. (t_d = 0.5 sec).

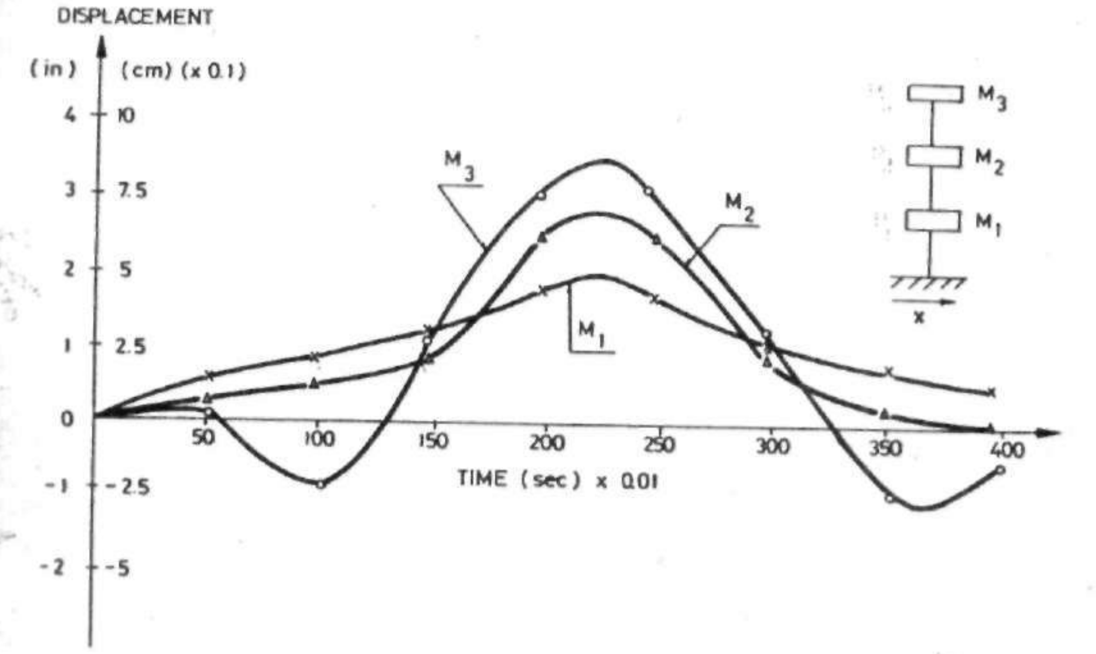


FIG. 5.8 RESPONSE OF MASSES DUE TO BASE DISPLACEMENT.

