TOLL BOOTH CONGESTION STUDIES AT THE LAGOS - IBADAN EXPRESSWAY

BY

NATHANIEL CHUKWUMA IKEM

A Dissertation Submitted To The Post-graduate School, Ahmadu Bello University, Zaria In Partial Fulfillment Of The Requirements For The Award Of Master Of Engineering (M.Eng.) Degree In Civil Engineering.

DEPARTMENT OF CIVIL ENGINEERING
FACULTY OF ENGINEERING
AHMADU BELLO UNIVERSITY, ZARIA

May, 1982

DECLARATION

This is to certify that this thesis has been composed by myself and that it is a record of my own research work carried out in the Highway and Transportation Section, Department of Civil Engineering, Ahmadu Bello University, Zaria, under the supervision of Dr. A. S. Alfa.

It has not been accepted in any previous application for a higher degree nor has any part of it been presented elsewhere for a degree or diploma.

Nathaniel Chukwuma Ikem.

17th May, 1982.

DIGITIZED

DEDICATION

This Thesis Is Dedicated

To

The Development Of Engineering Knowledge And Research In Nigeria; The Memory Of My Late Mother; All Members Of Ikem Family And All Those Who Suffered One Deprivation Or The Other During The Period Of This Research.

N. C. Ikem.

ABSTRACT

The problems of organization to meet the rising tide of traffic congestion, patron complaint and increased costs appear, for some time now, to characterize the operation of vehicular toll collection at the Lagos - Ibadan expressway toll booth.

It is the principal purpose of this research to study the weekend peak traffic delays at this facility by analyzing the queueing problem. This issue is addressed here in order to evaluate the level of service given to patrons and to seek a better understanding of the factors contributing to the poor toll booth service and the means by which service could be improved.

Two policemen constitute a set of parallel servers and vehicles leave them for the toll collectors who constitute another set of six parallel servers. Thus a tandem queueing situation, in which capacity restrictions are imposed, is identified for the system.

Specifically, it is a truncated 2-stage serial queue. It has two parallel servers at the first stage and six parallel servers at the second stage. Vehicles are of different types and the servers are different from one another while order of service is first come first served.

Data is collected and analyzed using explanatory and confirmatory statistical techniques in order to suggest possible theoretical models that can persimoniously describe the behaviour of the variables of interest for the system. In particular the possibility of a poisson arrival and an exponential service time for all channels at both stages is fully explored and the possibility for stationary ineuts and aggregation of data is surveyed through statistical methods. A one-way analysis of variance and curve fitting using the Kolmogorov-Smimov (K-S) test, are carried out on sets of the data and the poisson/exponential assumptions are fairly justified. A computer programme is written for use with the curve fitting technique.

An approximate low fidelity analytical model with the general capability of predicting the

'steady state' performance of the toll booths under a wide range of different physical and operational condition Is then sought to provide operational solutions to the problem.

A survey of candidate models from existing queueing literature suggests an appropriate base model which is modified and adopted for this study. The expected waiting time, mean number in line (excluding the first queue), the customer loss ratio and all other measures of system effectiveness which are of operational interest are then obtained from the model equations using a computer programme designed to evaluate these performance measures.

These system effectiveness measures constitute the basic model output and to validate this analytic/predictive model some of these predicted measures are compared with those measured under same conditions of input.

Finally, a few numerical results are tabulated and a graphical illustration shown. Recommendations are then made on how to improve operations and the thesis concludes by touching upon questions yet unsolved.

LIST OF SYMBOLS

The following is a list of the essential symbols used in this thesis together with their mathematical and operational meanings:

X,Y,Z	:	Usually used to denote general random variable.
M	:	Refers to Poisson-distributed events or to exponentially distributed intervals.
G	:	General distribution.
GI	:	General distribution with independence assumptions.
D	:	Constant distribution.
E_K	:	Erlang type K(K = 1,2...).
C	:	Number of parallel channels; also number of parallel channels in the 1st stage queue (1,2--- ∞).
S	:	Number of parallel channels in the 2nd stage queue (1,2--- ∞).
N	:	Maximum queueing length restriction for second stage queue.
n	:	Number of observations in a data set.
L	:	General restriction on system capacity (mostly in Chapter 3); also random variable that defines the number of 1st stage servers that are busy (mostly in Chapter 4).

FIFO : First in, First out.

LIFO : Last in, First out.

SIRO : Service in random order.

PSPO : Preemptive Service Priority Order.

NPPS : Non-preemptive Priority Service.

GD : General discipline.

C.V. : Coefficient of variation.

σ : Standard deviation.

T : Observed inter-arrival/service time variable.

M : Observed absolute frequency.

C_I : observed cummulative frequency.

FT : Expected cummulative frequency.

CDIFF: K-S statistic.

$\bar{T}$: Mean inter-arrival/service time variable.

ρ : Traffic intensity or utilization factor, usually the ratio of the input rate to the service rate.

λ : Arrival rate.

μ : Service rate.

P_n : Steady-state probability of having n units in the system.

K : Random variable that defines the number of vehicles in the second stage queue.

LL : Mean number of vehicles in system.

Lq : Mean number of vehicles in queue.

W : Mean waiting time in system.

Wq : Mean waiting time in queue.

$P_{(i\ j)}$: State probabilities of having i vehicles in the second stage queue or of having i of the first stage service channels being blocked while j second stage channels are busy.

h : Random variable that defines the number of second stage servers that are busy.

P_{oo} : Probability of no vehicle in the system.

a : Total traffic offered per service time.

a' : Total traffic carried per service time.

ρ' : Channel utilization factor.

LR : Customer loss ratio.

P_{oh} : State probability of second stage servers being engaged with no vehilce in queue.

P_{bls} : First stage blocking probability.

PKS : State probability of K-vehicles in the queue while all second stage servers are busy.

μ_1 : Rate of service at first stage channel.

μ_2 : Rate of service at second stage channel.

β : Fixed probability with which a vehicle leaves Stage 1 and gets to Stage 2.

α : Percentage level of significance.

SPSS : Statistical package for social sciences.

ANOVA: Analysis of variance.

K-S : Kolmogorov-Smirnov statistic.

χ^2 : Chi-square statistic.

D_{α}^{n} : Critical K-S statistic for n observed data points.

CONTENTS

		Page
DECLARATION		i
DEDICATION		ii
ABSTRACT		iii
ACKNOWLEDGEMENT		vi
LIST OF SYMBOLS		vii
LIST OF TABLES		xiv
LIST OF FIGURES		xvii
LIST OF PLATES		xviii
CHAPTER 1: INTRODUCTION		1
1.1.	Purpose and Scope of Investigation ..	1
1.2.	A Survey of Existing System and Organizational Strategies	6
1.2.1.	Brief Description of System, System Constraints and All Factors Affecting the System Performance	6
1.2.2.	The Business of Toll Collection, Traffic Control, Optimization and Scheduling of Staff	11
1.2.3.	Flowchart and Sketches	14
1.2.4.	Identification of System Variables ..	14
1.2.5.	Discussion and Conclusion	18
1.3.	Brief Review of Queueing Theory ..	19
1.3.1.	Definitions, Background, Applications and Limitations	19
1.3.2.	Associated Measures of Effectiveness ..	20
1.3.3.	Inbalance of Theory and Practice ..	22

		Page
1.4.	Description of Queueing Problems ..	23
1.5.	Characteristics of Queueing Processes..	23
1.5.1.	Arrival Pattern of Customers	24
1.5.2.	Service Pattern of Servers	25
1.5.3.	Queue Discipline	25
1.5.4.	System Capacity	26
1.5.5.	Number of Service Channels and Stages..	26
1.5.6.	Classification of Queues	27
1.6.	An Overview of Probability Theory, Markov Processes and Statistical Distribution Models	28
1.7.	Mathematical Methods Applied to Queueing Theory	31
1.8.	Computer Methods	32
1.9.	Models of Queueing Processes	32
1.9.1.	The Nature of Queueing Models	32
1.9.2.	Assumptions, Approximations and the Nature of Operational Solutions	33
1.9.3.	The Steady State Assumption	36
CHAPTER 2:	DATA COLLECTION AND ANALYSIS ..	38
2.1.	Introduction	38
2.2.	Technique and Organization for Data Collection	38
2.3.	Difficulties and Limitations	43
2.4.	Preliminary Survey and Analysis ..	45
2.5.	Detailed Data Collection and Analysis	50
2.5.1.	Introduction	50
2.5.2.	Organization and Strategy for Detailed Data Collection	50

Page

2.5.3. Data Analysis 52

2.5.3.1. The Approximate Techniques and Parameter Estimation 53

2.5.3.2. Statistical Tests 83

2.5.3.2.1. The One-Way Analysis of Variance Tests 84

2.5.3.2.2. Curve Fitting Using the K-S Technique 87

2.5.4. Results and Conclusions on Data Analysis 89

2.5.5. Selecting and Justifying the Model for the Service and Arrival Patterns 92

CHAPTER 3: LITERATURE REVIEW, SEARCH FOR, AND SELECTION OF ADEQUATE SYSTEM MODEL .. 99

3.1. Introduction 99

3.2. Classes of Queueing Models 100

3.3. Discussion of Existing Queueing Models 100

3.3.1. Models of Single Channel Processes .. 100

3.3.2 The Multiple-Channel Queueing Models .. 102

3.3.3. Models of Serial and Cyclic Processes 105

3.3.4. Simulation Models 115

3.4. Selection of Potential Candidate Models 116

3.5. Final Model Selection 120

CHAPTER 4: EVALUATION OF SELECTED MODEL .. 124

4.1. Theoretical Considerations and Model Structure 124

4.1.2. Assumptions 124

4.1.3. Problems Associated with Justifying the Use of this Model and Pertinent Facts About the System 130

Page

4.1.4. Arguments, Simplifications and Modifications Necessary to Justify the Applicability of this Model 131
4.1.5. Theoretical Aspects and Derivations .. 135
4.2. Evaluation of Model Equations and Validation of Model 139
4.2.1. Computer Approach 139
4.2.2. The Inputs to the Programme 143
4.2.3. Types of results obtained and Presentation of these results 149
4.2.4. Choice of System Effectiveness Measures 158
4.3. Model Validation 159

CHAPTER 5: RESULTS, COMMENTS, RECOMMENDATIONS AND CONCLUSION 163
5.1. Discussion and Comments 163
5.2. Recommendations 170
5.3. Suggestions for Further Research .. 175
5.4. Conclusions 177
APPENDICES 180
BIBLIOGRAPHY 211

LIST OF TABLES

			Page
Table	1.1.	Table of Queueing Notations	28
"	2.1.	Mean Inter-Arrival/Service Time Data for Lane B	62/63
"	2.2.	Mean Inter-Arrival/Service Time Data for Lane A	64/65
"	2.3.	Mean Inter-Arrival/Service Time Data for Lane 1	66
"	2.4.	Mean Inter-Arrival/Service Time Data for Lane 2	67/68
"	2.5.	Mean Inter-Arrival/Service Time Data for Lane 3	69
"	2.6.	Mean Inter-Arrival/Service Time Data for Lane 4	70
"	2.7.	Mean Inter-Arrival/Service Time Data for Lane 5	71
"	2.8.	Mean Inter-Arrival/Service Time Data for Lane 6	72
"	2.9.	ANOVA Table for Fridays at Police Queue	73
"	2.10.	ANOVA Table for Saturdays at Police Queue	73/74
"	2.11.	ANOVA Table for Friday and Saturday Combined at the Police Queue ..	74
"	2.12.	ANOVA Table for Fridays at Toll Booth Queue	75/76
"	2.13.	ANOVA Table for Saturdays at Toll Booth Queue	77/78
"	2.14.	ANOVA Table for Fridays at the Car Lanes	79

		Page
Table 2.15.	ANOVA Table for Saturday at the Car Lanes	79/80
" 2.16.	ANOVA Table for Fridays and Saturdays Combined at the Car Lanes	80
" 2.17.	ANOVA Table for Fridays and Saturdays Combined at the Bus Lanes	81
" 2.18.	ANOVA Table for Fridays at the Police Queue based on Periods	82
" 2.19.	ANOVA Table for Fridays and Saturdays Combined at the Police Queue based on Periods	82/83
" 2.20.	ANOVA Results Table	93-96
" 2.21.	K-S Goodness-Of-Fit Test Results ..	97
" 4.1.	System States for the Makino Model ..	126
" 4.2.	Observed and Predicted Queue Length Probabilities for Trucks	151
" 4.3.	Observed and Predicted Queue Length Probabilities for Buses	152
" 4.4.	Observed and Predicted Measures of Effectiveness for Cars, Buses and Trucks	153
" 4.5.	Values of Facility Utilization against Some System Measures	161
" 4.6.	Predicted System Measures for Some Suggested Options	162
" 4.7.	Proportion of each Vehicle Class Arriving in Police Queue	147

Page

Table 4.8. Proportion of each Vehicle Class including those making U-turns in the Toll Booth Queues 148

" 4.9. Service Time Data for Toll Booth Channels 149

" 4 10 Comparative Table for the Different Vehicle Classes using Some Predicted Effectiveness Measures 163

Appendix 1 Tables for Samples of Raw Data .. 180

Appendix 3a. Table of Proportions of Vehicles Arriving in each of the Toll Booth Lanes .. 201

LIST OF PLATES

Plate		Page
1.1.	Typical Off-Peak Situation to show Toll Booth Arrangements	17
1.2.	Near-Peak Traffic Situation in the Vicinity of the Police Point	17
1.3.	Typical Parking Facility Located after the Toll Booths	17

CHAPTER 1

INTRODUCTION

1.1. PURPOSES AND SCOPE OF INVESTIGATION

As societies become more complex, it seems to become more and more difficult for any one to avoid an involuntary participation in queueing processes and we all must have experienced the annoyance and frustration of having to wait in line sometimes. For example, people have to wait in barbershops to get a haircut and in the aircraft industry airplanes sometimes have to wait for either landing or take-off operations.

In waiting-line situations, problems arise because of either (1) too much demand on the facilities or inadequate service facilities leading to excess of waiting time or (2) too little demand, in which case there is too much idle facility time or too many facilities. Both of these cases constitute undesirable queueing situations; for while the first is undesirable to the customer, the second is undesirable to the queueing system manager. There is therefore a need for both the customer and the manager to understand and possibly estimate queues and their subtleties.

One of the most disturbing types of queueing processes is the vehicular traffic congestion which has so characterized life in much of our urban and suburban environments. This is because, together with its visual intrusion, air and noise pollution, it constitutes a major factor in the undesirable decay of the environment with which man is striving to co-exist in positive harmony.

This environmental problem is recognized as an undesirable by-product of highway and transportation facilities development and if this environment is to be modified, it must be understood in a quantitative sense through established analytical procedures. Conclusively, congestion is inefficient in the use of resources and is detrimental to the environment adjacent to a facility. It is therefore always the ultimate goal in the planning, design and operation of facilities to move in the direction of automatic methods of keeping the service force in correct balance with demand requirements in order to achieve optimum conditions within the limitations of the non-controllable variables in operation.

The business of vehicular toll collection and the attendant weekend traffic delays at the Lagos-Ibadan expressway toll booth falls into the class of problems

discussed above and the arguments of the above paragraphs provide the motivations for the study of this problem which is addressed in this research. The congestion gets to disturbing levels as evidenced from the number of vehicles in the queue while the problems of organization to meet the rising tide of this traffic congestion, patron complaints and increased costs appear enormous.

A complete solution of this congestion problem calls for an analysis of the queueing system followed by the optimization considerations that would lead to the development of a more precise method of controlling expenses and service while at the same time providing well-spaced reliefs to the toll collectors.

It is, however, the central aim of this research to analyze the queueing system and determine some system effectiveness measures which provide indices to a better understanding of the factors contributing to unsatisfactory toll booth service.

Standard operations research techniques of problem definition, model construction, model solution and validation were employed in the analysis. Therefore, in the analysis of the operational problems of this queueing situation, the following steps were taken:

(a) expressions were found for the short-time fluctuations of arrivals and services in terms of probabilities, (b) from these and the structure of the system, state probabilities were computed and (c) expected values of the various quantities of practical interest known as effectiveness measures were calculated.

In summary, the thesis is concerned with the analysis of the stochastic behaviour of a system of tandem queueing stations in which capacity restrictions are imposed on the queue lengths except the first one. Specifically, a 'steady state' truncated serial queue with two stages and multiple-booth servers is formulated. The customers are heterogeneous and the servers are also heterogeneous in this queueing system.

An approximate analytical model with the general capability of predicting the 'steady state' performance of the toll booths under a wide range of physical and operational conditions was then sought in order to provide an operational solution to the problem.

An examination of existing queueing literature and methods was carried out and the model developed by Makino (1964) was selected as the base model for this research. The base model resembles the physical system

under investigation but being an idealized representation of it, it had to be modified in order to reflect the real life problems at the study site.

Explanatory and confirmatory statistical techniques in form of graphs and Kolmogorov-Smirnov (K-S) test were explored to fairly justify the use of Poisson inputs and exponential service times at each of the two stages of the queueing system. Similarly, statistical tests to establish stationarity of solutions and aggregation of data were carried out using particularly the one-way analysis of variance test.

Starting from the 'steady state' difference equations and probabilities established in the base model, the mean number in queue and in system, the mean waiting time in queue and in system, the customer loss ratio and other system measures of operational importance were calculated in a straight forward manner for the toll booths queues. No attempt was made to fully determine the waiting-time distribution but a computer programme was written to help in the evaluation of some of these system performance measures.

Practical application of queueing theory, which appearsto receive little or no attention from operations

researchers, is fully discussed here and as it is one of the objectives of this research to see how the near perfect model idealization of real life problems can be made to predict life situations, the analytic/predictive model established was usedto predict some system measures which were then compared with the observed values.

The basic model output is this prediction of toll booth performance measures in terms of the decision variables of the system.

Analytical results are tabulated and graphical illustrations are shown while recommendations for improved operations and areas for further research are pointed out.

1.2. A SURVEY OF THE EXISTING SYSTEM AND ORGANIZATIONAL STRATEGIES

1.2.1. Brief Description of System, System Constraints and all Factors Affecting The System Performance

The study site is the multi-million naira Lagos - Ibadan expressway that houses twelve toll collection booths. The expressway is a dual carriage-way with two lanes in each direction. The two lanes in each direction empty into six booth stations (via six traffic lanes)

where motorists have to pay tolls. To attend to the motorists are toll collectors who provide service by the sale of tickets at prefixed toll charges and motorists are normally required to carry the exact toll charges to avoid change problem. Service provided by the toll collectors could also be in the form of just collecting tickets from motorists if the motorists have their origin at the other end of the expressway. Three modes of operation at the booths are therefore possible for the whole expressway, depending on a vehicle's origin and destination. The sketch of Figure 1.2 shows these modes in detail.

The Lagos end of the expressway is selected for study and in particular the Northbound traffic situation is analyzed because much of the delay is experienced in this direction.

The following are some features of this system that were useful in considering possible system variables:

1. Two policemen at a time are imposed on the two lanes of the expressway to carry out mandatory checks on all vehicles.
2. A police station, a junk yard for accidented vehicles and parking facilities both before and after the toll booths are provided for vehicles with problems.

3. A grade-separated clover-leaf intersection exists a considerable distance behind the police point on the ever-busy Ikorodu road and this has necessitated the provision for U-turning vehicles immediately after the police point.

4. At peak periods, there is always a very long queue behind the police and motorists leave this queue for the toll plaza queue in front of the toll booths.

5. The queue lengths in front of the toll booths are always limited by the policemen who are always positioned in front of their station at an approximate distance of 250 metres away from the toll booths.

6. Sometimes the capacity of the toll booth queue is exceeded and this affects the rate of service at the police point or the rate of arrival to the plaza queue.

7. Motorists make cash payments and receive receipts which are to be surrendered at the end of the journey.

8. Different vehicle classes use the expressway and special booth stations are designated to different types of vehicle classes. Thus there is one lane for heavy trucks and trailers (designated Lane 1), two lanes for pick-up trucks, buses and mini-buses (designated Lanes 2 and 3) and three lanes for cars (designated Lanes 4, 5 and 6).

9. The level of toll charged is currently two naira for cars, four naira for buses and ten naira for trucks.

10. Provision is made for two-wheeled vehicles on the car lanes but observed proportion of these vehicles was less than two per cent throughout the period of study. These were then ignored in subsequent analysis.

11. Two queues in series are therefore identifiable for this system consisting of the police or register stage queue and the toll plaza or marker stage queue.

12. At both stages vehicles are served in the order of arrival and at the booths one collector and an assistant are always available.

13. There are four service possibilities at the police check point:

(a) a vehicle is given a normal check and discharged into the plaza queue;

(b) a vehicle is delayed and removed from the system for further scrutiny after normal service;

(c) a vehicle is held back for abnormally long checks after which it is released into the toll booth queue system. For the period it is held back, there are no arrivals to the plaza queue and this accounts for the incidence of zero back-up traffic on the toll lanes often observed inspite of the excessively long queue existing behind the police.

(d) A vehicle can be allowed to pass unchecked at the police point due to the random sampling technique often employed by the policemen in stopping and

checking vehicles.

14. Traffic is heaviest at weekends but more so towards the month-ends and during special events or public holidays.

15. Weather conditions such as rainfall and extremely hot daily temperatures seem to affect performance especially at the police points.

16. Currently, payment is supposed to be on a cash-only basis and although the authorities always insist that motorists pay in the exact toll charge, to avoid change problem, most times the motorists do not do so and considerable delays result from this behaviour.

17. Fraudulent drivers or motorists also exist who, in a bid to pay less money, attempt to use toll lanes not normally designated for them. This leads to arguments and increased delays with motorists being [illegible]ally required to park in the same limited parking facilities available.

18. Motorists at times grow impatient and either balk, renege or jockey from lane to lane which indicates their tendency to opt for queues of shorter length. For the truck lanes there is no option but for the trucks to stay put in their single lane; for other lanes, these phenomena are observed.

19. Operation and character of problems including input and service rates appear to vary with time of the day,

day of the week and week of the month.

20. Exposure of environment and especially the collector to automobile exhaust and other environmental impacts of such facilities.

21. Inadequate ticketing arrangements as tickets get finished at times resulting in serious patron complaints.

Each of the above factors was considered in deciding which system variables to include in the model selected for this system.

1.2.2. The Business of Toll Collection, Traffic Control, Optimization and Scheduling of Staff

It is observed that currently the problems of organization and management of the toll booth operations are fairly enormous. The traffic manager and his toll sergeants are therefore called upon, from time to time, to employ managerial principles in the face of conflicting objectives in arriving at an optimum level of operations. It is such problems of organization to meet the rising tide of traffic congestion, patron complaint and increased costs that have made this kind of research a vital necessity for sound future operations.

Currently, some of the strategies that have been tried or are still being tried together with operational

problems still being encountered are listed below;

1. When traffic is not very heavy (usually during week-days), some service points are closed to motorists leaving a few in operation. At weekends (Fridays and Saturdays) for which this study is concerned, traffic is usually very heavy. For such periods, all the second stage service channels are always in operation baring any unforeseen circumstances.

2. Precedence is generally given to the patrons at the expense of the toll collectors whenever conflicts arose. But it must be noted that toll collection is a rather nerve-wrecking job, so that extended working periods without a relief are very undesirable.

3. Operational problems causing delays to users are often caused by motorists negotiating for change with the toll collectors when they do not have the correct fare as stipulated by the cash-only payment rule. Consequently, more delays ensue both in the queue and right in the parking facility after the toll booths from where the motorists come to negotiate. Officials normally insist on correct fare and that motorists should not be served unless they have correct fare while some 'standby' workers are normally required to attend to these motorists. Additionally, officials and toll sergeants also have to do with unscrupulous motorists who, because of the change problem, park their vehicles and come back to claim change for an amount higher than what they gave out to collectors.

This is a real problem but with the increased media campaign to make motorists carry correct fare and possibly by legislation, this problem might be eliminated and this will obviously affect the service time distribution. The implication of this strategy of correct fare is that with the motorists carrying the correct fare, the organizers' objective may be to generate constant service time just like automatic machines will do, and by so doing, cut down on the waiting time of patrons [Gaver, (1953) and Saaty, (1961)].

4. Ticketing arrangements appear to be inadequate as tickets sometimes get finished and the presence of a large number of security men especially during change of shift because of money transfers, does not help matters at all. The manager supervises these aspects himself in order to minimize their effect.

5. In dealing with impatient drivers, fraudulent motorists and having to direct traffic at times to less busy channels, the toll booth assistants are very useful.

6. Normally a toll collector and one assistant are provided per booth and shift duties are organized. It is assumed that collectors have no duties other than attending to customers implying a complete server availability. All other demands on the collectors' time known as anciliary work are negligible.

2.3. Flowchart And Sketches

flowchart depicting events in the vicinity of the toll booths is shown in the figure of Appendix 4a and the general traffic disposition and layout is shown on the sketches in Figures 1.1 and 1.2. The photographs shown in Plates 1.1, 1.2 and 1.3 further portray these events.

Essentially, vehicles get on to the expressway via the clover-leaf, get service in the form of checking from any of the policemen in either Lanes A or B in the order of their arrival and depending on whether they are cars, trucks or buses, distribute into the toll booth lanes or make a U-turn back to Lagos. Finally, the motorists get served by the toll collectors, in the form of sale of tickets, in the order of their arrival into the system, and then depart from the system.

1.2.4. Identification of System Variables

A visual and preliminary survey of the study site revealed the critical periods of delay. for Fridays and Saturdays and it is for such periods that all the factors previously mentioned as affecting the system performance, were separately considered. This was to determine the extent of significance of each variable and it was only those veriables that were considered to

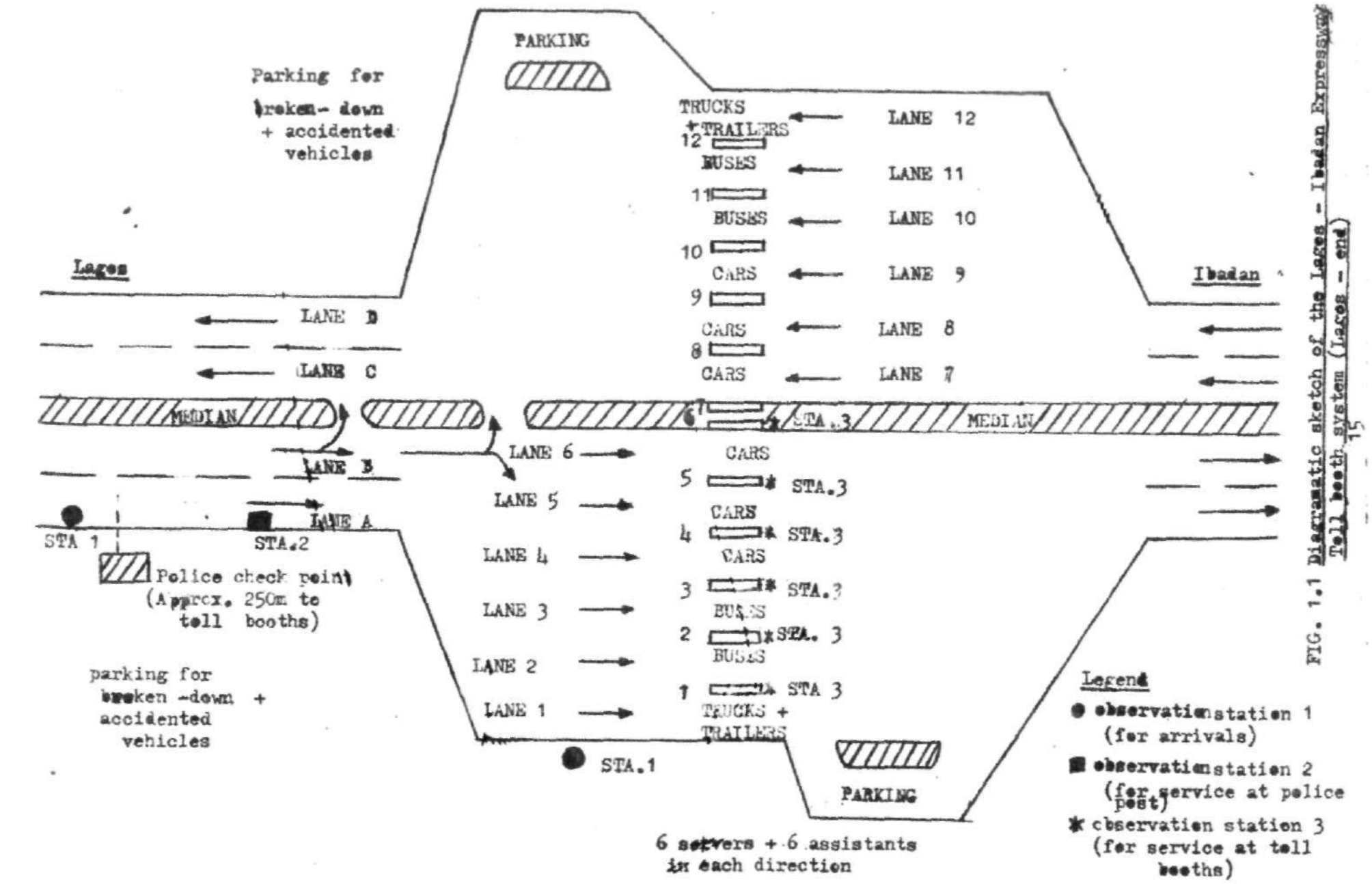
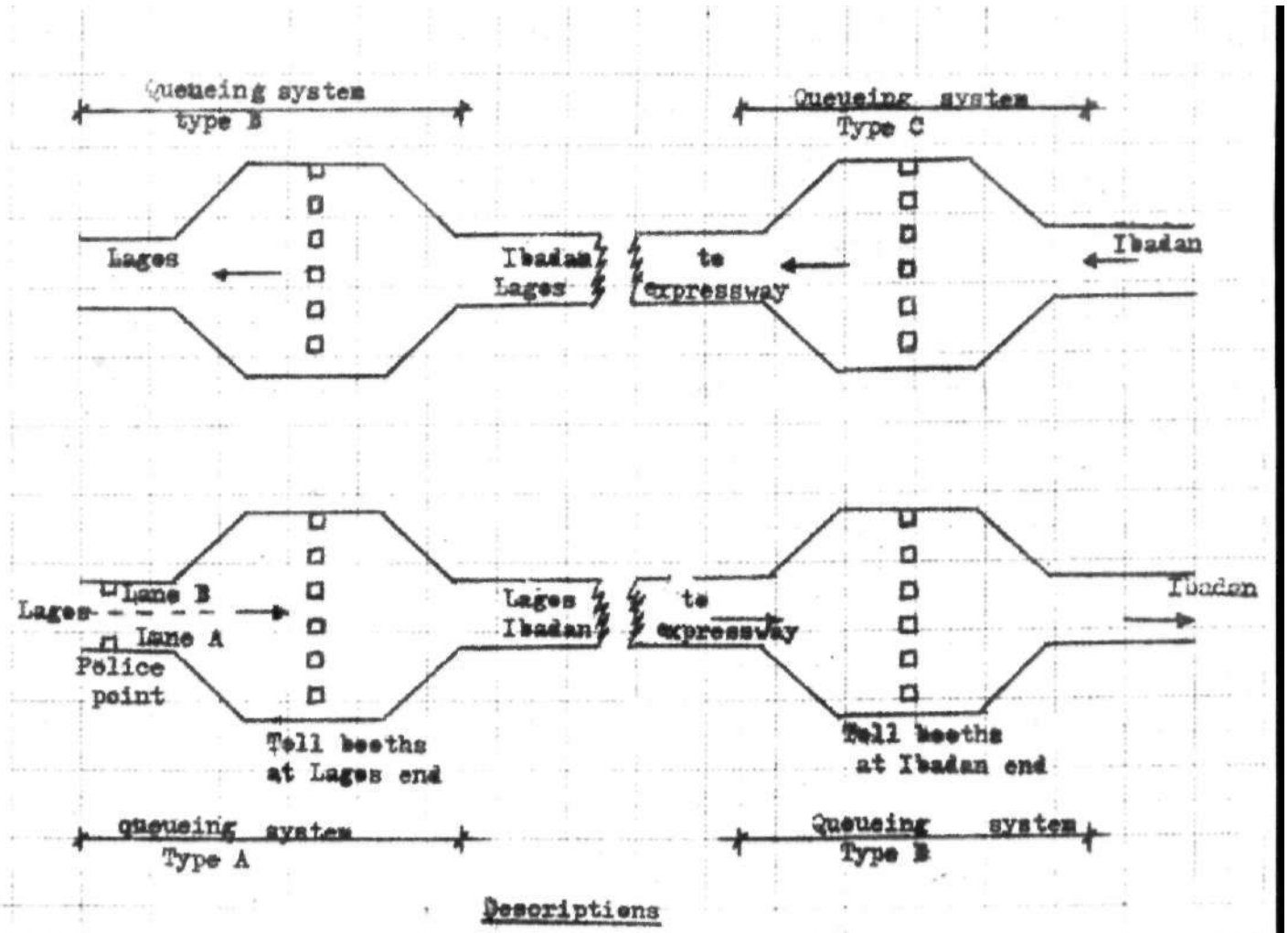


FIG. 1.1 Diagramatic sketch of the Lagos - Ibadan Expressway Toll booth system (Lagos - end)



Descriptions

TYPE A	TYPE B	TYPE C
Has the following features: 1. 2 queues in series 2. 2 servers in parallel (Police constraint) 3. 6 toll booth servers in parallel 4. Lagos end for North bound traffic 5. Mode of service-Motorists make payment and receive tickets	Has the following features: 1. No Police constraint 2. A single point service system with 6 toll booth servers in Parallel 3(a) Ibadan end for North bound traffic 3(b) Lagos end for South bound traffic 4. Mode of service-Motorists submit tickets at service point and are allowed to pass but occansionally motorists make payment and receive tickets if they have misplaced their tickets or if they have entered the expressway from a point other than the Lagos or Ibadan end	Has the following features: 1. Combines features of types A and B 2. No police constraint 3. A single point service system with 6 toll booth servers in parallel 4. Ibadan end for South bound traffic 5. Mode of service- Motorist make payment and receive tickets

FIG. 1.2 Diagramatic sketch with a description of the entire Expressway system and its modes of service



TYPICAL OFF - PEAK SITUATION TO SHOW THE ARRANGEMENT OF TOLL BOOTHS

PLATE 1.1



NEAR - PEAK TRAFFIC SITUATION IN THE VICINITY OF THE POLICE POST

PLATE 1.2



TYPICAL PARKING FACILITY LOCATED AFTER THE TOLL BOOTHS

PLATE 1.3

have significant effect on the system that were included in the final solution. Such inputs to the model included (1) the number of policemen and of toll booth servers; the maximum length of highway or capacity imposed on the toll booth queues. These are all subject to government policies and organizational strategies; (2) the rates of arrival into the queues, the rates of service at both stages and the probability of vehicles not making U-turns which are complete system parameters.

The effects of other factors and why they are not included in the model equation to be determined for this system are discussed in another chapter of this thesis.

1.2.5. Discussion and Conclusion

From this initial appraisal of the congestion problem in the vicinity of the toll booths, it is appreciated that the process of toll collection is a rather demanding one. Overall, therefore, there seems to be a need to reorganize the operations of the toll booth queueing system so as to stem the tide of patron complaints and poor levels of service which appear to characterize the present day operations as evidenced in the traffic delay to users.

The officials could be putting in their best but they seem to be battling with too many external factors. Solutions to this type of problems where many conflicting objectives are to be balanced cannot be completely achieved by the rule of thumb technique. A queueing analysis is therefore necessary as intuition could be a poor guide to solving such problems. At least, an analysis using the tool of queueing theory could yield some significant improvements in operation.

1.3. BRIEF REVIEW OF QUEUEING THEORY

1.3.1. Definitions, Background, Applications and Limitations

Queueing theory is mostly a mathematically descriptive theory which was developed to provide models which predict the behaviour of systems that attempt to provide service for randomly arising demands. Unlike most other techniques of operations research, it does not yield the optimum solution directly. Rather, it is a tool for analysis and the ultimate goal is to use the resulting measures of performance in some cost model so as to determine optimal procedures for the operation of the system.

For it to be worthwhile, a queueing situation must be studied over the period in which meaningful action is required with regard to congestion and the typical

procedure for dealing with such queueing problems are as follows:

(a) define and relate the variables of the situation in order to describe the problem;

(b) derive the associated distributions based on available data and using the appropriate statistical tests;

(c) use the distributions to develop the operating characteristics which describe the system as a whole;

(d) improve the performance of the system through the use of the appropriate decision models and based on the operating characteristics of the situation. However attempts, like this, to represent queueing processes by means of high fidelity mathematical models may not always be possible and even when they are, the models might become mathematically intractible. Hence, it might be necessary sometimes to use quite crude models that approximate the true situation in line with the morphological approach that is normally adopted in practical applications of queueing theory. Solutions from these queueing models, however, represent only a general evaluation of the operation because of the probabilistic approach normally adopted to cater for the irregularities that are often observed in arrival and service patterns.

1.3.2. Associated Measures of Effectiveness

The measures of effectiveness represent the indices

of performance or measures of effectiveness for the queueing system and since most queueing systems have stochastic elements, these measures are often random variables and their probability distributions or at the very least their expected values are normally expected to be found.

Generally, there are three system responses or performance measures of interest. These are: (1) some measure of the waiting time that a customer might be forced to endure; (2) an indication of the manner in which customers may accumulate; and (3) a measure of the idle time of the servers. These measures enable comparison between alternative solutions to be made and the choice of an appropriate measure of effectiveness for a particular situation must always be justified. It is the task of the queueing analyst to either determine the values of the appropriate measures of effectiveness for a given process or to design an optimal system. To do the former, the analyst must relate waiting delays, queue lengths, and such other measures to the given properties of the input stream and service procedures. On the other hand, for the design of a system, the analyst would probably want to balance customer waiting time against the idle time of the servers according to some inherent cost structure.

1.3.3. Imbalance Of Theory And Application

Queueing theory has been successfully applied to problems of telephone conversations, many areas of the aircraft industry, machine repair and so many other areas. However, it is recognized by several authors notably Lee (1966) and Saaty (1961), that a wide gap exists between the theory and application of queues and that there seems to be a lot of what will pass for queueing theory today in dozens of journals and texts while very little attention is paid to application in these same volumes.

This appearance of inactivity in the application of queueing theory is regrettable because queueing theory originated as a very practical subject whereas recent literature has been of little direct practical value. It is clear that the emphasis in the literature on the exact solution of queueing problems with clever mathematical techniques must become complementary to an equal emphasis on model building and the direct use of these techniques in management decision-making. Since most of these theoretical models must be applied to real life problems, it has become imperative that more research should be devoted to approximate solutions although this should not be taken to mean that the development of the practice of queueing theory should be restricted by a

lack of closed-form solutions that is presently the case. A problem solver cannot become frustrated with transform solutions and he must learn to put to work the developed theory.

1.4. DESCRIPTION OF QUEUEING PROBLEMS

A queueing system can be described as customers arriving for service, waiting for service if it is not immediate and eventually leaving the system after being served. These systems have some features that are common to all and from a consideration of these common features of queueing processes, the following primary factors could be said to pertain to any queueing problem:

1. the probability distribution of customer arrivals;
2. the probability distribution of service times;
3. the order in which customers receive service;
4. the number of servers or channels.

Generally, one is always interested in the effects of changes in any of these factors on some measures of effectiveness such as the waiting time distribution.

1.5. CHARACTERISTICS OF QUEUEING PROCESSES

A discussion of the basic characteristics of queueing processes and the issue of classification of queues follows in this section.

1.5.1. Arrival Pattern Of Customers

The moments in time at which individual customers or groups enter a queueing system are called arrival instants and the intervals between consecutive arrival instants are the inter-arrival times. The arrival pattern or input to a queueing system is often measured in terms of the average number of arrivals per some unit of time (mean arrival rate) or by the average time between successive arrivals (mean inter-arrival time). These quantities are clearly related and in some situations either one of these measures suffices in describing the system input. When arrivals occur in batches, instead of singly, both the time between successive arrivals and the number of customers in the batch are given probabilistic treatment.

In arrival processes, it is also necessary to know the reaction of a customer upon entering the system as he may decide to balk, renege or jockey in situations where there are two or more waiting lines. These three events characterize queues with impatient customers.

The manner in which arrival pattern changes with time is also important and a stationary arrival pattern is described as one that does not change with time (i.e. the form and the values of the parameters of the

probability distribution describing the input process are time-independent). When the pattern is not time-independent, it is said to be non-stationary.

1.5.2. Service Pattern Of Servers

The interval between the instant at which a customer begins to receive service and that at which this service ends, is know as his service time. Much of the discussion about arrival patterns is also appropriate in discussing service patterns. Service may be state-dependent as in the case of impatient customers in the arrival process while service and arrival processes are generally assumed to be mutually independent.

1.5.3. Queue Discipline

This refers to the manner by which customers are selected for service when a queue is formed. Most common of this is the first in , first out (FIFO) discipline. Others in common use are (1) the last in, first out (LIFO), (2) selection of service in random order independent of the time of arrival to the queue (SIRO); and (3) a variety of priority schemes where customers are given priorities upon entering the system. There are two general situations in priority schemes called the preemptive service priority order (PSPO) and the nonpreemptive priority service (NPPS).

The queue discipline is known to affect some aspects of system performance measures.

1.5.4. System Capacity

This refers to the physical limitation to the amount of waiting room in a queueing system, such that when the line reaches a certain length, no further customers are allowed to enter until space becomes available by a service completion. Such queues with limited waiting room can be viewed as ones with forced balking as customers are forced to balk if they arrive at a time when the queue size is at its limit. The limited waiting space may also be of an excessively long dimension when it is then referred to as infinite.

1.5.5. Number of Service Channels and Stages

The number of service channels refers to the number of parallel service stations which can serve customers simultaneously and it is generally assumed that the service mechanism of parallel channels operate independently of one another. When customers may, in fact, select or be directed to any of the channels, the channels are normally referred to as providing full availability while the case of restricted availability occurs when the channels are fully available with respect to some customers but not to others.

Another form of multi-channel arrangement is the serial type where a customer passes through many channels in turn and may have to wait to be served at each stage of the process. For some of this type of system, recycling or feedback may occur as in manufacturing processes where quality control inspections are performed after certain stages.

1.5.6. Classification of Queues

Based on the characteristics mentioned above for queues, they can be divided into different classes and this classification is now rather standard throughout the queueing literature. A queueing process is normally described by a series of symbols and slashes such as A/B/X:(Y/Z) where A indicates,in some way, the inter-arrival distribution, B the service pattern as described by the probability distribution for service time, X the number of parallel service channels, Y the restriction on service capacity, and Z the queue discipline. This classification was due, for the most part, to Kendal (1953) and Lee (1966). Other researchers have also made some modifications to this system of classification. In many situations, however, only the first three symbols are in use. The letters are nomally replaced with their standard symbols shown in the notation Table 1.1.

TABLE 1.1: TABLE OF QUEUEING NOTATIONS

CHARACTERISTICS	SYMBOLS	EXPLANATIONS
Inter-Arrival Time Distribution (A)	M	Exponential
	D	Deterministic
	E_K	Erlang Type K.
	GI	General Independent
Service-Time Distribution (B)	M	Exponential
	D	Deterministic
	E_K	Erlang Type K
	G	General
Number of Parallel Servers (X)	1,2---∞	
Restriction on System Capacity (Y)	1,,2---∞	
Queue Discipline(Z)	FIFO	First in, First out
	LIFO	Last in, First out
	SIRO	Service in Random Order
	PRI	Priority
	GD	General Discipline

1.6. AN OVERVIEW OF PROBABILITY THEORY, MARKOV PROCESSES AND STATISTICAL DISTRIBUTION MODELS

Statistics and probability theory have a wide

application in the area of queueing theory and the involvement of these concepts in queueing problems become much more appreciated when one remembers that the very nature and pattern of inputs and service, as used in the theory, are normally specified as probability distributions. The distribution of queue length, on which the derivation of most of the other parameters of the queueing system is based, is also probablistic in nature. The mathematics of the queueing theory is thus heavily weighted towards probability theory and a good knowledge of probability and statistical methods is therefore necessary for the theory and application of the queueing models. In queueing applications, a number of statistical distribution models have been widely used and the types that are most frequently encountered are as follows:

1. The negative exponential distribution, which has Markovian properties, is mathematically a most convenient form for the service time distribution to take. The reason for this is that the time remaining until the completion of a service is independent of the time for which service has already been in progress and this makes for much simplification in analysis.

2. The Pearson Type III distribution is used in many situations where the negative exponential is not adequate and the observed distribution does not differ markedly

from the negative exponential. In many of such cases, although the logarithmic normal distribution can also be fitted to such data, it is always much more convenient mathematically to use the Pearson Type III distribution.

The third fairly important type of service time distribution is the constant service distribution. This has a coefficient of variation equal to zero while the negative exponential distribution has a coefficient of variation of 100 per cent and the Pearson Type III distribution has values lying between these two extremes.

Normally, the above distributions can each be adopted to the inter-arrival intervals with similar reasoning and the Markovian property then makes it possible for the steady state probabilities to be written down easily.

The choice of an appropriate distribution model represents another issue of major consideration in applications. Normally, a statistical test is carried out on samples of data and if this goodness of fit test gives evidence that the Poisson-exponential assumption is mere suspect, three alternative lines of action can be pursued:

(a) Use Poisson-exponential model regardless, as an

approximation, provided the margin of failure is not wide.

(b) Search for other theoretical distributions and models that better accommodate the data; and

(c) Use the actual empirical distributions.

The alternative (b) might prove most undesirable for practical applications because of the mathematical complications that may result.

1.7. MATHEMATICAL METHODS APPLIED TO QUEUEING THEORY

The analysis of queueing problems often takes one into differential equations, difference equations, transform methods and complex variable theory. The transform technique is widely used and it employs the Laplace transform technique for functions of continuous variables and the geometric transform technique for functions of a discrete variable.

Normally, three procedures exist for solving the system of steady state equations in order to obtain the state probabilities. The first and probably the most straight-forward is an iterative procedure, the second involves generating functions and the third involves the concept of linear operators [Gross and Harris, (1974)]. Any of the methods may be more successful than the others depending on the particular model at hand.

1.8. COMPUTER METHODS

Digital computer methods are employed in queueing system analysis in the specific area of simulation as an alternative technique to the analytical method. Similarly, standard computer programmes exist and such programmes can also be developed to yield solution to queueing analytical models which would otherwise be very tedious or time consuming to solve manually. Most of these programmes are written in Fortran IV language except in the area of simulation where a choice is normally made from a number of specific computer languages applicable only to simulation. Computer programmes are thus ready tools in solving queueing problems and they are normally designed to yield all the system effectiveness measures as may be desired.

1.9. MODELS OF QUEUEING PROCESSES

1.9.1. The Nature Of Queueing Models

There is no science of queueing processes but a theory of queues exists which is concerned with the study of symbolic, mathematical models of queueing processes and which appears to be founded on a surprisingly slight body of empirical knowledge.

Normally, the mathematical model of a queueing process will consist of several symbolic relationships

between some primary factors that form the decision variables of the system and which include (a) the rate and pattern of arrival of customers, (b) the probability distribution of service times, (c) the order in which customers receive service, and (d) the number of servers or channels. It is the recognized characteristics of the queueing models as specified by the primary factors that are brought together in the model equation. It is therefore clear that the theory of queues is a general theory that is about the behaviour not of queueing processes themselves, but of generalized symbolic models of them.

1.9.2. Assumptions, Approximations and the Nature Of Operational Solutions

In the study of queueing situations, a great number of features call for a good sense of judgement in approximating to reality some of the observed events of the system which affect the mathematical modelling of the system.

Some of such features for which consideration is normally given are as follows: (a) stationarity and non-stationarity of inputs and outputs, (b) arrivals correlated with other aspects of the system, (c) arrivals in a continuous flow, (d) steady state solutions,

(e) transient solutions, (f) decomposition or splitting and combining of a Poisson stream, (g) full and partial availability, and (h) model validation.

This research has looked into some of these features that affect the particular system under study and at each stage of the study, it was necessary to make assumptions and approximations that would cater for observed features.

The practitioners of operations research maintain that these assumptions and approximations should be made in dealing with real life problems provided that they will reasonably simplify the system and lead to operational solutions that are discriminating enough to lead to a right decision. This simplification and approximation, implied in the operational solutions, becomes more necessary when one remembers that everyday queueing processes seem to be much more complicated than most of the existing queueing models. Thus, it is recognized that it is not always necessary to obtain solutions in the mathematician's sense, using very refined and rather complex models, when with a little bit of assumptions and approximations, closed form solutions of an approximate nature can be obtained for the same practical problem.

Once one accepts the necessity for approximations using fairly oversimplified models, then very much of the existing queueing models may become very useful for practical applications.

The three basic types of approximations that are suitable for queueing theory applications are:

1. the first type of approximation which is used to obtain operational solutions by applying models which are, a priori, the wrong ones but which are simple to deal with. Here it may be necessary to resort to simulation when the risk of making a wrong decision because of over simplification is unknown, or when the penalty for a wrong decision is very great;

(2) the second type of approximation becomes necessary when the cost of simulation becomes prohibitive. The principle here is that a process is simulated under various conditions, so that points are obtained on the response surfaces which are of interest. These are then subjected to statistical analysis with the aim of finding empirical relations between the various factors which can then be treated as a second order model of the process;

(3) this type is the classical one used in mathematics and physical sciences and is exemplified by two techniques:

(a) the expansion of the unmanageable functions in a formula in terms of power series, the higher terms of

which are ignorable in certain ranges of values of the variables (b) the substitution of initial values in the formula followed by iteration.

Thus in the application of queueing models either for planning the extent and character of new facilities or evolving strategies for manpower development in existing facilities or for comparing alternative forms of organization of servicing facilities, approximations, to a reasonable level, should be considered a legitimate tool.

1.9.3. The Steady State Assumption

The steady state assumption is most often made for queueing systems as a further aid towards a simplified solution. Normally, obtaining time-dependent or transient solutions of a queueing system is mathematically tedious for simple systems that one is compelled to suspect that for the very complicated real life problems, this approach may break down. The first working rule is therefore to assume a steady state. This could be justified for some types of queue as follows: (a) the number of occasions on which analysts are required to look at queueing process which are just starting up is quite small; (b) it is equally rare to be asked to study a process at the time of a change in the arrival or

service parameters; (c) usually one is not required to look into a queueing process until it has got into difficulties, and therefore by definition has been running for some time, (d) most practical questions about queueing processes have little to do with transient behaviour and always refer to normal working conditions which, by implication, is steady state condition; (e) the time-dependent solution is more mathematically involved.

However, there are quite genuine cases of practical importance concerning transient behaviour and for such solutions, it would be absurd to attempt to apply a precise time-independent solution. Decidedly, when the inputs are inhomogeneous in time or when the system is operating under a saturated state because of system parameters, a steady state solution cannot be possible. Also when either the servers or the customers or both are human-beings, starting-up periods are marked by such instability in the service-time distributions that accurate mathematical analysis almost becomes impossible. The arguments aside, it appears that the unarticulated, main reason for emphasizing the steady state solutions is the mathematical difficulty associated with transient solutions. It is, however, the view of most authors that transients should not be ignored in time-varying systems such as those encountered in the design of air traffic control procedures or mass transit systems. Besides, it is the premise of most authors that the transient behaviour of many systems is the most important characteristic of those systems.

CHAPTER 2

DATA COLLECTION AND ANALYSIS

2.1. INTRODUCTION

Having got an insight into the system performance level, characteristic interactions and possible variables of essence in the formulation of the system equations, this chapter sets out to quantify some of these variables and make decisions as to some of the staistical properties of the system.

The study carried out under this chapter can be classified as follows:

(a) Preliminary survey and analysis;

(b) detailed data collection and analysis;

(c) selecting and justifying a model.

2.2. TECHNIQUE AND ORGANIZATION FOR DATA COLLECTION

For the purpose of data collection, the study site was divided into four zones namely: (a) area before the police station, (b) just at the police point, (c) at the toll plaza (i.e. area covering the six toll booth lanes), and (d) at the toll booths. The observation stations are as marked on the layout sketch of Figure 1.1 and the data collected from each station is supposed to give the queue characteristics of the particular zone. The flow pattern of vehicles is in accordance with the

flowchart of Appendix 4a and the layout sketches of Chapter 1 while normally, the data collectors were positioned so as not to interrupt the traffic in any way. In particular, this was to guarantee that the motorists are not conscious of the fact that they were being observed.

Data was collected in two phases: (1) the preliminary survey and sample data gathering which is to indicate how to organize for detailed data collection, the saturation and under-saturation of facility, an understanding of the system and a general idea of the inter-arrival and service time distributions that would enable the modeling aspect of the research to commence and (2) the detailed data collection which is to supply detailed information about the system so as to confirm the distributions earlier assumed. Both involved the use of human toll collectors and are subjected to the same set of organization except that the preliminary data stage has a scantier outline.

The full outline of the methods, techniques and organization employed for this data collection is as follows:

1. the data was collected through the months of July, August, October and November, 1981 and the last set of data was collected in January, 1982. The collectors

were normally required to collect data for normal traffic flows and not for such occasions as queue breakdowns and the like. This non-collection of data for all conditions coupled with unpredictable weather and such other factors resulted in missing data in some of the cases.

2. a duration of 20 or 30 minutes was selected for most of the data but 5 minutes intervals for 20 or 30 minutes period was selected for the vehicle accumulation studies. For the counting distribution, a 30 seconds interval for a period of 20 or 30 minutes was also selected. However, the periods of observation in the field were a little bit more or less than the selected values occasionally, depending on the experience of the collector and sometimes on weather or police interruption.

3. all the data were collected manually, using human data collectors who were speedily trained up for the job. The collection for a particular day was done in two stages known as the police queue stage and the toll booth queue stage and the collectors were allocated in such a way that all data necessary for each zone of the study site where the collector is situated were collected. All relevant data about a stage is normally collected before they all move together to the next stage with a period of 30 minutes break being allowed between the stages to make for co-ordination and cross-checking of results.

change problem were collected by noting the number of such cases that occurred during the study period at the toll booth stations; (i) the vehicle distribution study was carried out to show the pattern of distribution of vehicles into the toll booth lanes as the vehicles left the police point. This was accomplished by two collectors each to one of the police lanes who counted the vehicles as they arrived into the lanes for normal periods of study. A note was also made of all the U-turning vehicles after the police point.

Together, these constitute the full range of data sets that were used at one stage or the other in this research.

6. The result of preliminary surveys together with the recommendations by Pignataro (1973) suggested a study period of 12 hours or as close to it as possible. The period of study was therefore scheduled to be from 7.00 a.m. to 7.00 p.m. subject to several constraints. Four rounds of data, each of about 3 hours duration were expected to be collected at 7.30 a.m., 10.30 a.m., 1.30 p.m. and 4.30 p.m. but this was very difficult to achieve almost for all time during this exercise because of several factors. All through, the exact periods at which data were collected were selected so as to avoid special event conditions such as holidays, unusual weather, queue

breakdowns and such other adverse system constraints because this study is not for the specific purpose of studying the effects of these special event conditions. This is very important as it is the aim here to investigate the system only under situations that could be called normal. Data collection was therefore suspended wherever there was a lot of interference between the system and the process of data collection. For example, the periods of induced idleness of either the police or the toll collectors - although not frequent - was not included in the study.

This elimination of wholly abnormal periods from the study period, as has been pointed out, results in a lot of missing data but it at least guarantees that some factors are fixed through the months covered by this exercise.

Only weekends, especially Fridays and Saturdays, are considered important for this study, since there is scanty traffic on Sundays and weekdays and there is then no reason to suggest any troublesome situations, especially for the Lagos-Ibadan traffic which is the focal point in this research.

2.3. DIFFICULTIES AND LIMITATIONS

The difficulties and limitations encountered

in the execution of this stage of the study are so varied and numerous that they could mar the results if not properly controlled.

Some of the major difficulties and limitations are as follows:

1. the process of data collection is a rather tedious one for this system characterized by so many varying conditions or apparent instability. A lot of hazard is imposed on the collectors and it is considered that the technique of using human collectors and timing devices is not adequate for this study. Rather, other techniques such as the use of mechanical counters or the analysis in a traffic laboratory of films taken at the study site should have been explored but for lack of necessary resources;

2. the interruption of counts by policemen and ministry officials was very frequent;

3. there was the issue of financial constraint to contend with;

4. adverse weather conditions such as rainfall and extremely hot daily temperatures did affect the exercise as either the policemen and or the data collectors had to abandon their post for this period;

5. the individual skill and experience of the data collectors many times lead to rejection of whole data sets and results.

2.4. PRELIMINARY SURVEY AND ANALYSIS

The purpose of this section is to identify periods of near constant passenger flow for the system and to try and specify the peak period of operation within the possible 12 hours period that is suggested for this study. Furthermore, by observing actual traffic flows and analyzing a few sets of data approximately, it was possible to suggest candidate models for the service and arrival patterns which stimulated the queueing model selection for the system. This also gave an insight into the facility saturation or under-saturation and the types of data necessary for detailed analysis.

The first step involved a visit to the study site to collect sample data relating to the variation of traffic on hourly basis for the system. A visual appreciation of the arrival and service mechanisms was also made and a few field notes taken. From this, graphs similar to Figures 2.1 and 2.2 were roughly drawn to show periods of high traffic intensity and thus identify the peak period. From the observations also it appears that an exponential assumption for both arrival and service mechanisms could be appropriate. Hence, since there are many queueing models available if one can justify the use of Poisson arrivals and exponential service process, the study will concentrate

on testing the validity of the exponential distribution as a model for service/inter-arrival times. Testing for the exponential has been chosen instead of poisson not only because of convenience, but because of the arbitrariness involved in selecting the time interval for the Poisson counts.

The second stage of the preliminary survey involved collecting a few sets of data on inter-arrival time and service time for both the police and the toll booth queues in the manner already discussed and within the peak period which can be taken from the discussion above to be from 11:00 to 16.00 hours of the day. Four collectors were used for this exercise which covered Fridays, Saturdays and Sundays and the event flow-chart of Appendix 4a resulted from this preliminary stage study.