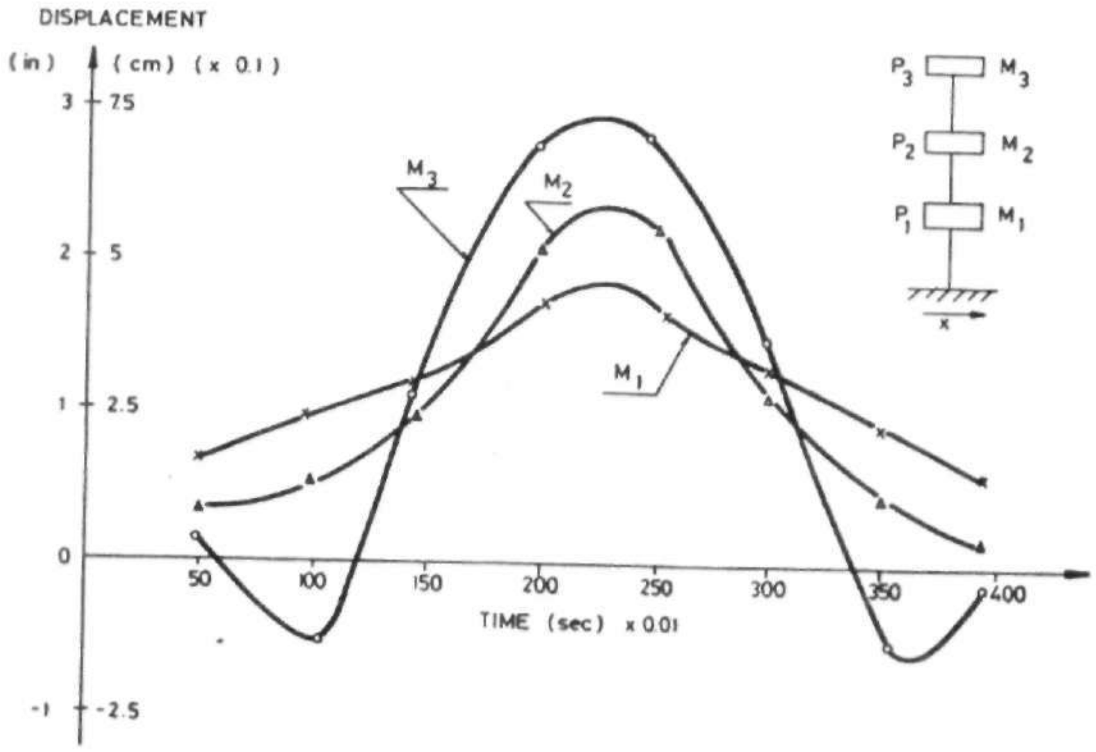


FIG. 5.9 RESPONSE OF MASSES DUE TO AIR PRESSURE AND GROUND (BASE) DISPLACEMENT

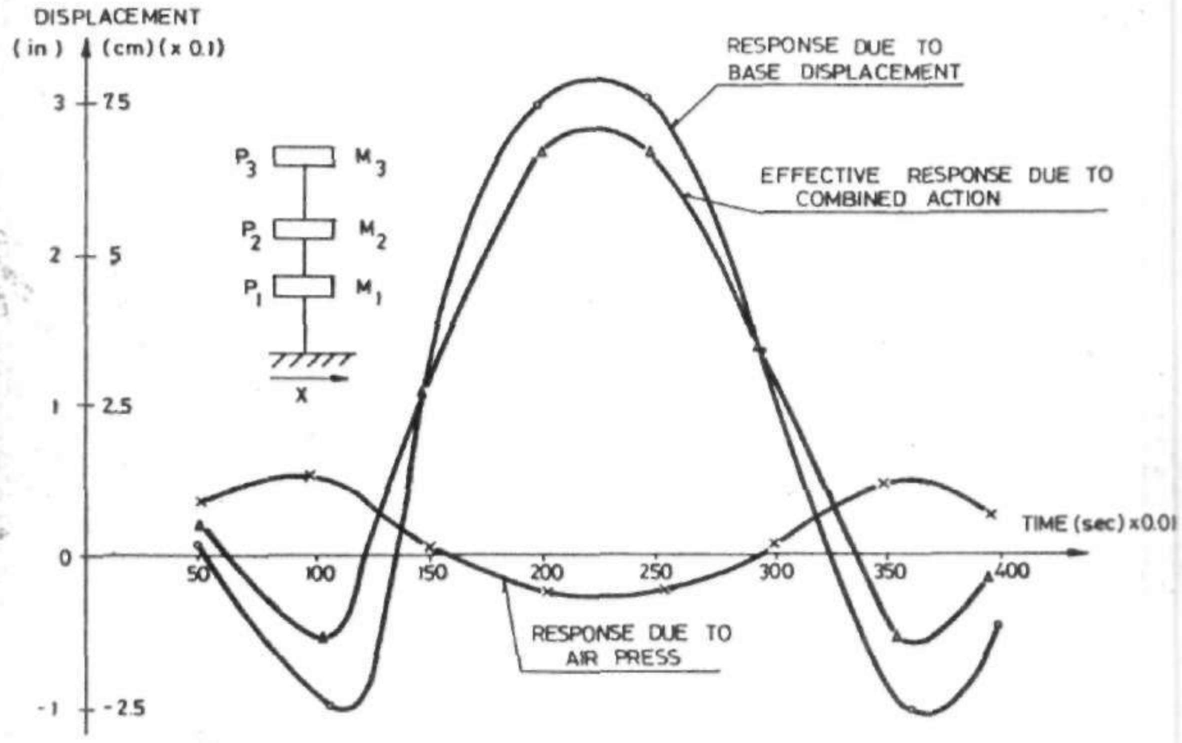


FIG. 5.10 COMPARATIVE RESULTS FOR RESPONSE OF M_3

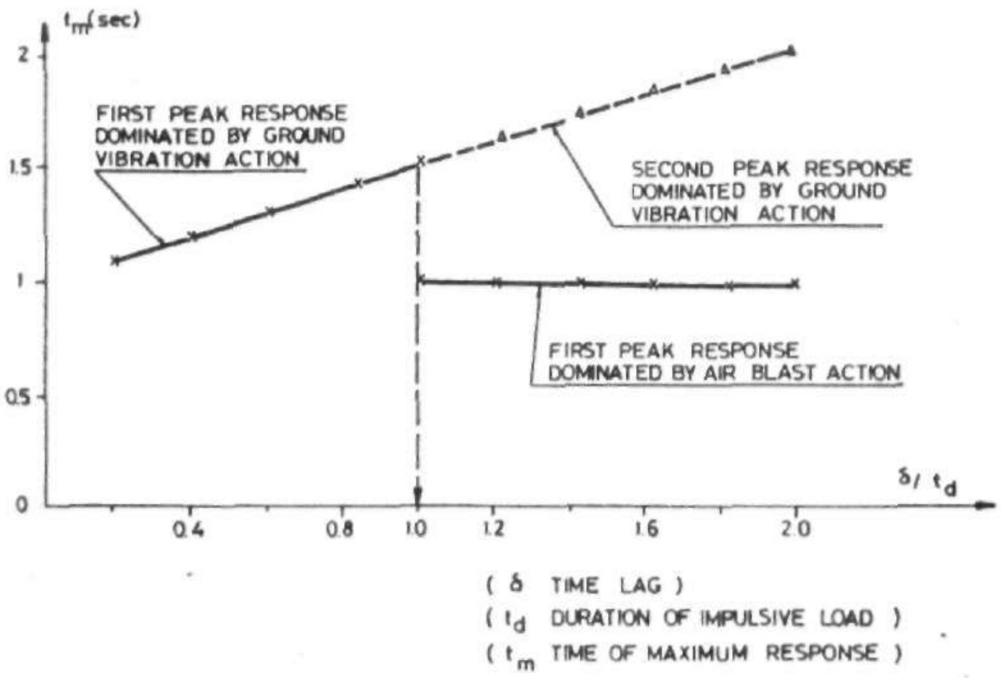


FIG. 5.11 VARATION OF TIME OF MAXIMUM RESPONSE WITH RATIO OF TIME LAG AND DURATION OF PULSE.

CHAPTER SIX

DISCUSSION AND CONCLUSION

6.1 DISCUSSION

Being able to evaluate the displacements of a structure resulting from some applied disturbance is very important in analysis. It is common practice to put limits on the amount of displacement that a structure should experience. There are cases when excessive displacements could limit the usefulness of a structure. Displacements are therefore a useful measure of structural response. The failure of a structure is sometimes defined in terms of displacements.

A structure which is subjected to dynamic loads responds in n-modes of vibration if the structure has n-degrees of freedom. In an idealised structure the total response of the lumped masses is influenced by the different modes of vibration and certain modes dominate the response. In this study, it is indicated that the fundamental mode of vibration - i.e. the first mode of vibration - dominates the total response Fig. 5.6. The fundamental mode therefore defines the fundamental period of vibration of the structure.

The close-coupled system used in the analysis has helped to indicate points of maximum response Figs. 5.5 & 5.8. The analysis has shown that for any multi-storey structure subjected to dynamic loads, maxiumum response - displacements - are expected at the top floor level. In all, the structure behaves like a cantilever system subjected to a one directional loading system. This may be due to the reduction in the resultant stiffness as we move further from the fixed base. The results have helped to specify points at which failure signs on structure could be expected earliest when such a structure is subjected to dynamic loading.

It is sometimes realized that the frequency of vibration at the inception of failure corresponds to one different from the fundamental mode. This may correspond to the second or third mode of vibration. The point of earliest failure could be predicted by looking at the eigenvectors corresponding to that mode. The eigenvectors indicate points where maximum response is expected for a particular mode of vibration. Lumped masses have assumed mass positions for the distributed mass system of a real structure. The eigenvectors are related to the lumped masses, thus a study of the behaviour of a structure in a certain mode of vibration can rightly be predicted by looking at the eigenvectors (characteristic shapes) in the mode.