In the data analysis stage, the method of moment procedure (Giffin, 1978) was adopted and the estimates of the parameters of the proposed theoretical curve (the exponential) were obtained by matching the moments of the curve against corresponding sample moments. In brief, histograms of the observed inter-arrival and service times were plotted in the manner of Figures 2.3 to 2.10 and these were compared to the standard shapes of known distributions to see how reasonable the exponential

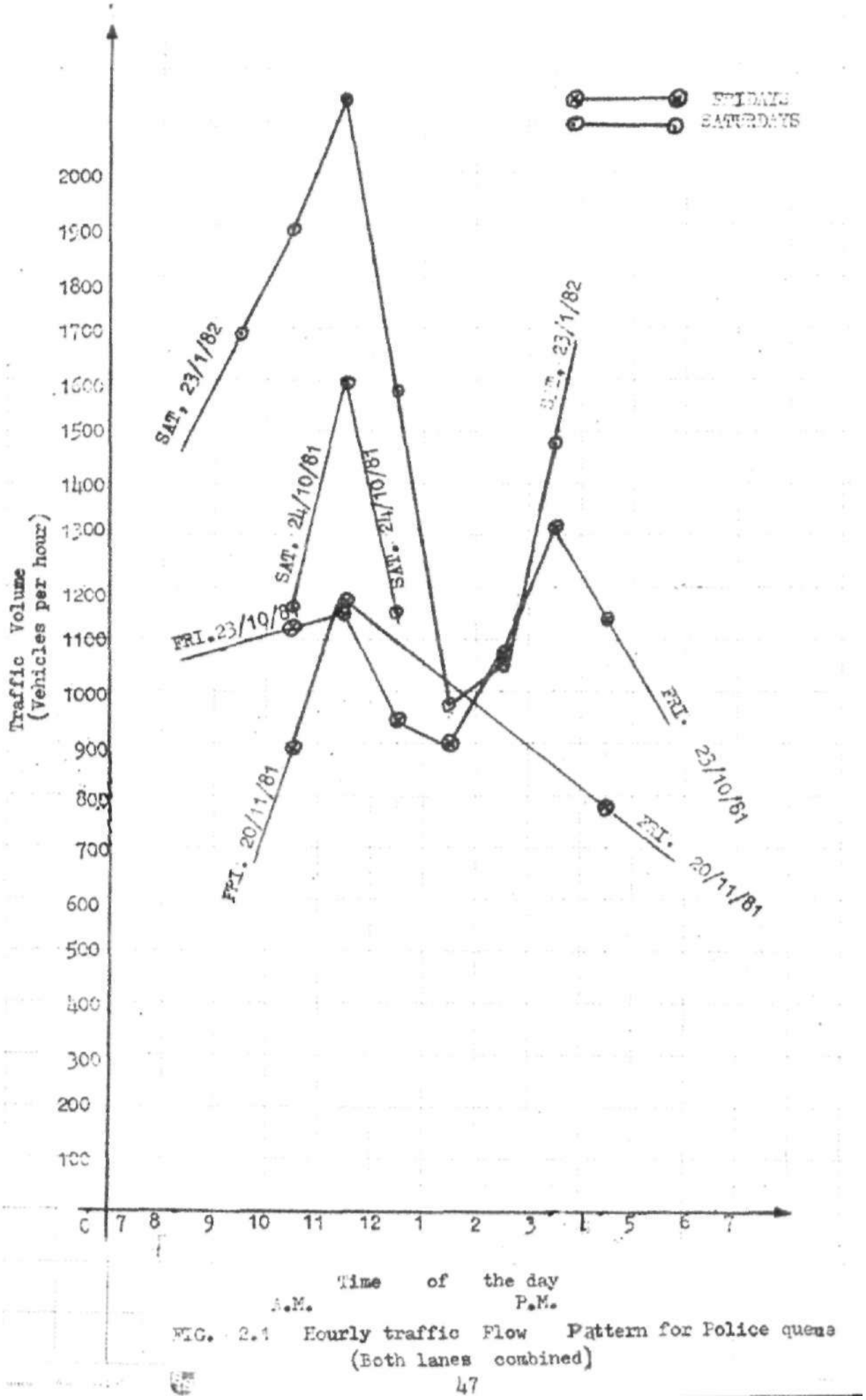


FIG. 2.1 Hourly traffic Flow Pattern for Police queue
(Both lanes combined)

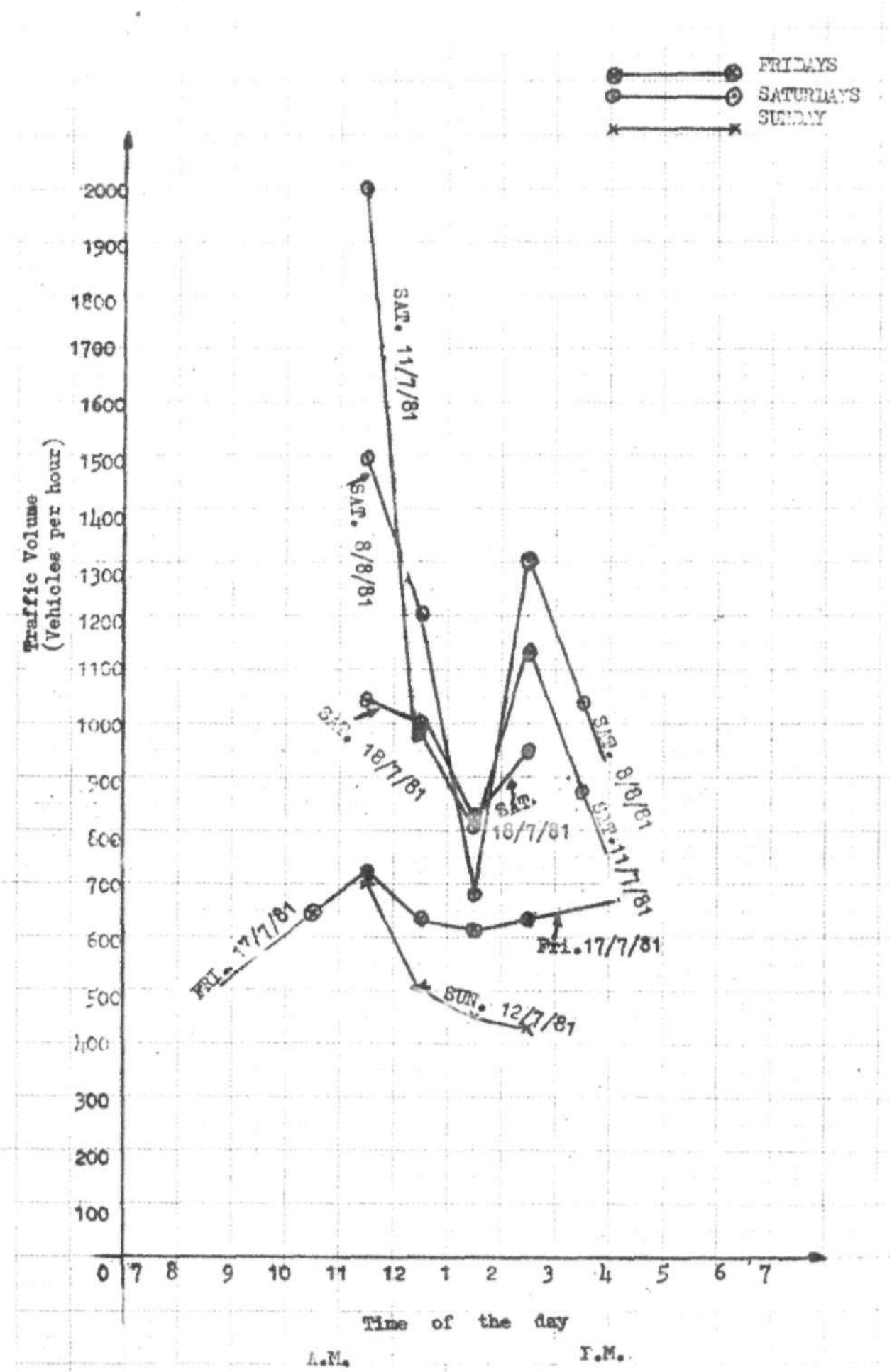


FIG. 2.2 Hourly traffic flow pattern for toll booth queue (all lanes combined)

assumption is for the system. This is then followed by moment matching and a visual comparison of the observed and theoretical distributions. The graphical tool (as applied here) is very useful in the tedious chore of identifying patterns and models in large data sets and it will be fully explored in all aspects of this research.

Another quantity of importance at this stage of the study is the coefficient of variation (C.V.)which is the ratio of the standard deviation to the sample mean of the distribution. This value is known to be equal to unity for the exponential, much less than one or tend to zero for constant time distribution and lie between zero and unity for the gamma distribution. The coefficient of variation was thus determined for the inter-arrival time and service time at both stages of queueing in the system by first estimating the mean and the standard deviation from observed data. These values were tabulated in the manner of Tables 2.1 to 2.8 and together with the plotted histograms, they showed how much the exponential assumption could be taken to be valid. From these results therefore, it was decided to go ahead with assumptions for the negative exponential distribution for both inter-arrival time and service time, since the results were not too poor for the practical and operational solution that are of interest in this research. The validity of the exponential assumption

was left to the confirmatory statistical analysis in the detailed data collection and analysis stage.

The utilization factors (ratio of service time to inter-arrival time) at both stages of the queueing system were also calculated for a few sets of the data so as to give an indication of how saturated the system is.

2.5. DETAILED DATA COLLECTION AND ANALYSIS

2.5.1. Introduction

This section involves the final and more detailed collection and analysis of data for the estimation of the statistical properties of the input and service times in sufficient details. This analysis enables the choice of an appropriate model to be made. Graphical methods and statistical techniques are extensively employed and such issues as curve fitting, stationarity of inter-arrival time and service time distributions, variations of arrival between lanes and variations of service between channels are addressed. Finally, a probability distribution model is selected for both arrival and service patterns while an attempt is made to justify the selection.

2.5.2. Organization And Strategy For Detailed Data Collection

Here, more accurate and varied data than was used in the preliminary study is collected and days and times

(of the day) are varied as much as possible to enable any changes of performance during the selected period of study to be noticed. The full range of data as earlier enumerated were collected through the months mentioned and samples of the new data for some of these cases are shown in Appendix 1. The techniques and organization for this stage of the study is also as earlier outlined with more stop-watches and digital watches being acquired.

Briefly, more data were collected to establish the pattern of traffic flows and periods of congestion for Fridays, Saturdays and Sundays, having concluded that other days -- Monday through Thursday -- are of no trouble-some consequence for this queueing problem. The data collected were plotted for some of these days as in Figures 2.1 and 2.2 which show a similarity in pattern for Fridays and Saturdays. However, after this stage, it was decided, based also on observation, to drop off Sundays and concentrate on Fridays and Saturdays. The period of interest for the study was also identified to vary from somewhere around 10.00 hours to 16.00 hours. Based on calculations and observations, the proportion of two-wheeled vehicles was found to be negligible for consideration in this study. Data on them was therefore not extensively collected.

Following the above discussion, therefore, data were collected wherever it was necessary for the different parts of the system while the collectors were forced to suspend collection when conditions were not conducive.

2.5.3. Data Analysis

The analysis involved the use of both the approximate techniques and the more rigorous statistical tests. In particular, the potentials of the One-way analysis of variance (ANOVA) in establishing the stationarity of distributions, the homogeneous nature of inputs and outputs, and the significant or non-significant variations between operating conditions on any chosen set of lanes or channels in the system were explored. Also a computer aided approach is extensively used for (1) execution of the ANOVA; (2) frequency distribution analysis that is vital to the plotting of the histograms and to the curve fitting procedure; (3) the curve fitting technique. A small computer programme was written for use with the curve fitting technique only because of the large set of data involved in the analysis.

The various stages in the analysis are as follows:

2.5.3.1. The Approximate Techniques and Parameter Estimation

Here graphical methods and the method of coefficient of variation earlier mentioned were employed in a much more extensive way in order to show the merits of the exponential assumptions for the arrival and service mechanisms.

For the graphical procedure, a selected number of the data sets were analyzed using the frequency option of the Standard Statistical Package for Social Sciences (SPSS, 1976) and then histograms plotted as shown in Figures 2.3 to 2.10. When compared, visually, with existing distributions, it can be seen that the histograms suggest that an exponential distribution could approximate all the sets of data. This implies that the graphical method supports the exponential assumption for both inter-arrival and service time distributions for Friday and Saturday data.

The coefficient of variation technique involved determining the coefficient of variation for the distributions of service and inter-arrival times. This was achieved by estimating the mean inter-arrival times, the mean service times and the standard deviations for service times and inter-arrival times from the data sets. The procedure and steps involved in determining these values are outlined in the sample calculation contained in Appendix 2.

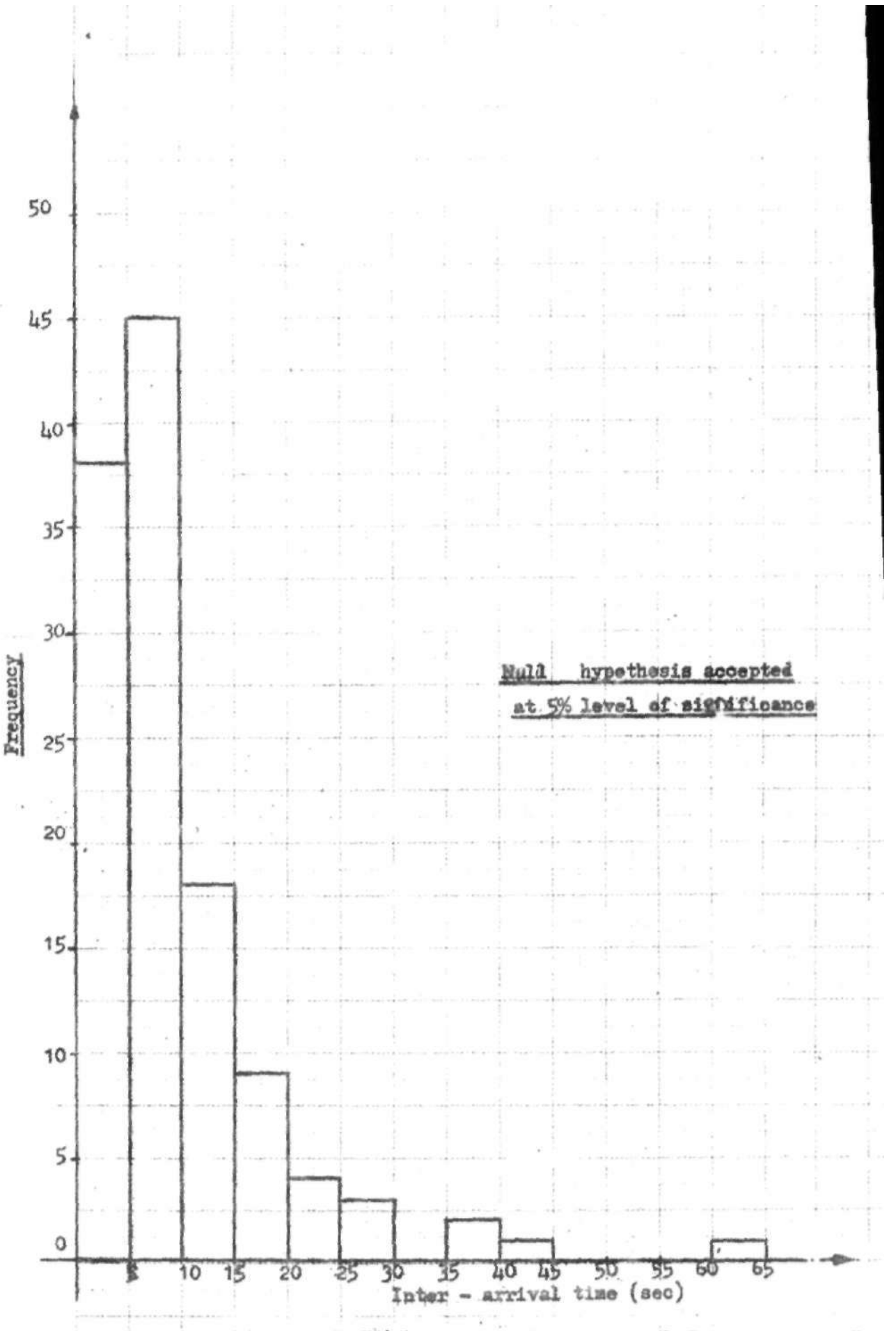


Fig. 2.3 Histogram of inter-arrival Times for lane B (Police queue)

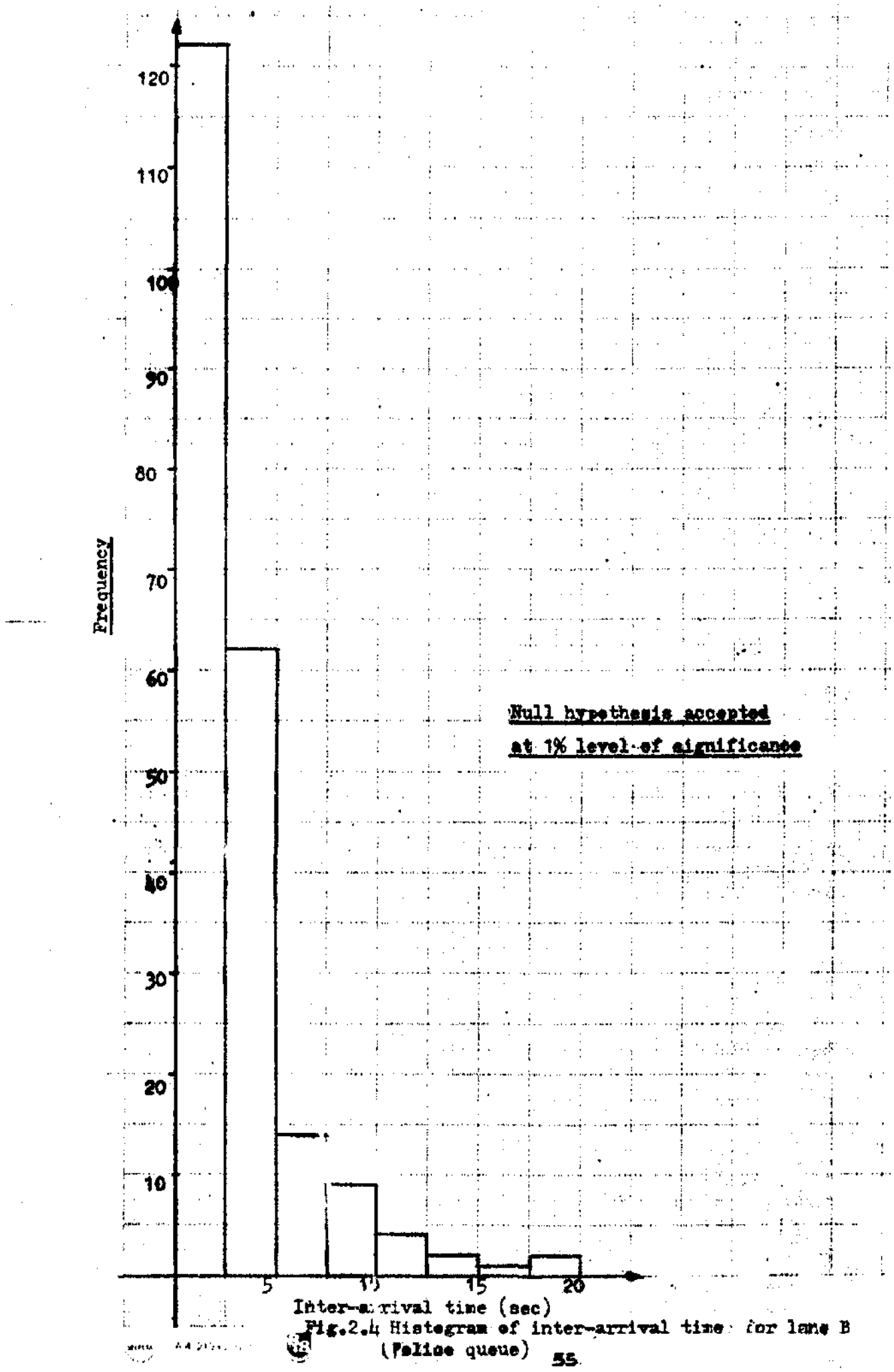


Fig.2.4 Histogram of inter-arrival time for lane B (Police queue)

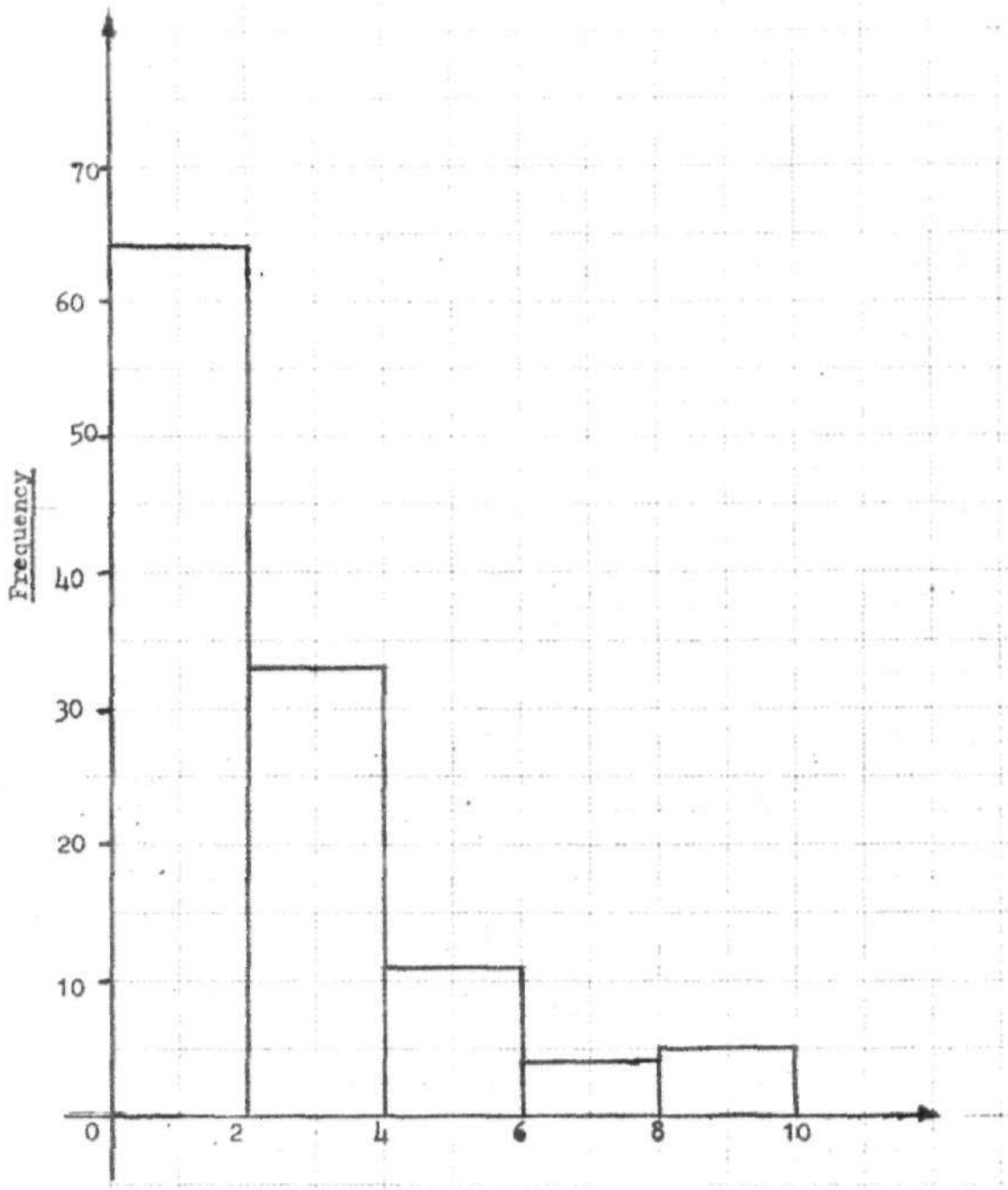


FIG. 2.5 Histogram of service times for Lane A (Police Queue)

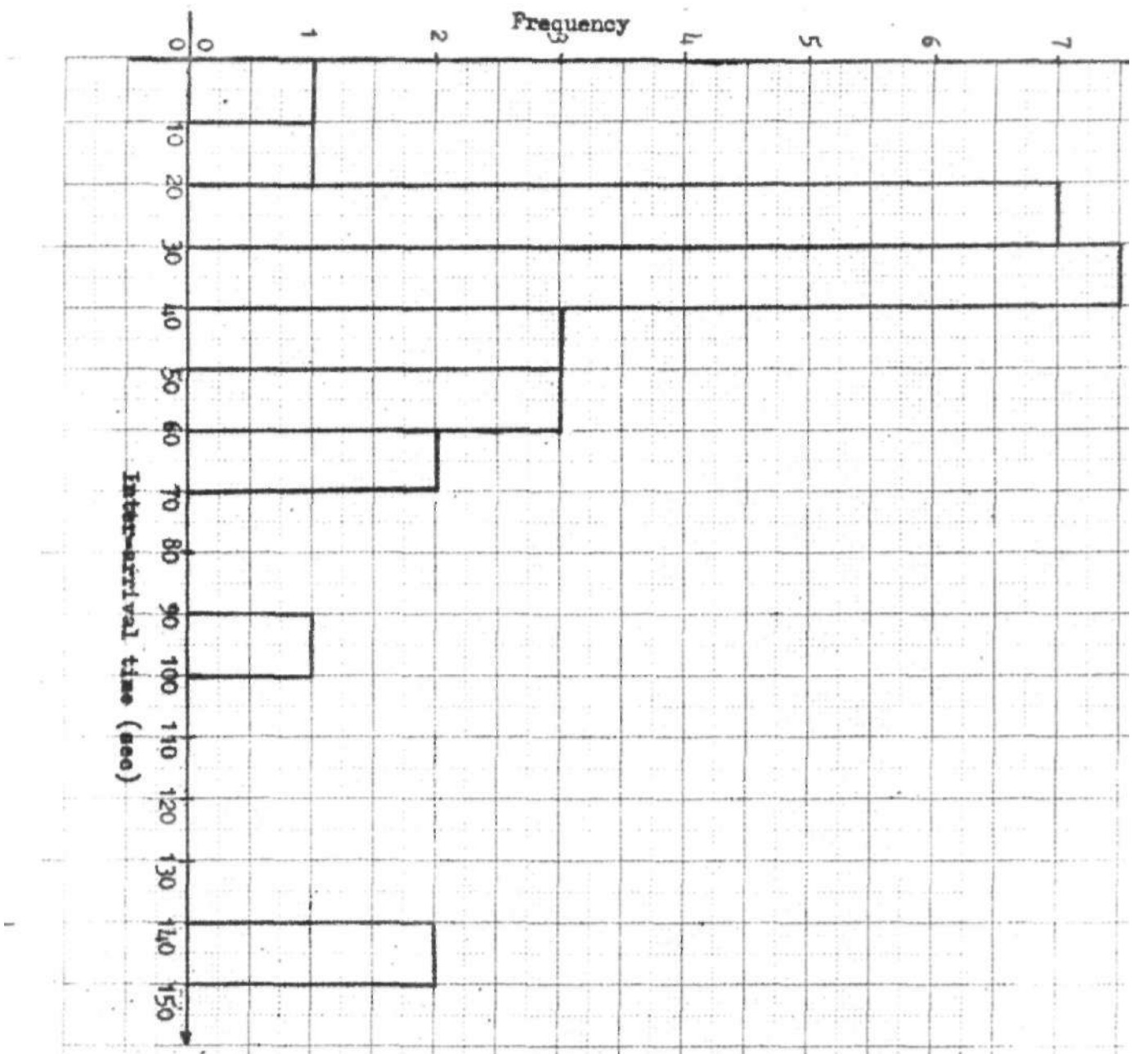


Fig. 2.6 Histogram of inter-arrival times for lane 1 (Toll booth queue)

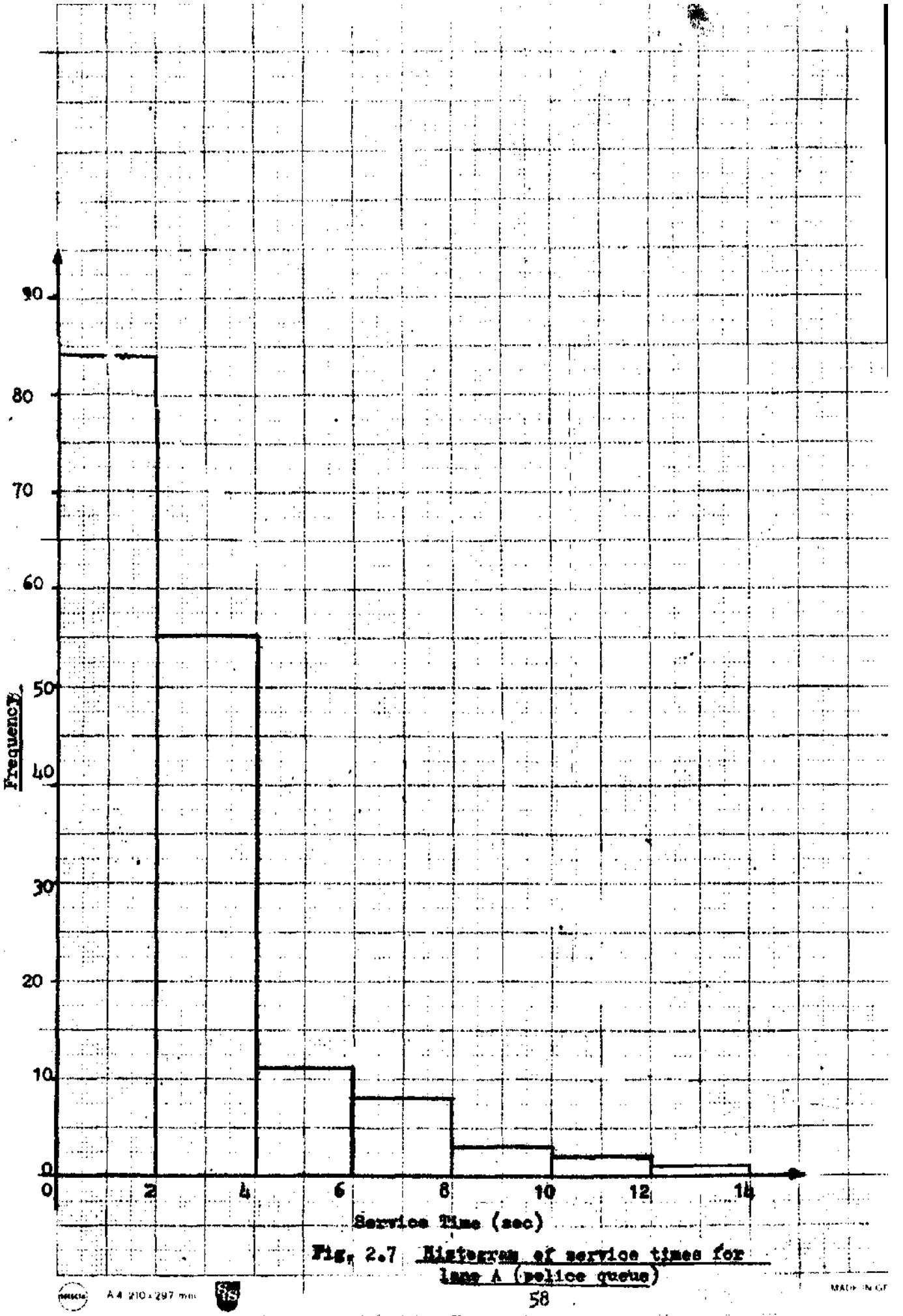


Fig. 2.7 Histogram of service times for lane A (police queue)

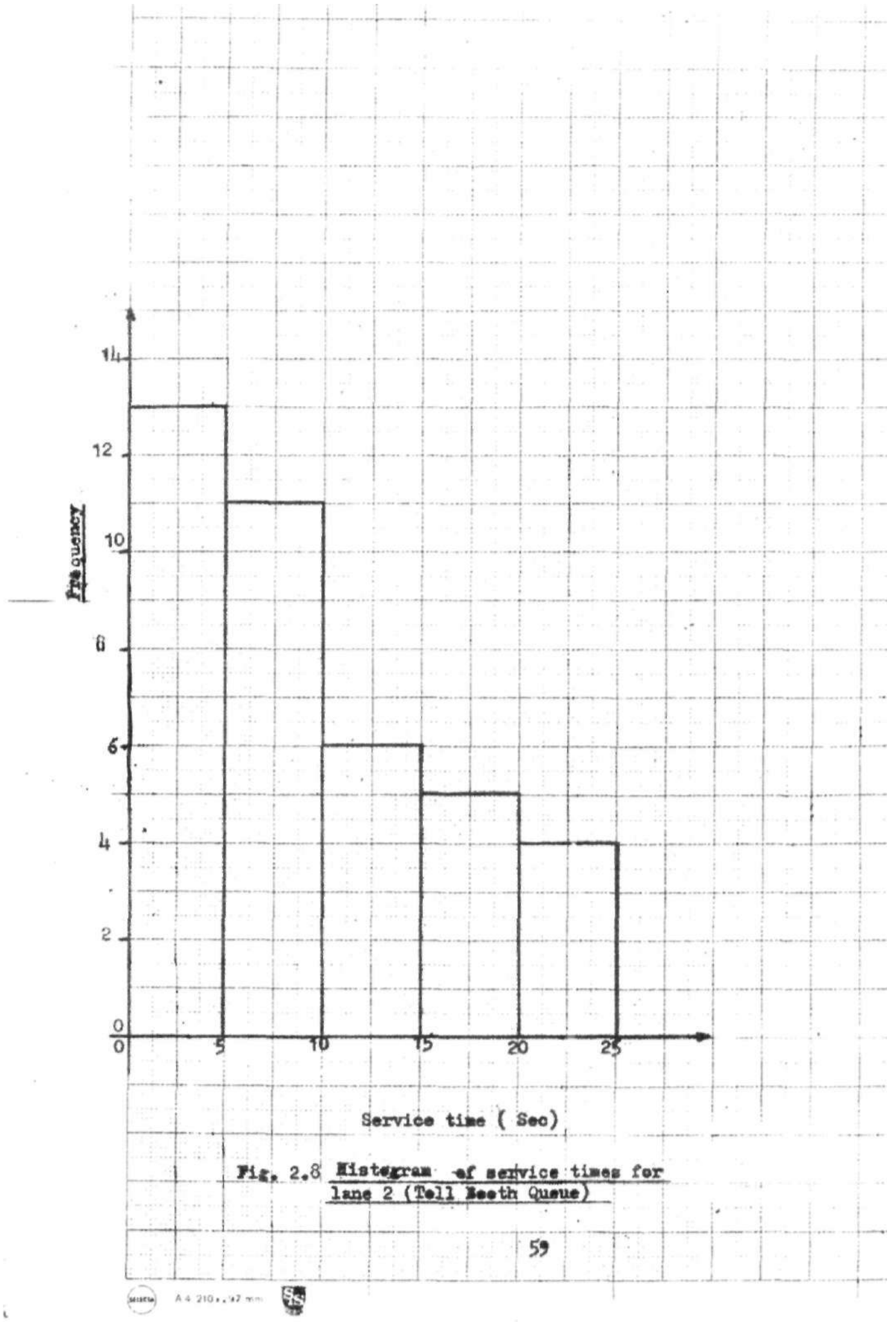


Fig. 2.8 Histogram of service times for lane 2 (Toll Booth Queue)

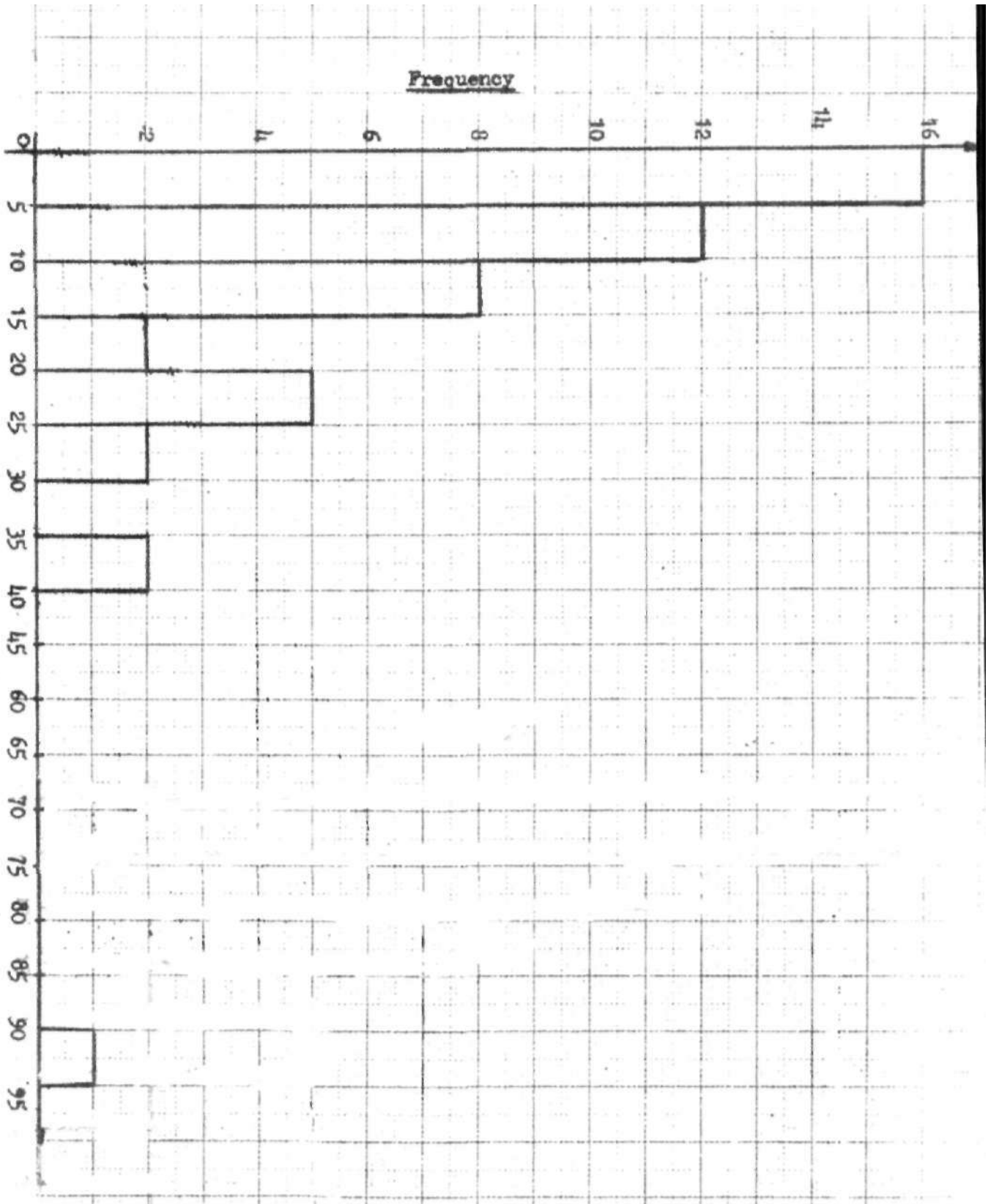


Fig. 2.9 Histogram of service Times for lane 4 (Toll Booth queue)

A 4 210 x 297 mm
MADE IN GERM.

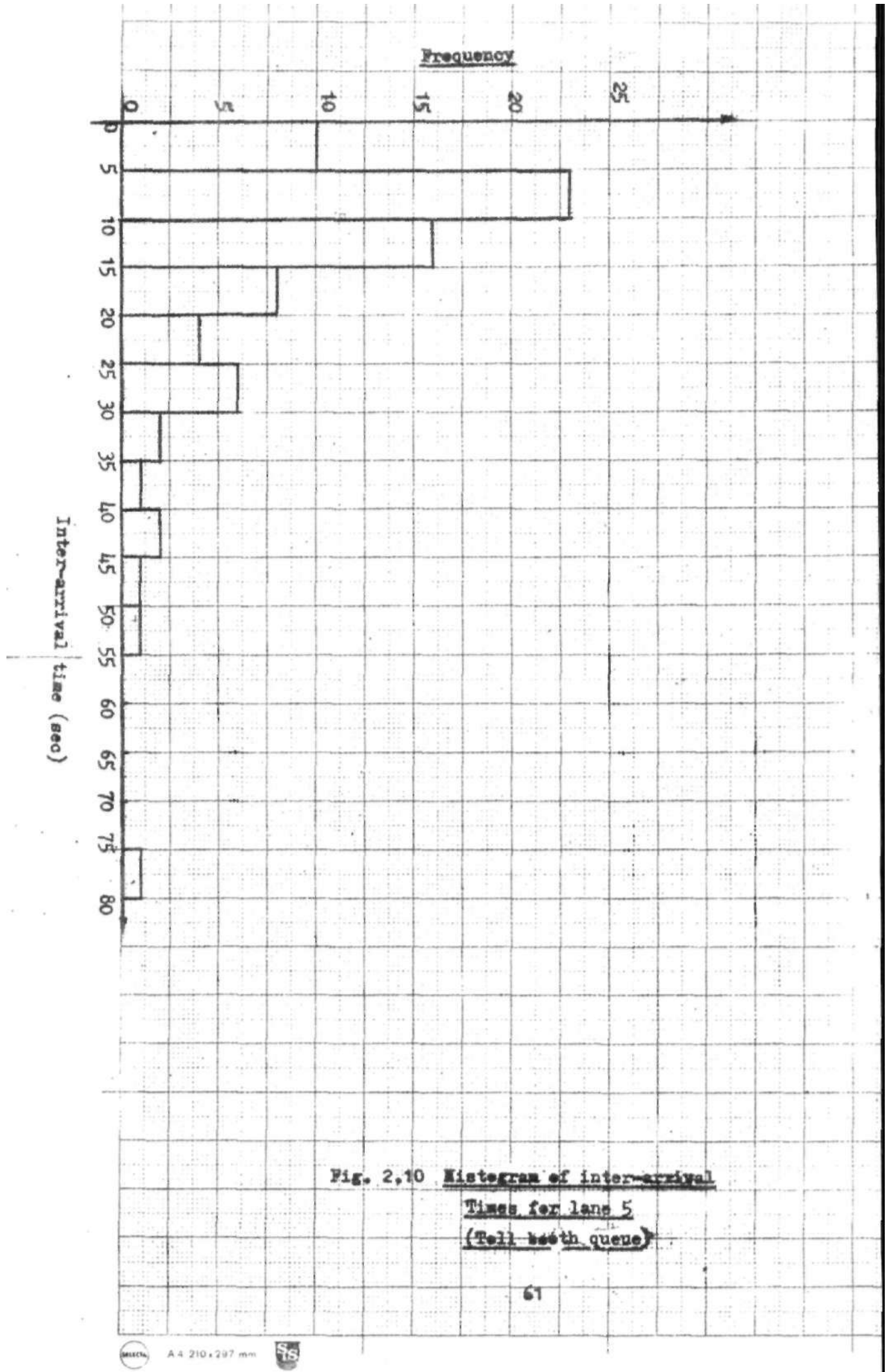


Fig. 2,10 Histogram of inter-arrival Times for lane 5 (Toll booth queue)

TABLE 2.1: DATA ANALYSIS OF SERVICE AND INTER-ARRIVAL TIMES FOR LANE B (POLICE QUEUE)

FRIDAY CASE

DATE	STARTING TIME	DURATION	MEAN INTER-ARRIVAL TIME (Sec.)				MEAN SERVICE TIME (Sec.)				COEFF. OF VARIATION (C.V.)		UTILIZATION FACTOR $\rho = t_s/t_a$
		(Mins)	T_a	T_c	T_b	T_t	T_{sa}	T_{sc}	T_{sb}	T_{st}	Inter Arr. Time	Serv. Time	
23/10/81	1100	30/30	6.0		-	-	3.8	-	-	-	.86	.65	.63
"	1100	30/30	-	7.4	-	-	-	3.8	-	-	1.1	.64	.51
"	1100	30/30	-	-	100.1	-	-	-	3.4	-	1.3	.57	.03
"	1100	30/30	-	-	-	66.2	-	-	-	4.4	1.1	.65	.07
"	1312	30/30	8.1	-	-	-	3.63	-	-	-	.96	1.2	.45
"	1425	30/27	7.2	-	-	-	3.5	-	-	-	1.1	2.2	.49
"	1425	30/27	-	8.6	-	-	-	2.95	-	-	1.4	.78	-
"	1425	30/27	-	-	98.3	-	-	-	6.9	-	1.1	3.3	-
"	1425	30/27	-	-	-	66.7	-	-	-	2.6	.79	.73	.04
"	1646	30/13	6.0	-	-	-	2.65	-	-	-	1.32	1.4	.44
20/11/81	1050	20/	7.1	-	-	-	-	-	-	-	1.1	-	-
"	1535	20/20	9.2	-	-	-	8.5	-	-	-	.88	.52	.92
8/ 1/82	1230	20/	10.1	-	-	-	-	-	-	-	.89	-	-
"	"	20/	-	10.4	-	-	-	-	-	-	.91	-	-
"	"	20/	-	-	354.7	-	-	-	-	-	1.4	-	-

TABLE 2.2.: DATA ANALYSIS OF SERVICE AND INTER-ARRIVAL TIMES FOR LANE A (POLICE QUEUE)

FRIDAY CASE

DATE	STARTING TIME	DURATION (Mins)	MEAN INTER-ARRIVAL TIME (Sec.)				MEAN SERVICE TIME (Sec.)				COEFF. OF VARIATION (C.V.)		UTILIZATION FACTOR $\rho = t_s/t_a$
			T_a	T_c	T_b	T_t	T_{sa}	T_{sc}	T_{sb}	T_{st}	Inter Arr. Time	Serv. Time	
23/10/81	1059	30/31	6.2	-	-	-	3.1	-	-	-	.77	.72	.5
"	1059	30/31	-	9.3	-	-	-	2.7	-	-	1.0	.70	.29
"	1059	30/31	-	-	40.9	-	-	-	5.5	-	1.65	.75	.13
"	1059	30/31	-	-	-	45	-	-	-	3.6	1.10	.67	.08
"	1315	30/31	7.2	-	-	-	6.0	-	-	-	1.13	.90	.83
"	1425	30/31	5.9	-	-	-	4.3	-	-	-	1.1	.64	.73
"	1425	30/31	-	8.0	-	-	-	4.2	-	-	1.2	.67	.53
"	1425	30/31	-	-	61.8	-	-	-	4.2	-	2.0	.53	.07
"	1425	30/31	-	-	-	46.5	-	-	-	4.8	1.4	.42	.10
"	1645	8/	5.9	-	-	-	-	-	-	-	.83	-	-
20/11/81	1050	20/23	8.1	-	-	-	4.4	-	-	-	.71	.84	.54
"	1535	20/20	8.9	-	-	-	16.8	-	-	-	.80	.91	1.90
8/ 1/82	1230	20/20	8.1	-	-	-	10.8	-	-	-	.96	1.3	1.33
"	1230	20/20	-	9.9	-	-	-	12.5	-	-	1.4	1.2	1.26
"	1230	20/20	-	-	69.6	-	-	-	4.25	-	.89	.52	.06
"	1230	20/20	-	-	-	62.5	-	-	-	-	1.2	-	-

SATURDAY CASE (TABLE 2.2. CONTD.)

DATE	STARTING TIME	DURATION (Mins)	MEAN INTER-ARRIVAL TIME (Sec.) T_a	T_c	T_b	T_t	MEAN SERVICE TIME (Sec) T_{sa}	T_{sc}	T_{sb}	T_{st}	COEFF. OF VARIATION (C.V.) Inter Arr. Time	Serv. Time	UTILIZATION FACTOR $\rho = T_s/T_a$
17/10/81	0958	32/7	8.7	-	-	-	-	-	-	-	.92	-	-
"	0958	32/	-	13.2	-	-	-	-	-	-	1.1	-	-
"	0958	32/	-	-	4.1	-	-	-	-	-	1.38	-	-
"	0958	32/	-	-	-	48.5	-	-	-	-	1.1	-	-
"	1058	30/31	7.6	-	-	-	23.1	-	-	-	.92	.60	3.04
"	1058	30/31	-	10.1	-	-	-	16.0	-	-	1.1	1.10	1.6
"	1058	30/31	-	-	48.3	-	-	-	28.3	-	1.5	.30	.59
"	1058	30/31	-	-	-	74.1	-	-	-	-	.77	-	-
24/10/81	1055	30/25	4.8	-	-	-	4.5	-	-	-	.78	.43	.94
"	1055	30/25	-	6.1	-	-	-	4.5	-	-	.96	.47	.74
"	1055	30/25	-	-	47.5	-	-	-	4.7	-	1.4	.19	.10
"	1055	30/25	-	-	-	48.3	-	-	-	4.3	1.2	.41	.09
"	1143	27	-	-	-	-	4.7	-	--	-	-	.73	-
21/11/81	1305	20/20	17.0	-	-	-	14.5	-	-	-	.86	.93	.85
23/ 1/82	0930	/20	-	-	-	-	2.9	-	-	-	-	.70	-
"	0930	/20	-	-	-	-	-	2.82	-	-	-	.71	-
"	0930	/20	-	-	-	-	-	-	3.8	-	-	.58	-
"	0930	/20	-	-	-	-	-	-	-	2.75	-	.62	-
"	1035	/20	-	-	-	-	8.22	-	-	-	-	1.3	-
"	1035	/20	-	-	-	-	-	3.8	-	-	-	.92	-
"	1035	/20	-	-	-	-	-	-	13.3	-	-	1.1	-

TABLE 2.3.: DATA ANALYSIS OF SERVICE AND INTER-ARRIVAL TIMES FOR TRUCK LANE 1 (TOLL BOOTH QUEUE)

DATE	START-ING TIME	DURA-TION (Mins)	MEAN INT. ARR. TIME (Sec) T_t	MEAN SERV. TIME (Sec) T_{st}	COEFF. OF VARIATION (C.V.) Int.Arr. Time	COEFF. OF VARIATION (C.V.) Serv. Time	UTILIZATION $\rho = t_s/t_a$
FRIDAY CASE							
10/ 7/81	1500	/37	-	16.7	-	.78	-
17/ 7/81	1110	30/	52.2	-	-	"	-
"	1130	30/	30.8	-	-	-	-
"	1220	30/	38.7	-	-	-	-
"	1320	30/	57.1	-	-	-	-
"	1350	30/	85.7	-	-	-	-
"	1525	30/19	60.0	15.5	-	.63	.26
20/11/81	1218	20/	51.6	-	.71	-	-
"	1453	/18	-	20.2	-	.61	-
27/11/81	1140	/20	-	19.0	-	.49	-
"	1425	20/20	33.5	25.0	.69	1.7	.75
SATURDAY CASE							
11/ 7/81	1153	30/	26	-	-	-	-
"	1250	30/	45	-	-	-	-
"	1350	30/	48.6	-	-	-	-
"	1435	30/	43.9	-	-	-	-
"	1535	30/	33.2	-	-	-	-
18/ 7/81	1155	20/	31.6	-	-	-	-
"	1243	20/	36.4	-	-	-	-
"	1332	20/	57.1	-	-	-	-
"	1402	20/	50.0	-	-	-	-
"	1420	20/	36.4	-	-	-	-
"	1500	19/	48.0	-	-	-	-
8/ 8/81	1155	28/48	55.9	12.6	.90	.90	.23
"	1250	/39	-	15.6	-	.73	-
17/10/81	1144	37/	58.9	-	.74	-	-
"	1529	/29	-	17.2	-	.58	-
21/11/81	1227	/18	-	10.0	-	.81	-
"	1345	/20	-	13.9	-	.74	-
23/ 1/82	1133	20/	52.2	-	-	-	-
"	1320	19/20	48.6	21.8	.66	.68	.45
"	1400	/20	-	26.2	-	.67	-
"	1445	20/	46.2	-	-	-	-

TABLE 2.4.: DATA ANALYSIS OF SERVICE AND INTER-ARRIVAL TIMES FOR BUS LANE 2 (TOLL BOOTH QUEUE)

FRIDAY CASE

DATE	STARTING TIME	DURATION (Mins)	MEAN INT. ARR. TIME (Sec.) T_b	MEAN SERVICE TIME (Sec) T_{sb}	COEFF. OF VARIATION (C.V)		UTILIZATION $\rho = t_s/t_a$
					Inter Arr. Time	Serv. Time	
17/ 7/81	1110	30/	75.0	-	-	-	-
"	1130	30/	100.0	-	-	-	-
"	1155	/20	-	21.1	-	.65	-
"	1220	30/	60.0	-	-	-	-
"	1320	30/	92.3	-	-	-	-
"	1350	30/	133.3	-	-	-	-
"	1553	20/18	66.7	17.2	-	.97	.26
20/11/81	1218	20/	54.8	-	.97	-	-
"	1345	/20	-	7.1	-	.71	-
"	1453	/20	-	11.2	-	.91	-
27/11/81	1140	/20	-	7.7	-	.38	-
"	1425	20/20	73.1	7.7	1.4	.77	.11

SATURDAY CASE (TABLE 2.4 CONTD.)

DATE	STARTING TIME	DURATION (Mins)	MEAN INT-ARR. TIME (Sec.) T_b	MEAN SERVICE TIME (Sec.) T_{sb}	COEFF.OF VARIATION (C.V.)		UTILIZATION $\rho = t_s/t_a$
					Inter Arr. Time	Serv. Time	
11/ 7/81	1153	30/	20.0	-	-	-	-
"	1250	30/	66.7	-	-	-	-
"	1350	30/	50.0	-	-	-	-
"	1435	30/	56.3	-	-	-	-
"	1525	30/	36.5	-	-	-	-
18/ 7/81	1155	20/	26.1	-	-	-	-
"	1220	20/20	33.3	14.4	-	.72	.43
"	1332	20/	70.6	-	-	-	-
"	1402	20/	57.1	-	-	-	-
"	1420	20/	34.3	-	-	-	-
"	1525	20/20	30.8	14.0	-	.94	.46
8/ 8/81	1155	30/	20.1	-	-	.94	-
"	1330	/31	-	14.5	-	.91	-
17/10/81	1150	38/	29.0	-	.77	-	-
"	1306	30/	28.9	-	-	-	-
"	1450	/30	-	9.5	-	.75	-
"	1529	28/28	43.7	10.0	1.1	.66	.23
21/11/81	1227	/20	-	7.7	-	1.00	-
"	1345	/20	-	8.5	-	.80	-
23/ 1/82	1133	20/	32.4	-	-	-	-
"	1320	/19	-	10.8	-	-	-
"	1400	/20	-	13.5	-	-	-
"	1445	20/	42.9	-	-	-	-

TABLE 2.5.: DATA ANALYSIS OF SERVICE AND INTER-ARRIVAL TIMES FOR BUS LANE 3 (TOLL BOOTH QUEUE)

DATE	STARTING TIME	DURATION (Mins)	MEAN INT. ARR.TIME (Sec.) T_b	MEAN SERV. TIME (Sec) T_{sb}	COEFF. OF VARIATION		UTILIZATION $\rho = \bar{t}_s / \bar{t}_a$
					Inter Arr. Time	Serv. Time	
FRIDAY CASE							
17/ 7/81	1130	30/	52.2	-	-	-	-
"	1220	30/	100.0	-	-	-	-
"	1316	30/20	46.2	13.8	-	.64	.30
"	1320	30/	109.1	-	-	-	-
"	1350	30/	85.7	-	-	-	-
"	1619	30/19	28.6	17.1	-	.74	.60
20/11/81	1245	20/	33.7	-	1.2	-	-
"	1345	/20	-	15.8	-	.73	-
"	1453	/20	-	9.0	-	.70	-
27/11/81	1140	/20	-	10.6	-	.68	-
"	1425	20/20	26.1	9.4	.90	.81	.36
SATURDAY CASE							
11/ 7/81	1153	30/	14.1	-	-	-	-
"	1250	30/	27.3	-	-	-	-
"	1350	30/	45.0	-	-	-	-
"	1435	30/	25.0	-	-	-	-
"	1535	30/	48.9	-	-	-	-
18/ 7/81	1155	30/	48.0	-	-	-	-
"	1243	30/20	26.7	14.1	-	.72	.53
"	1332	30/	37.5	-	-	-	-
"	1402	30/	30.8	-	-	-	-
"	1420	30/	44.4	-	-	-	-
"	1550	30/20	36.4	14.5	-	.72	.40
8/ 8/81	1155	28/	21.4	-	.87	-	-
"	1435	/27	-	11.9	-	.86	-
17/10/81	1150	37/	37.2	-	.85	-	-
"	1529	26/29	37.7	15.9	1.4	.75	.42
21/11/81	1155	/22	-	9.5	-	1.2	-
"	1345	/25	-	5.7	-	1.0	-
23/ 1/82	1133	20/	31.6	-	-	-	-
"	1320	18/20	34.0	7.1	.73	.83	.83
"	1400	/18	-	16.1	-	.61	-
"	1445	20/	54.5	-	-	-	-

TABLE 2.6.: DATA ANALYSIS OF SERVICE AND INTER-ARRIVAL TIMES FOR CAR LANE 4 (TOLL BOOTH QUEUE)

DATE	STARTING TIME	DURATION (Mins)	MEAN INT. ARR. TIME (Sec.) T_c	MEAN SERVICE TIME (Sec.) T_{sc}	COEFF. OF VARIATION (C.V.) Inter Arr. Time	COEFF. OF VARIATION (C.V.) Serv. Time	UTILIZATION $\rho = \bar{t}s/\bar{t}a$
FRIDAY CASE							
17/ 7/81	1110	30/	24.5	-	-	-	-
"	1130	30/	38.7	-	-	-	-
"	1220	30/	70.6	-	-	-	-
"	1320	30/	120.0	-	-	-	-
"	1340	/20	-	13.14	-	.95	-
"	1350	30/	92.3	-	-	-	-
"	1645	30/21	21.4	10.0	-	.88	.47
20/11/81	1245	20/	25.2	-	1.2	-	-
"	1345	/20	-	13.3	-	1.14	-
27/11/81	1354	20/20	13.7	8.5	.95	.50	.62
"	1207	/20	-	7.7	-	.87	-
SATURDAY CASE							
11/ 7/81	1153	30/	8.9	-	-	-	-
"	1250	30/	18.9	-	-	-	-
"	1350	30/	34.6	-	-	-	-
"	1435	30/	14.3	-	-	-	-
"	1535	30/	27.0	-	-	-	-
18/ 7/81	1155	20/	17.6	-	-	-	-
"	1307	30/20	24.5	9.4	-	.97	.38
"	1402	20/	20.7	-	-	-	-
"	1420	20/	42.9	-	-	-	-
"	1443	20/	20.3	-	-	-	-
"	1615	30/19	26.1	8.7	-	.89	.33
8/ 8/81	1105	26/	9.4	-	.75	-	-
"	1535	/30	-	7.8	-	.65	-
17/10/81	1150	23/	23.7	-	.97	-	-
"	1430	34/30	22.6	9.7	.67	.79	.43
21/11/81	1155	/20	-	21.5	-	.77	-
23/ 1/82	1133	20/	10.5	-	-	-	-
"	1250	20/20	23.1	13.7	.59	.56	.59
"	1400	/20	-	13.6	-	1.3	-
"	1445	20/	24.0	-	-	-	-

TABLE 2.7.: DATA ANALYSIS OF SERVICE AND INTER-ARRIVAL TIMES FOR CAR LANE 5 (TOLL BOOTH QUEUE)