This study intends to find the relative contribution of base excitation loads and air blast pressure loads from a source of rock blast on a structure. In either case the response of the different masses is of the same pattern. The top floor masses deflect largest from the initial position. The magnitude of response of course differs. The total response is dominated by the base excitation loads. The contribution of air blast loads is so small, but significantly the effect is to reduce the magnitude of the maximum response.

Complaints from residents around areas of quarry operations are mainly due to the noise produced during a rock blast. This has made Mining Engineers develop various methods of reducing this noise. The noise arises from the magnitude of the air blast pressure. The research showed that even though air-blast pressure makes residents uncomfortable it causes a decrease in the maximum response. The decrease even though small still exists. It must be realised that structural failure from dynamic forces might not necessarily be due to large deflections but sometimes due to resonance. Thus if the frequency of any of the inputs corresponds to that of any component of a structure, we expect failure. Overall the plots show greater numerical response values from base excitation than from air blast pressure.

6.2 CONCLUSION

The primary concern of this study was to generate information on damage response measures. The following conclusions can therefore be made:-

(a) A structure responding to dynamic loads responds in different modes of vibration of a number equal to the number of degrees of freedom.

(b) The structural response is dominated by the fundamental mode of vibration which in most cases is the mode responsible for dynamic structural damage.

(c) Maximum response is mainly due to base excitation loads. The soil medium transmits vibration waves readily, therefore efforts should be made to reduce the amount of ground vibration that is transmitted to a nearby structure. This may be done by excavating a trench round the foundation of the structure.