DATE	STARTING TIME	DURATION (Mins)	MEAN INT. ARR. TIME (Sec.) T_c	MEAN SERVICE TIME (Sec.) T_{sc}	COEFF. OF VARIATION (C.V.) Inter Arr. Time	COEFF. OF VARIATION (C.V.) Serv. Time	UTILIZATION $\rho = t_s/t_a$
FRIDAY CASE							
10/ 7/81	1153	/24	-	14.6	-	.84	-
17/ 7/81	1110	30/	19.7	-	-	-	-
"	1130	30/	23.1	-	-	-	-
"	1220	30/20	17.9	10.0	-	.68	.56
"	1320	30/	20.7	-	-	-	-
"	1350	30/	22.2	-	-	-	-
"	1708	30/20	13.2	8.3	.78	.63	-
20/11/81	1315	20/	19.0	-	.84	-	-
"	1345	/20	-	9.2	-	.90	-
"	1453	/20	-	6.2	-	.76	-
27/11/81	1207	/20	-	8.0	-	.73	-
"	1320	20/20	14.6	7.5	.90	.57	.51
SATURDAY CASE							
11/7 /81	1153	30/	7.4	-	-	-	-
"	1250	30/	13.2	-	-	-	-
"	1350	30/	16.8	-	-	-	-
"	1435	30/	12.0	-	-	-	-
"	1525	30/	13.5	-	-	-	-
18/ 7/81	1155	20/	11.3	-	-	-	-
"	1243	20/	12.9	-	-	-	-
"	1332	30/20	14.1	7.9	-	.69	.56
"	1402	20/	13.0	-	-	-	-
"	1420	20/	13.6	-	-	-	-
"	1638	30/20	17.4	9.2	-	.67	.53
8/ 8/81	1105	28/	9.6	-	.57	-	-
17/10/81	1150	22/	10.6	-	1.1	-	-
"	1420	34/	28.2	-	.58	-	-
21/11/81	1155	/20	-	7.8	-	.56	-
"	1345	/30	-	15.4	-	.71	-
23/ 1/82	1133	20/	11.0	-	-	-	-
"	1250	20/20	15.9	7.4	.85	.65	.47
"	1400	/20	-	7.9	-	.55	-
"	1445	20/	15.6	-	-	-	-

TABLE 2.8.: DATA ANALYSIS OF SERVICE AND INTER-ARRIVAL TIMES FOR CAR LANE 6 (TOLL BOOTH QUEUE)

DATE	STARTING TIME	DURATION (Mins)	MEAN INT. ARR. TIME (Sec.) T_c	MEAN SERVICE TIME (Sec.) T_{sc}	COEFF. OF VARIATION (C.V.) Inter Arr Time	COEFF. OF VARIATION (C.V.) Serv. Time	UTILIZATION $\rho = t_s/t_a$
FRIDAY CASE							
17/ 7/81	1110	30/	18.2	-	-	-	-
"	1130	30/	21.4	-	-	-	-
"	1247	30/19	19.1	10.2	-	.82	.53
"	1320	30/	28.0	-	-	-	-
"	1350	30/	24.0	-	-	-	-
"	1739	30/36	17.1	11.9	-	.88	.70
20/11/81	1315	20/	20.4	-	1.3	-	-
"	1453	/20	-	9.3	-	.84	-
27/11/81	1250	20/20	21.1	7.6	1.1	.68	.36
SATURDAY CASE							
11/ 7/81	1153	30/	6.7	-	-	-	-
"	1250	30/	14.0	-	-	-	-
"	1350	30/	13.7	-	-	-	-
"	1435	30/	12.7	-	-	-	-
"	1535	30/	19.0	-	-	-	-
18/ 7/81	1155	20/	8.5	-	-	-	-
"	1243	20/	15.8	-	-	-	-
"	1332	20/	16.4	-	-	-	-
"	1402	30/20	14.8	9.4	-	.91	.63
"	1420	20/	16.4	-	-	-	-
"	1702	30/20	17.9	9.6	-	.96	.54
8/ 8/81	1105	33/	9.3	-	1.0	-	-
17/10/81	1150	33/	11.6	-	1.1	-	-
"	1430	29/30	20.7	12.8	.87	.62	.62
"	1630	/30	-	11.7	-	.77	-
21/11/81	1155	/22	-	18.2	-	1.2	-
"	1345	/30	-	12.2	-	1.0	-
23/ 1/82	1133	20/	10.2	-	-	-	-
"	1250	20/20	17.5	10.6	.81	.83	.61
"	1400	/20	-	10.4	-	.68	-
"	1445	20/	22.6	-	-	-	-

TABLE 2.9a.: ANALYSIS OF VARIANCE TABLE

DAY OF WEEK: FRIDAYS

TYPE: ARRIVAL BY LANE/ARRIVAL BY TIME

LOCATION: POLICE QUEUE

(INTER-ARRIVAL TIME IN SECONDS)

LANE \ TIME	(23/10/81) 10 - 11 a.m.	(23/10/81) 1 - 2 p.m.	(23/10/81) 2 - 3 p.m.	(23/10/81) 4 - 5 p.m.	(20/11/81) 10 - 11 a.m.	(20/11/81) 3 - 4 p.m.	(8/1/82) 12 - 1 p.m.
A	6.2	7.2	5.9	5.9	8.1	8.9	8.1
B	6.0	8.1	7.2	6.0	7.1	9.2	10.1

TABLE 2.9b.:

TYPE: SERVICE BY LANE/SERVICE BY TIME

(SERVICE TIME IN SECONDS)

LANE \ TIME	(23/10/81) 10 - 11 a.m.	(23/10/81) 1 - 2 p.m.	(23/10/81) 2 - 3 p.m.	(23/20/81) 4 - 5 p.m.	(20/11/81) 10 - 11 a.m.	(20/11/81) 3 - 4 p.m.	(8/1/82) 12 - 1 p.m.
A	3.1	6.0	4.3	-	4.4	16.8	10.8
B	3.8	3.6	3.5	2.7	-	8.5	-

TABLE 2.10a.: ANOVA TABLE

DAY OF WEEK: SATURDAYS

TYPE: ARRIVAL BY LANE/ARRIVAL BY TIME

LOCATION: POLICE QUEUE

(INTER-ARRIVAL TIME IN SECONDS)

LANE \ TIME	(17/10/81) 9 - 10 a.m.	(24/10/81) 10 - 11 a.m.	(24/10/81) 11 - 12 p.m.	(21/11/81) 1 - 2 p.m.	(23/1/82) 9 - 10 a.m.
A	7.6	4.8	-	17.0	-
B	7.0	6.8	6.3	8.2	4.6

TABLE 2.10b.:

TYPE: SERVICE BY LANE/SERVICE BY TIME

(SERVICE TIME IN SECONDS)

LANE \ TIME	(17/10/81) 9 - 10 a.m.	(24/10/81) 10 - 11 a.m.	(24/10/81) 11 - 12 p.m.	(21/11/81) 1 - 2 p.m.	(23/1/82) 9 - 10 a.m.	(23/1/81) 10 - 11 a.m.
A	23.1	4.5	4.7	14.5	2.9	8.2
B	9.4	2.71	3.5	9.0	12.6	-

TABLE 2.11a.: ANOVA TABLE

DAY OF WEEK: FRIDAY & SATURDAY

TYPE: ARRIVAL BY LANE/ARRIVAL BY TIME

LOCATION: POLICE QUEUE

(INTER-ARRIVAL TIME IN SECONDS)

LANE \ TIME	(23/10/81) 10 - 11 a.m	(23/10/81) 1 - 2 p.m.	(23/10/81) 2 - 3 p.m.	(23/10/81) 4 - 5 p.m.	(20/11/81) 10 - 11 a.m	(20/11/81) 3 - 4 p.m.	(8/1/82) 12 - 1 p.m.	(17/10/81) 9 - 10 a.m	(24/10/82) 10 - 11 a.m.	(21/11/81) 1 - 2 p.m.	(24/11/81) 11 - 12 p.m	(23/1/82) 9 - 10 a.m
A	6.2	7.2	5.9	5.9	8.1	8.9	8.1	7.6	4.8	17.0	-	-
B	6.0	8.1	7.2	6.0	7.1	9.2	10.1	7.0	6.8	8.2	6.3	4.6

TABLE 2.11b.: ← Fridays | Saturdays →

TYPE: SERVICE BY LANE/SERVICE BY TIME

(SERVICE TIME IN SECONDS)

LANE \ TIME	(23/10/81) 10 - 11 a.m	(23/10/81) 1 - 2 p.m.	(23/10/81) 2 - 3 p.m.	(23/10/81) 4 - 5 p.m.	(20/11/21) 10 -111 a.m	(20/11/81) 3 - 4 p.m.	(8/1/82) 12 - 1 p.m.	(17/10/81) 9 - 10 a.m	(24/10/82) 10 - 11 a.m	(21/11/81) 1 - 2 p.m.	(24/11/81) 11 - 12 p.m	(23/1/82) 9 - 10 a.m	(23/1/82) 10 - 11 a.m
A	3.1	6.0	4.3	-	4.4	16.8	10.8	23.1	4.5	14.5	4.7	2.4	8.2
B	3.8	3.6	3.5	2.7	-	8.5	-	9.4	2.7	9.0	3.5	12.6	-

 ← Fridays | Saturdays →

TABLE 2.12a.: ANOVA TABLE

DAY OF WEEK: FRIDAY

TYPE: ARRIVAL BY LANE/ARRIVAL BY TIME

LOCATION: TOLL BOOTH QUEUE

(INTER-ARRIVAL TIME IN SECONDS)

LANE \ TIME	(17/7/81) 11 - 12 p.m.	(17/7/81) 12 - 1 p.m.	(17/7/81) 1 - 2 p.m.	(17/7/81) 3 - 4 p.m.	(17/7/81) 4 - 5 p.m.	(17/7/81) 5 - 6 p.m.	(20/11/81) 12 - 1 p.m.	(20/11/81) 1 - 2 p.m.	(20/11/81) 2 - 3 p.m.	(27/11/81) 11 - 12 p.m.	(27/11/81) 2 - 3 p.m.	(27/11/81) 1 - 2 p.m.
1	30.8	38.7	57.0	60.0	-	-	51.6	-	-	-	33.5	-
2	87.5	60.0	113.0	66.7	-	-	54.8	-	-	-	73.1	-
3	52.2	100.0	46.2	-	28.6	-	33.7	-	-	-	26.1	-
4.	31.5	70.6	106.0	-	21.4	-	25.2	-	-	-	13.7	-
5.	21.4	17.9	21.4	-	13.2	-	-	19.2	-	-	-	14.6
6.	19.8	19.1	26.0	-	-	17.1	-	20.4	-	-	-	21.1

TABLE 2.12b.:

TYPE: **SERVICE BY** LANE/SERVICE BY TIME

(SERVICE TIME IN SECONDS)

LANE \ TIME	(17/7/81) 11 - 12 p.m.	(17/7/81) 12 - 1 p.m.	(17/7/81) 1 - 2 p.m.	(17/7/81) 3 - 4 p.m.	(17/7/81) 4 - 5 p.m.	(17/7/81) 5 - 6 p.m.	(20/11/81) 12 - 1 p.m.	(20/11/81) 1 - 2 p.m.	(20/11/81) 2 - 3 p.m.	(27/11/81) 11 - 12 p.m.	(27/11/81) 2 - 3 p.m.	(27/11/81) 1 - 2 p.m.
1	-	-	-	15.5	-	-	-	-	20.2	19.0	25.0	-
2	21.1	-	-	17.2	-	-	-	7.1	11.2	7.7	7.7	-
3.	-	-	13.8	-	17.1	-	-	15.8	9.0	10.6	9.4	-
4	-	-	13.1	-	10.0	-	-	13.3	-	7.7	8.5	-
5	-	10.0	-	-	8.3	-	-	9.2	6.2	8.0	-	7.5
6	-	10.2	-	-	-	11.9	-	-	9.3	-	-	7.6

TABLE 2.13a.: ANOVA TABLE

DAY OF WEEK: SATURDAY

TYPE: ARRIVAL BY LANE/ARRIVAL BY TIME

LOCATION: TOLL BOOTH QUEUE

(INTER-ARRIVAL TIME IN SECONDS)

LANE	TIME	11/7/81	18/7/81	8/8/81	17/10/81	23/1/82
1	11-12	-	-	-	-	52.2
	12-1	26.0	35.0	55.9	58.9	-
	1-2	45.0	57.1	-	-	48.6
	2-3	48.6	43.2	-	-	46.2
	3-4	43.9	48.0	-	-	-
	4-5	33.2	-	-	-	-
2	11-12	-	-	-	-	32.4
	12-1	20.0	29.7	20.1	29.0	-
	1-2	66.7	70.6	-	28.9	-
	2-3	53.2	45.7	-	-	42.9
	3-4	43.4	30.8	-	43.7	-
	4-5	33.2	-	-	-	-
3	11-12	-	-	-	-	31.6
	12-1	37.4	21.4	-	37.2	-
	1-2	37.5	-	-	-	34.0
	2-3	37.6	-	-	-	-
	3-4	-	-	-	37.7	54.5
	4-5	36.4	-	-	-	-
4	11-12	8.9	17.6	9.4	-	10.5
	12-1	-	-	-	23.7	-
	1-2	18.9	24.5	-	-	23.1
	2-3	24.5	20.3	-	22.6	24.0
	3-4	27.0	-	-	-	-
	4-5	-	26.1	-	-	-
5	11-12	7.4	-	9.6	-	11.0
	12-1	-	12.1	-	10.6	-
	1-2	13.2	14.1	-	-	15.9
	2-3	14.4	13.3	-	28.2	15.6
	3-4	13.5	-	-	-	-
	4-5	-	17.4	-	-	-
6	11-12	6.7	-	9.3	-	10.2
	12-1	-	12.5	-	11.6	-
	1-2	14.0	16.4	-	-	17.5
	2-3	13.2	15.3	-	20.7	22.6
	3-4	-	-	-	-	-
	4-5	-	-	-	-	-
	5-6	-	17.9	-	-	-

TABLE 2.13b.:

TYPE: SERVICE BY LANE/SERVICE BY TIME

(SERVICE TIME IN SECONDS)

LANE \ TIME	18/7/81						8/8/81				17/10/81			21/11/81		23/1/82	
	12-1	1-2	2-3	3-4	4-5	5-6	12-1	1-2	2-3	3-4	2-3	3-4	4-5	12-1	1-2	1-2	2-3
1	14.3	-	-	18.0	-	-	12.6	15.6	-	-	-	17.2	-	10.0	13.9	21.8	26.2
2	14.4	-	-	14.0	-	-	-	14.5	-	-	-	10.0	9.5	7.7	8.5	10.8	13.5
3	14.1	-	-	-	14.5	-	-	-	11.9	-	-	15.9	-	9.5	5.7	7.1	16.1
4	-	9.4	-	-	8.7	-	-	-	-	7.8	9.7	-	-	21.5	-	13.7	13.6
5	-	7.9	-	-	9.2	-	-	-	-	-	-	-	-	7.8	15.4	7.4	7.9
6	-	-	9.4	-	-	9.6	-	-	-	-	12.8	-	11.7	18.2	12.2	10.6	10.4

TABLE 2.14a.: ANOVA TABLE

DAY OF WEEK: FRIDAY

TYPE: ARRIVAL BY LANE/ARRIVAL BY TIME

LOCATION: TOLL BOOTH QUEUE FOR CARS

(INTER-ARRIVAL TIME IN SECONDS)

PERIOD / LANE	I 10-12p.m.	II 12-2 p.m.	III 2-4 p.m.	IV 4-6 p.m.
4	31.5	88.3	21.4	-
5	21.4	19.7	13.2	-
6	19.8	22.6	-	17.1

TABLE 2.14b.:

TYPE: SERVICE BY LANE/SERVICE BY TIME

(SERVICE TIME IN SECONDS)

PERIOD / LANE	I 10-12p.m.	II 12-2 p.m.	III 2-4 p.m.	IV 4-6 p.m.
4	-	13.1	10.0	-
5	-	10.0	8.3	-
6	-	10.2	-	11.9

TABLE 2.15a.: ANOVA TABLE

DAY OF WEEK: SATURDAY

TYPE: ARRIVAL BY LANE/ARRIVAL BY TIME

LOCATION: TOLL BOOTH QUEUE FOR CARS

(INTER-ARRIVAL TIME IN SECONDS)

PERIOD / LANE	I	II	III	IV
4	17.6	24.5	20.3	26.1
5	-	13.1	13.3	17.4
6	14.5	15.3	-	17.9

TABLE 2.15b.:

TYPE: SERVICE BY LANE/SERVICE BY TIME

(SERVICE TIME IN SECONDS)

PERIOD / LANE	I 10-12 p.m.	II 12-2 p.m.	III 2-4 p.m.	IV 4-6 p.m.
4	-	9.4	-	8.7
5	-	7.9	-	9.2
6	-	9.4	-	9.6

TABLE 2.16a.: ANOVA TABLE

DAY OF WEEK: SATURDAY & FRIDAY

TYPE: ARRIVAL BY DAY

LOCATION: TOLL BOOTH QUEUE FOR CARS

(INTER-ARRIVAL TIME IN SECONDS)

LANE / DAY	4	5	6
$Friday_1$ (July)	21.4	13.2	17.1
$Friday_2$ (Nov.)	25.2	19.0	20.4
$Friday_3$ (Nov.)	13.7	14.6	21.1
$Saturday_1$ (July)	24.5	14.1	15.9
$Saturday_2$ (Oct.)	22.2	28.2	20.7
$Saturday_3$ (Jan.)	24.0	15.8	17.5

TABLE 2.16b.:

TYPE: SERVICE BY DAY

(SERVICE TIME IN SECONDS)

LANE / DAY	4	5	6
$Friday_1$ (July)	10.0	8.3	11.9
$Friday_2$ (Nov.)	13.3	9.2	9.3
$Friday_3$ (Nov.)	8.5	7.5	7.6
$Saturday_1$ (July)	9.4	7.9	9.4
$Saturday_2$ (Oct.)	9.7	-	12.8
$Saturday_3$ (Jan.)	13.6	7.7	10.6

TABLE 2.17a: ANOVA TABLE

DAY OF WEEK: FRIDAYS & SATURDAYS

TYPE: ARRIVAL BY DAY

LOCATION: TOLL BOOTH QUEUE FOR BUSES

(INTER-ARRIVAL TIME IN SECONDS)

DAY \ LINE	2	3
$Friday_1$ (July)	66.7	46.2
$Friday_2$ (Nov.)	54.8	33.7
$Friday_3$ (Nov.)	73.1	26.1
$Saturday_1$ (July)	29.7	37.4
$Saturday_2$ (Oct.)	43.7	37.7
$Saturday_3$ (Jan.)	42.9	54.4

TABLE 2.17b.:

TYPE: SERVICE BY DAY

(SERVICE TIME IN SECONDS)

DAY \ LINE	2	3
$Friday_1$ (July)	17.2	13.8
$Friday_2$ (Nov.)	7.1	15.8
$Friday_3$ (Nov.)	7.7	9.4
$Saturday_1$ (July)	14.4	14.1
$Saturday_2$ (Oct.)	10.0	15.9
$Saturday_3$ (Jan.)	13.5	16.1

TABLE 2.18a.: ANOVA TABLE

DAY OF WEEK: FRIDAY

TYPE: ARRIVAL BY LANE/ARRIVAL BY TIME

LOCATION: POLICE QUEUE

(INTER-ARRIVAL TIME IN SECONDS)

PERIOD / LANE	I 10-12 pm	II 12-2 p.m.	III 2-4 p.m.	IV 4-6 p.m.
A	6.2	7.2	5.9	5.9
B	6.0	8.1	7.2	6.0

TABLE 2.18b.:

TYPE: SERVICE BY LANE/SERVICE BY TIME

PERIOD / LANE	I 10-12 pm	II 12-2 p.m.	III 2-4 p.m.	IV 4-6 p.m.
A	3.1	6.0	4.3	-
B	3.8	3.6	3.5	2.7

TABLE 2.19a.: ANOVA TABLE

DAY OF WEEK: FRIDAYS & SATURDAYS

TYPE: ARRIVAL BY DAY

LOCATION: POLICE QUEUE

(INTER ARRIVAL TIME IN SECONDS)

LANE / DAY	A	B
$Friday_1$ (Oct.)	6.2	6.0
$Friday_2$ (Nov.)	8.1	7.1
$Friday_3$ (Jan.)	8.1	10.1
$Saturday_1$ (Oct.)	7.6	7.0
$Saturday_2$ (Nov.)	17.0	8.2
$Saturday_3$ (Jan.)	-	4.6

TABLE 2.19b.;

TYPE: SERVICE BY DAY

(SERVICE TIME IN SECONDS)

DAY \ LANE	A	B
$Friday_1$ (Oct.)	3.1	3.8
$Friday_2$ (Nov.)	4.4	-
$Friday_3$ (Jan)	10.8	-
$Saturday_1$ (Oct.)	23.1	9.4
$Saturday_2$ (Nov.)	14.5	9.0
$Saturday_3$ (Jan.)	2.9	12.6

Also from corresponding sets of mean inter-arrival and service times, the traffic intensity or utilization factor for each channel in both stages of the queueing system was determined. Tables 2.1 to 2.8 also show these calculated values. Compared with acceptable ranges for the exponential, the range of values (in these tables) of the coefficient of variation for Fridays and Saturdays seem to further suggest the exponential distribution as a good approximation for the inter-arrival and service time distributions. More rigorous statistical techniques are therefore explored in the next section to validate this exponential assumption.

2.5.3.2. Statistical Tests

Two different types of tests were carried out here with the purpose of establishing more properties of the system, the possibility or otherwise of aggregating data

and a final statistical confirmation or otherwise of the exponential assumption. The techniques employed are as follows:

1. the one-way analysis of variance (ANOVA) tests using the relevant SPSS options;
2. curve fitting and hypothesis testing.

The ANOVA tests were carried out before the curve fitting tests so as to make use of the ANOVA results in reducing the number of data sets necessary for the curve fitting tests.

2.5.3.2.1. The One-Way Analysis of Variance Tests

The basic theory of the one-way analysis of variance is as given in Freund and Walpole (1980). This analysis allows users to statistically test whether the means of subsamples into which the sample data are broken are significantly different from each other. In summary, it implies a generalization of the hypothesis tests concerning differences between two means and decides whether observed differences among more than two sample means can be attributed to chance or whether there are real differences among the means of the populations sampled. The different populations are assumed to have different means but a common variance and more formally put, users are testing the null hypothesis (H_o): $\mu_1 = \mu_2 = \mu_3 = \ldots \mu_K = \mu$ against the alternative that they are not equal; where μ_i

is the mean of the i^{th} population. If the means are not found to be significantly different, users cannot reject the hypothesis that the true subpopulation means are equal and that the deviations which occur are the result of sampling error. Conversely, if it is found that the means are significantly different, users can reject the hypothesis that the true subpopulation means are equal.

For the purpose of this research, the values of the mean inter-arrival times and mean service times determined in the previous section and shown in Tables 2.1 to 2.8 were used for the tests to be described below. In so doing, it has been assumed that the arrival and service rates within any given hour are constant, the minute to minute variations being ignored. For each of the tests described below, two possible approaches were adopted each corresponding to a set of ANOVA tables that were the inputs to the computer. In the first approach or first set of data from the tables, the interval of one hour was selected as the unit within which variation can be neglected. In the second approach using another set of data from the tables, the period of study is divided into four subperiods with due consideration to the traffic flow pattern of Figures 2.1 and 2.2. The basic assumption here is that there is not much variation within these periods.

The tests carried out were as follows:

1. For the police queue:

(a) Arrival by lane for Fridays and Saturdays

(b) Service by lane for Fridays and Saturdays.

These two tests sought to establish whether there is a significant difference between the means of arrival rates or service rates on both police lanes of the expressway. This was done for Fridays and Saturdays taken separately and when both data from Friday and Saturday are considered.

(c) Arrival by time for Fridays and Saturdays

(d) Service by time for Fridays and Saturdays.

These two tests sought to establish the stationarity or otherwise of the arrival and input distributions. It is to find out if there is any significant difference between the tested values from time to time or if the existing difference could be taken to be statistically insignificant. This is carried out for Fridays and Saturdays separately and also when data from both days are combined.

(e) Arrival by day

(f) Service by day.

These two tests sought to further confirm if Saturday and Friday data can be taken to be the same or significantly different from each other. If there is no significant difference, then data can be aggregated.

2. For the toll booth queues, the test types are exactly the same as the above. The only difference is that first, the six lanes are taken together instead of two lanes as

discussed for the police queues. Also the same number of tests are repeated for the case of three lanes when only the car lanes are considered and for the case of two lanes when only the bus lanes are considered.

Some of the list of ANOVA tables used for this analysis are given in Tables 2.9 to 2.19 while the full list of results from this test are tabulated and shown in Table 2.25.

2.5.3.2.2. Curve Fitting Using the K-S Technique

Based on the results of the ANOVA tests, some sets of the data were selected for statistical validation of the negative exponential in such a way as to avoid duplication of results. Thus for lane combinations where ANOVA has shown that there is not much risk in assuming aggregation of data, or mean service or arrival rates, it was needless duplicating the curve fitting experiment once results are known for one of such identical situations. Altogether, twenty-three sets of data were analyzed and the results are shown in Table 2.21 for all the cases. The results indicate a positive confirmation for the negative exponential although rather weakly in some cases.

The K-S technique known fully as the Kolmogorov-Smirnov goodness-of-fit test has been selected in place of the more conventional and traditional Chi-Square test because of the following arguments.

(a) The Chi-Square test is versatile and can be applied simply to test any distributional assumption without having to know the values of the distribution parameters. However, its major drawbacks are its lack of sensitivity in detecting inadequate models when few observations are available and the frequent need to arrange the data into arbitrary intervals, the magnitude of which can affect the outcome. In general, therefore, it is required that the number of classes be equal to or greater than five and that the calculated theoretical frequency value also be equal to or greater than five if this technique is to be used.

(b) The K-S test on the other hand, is another widely used test and its advantage over the Chi-Square (χ^2) test is that it is not necessary to divide the data into intervals. This is because the values are used for the test as they are observed and hence the problems associated with the Chi-Square approximation for small class numbers or small calculated theoretical frequencies would not appear with K-S test.

(c) Although a controversy exists as to which is more powerful of the K-S and χ^2 tests, there is the general feeling that the Kolmogorov test is probably more powerful in most situations [Conover, (1971)].

In consideration of all the above and the fact that a lot of the data collected for this research could be called small sample sizes, there was no hesitation in adopting the K-S technique.

The basic procedure for the K-S test is as given by Ang and Tang (1975) and because of the large set of data to be analyzed, a computer programme was written to evaluate the K-S statistic for each data point. For this test, the raw data has been transformed into a more useful form using the frequency option of the Statistical Package for Social Sciences Computer Programmes. The basic logic of the K-S technique is as shown in the flowchart of Appendix 4b and the computer programme is also attached

2.5.4. Results and Conclusions on Data Analysis

Although other distributions - notably the Erlang and the lognormal - can be fitted to the observed sets of data, the approximate tests using graphical methods and the coefficient of variation criteria indicate that the negative exponential distribution is acceptable for further confirmation using statistical tests. The negative exponential distribution might not be the best fitting of all the possible distribution models but the mathematical tractability of the system queueing models resulting from it made its case a very strong one, at least for approximate solutions.

The traffic flow pattern shows two defined peaks but the ANOVA analysis carried out suggests that within the study period the variations of mean arrival and service rates for individual lanes and also between lanes can be considered to be statistically insignificant for both the police and the toll booth queues. Specifically, the

following are the results of the ANOVA analysis as can be seen from Tables 2.20 a and 2.20 b.

(a) Except in a few cases, it was found that the variations in mean input and output are not significant and that the assumption of stationarity of the distribution patterns for both arrivals and departures is acceptable at 5 per cent level of significance in both cases and for both stages of the queueing system.

(b) For the police queue, there is no significant variation in the arrival and service rates for both lanes and for both Fridays and Saturdays. Distributions are also time independent. This then means that data can be aggregated for Fridays and Saturdays and also for the expressway lanes A and B designated in this study as the police lanes.

(c) The assumption of equal mean rates of arrival on the six toll booth lanes when taken together cannot be accepted even at one per cent level of significance. Similarly, the equality of mean service rates on all channels taken together was rejected. However, when the lanes for the different vehicle classes were considered separately, it was concluded that the equality of mean arrival or service rates for the bus channels were statistically acceptable at 5 per cent level of significance. In the case of the car lanes, however, the equality of mean service rates for channels was acceptable while the results show that the mean rates of arrival into each of the lanes could be regarded as different for all practical purposes. As regards the stationarity of the arrival

and service mechanisms, the toll booth operation on all the lanes indicate positively that stationarity conditions exist throughout.

(d) The results in Table 2.20b are based on the assumption that within each of the four sub-periods into which the study period has been partitioned, no variations in arrival or service rates exist. It is therefore necessary to note that for all the cases examined, the results tend to confirm the results earlier based on the assumption of non-variation within one hour periods for input and outputs. In particular the non-significant difference between Friday and Saturday data are established more firmly following the results of the 'Arrival by day' and 'Service by day' analysis. Similarly the non-variation between certain months is established.

Table 2.21 shows the results of the twenty-three sets of data analyzed for exponential validation via the K-S test technique. Based on the ANOVA test, the K-S test has been missed out on those lanes where results for lanes that are statistically similar to it exist. The tests show that for all practical purposes, all arrivals and services on each of the lanes within the system have Markovian characteristics which make the job of solving the queueing problem much easier. It was not necessary to analyze all aspects of the data concerning any lane following the results of the ANOVA tests cited earlier. No analysis has therefore been carried out for lane 3 since it is in all respects statistically the same as lane 2. Similarly only the inter-arrival time data

for lanes 5 and 6 were analyzed, the service time distribution being the same as that of lane 4.

2.5.5. Selecting And Justifying The Model For The Service And Arrival Patterns

In finally selecting and justifying the exponential assumption for the queueing system of the Lagos-Ibadan expressway, it is appreciated that:

(i) the system involves a serial queueing process which becomes less mathematically attractive even for moderate system complexities;

(ii) a simulation approach is not intended for this study;

(iii) most of the models contained in queueing literature to be examined for the selection of an adequate queueing model for the system are based on the exponential assumption;

(iv) an operational solution is what is being sought for the system;

(v) other options such as probing for the best fitting curve or the use of the observed distribution could be pursued;

(vi) it is not the purpose of this research to probe for the best fitting theoretical curve or to use an observed distribution that would complicate the system queueing model when other easier to manage and approximate fitting distributions exist;

TABLE 2.20a.: ANOVA RESULTS TABLE

PART OF SYSTEM	DAY OF WEEK	TYPE OF TEST	LEVEL OF SIGNIFI-CANCE	COMMENTS
2-Lane Express-way (Police Queue)	Friday	Arrival By Lane	.290	Accept null hypo-thesis at 5% le-vel that there is no significant difference bet-ween arrivals on both lanes.
	Saturday	"	.337	
	Fri.&Sat.	"	.674	
2-Lane Express-way (Police Queue)	Friday	Arrival By Time	.04	Accept at 1% level that input is time homoge-neous.
	Saturday	"	.415	
	Fri.&Sat.	"	.160	
2-Lane Express-way (Police Queue)	Friday	Service By Lane	.249	Accept at 5% level that ser-vice rates are same for both lanes.
	Saturday	"	.585	
	Fri.&Sat.	"	.225	
2-Lane Express-way (Police Queue)	Friday	Service by Time	.184	Accept at 5% level that ser-vice rates on both lanes are time homogeneous
	Saturday	"	.348	
	Fri.&Sat.	"	.250	
6-Lane Toll Booth Queue	Friday	Arrival By Lane	.001	Reject null hypo-thesis. Arrival rates differ for the six lanes
	Saturday	"	.001	
	Fri.& Sat.	"	.001	
6-Lane Toll Booth Queue	Friday	Arrival By Time	.331	Accept null hypo-thesis at 5%. This implies time homo-geneous inputs or stationarity of arrival patterns.
	Saturday	"	.650	
	Fri.&Sat.	"	.181	

TABLE 2.20a.:(contd.)

6-Lane Toll Booth Queue	Friday Saturday Fri.&Sat.	Service By Lane " "	.001 .028 .001	Reject null hypothesis that mean service rates are same for six channels.
6-Lane Toll Booth Queue	Friday Saturday Fri.&Sat.	Service By Time " "	.677 .939 .963	Accept at 5% level that mean service rate is the same at all times. This implies homogeneous outputs or stationarity.
3-Lane Toll Booth Queue For Cars	Friday Saturday Fri.&Sat.	Arrival By Lane " "	.08 .009 .001	Reject null hypothesis that mean rates of arrival are same for all the car lanes.
3-Lane Toll Booth Queue For Cars	Friday Saturday Fri.&Sat.	Service By Lane " "	.172 .390 .051	Accept at 5% level that the mean rates of service are the same for all car channels.
2-Lane Toll Booth Queue For Buses	Friday Saturday Fri.&Sat.	Arr./Ser. By Lane " "	.077/.828 .400/.849 .045/.721	Accept at 5% level that the mean rates of arrival and Service are same for both bus lanes.

TABLE 2.20b.: ANOVA RESULTS TABLE
(BASED ON DEFINED PERIODS)

PART OF SYSTEM	DAY OF WEEK	TYPE OF TEST	LEVEL OF SIGNIFI-CANCE	COMMENTS
2-Lane Express-way (Police Queue)	Fri.&Sat.	Arrival By Day	.320	Accept null hypo-thesis at 5% level that there is no significant difference bet-ween Friday and Saturday data.
2-Lane Express-way (Police Queue)	Fri.&Sat.	Service By Day	.063	Accept at 5% level that the mean rates of service are the same for Fridays and Saturdays.
3-Lane Toll Booth Queue For Cars	Fri.&Sat.	Arrival By Day	.345	Accept at 5% that arrivals at each of the car lanes is not signifi-cantly different for Frid. & Sat.
3-Lane Toll Booth Queue For Cars	Fri.&Sat.	Service By Day	.386	Accept at 5% level that ser-vice is not toc different for the car channels on Fridays & Sat.
2-Lane Toll Booth Queue For Buses	Fri.&Sat.	Arrival By Day	.796	Accept at 5% level that arri-val is same for Fridays and Saturdays.

TABLE 2.20b.:(contd.)

2-Lane Toll Booth Queue For Buses	Fri.&Sat.	Service By Day	.405	Accept at 5% level that service rates are significantly same for both days.
2-Lane Express-way (Police Queue)	Friday Saturday	Arrival By Lane "	.413 .120	Accept at 5% level that arrival is same for both lanes and for both days
2-Lane Express-way (Police Queue)	Friday Saturday	Arrival By Time "	.119 .266	Accept at 5% level that there is stationarity input or arrivals for the 4 periods considered.
2-Lane Express-way (Police Queue)	Friday Saturday	Service By Lane	.219 .161	Accept null hypo. at 5% that service rates are same for both lanes for both days.
2-Lane Express-way (Police Queue)	Friday Saturday	Service By Time	.499 .434	Stationarity of service rates accepted for both lanes and for both days.
2-Lane Toll Booth Queue For Buses	Friday Saturday	Arr./Ser. By Lane "	.117/.219 .405/.161	Accept null hypo. that mean rates of arrival and service are same for both lanes.
3-Lane Toll Booth Queue For Cars	Friday Saturday	Arr./Ser. By Lane "	.246/.403 .026/.409	Accept null hypo. that arrivals & service are not significantly different for all lanes.
3-Lane Toll Booth Queue For Cars	Friday Saturday	Arrival By Time "	.646 .766	Accept at 5% that mean arrivals are not different for the different periods.

TABLE 2.21: K-S GOODNESS-OF-FIT TEST RESULTS

PART OF SYSTEM	TEST FOR INTER-ARRIVAL TIME DISTRIBUTION	TEST FOR SERVICE TIME - DISTRIBUTION	COMMENTS
	Friday Data (23/10/81)		Null hypothesis accepted at 1% level.
Lane A (Police Queue)	-	Friday Data (24/10/81)	Null hypothesis just rejected at 1% level
		Saturday Data (23/1/82	Null hypothesis accepted at 5% level.
Lane B (Police Queue	Friday (8/1/82)	-	Null hypothesis okay at 5% level.
	Saturday (24/10/81)		Null hypothesis accepted at 1% level.
	Friday (23/10/81)		Null hypothesis just rejected at 1% level.
Lane 1 (For Trucks)	Saturday (8/8/81) Friday (27/11/81)		Accept null hypothesis at 5% level. Null hypothesis okay at 5% level.
		Saturday (17/10/81)	Null hypothesis accepted at 5% level.
		Friday (27/11/81)	Null hypothesis accepted at 1% level.
Lane 2 (For Buses)	Saturday (17/10/81) Friday (27/11/81)		Null hypothesis just rejected at 1% level. Null hypothesis accepted at 5% level.
		Saturday (17/10/81)	Null hypothesis accepted at 5% level.
		Friday (27/11/81)	Null hypothesis accepted at 1% level.
Lane 3 (For Buses)	=	=	Same as for Lane 2.
Lane 4 (For Cars)	Friday (20/11/81)		Null hypothesis accepted at 5% level.
		Friday (20/11/81	Null hypothesis accepted at 5% level.
		Saturday (23/1/82)	Null hypothesis accepted at 5% level.
		Saturday (23/1/82)	Null hypothesis accepted at 1% level.
Lanes 5 and 6 (For cars)	Friday (27/11/81)& Saturday (23/1/82)		Null hypothesis accepted at 5% level for both lanes and for both days.

(vii) unless the distribution of the variable is either exponential or deterministic, it might not be possible to give a strong theoretical justification for the choice of a particular distribution.

Based on the results of the analysis done so far, the distributions seem to be far from the deterministic assumption and so considering the arguments raised here, enough justification could be shown for selecting the negative exponential to describe the arrival and service processes for the Lagos-Ibadan expressway toll booth queues.

Having selected the negative exponential to describe the arrival and service distribution patterns for this queueing system, the next step will be to search for a suitable model for analyzing the queueing system.

This very important stage in this study is carried out in the following chapter.

CHAPTER 3

LITERATURE REVIEW, SEARCH FOR, AND SELECTION OF ADEQUATE SYSTEM MODEL.

3.1 INTRODUCTION

It is the purpose of this chapter to review the existing approaches to, and models for, the analysis of queueing systems with particular reference to tandem queueing systems. In studies of the type addressed in this thesis, it is assumed in most cases that a model of the system exists in queueing literature. It is then only necessary to search for potential candidate models, evaluate these models against the background of what is required for the system, select the model that best suits and describes the system and finally modify this base model to reflect a complete reality of the system.

This procedure is adhered to for this research and following the first part which involves a review of existing models, a suggested list of potential candidate models are presented for further consideration. Model specifications are then obtained and a model selection carried out.

This review is thus intended to provide a basis from which the study framework could be developed although a continuing literature review is maintained throughout the period of this study.

This implies that an in-depth review of literature pertaining to each stage of the research was undertaken during the course of this study.

3.2 CLASSES OF QUEUEING MODELS.

In queueing literature, most of the analytical models existing can be classified into:

(a) Single server queueing models for single stage service.

(b) Multiple server queueing models for single stage service.

(c) Serial and cyclic queueing models.

Imposing limited waiting room on any of these models normally means further classification. Also queues could be classified as Poisson or non-Poisson depending on the input and service time distributions. Several other types of classification are also possible depending on the researcher and the purpose of the study.

In this literature survey, the queueing models will be discussed according to whether they are single, multiple or serial server and the other cases will be mentioned where necessary.

3.3 DISCUSSION OF EXISTING QUEUEING MODELS

3.3.1 Models of single-channel processes

This is the class of queueing models that has been most studied by researchers and writers and queueing literature continues to grow with researches that continue to explore new fields in this near saturated area.

This is because they are more fundamental and form the basic building blocks upon which the relatively more complex models are based. Most common queueing situations are however not single server queues and where they are, they are soon modified to the two-channel system when the facility gets into trouble.

Of all the single server queueing models available, the M/M/1 : (∞/FIFO) model remains the simplest and it is quite easy to treat mathematically. Hence it, alone, has attracted the greatest number of research materials. Many authors, Saaty (1961), Cox and Smith (1961), Lee (1966), Prabu (1965), Taha (1971), Giffin (1978) and a host of other notably good authors of queueing literature, present excellent approaches for the formulation and solution of this class of queueing problems. Gaver, (1953) has derived the relationship between the distribution of customer-servicing times and that of customer-waiting times for this class of models and has shown mathematically that this holds for a large class of servicing time distributions and that if the right sort of alteration is made in service-time distributions, considerable reduction in customer waiting times can be made. New researches continue to yield results for this class of models and recently Labetoulle, et. al, (1981) has analysed the M/M/1 model with feedback and obtained the distribution of inter-input, inter-output and inter-feedback times.

LIBRARY RECEIVED

Generally, because of its aid in understanding all other queueing models, the mathematics for this model is usually set out in considerable detail in most of the existing literature. Cox and Smith,(1961) and Morse, (1958) present excellent methods for solving for the transient state conditions while Giffin,(1978) suggests the use of the system balance arguements and a transition rate flow diagram to quickly construct steady state equations.

3.3.2 The multiple-channel queueing models.

These are characterized by many parallel servers attending to customers or units who are all served from a single line. The principal difference between the multiple and single server queueing processes is that the former exhibits a greater variety of queue disciplines: the possibilities of unusual customer behaviour are more often realised and except for a few recent attempts, classical multiple-channel models mainly assumed the operation of a FIFO rule.

Of all the existing multi-channel models, the simplest is the m/m/C: (∞/FIFO) and is a logical extension of the M/M/1: (∞/FIFO) model. It has attracted more research materials than any other model in this class, possibly because of its mathematical tractability, and its steady state equations are easily set out using the balance arguements of Giffin, (1978).

The main assumptions for this class of models are:

1. Customers arrive in Poisson fashion at a mean rate λ
2. There are C identical servers or channels in parallel.
3. The service-time distribution for each channel is exponential with mean $1/\mu$.
4. Servers are non-cooperative in the sense that one customer is never attended by more than a single server.
5. Arriving customers enter the first available open channel. When all channels are full, they form a single queue with unlimited waiting space from which they are served in FIFO fashion. The possibility of disintegretion - that is that the single queue would break down into several queues and the associated phenomena of jockeying, preferential queueing and so on - was not contemplated.
6. If there are two or more free channels, the arriving customer goes to any one of them and exercises no preference for any channel or group of channels.
7. As soon as a channel falls free, the customer who has been queueing longest goes into it instanteneously.

The truncated M/M/C: (L/FIFO) model follows directly from the M/M/C: (∞/FIFO) model on making the limited waiting room assumption and is by far more general and more useful.

Saaty,(1961) and Lee,(1966) show how the waiting-time distribution can be determined for this class of models.

It appears that less research has been devoted to the multiple-server queues, most likely, because of its relatively more complex structure even when some simplifications are made. Hence not too many useful models of this type have been extensively studied in queueing literature. These are fully discussed in texts on queueing literature notably Saaty,(1961) and Giffin,(1978). More research is however going on in this direction although one expects it to attract more researchers. Burke,(1956) has determined the output from this type of model when input is Poisson and service is negative exponential. Gumbel,(1959) has also considered a multiple-channel system with non-homogeneous servers and showed that an 'equivalent' system with homogeneous servers does not exist except for the sake of simplification in engineering problems. Also, Hunter,(1971) has studied the case of two queues each with a limited waiting space, an independent negative exponential service times and arrivals from a correlated bivariate Poisson distribution. He obtained steady state equations and a study of the correlation between input distributions and the correlation of the queue size distributions gave rise to some general conjectures concerning these relationships.

3.3.3 Models of serial and cyclic processes.

Many real world problems exist in which two or more queues are arranged in series with the output from one queueing process, or part of it being the input into another. When the ends are joined to form a loop, so that customers rotate through the sytem, the system is called a cyclic queueing system.

Very few models of these processes appear in operational research literature due probably to its higher order of complexity compared to other classes of queueing systems. Before a review of the existing literature for this class of models is carried out it will be necessary to discuss two important features of these systems. These are the phenomenon of blocking and the associated concept of maximum utilization and the output from Poisson queues.

(a) Blocking

The blocking of a facility is that situation that occurs when a unit has completed service in one stage but cannot proceed because the next stage queue is at its maximum length. The phenomenon is therefore a feature of serial queues which occurs only when a limited queues is allowed at the second or subsequent stages and no unit is allowed out of the system immediately after the first stage without going through the second or other stages.

For such blocked situations, the maximum possible utilization which is unity for a single stage of service will be reduced and the first stage is then said to become blocked, or not available for service on incoming units. This blocking phenomenon results in an effective service rate for the first stage that is lower than the actual service rate of the stage. The effective service rate is given by the product of the original service rate and a factor equal to the fraction of time that the first stage is unblocked. For steady-state operation therefore, the maximum allowable arrival rate is less than this effective service rate, and this explains the reduction in maximum possible utilization as earlier pointed out. The situation becomes worse if more than two stages are operated in sequence. Blocking can occur in all stages except the last stage, and the blocking in each stage will have an effect on all preceding stages. The maximum possible utilization will then decrease still further.

In the general N-stage problem, blocking occurs more frequently in the first stage than in any succeeding stage, and the maximum possible utilization for the first stage is the maximum possible utilization for the entire system. Hence the first stage maximum utilization is also the maximum utilization of the system.

(b) The output of a queue.

This is an important question to address in studying tandem queues where the distribution of output from one channel then comprises the input into the subsequent channel.

Burke (1956), Reich (1957) and others have studied this problem. Their studies have shown that the output of a steady state queue is independent of the type of service discipline used. Burke's work (1957), showed for the steady state the independence of time intervals between departures for the system M/M/C. For this system, the departures satisfy a Poisson process with the same parameter as that of the input distribution - a fact previously known on intuitive grounds based on steady-state arguments. He also showed the independence of the inter-departure interval and the state of the system at the end of the interval. The output distribution at any time t depends only on the state of the system at t and not on any previous states.

Reich, using different methods, has shown that it is not generally reasonable to expect the outputs to match the inputs even in the steady state. He also gave the partial converse to Burke's result: that for a single-channel queue, Poisson arrivals and departures imply an exponential service-time distribution or a step function at zero.

Lee (1961) reports that it has been shown rigorously that it is only when an infinite-size queue is tolerated that the Poisson result for the output is correct. Thus he gave the infinite system capacity and exponential distribution of service time as the jointly necessary and sufficient conditon for the independence of the inter-departure intervals and independence of the queue length left by a departing unit from the interval since the previous departure.

Most text-books of queueing literature notably Saaty,[39] Taha,[41] Lee,[30] Morse[35] and a host of others report on the current state of developments in the area of serial and cyclic queues and call for further research in this area. The queueing structure looks rather complicated to be very attractive to researchers. Most of the analytical work that has been done for queues with a number of facilities in series has therefore been restricted to Poisson arrivals and exponential service times with ordered queue discipline: once in the system, a unit either stays on for service through to the last phase or is able to go to the proper phase for service. Such studies have also almost entirely concentrated on deriving steady state solutions.

The only model of a serial process that has received much considerable attention by mathematicians is that comprising an $M/M/C_1$: (∞ /FIFO) feeding into $M/M/C_2$: (∞ /FIFO) system.

Such studies have concentrated on taking a specific number of stages for such studies but Jackson, (1957) has taken the study of this kind of queue to the ultimate by solving for the general case of K distinct stages. His work does not only concern the general case but has produced formulas which are eminently usable [Lee (1961)]. In practice it is generally easy to solve these type of queueing situations using Jackson's results provided the two queues can be considered independent of each other. The general and particular conditons under which the queues can be so considered stem from the works of Burke, (1956) and Reich, (1957) but in practice, they are considered independent if the space between them is large enough and this could be taken to be so if the probability of an arrival being turned away is equal to or less than one per cent.

Jackson, (1954) has methodically studied the problem of tandem queues in the steady state with Poisson assumptions (different exponential distribution for each phase), giving the probability of various numbers of items at the different phases together with the average number and the waiting-time distribution in each phase.

Jackson, (1957) permits poisson arrivals to each phase both from the system and from outside. A unit arrives at different phases with different probabilities.

The service distributions are exponential and each phase consists of several parallel channels. He derives an expression for the probability of a given number of customers being in some phase, for Poisson input, exponential service times, no leaving of system until all phases are entered beginning at the first phase.

By assuming a different exponential distribution for waiting longer than a prescribed time at each channel in a collection of channels in series, Nelson,(1958) has computed the probability of waiting longer than a prescribed time at all channels.

Debaun and Katz (1959), using Chi-square approximation to the sum of exponentials, have simplified Nelson's computations.

Hunt (1956), by assuming Poisson input, exponential service times, no leaving of system until all channels are entered, beginning at the first channel, has studied utilization (ratio of mean arrival rate to mean service rate). Four particular cases of service facilities in series are treated involving infinite storage space between stages, no storage space between stages,finite storage between stages and the case of un-paced belt-production line. In the last case the probability of completion of service in time t after the line moves is one of the results computed.

A comparison is given for considering a two-stage case and the maximum possible utilization is obtained for all cases discussed. Any or all of the stages may be in operation simultaneously. The mean numbers of units in the system as a measure of effectiveness for comparison purposes were determined. The maximum possible utilization was also obtained, in order to show the effects of inter-stage storage space. When no queues are allowed before subsequent channels, blocking was considered to have occured and effective service rate is then the product of the given service rate and the fraction of time during which the channel is unblocked.

This represents the first real attempt to model serial queueing processes with capacity restrictions and the assumptions made were:

(a) In all cases distribution of arrival to the first stage is assumed to be Poisson and service is exponential at all stages.

(b) Queues operate on a first-come, first served basis with no defections.

(c) Service commences instantaneously when a unit arrives at an empty stage, and units are transferred instantaneously from one stage to the next on completion of service.

Koenigsberg (1960) considered a closed system with two stages, the first a repair stage and the second, an operating stage.

There are N machines in the system, of which a maximum of A can be operating at any one time (A operators). When breakdowns occur at a mean rate μ_2 the machines enter the repair stage which has M parallel servers (M repairmen), who service the machines at a mean rate μ_1. The mean rates μ_1 and μ_2 are the parameters of an exponential distribution, and the numbers N, A, M and μ_2/μ_1, define the output of the system. When A = N, the problem becomes identical to the Swedish machine problem for which tables are already available. Here however is a 2-stage cyclic queue in which machines move through a closed loop and mechanics are 'fixed'. Service and arrivals are exponential and as usual the arrival rate at the repair facility is proportional to the number of machines in service. Standby equipment is available in this case to replace units under repair and the effects of such equipment on the efficiency of the system is evaluated. Thus there are N machines which move through the cycle. In stage 1, there are M repairmen; machines that have completed service at stage 1 move on to stage 2, which is the productive or working stage. The number of operators is A which is less than N. If $A < N$ repaired machines may join queue where they wait. This later case is one in which spare and standby machines are available. Three measures of utilization (efficiency) factors of interest were determined viz: the utilization of the repair facilities, the utilization of the operator and the utilization of the machines.

Makino (1964) considers three classes of serial queues: (1) two single service stations in series (2) three single service stations in series and (3) a two stage multiple server queue. Capacity restrictions are imposed upon all the stages except at the first stage. Under the assumptions made, the mean passage time which gives a measure of the influence of blocking in the systems is calculated for the exponential distribution and for the Erlang distribution of service times in some cases. The output distributions are also calculated for these different systems.

Assumptions for this model (1) there is an infinite waiting line in front of the first stage service station which guarantees that there is always a customer available for service (2) the next customer occupies the service station as soon as the first stage service station becomes empty. So it is not necessary to consider the empty state in the first stage service station. The infinite assumption is made for the first stage service station because it is considered that prolongation of the mean Passage Time by blocking effects is most significant in the first stage service station with infinite input.

All general assumptions made by Hunt, (1956) also apply to this model.

Hillier and Boling (1966) considered and got results for a serial queue of N service channels in which limited waiting space except at the first queue are imposed. Each channel has either an exponential or Erlangian service-time distribution. Thus the input process is such that the first stage queue is never empty. The steady state output rate and mean number in the system (excluding the first queue are obtained). Two methods of achieving this - a relatively efficient and then an exceptionally efficient one in the case of exponential holding time - are developed. Thirdly extensive new numerical results are obtained.

They present an excellent survey of existing models of queueing systems where capacity restrictions are imposed and their purpose is to present some new extensions of Hunt's work, both in terms of numerical results and numerical procedures that make it possible to analyse much larger systems having either exponential or Erlang service times.

The general assumptions for this model which are also applicable to those of Hunt and Makino are as follows:

(a) The queueing system consists of N service channels in series and in steady-state condition.

Thus every customer must be processed through each of the N single-server service facilities in the same fixed sequence.

(b) Service times at the jth facility are Erlangian with shape parameter k_j (j= 1,2 --- N), at all stations and a mean $1/\mu_j > 0$ although k_j will be restricted to one (i,e an exponential distribution) in certain cases.

(c) All service times are independent.

(d) There is always a customer available to be processed at the first facility.

(e) A limited number of customers is allowed before each server except at the first stage where it is unlimited. Thus, if a customer completes service at the (j - 1)st facility when the queue before the jth facility is full, that customer must be held at the (j - 1)st facility until the queue is no longer full. Because of such blocking, the mean output rate of the system will be less than the smallest mean service rate.

Recently Sharma,(1973) has considered a serial queue that is quite general for finite queues. He discusses the stationary behaviour of an arbitrary number r of queues in series, with multi-server service facilities at each queue and finite space between stations. Poisson input and exponential service times have been assumed.

By assigning suitable values to the constants in this model, one can find results both for phase type service, as well as service with waiting allowed before different phases. Steady-state queueing probabilities are obtained and system is solved for the case where there is only one server at each station. This demonstration with the case of one server reveals the tedious mathematical manipulations involved for this seeming simple system and this could reduce the range of applicability of the model.

3.3.4 Simulation models.

Although not much of simulation techniques applied to queueing processes are reported in the literature on queues many text-books, Lee (1961) Gross and Harris (1974) and many journal authors recommend that they should be used whenever they can be justified.

The simulation models offer a means of studying queueing situations which cannot be handled analytically because of the complication of the system or its mathematical intractability. They also come into use when the characteristics of the input and service mechanisms do not permit stability or when the nature of the queue discipline is not easy to model. A combination of any of these factors will also justify it.

(1974)

Gross and Harris report a case of the application of simulation to toll booth traffic and it is suggested that when serial queues are fairly complicated or when the distributions are not exponential as normally assumed, then the only course of action is simulation. Given the system under study, with its complication in character of customers, other operating conditions and configuration, one is bound to include the simulation modelling technique (at least in the vicinity of the toll booth where traffic behaviour may not comply with steady state behaviour) as one of the approaches to solving this queueing problem.

3.4 SELECTION OF POTENTIAL CANDIDATE MODELS.

From the foregoing discussion of existing queueing models, it is the objective here to consider them further in brief detail and in relation to the existing system problem that is to be solved.

First, the single server queueing models are seen to be unfit for further consideration since the queueing problem is definitely too far from that of a simple single server model. To model the system in this way would just amount to unjust over simplification in the face of other more suitable techniques. Such a model will also not be flexible for all the manipulations that are vital in studying the system.

The multiple channel queues have far more practical applicability than the single server queues and this is well demonstrated in this study.

The two queues which constitute the serial arrangement could be modelled as multiple-channel queues after making valid assumptions and they are flexible enough for manipulation. Given the assumptions normally made for this class of models and the fact that evidence exists that the arrivals on all lanes and services in all the channels could be fairly represented by Poisson and negative-exponential distributions respectively, it is then an easy matter to propose an M/M/2: (∞/FIFO) model for the queues behind the Police. Also depending on the assumptions to be made about the toll booth queue as regards dishomogeneity of service at the channels, vehicle class differences and customer minute to minute behaviours, this queueing situation can be represented by a M/M/C: (∞/FIFO) or a M/M/C: (L/FIFO) model.

Also depending on the assumption made about the Police services (whether fully active, traffic 'cop' type of service or immigration type control) in modelling the entire system, a single multiple channel queue may be formulated or the two queues might be considered independent of each other and evaluated as two different independent multiple channel queues.

The multiple channel model is therefore a suitable model for further consideration either on its own or in a serial queueing arrangement.

Of all the serial and cyclic queueing models proposed three appear to be particularly appleaing and warrant further consideration. The first is the two-stage model proposed by Makino which appears to represent the exact configuration of the system without too much over simplifying assumptions. The model appears to possess the further attribute of being easy to manipulate. The model can also be used following the assumptions implicit in its formulation for fairly varying traffic conditions. It can be used also to study the organization of the entire system by manipulating the system variables. Having accepted the approximate exponential behaviour of both traffic and servers; the approximate equality of service channels when the different vehicle classes are considered separately and the statistical possibility of near equality of service rates for the Police it is quite an easy and reasonable thing to consider this model further.

The second candidate model to be proposed in this class is the cyclic queue model of Koenigsberg, (1960). It is proposed here to apply the general machine interference model with standby generators, operators and service men to the traffic situation in the vicinity of the toll booth. It includes the policemen as repairmen and the vehicles as the breakdown machines the assumption being made that for peak traffic operations, this system like the machine problem has a limited number of vehicles always present in between the Police and toll booths.

Assumptions have to be made however about the output of the system vis-a vis the input to the police queue in terms of equality. This is the main draw back of the model but aside from this it has most of the attributes of the Makino model and the system satisfies fairly well all the assumptions made about the model.

The third type of model in this class is the serial queueing model of Jackson,(1958) where a M/M/C: (∞/FIFO) model feeds into a M/M/C: (∞/FIFO) model. This is a model with sufficient merit also to justify it for further consideration. The system is studied for the different vehicle classes separately and since the characteristics of the system are such that the Markovian assumptions are fairly fulfilled and service discipline can be accepted to be the FIFO type, there will be no difficulty in applying this model. It also has the advantage that, at steady state and for independence of the queues excellent charts exist as in Lee (1961), for reading off some effectiveness measures. Its major draw back is that it does not consider the effect of inter-stage blocking due to the limited capacity between the service stations. However from the work of Burke, Reich and Finch and the fact that authors (e.g Lee, 1961) suggest independence at a probability equal to or less than 0.01 of a customer being rejected, this model could be applied without much error of assumption.

The model proposed by Hunt is rather too limited in scope for the system under investigation and would require a lot of modification before it can be used. Sincerely the model developed by Hillier and Boling does not represent the system much and the mathematical methods of evaluation and analysis may not be very flexible in terms of the modified system.

Sharma's model,(1973) has a lot of potentials as it is a fairly good representation of the system like the Makino and the Koenigsberg models but unlike them, it does not have the near mathematical simplicity that is so vital for practical application. Herein lies its major draw back and reason why it is not considered further.

Finally, the simulation modelling technique always comes in for serious consideration in very difficult situations when analytical approach is near impossible. As things stand, there does not seem to be much problem in getting and applying an appropriate model at least from the point of view of an a priori operational solution that is always appealing to a queueing analyst.