It should be noted that most complaints associated with conventional blasting ground motions involve relatively minor items, such as hairline cracks in mansory, gypsum wallboard or plaster walls and occasional breakage of windows, and not failure of the primary structure. It follows that the elastic range analysis as used herein is of primary importance for building response to dynamic loads due to quarry blastings.

6.3 SUGGESTIONS

The work above has been done using the assumption that the structure is linearly elastic. The following suggestions are made for further work:-

(a) The structure to be considered should be non-linear but elastic.

(b) A research effort geared towards reducing the computed response characteristics to prescribed safe distances from blasting sites for residential structures.

(c) The response pattern of residential structures towards alternating and multi-directional blasting sites.

CHAPTER SEVEN

USERS GUIDE

The results presented in this thesis are based on the computer programme attached to this thesis. It is therefore important to give guidelines on how to use the programme.

There are many computer programmes written by computer analysts for the purpose of obtaining solutions to specific problems. The nature of the problem and the accuracy required will determine the type of programme that will be most useful. The computer programme presented in this thesis is written in FORTRAN Language. The following pages will explain how it can be used.

7.1 THE SUB-PROGRAMMES

This computer programme is written for the purpose of obtaining solutions to a close coupled undamped dynamic system. The three lumped masses connected by massless springs link the masses to a base Fig. 4.1. The solution to such a dynamic problem is obtained by going through a sequence which should be adhered to in order to obtain the results presented in chapter five. The programme is so arranged as to obtain the desired results.

The programme performs the following operations:-

1. It inputs the physical properties of the structure such as the Youngs Modulus of the different members, the second moment of area, crossectional area of members and the number of joints involved.

2. It inputs the type of joint constraints and the coordinates of the joints with reference to a selected axis. All these are done in the subprogramme STAT.

3. The next inputs are the applied dynamic loads - their peak values and duration of application. This is done in the subprogramme DYNP.

4. The static properties of the structure are then calculated. The unconstrained stiffness matrix is first obtained in the subprogramme UNCON and the constrained stiffness matrix is derived from the unconstrained stiffness matrix in the subprogramme CON.

5. Dynamic properties of the structure such as the natural frequencies, the periods of vibration and the characteristic shapes are then calculated in the subprogrammes EIGVV and SUDOV.

Numbers 1 - 5 estimate the static properties and the natural dynamic properties of the structure.

6. The above information is then used in the subprogramme TOBAL to obtain the dynamic response due to the applied dynamic loads.

The results from the calculations in 1 - 6 are seperated as output in the order of the calculations so that the results could be checked in the sequence they are used.

7.2 INPUT RECORD

The input data for the subprogrammes are arranged in a sequence given in Table 7.1.

It was not the intention to write an all purpose programme for dynamic response analysis, therefore the programme was tailored towards bving successful for the structure studied.

Table 7.1 should be able to help a user of this programme understand the input procedure. All other parameters that are internally calculated are printed out by a PRINT format which is closely guarded by the number of degrees of freedom involved in the computations.

This programme can be improved to be able to obtain

TABLE 7.1 INPUT RECORD ARRANGEMENT

DESCRIPTION	SYMBOL	FORMAT	No OF CARDS	UNITS
Number of Members	NM	I3	1	-
Number of Joints	NJ	I3	1	-
Member Properties:- Area / 2nd Moment of Area / Youngs Modulus	A / BI / E	3D15.5	3	M^2 / M^4 / Nm^{-2}
Joint Constraints '0' for free '1' for constrained x,y & z directions	ICON	3I5	NJ	-
Member Incidence Matrix	INC	2I5	NM	-
Joint Coordinates	X	2D15.5	NJ	M
Iteration values	P	3D15.5	1	-
Mass of Lumped Masses	AM	3D15.5	1	kg
Distribution factor of air pressure loads	AQ	3D15.5	1	-
Ground Vibration Props:- Peak Value of Acc. / Duration of Impulse / Wt of Explosive	XCA / T1 / WT	3D15.5	1	MS^{-1} / Sec / N
Air blast press. props:- Distance from source / Peak press value / Duration of impulse	DIST / BQ / T2	3D15.5	1	M / Nm^{-2} / Sec

solutions for more than three degrees of freedom. The main alteration will be in the estimation of eigenvalues and eigenvectors. The subprogramme EIGVV uses the 'deflation' method. The accuracy of this method is appreciative when the matrix involved is a 3 x 3 matrix. Where there is a larger matrix involved there will be a need to employ a different method of solution.

The dimensions of all the subprogrammes are closely linked with the number of degrees of freedom. The dimensions could be adjusted to suit the size of the characteristic matrix. The basic matrix equation being

$$A_c \, q_c = \lambda \, q_c \qquad .....(4.13)$$

Where A_c is the characteristic matrix.

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