3.5 FINAL MODEL SELECTION

From the foregoing considerations and selections of candidate models, it follows that a model to be used as base model for this study will be selected from the following:

the M/M/C: (∞/FIFO) model, the Koenigsberg model, the Jackson model and the Makino model.

(1) THE M/M/C: (∞/FIFO) MODEL.

The multi-channel model proposed here is modelled after the immigration control or the traffic 'cop' type of queue where the service effect at the first stage is considered to be negligible on the operations at the second stage. It amounts to a case of removing the Police and although it could be used for our purpose, it is on over simplification in the face of better alternatives. It is therefore dropped.

(2) THE KOENIGSBERG MODEL.

The Koenigsberg's model is also dropped on the grounds that a lot of assumptions have to be acceptable to a third party. The risk of a wrong assumption is not worth taking here moreso when a model that depicts the system much better than it does has also a much more better and easier mathematical approach.

(3) THE JACKSON MODEL

The Jackson model could have been used at least for its simplicity but the Makino model is also not too complicated and can be efficiently executed on an electronic computer. Moreso the Makino model would model the system as it is by taking into consideration the inter-stage blocking possibility.

(4) THE MAKINO MODEL

The major snag of the Makino model is that the author himself has not presented any practical results that are utimately usuable but at least one can modify the model from any stage of its development for practical applications.

Having considered the essential attributes of the Makino model against the background of the arguments so far stated, it is finally selected for this study. The Jackson model however provides a good alternative.

CHAPTER 4

EVALUATION OF SELECTED MODEL

4.1 THEORETICAL CONSIDERATIONS AND MODEL STRUCTURE

The structure of the Makino model selected for this study is represented by the diagramatic sketch of Figure 4.1.

There are C servers in Stage 1 known (for the purpose of this study) as the register stage. Units leaving these first stage service points either queue up in the limited waiting room provided or go straight for service in any of the S channels of service in Stage 2, known here as the marker stage.

4.1.2. Assumptions

The assumptions upon which the Makino model is based are as follows:

1. Customers arrive in Poisson fashion at a constant rate in both queues.
2. There are C identical servers in Stage 1 and S identical servers in Stage 2.
3. The service-times for each channel in both stages are independent with exponential distributions and the service rates have a common value of μ_1 for all first stage servers and μ_2 for all second stage servers.
4. Servers are non-cooperative at both stages in the sense that one customer is never attended to by more than a single server at a time.

5. For both stages, arriving customers enter the first available open channel and when all channels are full, they form a single queue with unlimited waiting space in the first queue and with limited waiting space in the second queue. From these single lines of queue, customers are drawn and served in a FIFO fashion with no defections.

6. If there are two or more free channels in any of the stages, the customer goes to any one of them and exercises no preference for any channel or group of channels.

7. There is always a customer available to be processed at the first facility and units are transferred instantaneously from one stage to the next on the completion of service.

Thus the model is restricted to only those states of the system in which all of the first stage stations are in service or in blocking and under all these assumptions Makino derived the steady state probabilities for the system.

Using his own notations, all the states identifiable for the system are as shown in Table 4.1.

Thus by defining P_{ij} as the probabilities that the system is in state i,j where

$$i = \begin{cases} 1,2,\text{----}K \ (K \leq N) = \text{no. of units in the limited waiting room for the second stage} \\ 1,2,\text{----}C \ (K > N) = \text{no. of blocked first stage channels} \end{cases}$$

and

$j = 1,2,\text{----}S$ = no. of second stage channels busy

TABLE 4.1.: SYSTEM STATES

STATE (i,j)	FIRST STAGE (NO. OF STATIONS IN BLOCKING)	NO. IN WAITING ROOM	SECOND STAGE (NO. OF STATIONS IN SERVICE)
(0, 0)	0	0	0
(0, 1)	0	0	1
⋮	⋮	⋮	⋮
(0, S)	0	0	S
(1, S)	0	1	S
(2, S)	0	2	S
⋮	⋮	⋮	⋮
(N, S)	0	N	S
(B_1, S)	1	N	S
(B_2, S)	2	N	S
⋮	⋮	⋮	⋮
(B_C, S)	C	N	S

N is equal to the maximum number of units allowed in the second stage queue.

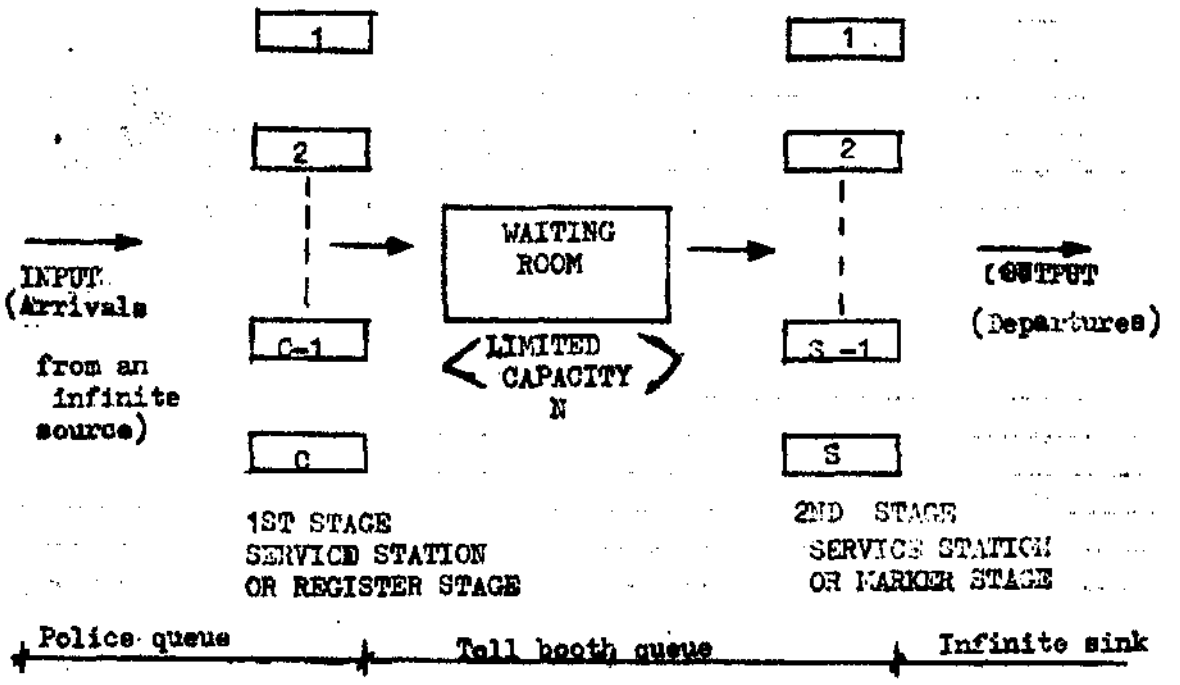


FIG. 4.1 Makin.e's Model Structure

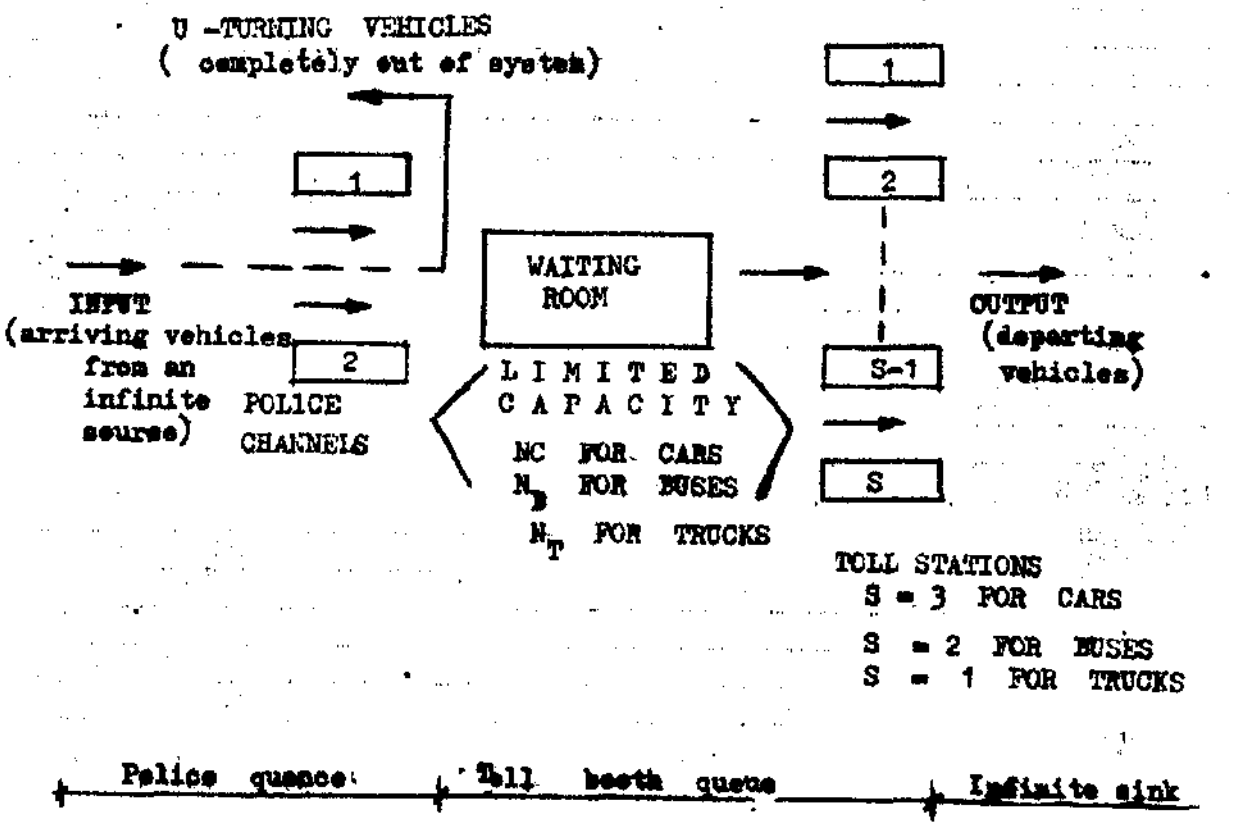


FIG 4.2 Proposed Model Structure

Makino obtained the following steady state results:

$$\mu_2 \cdot P_{0,1} - C\mu_1 \cdot P_{0,0} = 0$$

$$C\mu_1 \cdot P_{0,h-2} + h\mu_2 \cdot P_{0,h} - \{C\mu_1 + (h-1)\mu_2\} \cdot P_{0,h-1} = 0 \quad (\text{for } 2 \leq h \leq S)$$

$$C\mu_1 \cdot P_{0,S-1} + S\mu_2 \cdot P_{1,S} - (C\mu_1 + S\mu_2) \cdot P_{0,S} = 0$$

$$C\mu_1 \cdot P_{K-2,S} + S\mu_2 \cdot P_{K,S} - (C\mu_1 + S\mu_2) \cdot P_{K-1,S} = 0 \quad (\text{for } 2 \leq K \leq N)$$

$$C\mu_1 \cdot P_{N-1,S} + S\mu_2 \cdot P_{B_1,S} - (C\mu_1 + S\mu_2) \cdot P_{N,S} = 0$$

$$C\mu_1 \cdot P_{N,S} + S\mu_2 \cdot P_{B_2,S} - \{(C-1)\mu_1 + S\mu_2\} \cdot P_{B_1,S} = 0$$

$$\{C - (L-1)\}\mu_1 \cdot P_{B_{L-1},S} + S\mu_2 \cdot P_{B_{L+1},S} - \{(C-L)\mu_1 + S\mu_2\} \times P_{B_L,S} = 0 \quad (\text{for } 2 \leq L \leq C-1)$$

$$\mu_1 \cdot P_{B_{C-1},S} - S\mu_2 \cdot P_{B_L,S} = 0 \qquad \text{----------(4.1)}$$

where μ_1 = rate of service at the first stage service station

and μ_2 = rate of service at the second stage service station.

From these equations, the following results follow:

$$P_{0,h} = \frac{1}{h!}\left(\frac{C\mu_1}{\mu_2}\right)^h \cdot P_{00} \quad (\text{for } 0 \leq h \leq S), \qquad \text{----------(4.2)}$$

$$P_{K,S} = \frac{1}{S!}\left(\frac{C\mu_1}{\mu_2}\right)^S \cdot \left(\frac{C\mu_1}{S\mu_2}\right)^K \cdot P_{00} \quad (\text{for } 0 \leq K \leq N) \qquad \text{---(4.3)}$$

$$P_{B_L,S} = \left\{ \frac{1}{S!} \left(\frac{C\mu_1}{\mu_2}\right)^S \cdot \left(\frac{C\mu_1}{S\mu_2}\right)^N \cdot P_{00} \right\} x$$

$$\left(\frac{\mu_1}{S\mu_2}\right)^L \left\{ C(C-1)\text{----}(C-L+1) \right\}$$

(for $1 \leq L \leq C$) ----------(4.4)

Considering that

$$\sum_{h=0}^{S} P_{0,h} + \sum_{K=1}^{N} P_{K,S} + \sum_{L=1}^{C} P_{B_L,S} = 1$$

$$P_{00} = \frac{1}{\sum_{h=0}^{S} \frac{1}{h!}\left(\frac{C\mu_1}{\mu_2}\right)^h + \sum_{K=1}^{N} \left\{ \frac{1}{S!}\left(\frac{C\mu_1}{\mu_2}\right)^S \cdot \left(\frac{C\mu_1}{S\mu_2}\right)^K \right\}} \cdot \cdots$$

$$+ \sum_{L=1}^{C} \left[\frac{1}{S!}\left(\frac{C\mu_1}{\mu_2}\right)^S \left(\frac{C\mu_1}{S\mu_2}\right)^N \left(\frac{\mu_1}{S\mu_2}\right)^L \left\{ C(C-1)\text{----}(C-L+1) \right\} \right]$$

.....(4.5)

Equations (4.1) to (4.5) represent the basic equations of state probabilities of the Makino model.

The work done in this thesis is specifically based upon these equations and not on any other result of Makino. The system is looked upon more critically (considering the list of variables and factors enumerated in Chapter 1, which affect the system performance) against the background of assumptions made for the Makino model. Some modifications are then made on equations (4.1) to (4.5) so as to reflect the reality of the problem at hand and equations

for the system effectiveness measures are then derived from them.

4.1.3. Problems Associated With Justifying The Use Of This Model And Pertinent Facts About The System

Most of the factors, initially thought to be affecting the system considerably, were later found from the data collection and analysis stage to have negligible effect. They were therefore not considered further for inclusion as variables in the process of modifying the model equation as presented in this section of the thesis. In particular, the percentages of those constituting the change problem and the fraudulent cases were low enough to be neglected as factors. If anything, the small incidence of extended service time by these two factors may have contributed strongly to the observed exponential distribution of service at the toll points as assumed for the Makino model.

The question of customer impatience is equally a negligible factor when one considers the balking and reneging phenomenon but a high incidence of jockeying was observed for the bus and car lanes and consideration was given to this. The number of U-turning vehicles is also considerable to warrant its inclusion in the modified model.

Other characteristics of the system that must be accounted for in the modified model are:

1. heterogeneous customers
2. heterogeneous servers.

Furthermore, some of the basic assumptions in the Makino model (like most classical queueing models), do not apply to a real system involving human servers and customers like in the toll booth queueing system. The simplifications embodied in the model therefore constitute specification errors which cannot be avoided and must remain even with the modified model used for this study. The other four factors - jockeying, U-turning, heterogeneous customers and heterogeneous servers - are however, accounted for as discussed in the following section.

4.1.4. Arguments, Simplifications And Modifications Necessary to Justify Applicability of this Model

In accounting for the factors mentioned in the previous section it was, for most of the factors, necessary to either simplify the system or adjust the base model while in one case it was only necessary to argue that the behaviour of the system has already accommodated the factor. The factors are considered as follows:

(a) Jockeying: No modification was made to the model or the system in this case. It was not considered necessary because although there is a high incidence of jockeying, there is substantial argument [Gross and Harris (1974); Lee (1966); Burke (1956) and Giffin,(1978)] to support that under this condition the individual channel queues

are approximated by the single line queue assumption of the Makino model. A proof actually exists for the two channel case [(Lee (1966)] and therefore for the system under study, it might not be necessary to modify the model as the jockeying effect is absorbed by the system leading to simplifications.

(b) U-turning vehicles: One of the assumptions of the Makino model is that every unit passing through the first stage must get to the second stage. In the real system, however, the percentage of U-turning vehicles is high and this must be reflected in the modified model. This has been achieved in the modified model by introducing a factor into the original model equation. This factor is the fixed probability of a given type of vehicle leaving Stage 1 and getting to stage 2 without making a U-turn. It is an approximation but it is suitable for the level of exactness expected in this solution and follows the line of discussion in Saaty,(1961) on nonsymmetric queues; in Taha,(1971) and in Jackson,(1957).

(c) Heterogeneous Customers: There are different vehicle classes that differ both physically and in some aspects of their behaviour. The best way out of this problem was therefore to decompose the Poisson vehicle stream into that of the different classes ignoring the two-wheeled vehicles which have a negligible proportion in the vehicle stream. Hence the modified model was applied separately to the cases of the trucks, cars and buses. It was also necessary to determine the maximum queueing length allowed for

each vehicle class by using the measured distance on the field and the size of the different vehicle classes as given in the Highway Design Manual (1972). The following should therefore be included in the Makino model for each class:

Cars: C = 2; S = 3

Buses: C = 2; S = 2

Trucks: C = 2; S = 1

Giffin, (1978) indicates a procedure for decomposing and combining Poisson streams and this approach was adopted for this study.

(d) Heterogeneous Servers: The heterogeneous set of servers can be approximated by a homogeneous set only if the extent of dishomogeneity is small [Gumbel, (1959)]. Hence to further reduce the dishomogeneity between the servers at the toll booths, the different vehicle classes were considered separately. Statistical analysis has shown this disparity in service rates to be negligible in this case. Hence serving the customers by vehicle classes ensures that service times are identically and independently distributed as required in the Makino model.

Finally, it is necessary to modify the assumption that on completion of service a customer moves instantaneously to Stage 2 as contained in the Makino model to read: Upon completion of a service, the customer engages a Bernoulli switch and with fixed probability β, he moves instantaneously from Stage 1 to Stage 2; while with probability $1 - \beta$, he leaves the system completely.

4.1.5. Theoretical Aspects and Derivations

The modified model of the system to be studied is shown in the diagramatic sketch of Figure 4.2. Basically, the only difference between this model and the Makino model that affects the mathematical formulations is the number of turning vehicles.

It is therefore the purpose of this section to introduce the effect of this turning in the Makino equations 4.1 to 4.5 and then to obtain some measures of system effectiveness therefrom.

From the technique in the literature earlier cited, it is a trivial matter to show that it is only necessary to multiply the first stage rate of service by the probability factor to get the desired effect.

Mathematically, this implies that the value μ_1 in all the equations should be replaced with $\beta\mu_1$.

With this substitution, only the new equivalents of equations 4.2 to 4.5 will be restated here. These are the set of equations required for computing the system effectiveness measures.

Thus the equations of steady state probabilities are:

$$P_{0,h} = \frac{1}{h!}\left(\frac{C\beta\mu_1}{\mu_2}\right)^h . P_{00} \quad (\text{for } 0 \leq h \leq S) \quad \text{---(4.6)}$$

$$P_{K,S} = \frac{1}{S!}\left(\frac{C\beta\mu_1}{\mu_2}\right).\left(\frac{C\beta\mu_1}{S\mu_2}\right)^K . P_{00} \ (\text{for } 0 \leq K \leq N)$$

---(4.7)

$$P_{B_L,S} = \left\{\frac{1}{S!}\left(\frac{C\beta\mu_1}{\mu_2}\right)^S . \left(\frac{C\beta\mu_1}{S\mu_2}\right)^N . P_{00}\right\} \times$$

$$\left(\frac{\beta\mu_1}{S\mu_2}\right)^L \left\{C(C-1)\text{---}(C-L+1)\right\}$$

(for $1 \leq L \leq C$)

---(4.8)

and

$$P_{00} = \frac{1}{\sum_{h=0}^{S} \frac{1}{h!}\left(\frac{C\beta\mu_1}{\mu_2}\right)^h + \sum_{K=1}^{N}\left\{\frac{1}{S!}\left(\frac{C\beta\mu_1}{\mu_2}\right)^S . \left(\frac{C\beta\mu_1}{S\mu_2}\right)^K\right\}}$$

$$+ \sum_{L=1}^{C}\left[\frac{1}{S!}\left(\frac{C\beta\mu_1}{\mu_2}\right)^S . \left(\frac{C\beta\mu_1}{S\mu_2}\right)^N \left(\frac{\beta\mu_1}{S\mu_2}\right)^L \left\{C(C-1)\text{---}(C-L+1)\right\}\right]$$

---(4.9)

Define (i) $\alpha_C = \frac{C!}{C^C}$

(ii) $\alpha_S = \frac{S!}{S^S}$

(iii) $X = \frac{C\mu_1}{S\mu_2}$

(iv) $\rho = X\beta$

Then equation 4.6 implies

$$P_{0,h} = \frac{(S\beta X)^h}{h!} P_{00} \quad \text{---(4.10)}$$

(for $0 \leq h \leq S$)

Similarly by substitution

$$P_{K,S} = 1/\alpha_S (\beta X)^{S+K} P_{00} \quad \text{----------(4.11)}$$

(for $1 \leq K \leq N$)

and

$$P_{B_L,S} = \left\{\frac{S^S}{S!}(X/\beta)^S \cdot (\beta X)^N \cdot P_{00}\right\}\left\{\frac{(X\beta)^L}{C^L}\right\}\left\{C : \right\}$$

$$= \frac{\alpha_C}{\alpha_S}(X\beta)^{S+N+L} C^{C-L} P_{00} \quad \text{----------(4.12)}$$

(for $1 \leq L \leq C$)

Now let $R = \sum_{h=0}^{S} \frac{(SX/\beta)^h}{h.}$

and $S = \sum_{K=1}^{N} \frac{1}{\alpha_S}(X/\beta)^{S+K}$

= second term in the denominator for equation 4.9.

Similarly let the third term in the denominator for equation 4.9 be defined as

$$T = \sum_{L=1}^{C} \frac{\alpha_C}{\alpha_S}(X/\beta)^{S+N+L} C^{C-L}$$

then equation 4.9 can be written as

$$P_{00} = \frac{1}{R + S + T} \quad \text{----------(4.13)}$$

Now for this truncated system, the effective arrival rate representing those calling customers who actually arrived at toll plaza without being blocked defined by λ_e is calculated as

$$\lambda_e = \lambda(1 - P_{B_L,S}) = C\beta\mu_1(1 - P_{B_L,S})$$

where $\lambda_e < C\mu$ under steady state and $P_{B_L,S}$ is the blocking probability.

Hence

$$\text{Offered load} = a = \frac{C\beta\mu_1}{\mu_2} = S\beta X \quad \text{----(4.14)}$$

$$\text{Carried load} = a' = \left(\frac{C\beta\mu_1}{\mu_2}\right)(1 - P_{B_L,S})$$

$$= S\beta X(1 - P_{B_L,S}) \quad \text{----(4.15)}$$

$$\text{Traffic intensity} = \rho = \beta X \quad \text{----(4.16)}$$

and $$\text{Utilization factor} = \rho' = \beta X(1-P_{B_L,S}) \quad \text{----(4.17)}$$

The expected queue length is calculated from the definition of expected values and the technique of sum calculus as follows:

Probability of K units being in the queue for $0 \leq K \leq N$ is given by equation 4.7

Hence let L_q = mean no. in queue

then
$$L_q = \sum_{K=1}^{N} K P_{K.S}$$

$$= \frac{1}{S!}\left(\frac{C\beta\mu_1}{\mu_2}\right)^S P_{00} \cdot \sum_{K=1}^{N} K\left(\frac{C\beta\mu_1}{S\mu_2}\right)^K$$

$$= \frac{1}{S!}\left(\frac{C\beta\mu_1}{\mu_2}\right)^S P_{00}\left[\sum_{K=0}^{N} K \rho^K\right]$$

Now from the technique of sum calculus. we know that the following definite sum holds:

$$\sum_{M}^{N} n P^n = \left| \frac{P^n\left[n(P-1) - P\right]}{(P-1)^2} \right|_{n=m}^{n=N+1}$$

Therefore by substitution

$$L_q = \frac{1}{S!}\left(\frac{C\beta\mu_1}{\mu_2}\right)^S P_{00}\left\{\rho^K\left[\frac{K(\rho-1)-\rho}{(\rho-1)^2}\right]_{K=0}^{K=N+1}\right\}$$

which simplified further gives

$$L_q = \frac{1}{S!}\left(\frac{C\beta\mu_1}{\mu_2}\right)^S \frac{P_{00}}{(\rho-1)^2}\left[N\rho^{N+2}-(N+1)\rho^{N+1}+\rho\right]$$

and relevant substitutions ⇒

$$L_q = \frac{(\beta X)^S}{\alpha_S(\beta X-1)^2}\left[N(\beta X)^{N+2}-(N+1)(\beta X)^{N+1}+\beta X\right]P_{00}$$

----------(4.18)

Under steady state condition, it is known that the carried load (a') is equivalent to the mean no. of servers. Hence the expected number in the system ⇒

$$LL = L_q + a' \quad \text{----------(4.19)}$$

With the effective arrival rate, expected number in the system and the expected number in the queue now known, the mean waiting times are easily computed using Little s formulas (Giffin 1978).

Mean waiting time in queue ⇒

$$W_q = \frac{L_q}{\lambda_e} = \frac{L_q}{C\beta\mu_1(1-P_{E_L,S})} \quad \text{----------(4.20)}$$

and

Mean waiting time in system ⇒

$$W = LL/\lambda_e = W_q + \frac{1}{\mu_2} \quad \text{----------(4.21)}$$

The customer loss ratio represents the ratio of the time lost by the customer in the queue to the time usefully

spent there. It is given by

$$LR = W_q \cdot \mu_2 \quad \text{----------(4.22)}$$

This loss ratio is particularly important to customers requiring little service as in the case under study.

These equations for the evaluation of relevant system performance measures, together with the equations of steady state probabilities, constitute the full set of model equations.

A computer programme has been written for evaluating all these system performance measures.

4.2. EVALUATION OF MODEL EQUATIONS AND VALIDATION OF MODEL

4.2.1. Computer Approach

In the practical application of this model to a particular problem, the manual evaluation of the different measures of effectiveness could be very tedious as it is quite evident that in a practical situation the inputs could be so large as to render manual calculations either impossible or completely inefficient and time wasting as a method. Therefore a computer approach to the problem has been adopted for this research and the model has been programmed for completely automatic execution on a digital computer. The programme flowchart is shown in Appendix 4c and the complete computer programme and a sample of the computer output results for the particular work carried out in this research are also included.

```
MNF,I=QUMOD5,T,B=B,L=L.
```

```
 1.  000000B          PROGRAM QMODEL(DATQ,OUTF,TAPE5=DATQ,TAPE6=OUTF)
                C     THIS PROGRAM CALCULATES THE STATE PROBABILITIES AND THE
                C     RELATED MEASURES OF EFFECTIVENESS FOR A TRUNCATED TWO-STAGE
                C     MULTI-SERVER SERIAL QUEUE WITH POISSON ARRIVALS AND EXPONENTIAL
                C     SERVICE TIME
                C
                C     STATISTICAL TESTS ARE USED TO JUSTIFY THE APPROXIMATION OF MEAN
                C     ARRIVAL RATES,MEAN SERVICE RATES AND PROBABILITY DISTRIBUTIONS
                C     OVER TIME
                C
                C     THE QUEUE STATISTICS CALCULATED IN RELATION TO STAGE2 QUEUE ARE:
                C      1.  RHO=TRAFFIC INTENSITY OR RATIO OF UTILIZATION OF STAGE2 TO
                C            STAGE1
                C      2.  P00=PROBABILITY OF NO VEHICLE IN SYSTEM
                C      3.  LQ=MEAN NUMBER IN THE QUEUE
                C      4.  A= TOTAL TRAFFIC OFFERED PER SERVICE TIME
                C      5.  ADASH=TOTAL TRAFFIC CARRIED PER SERVICE TIME
                C      6.  RODASH=CHANNEL UTILIZATION FACTOR
                C      7.  WQ=MEAN WAITING TIME IN QUEUE
                C      8.  W=MEAN WAITING TIME IN SYSTEM
                C     9.  LL=MEAN NUMBER OF VEHICLES IN SYSTEM
                C     10.  LR=CUSTOMER LOSS RATIO
                C     11.  POH=STATE PROBABILITY OF SECOND STAGE SERVERS BEING ENGAGED
                C     12.  PBLS=FIRST STAGE BLOCKING PROBABILITY
                C     13.  PKS=STATE PROBABILITY OF K-VEHICLES IN THE QUEUE
                C            WHILE ALL SECOND STAGE SERVERS ARE BUSY
                C
                C     OTHER VARIABLES USED ARE:
                C        1.  TYPE=VEHICLE TYPE
                C        2.  C=NUMBER OF FIRST STAGE SERVERS
                C        3.  S=NUMBER OF SECOND STAGE SERVERS
                C        4.  MU1=RATE OF SERVICE AT THE FIRST STAGE CHANNELS
                C        5.  MU2=RATE OF SERVICE AT THE SECOND STAGE CHANNELS
                C        6.  BETA=FIXED PROBABILITY WITH WHICH A VEHICLE LEAVES STAGE 1
                C            FOR STAGE 2
                C        7.  R=COMPUTED VARIABLE FOR DETERMINATION OF P00
                C        8.  P=COMPUTED VARIABLE FOR DETERMINATION OF P00
                C        9.  T=COMPUTED VARIABLE FOR DETERMINATION OF P00
                C     ********************************************************
                C
 2.  004121B          INTEGER TYPE,C,S,H,H1
 3.  004121B          REAL LL(100),LQ,R(100)
 4.  004121B          DIMENSION POH(100),PBLS(100),PKS(100),ADASH(100),RODASH(100),
                     1WQ(100),W(100)
 5.  004121B        9 READ(5,10,END=999)TYPE,C,S,MU1,MU2,N,BETA
                C
                C     CALCULATION OF TRAFFIC INTENSITY
                C
 6.  005753B          ALPHAC=FLOAT(IFACT(C))/FLOAT(C**C)
 7.  005770B          ALPHAS=FLOAT(IFACT(S))/FLOAT(S**S)
 8.  006006B          X=FLOAT(C*MU1)/FLOAT(S*MU2)
                C
                C     INITIALISATION OF VARIABLES
                C
 9.  006013B          R=1.0
10.  006015B          T=0.0
11.  006016B          P=0.0
12.  006016B          P00=0.0
13.  006017B          DO 102 JJ=1,100
14.  006022B          POH(JJ)=0.0
15.  006026B          PBLS(JJ)=0.0
16.  006032B          PKS(JJ)=0.0
17.  006036B          LL(JJ)=0.0
18.  006042B          LR(JJ)=0.0
19.  006046B          ADASH(JJ)=0.0
20.  006052B          RODASH(JJ)=0.0
21.  006056B          WQ(JJ)=0.0
22.  006062B          W(JJ)=0.0
23.  006066B      102 CONTINUE
                C     ********************************************************
                C
                C     CALCULATION OF SYSTEM PARAMETERS
                C
24.  006071B          DO 105 H=1,S
```

```
25.    006100B     105 R=R+((S*BETA*X)**H)/FLOAT(IFACT(H))
26.    006124B         DO 106 K=1,N
27.    006133B     106 P=P+1/ALPHAS*(BETA*X)**(S+K)
28.    006156B         DO 107 L=1,C
29.    006166B     107 T=T+ALPHAC/ALPHAS*(BETA*X)**(S+N+L)*C**(C-L)
30.    006225B         P00=1/(R+P+T)
31.    006230B         H1=0
32.    006231B         K=0
33.    006232B     109 POH1=((S*BETA*X)**H1)/FLOAT(IFACT(H1))*P00
34.    006253B         K=K+1
35.    006254B         POH(K)=POH1
36.    006261B         H1=H1+1
37.    006263B         IF(H1.LE.S)GO TO 109
38.    006265B         DO 110 K=1,N
39.    006274B     110 PKS(K)=1/ALPHAS*(BETA*X)**(S+K)*P00
40.    006325B         DO 111 L=1,C
41.    006334B     111 PBLS(L)=ALPHAC/ALPHAS*(BETA*X)**(S+N+L)*C**(C-L)*P00
42.    006401B         A=S*BETA*X
43.    006403B         DO 112 IK=1,C
44.    006412B     112 ADASH(IK)=S*BETA*X*(1-PBLS(IK))
45.    006433B         RHO=X*BETA
46.    006433B         DO 113 IK=1,C
47.    006443B         RODASH(IK)=BETA*(*(1-PBLS(IK))
48.    006443B     113 CONTINUE
49.    006463B         LQ=(BETA*X)**S/(ALPHAS*(BETA*X-1)**2)*(N*((BETA*X)**(N+2))
              1-(N+1)*(BETA*X)**(N+1)+BETA*X)*P00
50.    006526B         DO 114 IK=1,C
51.    006536B         LL(IK)=ADASH(IK)+LQ
52.    006543B         WQ(IK)=LQ/((C*BETA*MU1)*(1-PBLS(IK)))
53.    006555B         W(IK)=WQ(IK)+1/FLOAT(MU2)
54.    006564B         LR(IK)=WQ(IK)*FLOAT(MU2)
55.    006571B     114 CONTINUE
56.    006574B         WRITE(6,117)
57.    006603B         KK=TYPE-2
58.    006604B         IF(KK)151,152,153
59.    006606B     151 WRITE(6,141)
60.    006615B         GO TO 142
61.    006616B     152 WRITE(6,143)
62.    006625B         GO TO 142
63.    006626B     153 WRITE(6,144)
64.    006635B     142 WRITE(6,145)C,S,MU1,MU2,N,BETA
C
CCCC    ***********************************************************
CCCC
CCCC    PRINT SYSTEM EFFECTIVENESS MEASURES
C
65.    006652B         WRITE(6,115)RHO,P00,LQ,A
66.    006664B         WRITE(6,131)(ADASH(I),I=1,C)
67.    006711B         WRITE(6,132)(RODASH(I),I=1,C)
68.    006736B         WRITE(6,133)(WQ(I),I=1,C)
69.    006763B         WRITE(6,134)(W(I),I=1,C)
70.    007010B         WRITE(6,135)(LL(I),I=1,C)
71.    007035B         WRITE(6,136)(LR(I),I=1,C)
72.    007062B         WRITE(6,137)(POH(I),I=1,S+1)
73.    007110B         WRITE(6,138)(PBLS(I),I=1,C)
74.    007135B         WRITE(6,139)(PKS(J),J=1,N)
75.    007162B         GO TO 9
77.    007162B     117 FORMAT(1H1,//10X,*RESULTS OF TRUNCATED TOLL BOOTH QUEUING SYSTEM*)
78.    007162B     141 FORMAT(26X,*FOR CARS*)
79.    007162B     143 FORMAT(26X,*FOR BUSES*)
80.    007162B     144 FORMAT(26X,*FOR TRUCKS*)
81.    007162B     145 FORMAT(/22X,*C=*,I2,*  S=*,I2,*  MU1=*,I5,*  MU2=*,I5,*  N=*,I3,*
              1  BETA=*,F7.4)
82.    007162B     115 FORMAT(///10X,*RHO    =*,F9.4/10X,*P00    =*,F9.4/10X,
              1*LQ     =*,F9.4/10X,*A      =*,F9.4)
83.    007162B     131 FORMAT(10X,*ADASH  =*,10F9.4)
84.    007162B     132 FORMAT(10X,*RODASH=*,10F9.4)
85.    007162B     133 FORMAT(10X,*WQ     =*,10F9.4)
86.    007162B     134 FORMAT(10X,*W      =*,10F9.4)
87.    007162B     135 FORMAT(10X,*LL     =*,10F9.4)
88.    007162B     136 FORMAT(10X,*LR     =*,10F9.4)
89.    007162B     137 FORMAT(10X,*POH    =*,11F9.4)
90.    007162B     138 FORMAT(10X,*PBLS   =*,10F9.4)
91.    007162B     139 FORMAT(10X,*PKS    =*,10F9.4)
92.    007162B     10  FORMAT(I1,2I2,2I3,I4,F7.4)
CCC
CCC      THIS PROGRAM IS COMPLETELY GENERAL WITH THE FOLLOWING EXCEPTIONS
CCC    C          1.   TYPE MUST NOT BE GREATER THAN 3
CCC                2.   N MUST NOT BE GREATER THAN 100
CCC
92.    007162B   999  STOP
93.    007165B        END
CCC     ***********************************************************
```

```
C
CC      FUNCTION IFACT(N) TO CALCULATE THE FACTORIAL OF NUMBERS AS
CC            SPECIFIED
C

1       0000000B          FUNCTION IFACT(N)
2.      0000000B          K=N
3.      0000001B          IFACT=1
4.      0000003B          IF(N.EQ.0 .OR. N.EQ.1)RETURN
5.      0000010B        1 IFACT=IFACT*K
6.      0000012B          K=K-1
7.      0000013B          IF(K.NE.0) GO TO 1
8.      0000014B          RETURN
9.      0000017B          END
C
CC          ***********************************************************
C
```

```
UNIVERSITY OF MINNESOTA FORTRAN COMPILER (VERSION 5.0 - 77/03/24) ON THE CY72 UNDER  N O S 1.1.0

                    MNF,I=CUFIT,T,B=B,L=L.

                    C       PROGRAM TO FIT NEGATIVE EXPONENTIAL DISTRIBUTION TO OBSERVED DATA
                    C       KOLMOGOROV-SMIRNOV(K-S)GOODNESS OF FIT TECHNIQUE USED
    1.    000000B           PROGRAM CUFIT(NATAL,OUTF,TAPE5=NATAL,TAPE6=OUTF)
                    C       T=OBSERVED INTER-ARRIVAL TIME/SERVICE TIME
                    C       M=OBSERVED ABSOLUTE FREQUENCY
                    C       C=OBSERVED CUMULATIVE FREQUENCY(%AGE)
                    C       FT=EXPECTED CUMULATIVE FREQUENCY
    2.    004121B           DIMENSION T(513),M(513),C(513),XMEAN(23),P(23),
                           1 FT(513),C1(513),CDIFF(513)
                    C
                    C       READ IN THE INPUT DATA
                    C
    3.    004121B           DO 76 J=1,513
    4.    012207B           READ(5,77,END=79)T(J),M(J),C(J)
    5.    012236B       76  CONTINUE
                    C
                    C       CALCULATION OF SIMPLE MEANS
                    C
    6.    012241B       79  STIME=0.0
    7.    012243B           K=0
    8.    012244B           IFREQ=0
    9.    012244B           DO 90 J=1,513
   10.    012250B           STIME=STIME+T(J)*M(J)
   11.    012257B           IFREQ=IFREQ+M(J)
   12.    012263B           IF(C(J).EQ.100.0)GO TO 95
   13.    012271B           GO TO 90
   14.    012273B       95  K=K+1
   15.    012275B           XMEAN(K)=STIME/FLOAT(IFREQ)
   16.    012303B           STIME=0.0
   17.    012304B           IFREQ=0
   18.    012305B       90  CONTINUE
   19.    012311B           DO 91 N=1,23
   20.    012314B       91  P(N)=1.0/XMEAN(N)
                    C
                    C       CALCULATION OF EXPECTED FREQUENCIES
                    C
   21.    012332B           KK=1
   22.    012333B           DO 92 L=1,513
   23.    012336B           FT(L)=1-EXP(-P(K)*T(L))
   24.    012356B           IF(C(L).EQ.100.0)KK=KK+1
   25.    012367B       92  CONTINUE
                    C
                    C       CALCULATION OF OBSERVED FREQUENCIES
                    C
   26.    012372B           DO 96 J=1,513
   27.    012374B           C1(J)=C(J)/100.0
                    C
                    C       CALCULATION OF K-S STATISTICS
                    C
   28.    012401B       96  CDIFF(J)=ABS(C1(J)-FT(J))
   29.    012412B           K=1
                    C
                    C       PRINT ALL RELEVANT STATISTICS
                    C
   30.    012413B           WRITE(6,101)
   31.    012422B           WRITE(6,102)P(1)
   32.    012431B           WRITE(6,103)
   33.    012437B           DO 98 I=1,513
   34.    012441B           WRITE(6,104)T(I),M(I),C1(I),FT(I),CDIFF(I)
   35.    012501B           IF(C(I).EQ.100.0)GO TO 105
   36.    012506B           GO TO 98
   37.    012510B   105       WRITE(6,115)
   38.    012517B           K=K+1
   39.    012520B           IF(K.GT.23)GO TO 98
   40.    012521B           WRITE(6,101)
   41.    012530B           WRITE(6,102)P(K)
   42.    012543B           WRITE(6,103)
   43.    012551B    98     CONTINUE
                    C
                    C       FORMAT STATEMENTS ARE AS FOLLOWS
                    C
   44.    012554B   102     FORMAT(30X,*LAMBDA/MEU=*,F8.4,*VEH.PER SECOND*)
   45.    012554B   103     FORMAT(///9X,*TIME      OBS.    OBS.      EXPTD.      K-S*/
                           119X,*FREQ.   CUM.     CUM.*/26X,*FREQ.     FREQ.*/)
   46.    012554B   104     FORMAT(8X,F5.1,5(,I4,3X,F6.4,3X,F6.4,3X,F6.4)
   47.    012554B   101     FORMAT(1H1,//25X,*GOODNESS-OF-FIT TEST*/)
   48.    012554B    77     FORMAT(29X,F3.0,,X,I3,24X,F5.1/)
   49.    012554B   115     FORMAT(////11X,'DATA FOLLOW NEGATIVE EXPONENTIAL DISTRIBUTION*)
   50.    012554B           STOP
   51.    012555B           END
```

4.2.2. The Inputs To The Programme

To use the programme, one would first of all have to determine the model inputs mentioned earlier. That is the rates of service at both stages, μ_1 and μ_2; the maximum queue length for the second stage queue N; the number of servers at both stages, C and S; and the fixed probability, β, of leaving stage 1 and getting to stage 2 without leaving the system, must first be determined. C and S are normally specified constants and the determination of service rates have been considered in the relevant section of data collection and analysis.

It is therefore the purpose of this section to show by calculations how the service rates and the values for waiting capacity and fixed probability can be determined from the results of data collection and analysis for use in a particular situation. For example, in this study, the following calculations were carried out:

(a) Calculation for the Maximum Waiting Room for each Vehicle Type

From Highway Design Manual, the overall length of vehicle types are as follows:

(i) Cars = 19 ft = 5.8 m

(ii) Buses = 30 ft = 9.2 m

(iii) Trucks = 50 ft = 15.3 m.

But distance between police check point and toll booths is approximately 250 m.

Hence

$$N_{car} \simeq \frac{250}{5.8} \simeq 43 \text{ cars.}$$

$$N_{Bus} \simeq \frac{250}{9.2} \simeq 27 \text{ buses.}$$

$$N_{Trucks} \simeq \frac{250}{15.3} \simeq 16 \text{ trucks.}$$

where N_{car}, N_{bus} and N_{trucks} are values of N for the different types.

(b) Calculation of Fixed Probability β

Now from Table 4.7

Mean α_1 = .0903 = Proportion of all vehicles that are trucks.

Mean α_2 = .1441 = Proportion of all vehicles that are buses.

Mean α_3 = .7587 = Proportion of all vehicles that are cars;

and from Table 4.8

Mean α_U = .3778 = Proportion of all vehicles turning.

The proportion of U-turning vehicles that is of type i was not determined on the field and so it had to be estimated approximately. Hence from approximate estimates indicated in Appendix 3b, the proportion of U-turning vehicles that are trucks = α''_1 = .0925.

Similarly for buses α''_2 = .0619

and for cars α''_3 = .8249.

It is therefore clear that cars constitute the largest number of turning vehicles.

Next to be determined is the probability of a vehicle making a U-turning given that it is of a particular class. Taking the car for example,

$$\text{Reqd. is } \frac{P(C \cap U)}{P(C)} = P(U/C)$$

where P(U/C) = probability of a vehicle making a U-turn given that it is a car

$$= \frac{\text{total no. of cars in turning stream}}{\text{total no. of cars in system}}$$

$$\Rightarrow P(U/C) = \text{Probability of U-turning for cars} = \frac{.8249 \times .3778\ K}{.7587\ K} = .411$$

where K = total number of all vehicle classes in police queue.

Hence probability of getting to Stage 2 from Stage 1

$$= 1 - P(U/C) = 1 - .411 = .589$$

$$\text{i.e. } \beta_{cars} = \underline{\underline{.589}}$$

Similarly β for buses $\beta_{bus} = \underline{\underline{.838}}$

and β for trucks $\beta_{trucks} = \underline{\underline{.613}}$

(c) Calculations for First Stage Service Rate

Typical values of service times in each lane

$\Rightarrow$ for Lane A = 2.9 sec. for 117 observations.

and for Lane B = 1.04 sec. for 169 observations.

Hence weighted mean = 1.8 sec for both lanes.

This implies that μ_1 = 2000 vehicle/hour for the mixed population of vehicles. Applying the decomposition technique for exponential random variables (Giffin 1978)

$\Rightarrow \mu_{1_{cars}}$ for cars only = .7587 x 2000 veh./hour

= 1517 veh./hour.

Similarly for buses $\mu_{1_{buses}}$ = .1441 x 2000 = 288 veh./hour.

and for trucks $\mu_{1_{trucks}}$ = .0903 x 2000 = 181 veh./hour.

(d) Calculations For Second Stage Service Rate

Using the ANOVA deductions in Chapter 2; then from Table 4.9

(i) Mean service time of trucks = 16.6 sec.

$\Rightarrow \mu_{2_{trucks}}$ = 217 veh./hour.

(ii) Mean Service time in Lane 2 = 11.43 sec.

$\Rightarrow$ 315 veh./hour

Mean service time in Lane 3 = 11.85 sec.

$\Rightarrow$ 304 veh./hour

and weighted mean = 310

or $\mu_{2_{buses}}$ = 310 veh./hour.

Similarly for cars

(iii) Mean service time in Lane 4 = 12.1 sec.

$\Rightarrow$ 298 veh./hour.

Mean service time in Lane 5 = 9.27 sec.

$\Rightarrow$ 388 veh./hour.

and Mean service time in Lane 6 $\Rightarrow$ 11.86 sec.

$\Rightarrow$ 304 veh./hour.

Therefore the weighted mean rate of service for cars

= $\mu_{2_{cars}}$ = 326 veh./hour.

These values as they occur within each class of vehicles do not show great variability and can be considered identical for each server within the class (Gumbel, 1959).

TABLE 4.7.: PROPORTION OF EACH VEHICLE CLASS IN THE POLICE QUEUE BASED ON ACTUAL COUNTING

DAY OF WEEK	PERIOD	NUMBER OF				TOTAL NO.	PROPORTION OF			
		TRUCKS	BUSES	CARS	AUTO-CYCLES		TRUCKS (α_1)	BUSES (α_2)	CARS (α_3)	AUTO-CYCLES (α_4)
Saturday	1130-1150	113	235	974	17	1339	.0844	.1755	.7274	.0126
Saturday	1230-1250	23	28	169	-	220	.1045	.1273	.7682	-
Saturday	0958-1018	44	80	371	5	500	.0880	.1600	.7420	.0100
Saturday	1055-1115	49	66	464	3	582	.0842	.1134	.7973	.0052
						Mean	.0903	.1441	.7587	-

TABLE 4.8: PROPORTION OF EACH VEHICLE CLASS INCLUDING THOSE MAKING U-TURNING IN TOLL BOOTH QUEUE BASED ON ACTUAL COUNTING

DAY OF WEEK	PERIOD	LANE						U-TURNING VEHICLES	TOTAL	PROPORTION OF			
		1	2	3	4	5	6			TRUCKS (α_T)	BUSES (α_B)	CARS (α_C)	U-TURN VEH. (α_U)
Friday	1159-1219	48	28	27	25	81	50	316	575	.0835	.0956	.2713	.5496
Friday	1514-1534	35	43	35	37	62	77	368	657	.0533	.1187	.2679	.5601
Saturday	1233-1253	41	47	50	66	93	87	191	575	.0713	.1687	.4278	.3322
Friday	1123-1143	28	21	19	30	35	31	216	380	.0737	.1053	.2526	.5684
Friday	0945-1005	7	36	34	27	41	53	244	442	.0165	.1584	.2738	.5520
Friday	1007-1027	10	30	32	33	36	30	213	384	.0260	.1615	.2578	.5547
Saturday	1031-1051	17	27	32	32	52	73	171	404	.0421	.1460	.3886	.4233
								Mean for Saturdays only		.0567	.1574	.4082	.3778

TABLE 4.9.: SERVICE TIME DATA FOR TOLL BOOTH CHANNELS

DAY OF WEEK	PERIOD	SERVICE TIMES (Sec.)					
		LANE 1	LANE 2	LANE 3	LANE 4	LANE 5	LANE 6
Saturday	1155-1215	14.3	14.4	14.1	-	-	-
Saturday	1307-1327	-	-	-	9.4	7.9	9.4
Saturday	1500-1520	18.0	14.0	14.5	-	-	-
Saturday	1615-1635	-	-	-	8.7	9.2	9.6
Saturday	1155-1215	12.6	14.5	-	-	-	-
Saturday	1535-1555	-	-	11.9	7.8	-	-
Saturday	1250-1310	15.6	10.0	15.9	-	-	11.7
Saturday	1435-1455	-	9.5	-	9.7	-	12.3
Saturday	1529-1549	17.2	7.7	9.5	21.5	7.8	18.2
Saturday	1450-1510	-	8.5	5.7	-	-	12.2
Saturday	1227-1247	10.0	10.8	7.1	13.7	15.4	-
Saturday	1345-1405	13.9	-	-	-	-	-
Saturday	1320-1340	21.8	-	-	-	7.4	10.6
Saturday	1400-1420	26.2	13.5	16.1	13.6	7.9	10.4
MEAN		16.6	11.43	11.85	12.1	9.27	11.86

4.2.3. Types Of Results Obtained and Presentation of These Results

Having obtained the inputs, it was then necessary to generate a table of data for use with the programme. This data was generated with variations of the inputs in order to study the system under varied conditions of input. A thorough consideration was given to this, considering that results generated will depend on the variations of input to the computer and that each set of data will give a different

system response depending on what an investigator is interested in.

Thus the following sets of data were generated for each vehicle type in this study:

(a) Validation data: Here, inputs are selected such that the results obtained can be used to show how reasonably close the model approximates the realities of the system. Table 4.4 contains some results of such validation.

(b) Working tables or curves data: The data here are so selected so that some of the predicted system measures can be linked to the others either through graphs or tables By so doing, curves or tables which could serve as a working tool are prepared as practical guide in the study of the system. For this study the utilization factor has been selected as the independent variable and its relationship with some other measures like the mean waiting time in queue, customer loss ratio and the mean queue length is plotted in the graphs of Figure 4.3 and Figure 4.4. This method could be used to produce extensive practical results such that once a value of system measure is known through field measurements, other system measures can then be determined. Table 4.5 shows some more of these results.

(c) Data useful for design of optimum solutions: Here, so many options of the facility organization are suggested and enough data is generated to see how these different options compare with each other. The aim here is to use the results generated by these options to build up tables or graphs that are useful in the determination of optimum solution. For

TABLE 4.2.: OBSERVED AND MODEL PREDICTED QUEUE LENGTH PROBABILITIES FOR TRUCKS

OBSERVED QUEUE LENGTH (X)	OBSERVED FREQUENCY (F_O)	OBSERVED PROBABILITY P(X)	PREDICTED PROBABILITY P(X)	EXPECTED FREQUENCY (F_E)
0	0	.000	.084	5
1	2	.034	.043	3
2	6	.102	.044	3
3	3	.051	.045	3
4	3	.051	.046	3
5	0	.000	.047	3
6	5	.085	.048	3
7	10	.169	.050	3
8	8	.136	.051	3
9	10	.169	.052	3
10	6	.102	.053	3
11	4	.068	.054	3
12	1	.017	.055	3
13	1	.017	.057	3
14	0	.000	.058	3
15	0	.000	.059	3
16	0		.061	4
17	0	.000	.062	4
18	0	.000	.032	2
Σ	59	1.000	1.000	60

TABLE 4.3.: OBSERVED AND MODEL PREDICTED QUEUE LENGTH PROBABILITIES FOR BUSES

OBSERVED QUEUE LENGTH (X)	OBSERVED FREQUENCY (F_O)	OBSERVED PROBABILITY (F(X)	PREDICTED PROBABILITY P(X)	EXPECTED FREQUENCY (F_E)
0	13	.220	.470	28
1	0	.000	.118	7
2	1	.017	.092	5
3	0	.000	.071	4
4	0	.000	.056	3
5	0	.	.043	3
6	1	.017	.034	2
7	2	.034	.026	2
8	0	.000	.020	1
9	0	.000	.016	1
10	2	.034	.012	1
11	1	.017	.010	1
12	2	.034	.010	1
13	3	.051	.006	1
14	3	.051	.005	0
15	3	.051	.004	0
16	2	.034	.003	0
17	2	.034	.002	0
18	2	.034	.002	0
19	0	.000	.001	0
20	3	.051	.001	0
21	0	.000	.001	0
22	2	.034	.001	0
23	1	.017	.001	0
24	1	.017	.000	0
25	4	.068	.000	0
26	1	.017	.000	0
27	0	.000	.002	0
28	3	.051	.000	0
29	0	.000	.000	0
30	2	.034	.000	0
31	1	.017	.000	0
32	1	.017	.000	0
33	1	.017	.000	0
36	2	.034	.000	0
M	59	1.000	1.000	60

TABLE 4.4.: OBSERVED AND MODEL PREDICTED MEASURES OF EFFECTIVENESS

VEHICLE TYPE	OBSERVED UTILIZATION $\rho'_{OBS.}$	PREDICTED UTILIZATION $\rho'_{PRED.}$	OBSERVED WAITING TIME IN QUEUE $W_{q\ OBS.}$	PREDICTED WAITING TIME IN QUEUE $W_{qPRED.}$ (Sec.)	OBSERVED CUSTOMER LOSS RATIO LR_{OBS} (Sec.)	PREDICTED CUSTOMER LOSS RATIO $LR_{PRED.}$
Cars	.560	1.290	62.650	46.100	5.900	4.170
Buses	.210	.780	66.800	17.640	5.600	1.530
Trucks	.450	.990	104.300	123.840	4.780	7.46

this study, the options considered are shown in the results Table 4.6 and here: (a) the various combinations of numbers of servers at both stages have been studied; (b) the maximum number of waiting customers has also produced a set of options - this will show in particular what effect waiting room has on blocking; (c) with everything fixed, the fixed probability of vehicles getting to Stage 2 has also been varied to produce the last set of options in order to study particularly the effect of removing the U-turning point (at least to another place).

All results are presented graphically or in a tabular form throughout and should be useful decision tools in thinking about improvement. The comparative Table 4.10 is an example of such a table of results showing measures that describe the system performance as it relates to cars, trucks and buses in the present organized state of the facility.

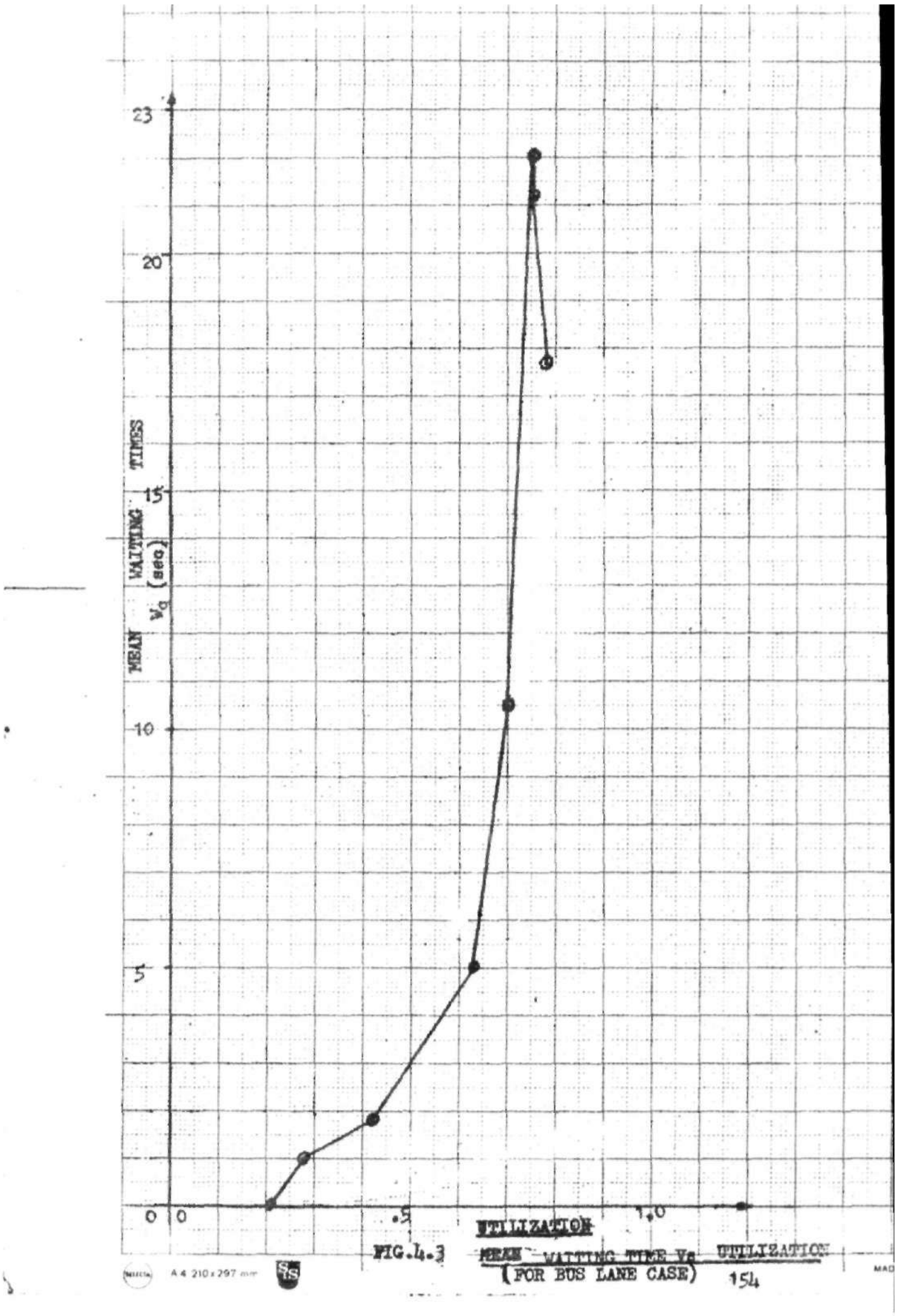


FIG.4.3 MEAN WAITING TIME Vs UTILIZATION (FOR BUS LANE CASE)

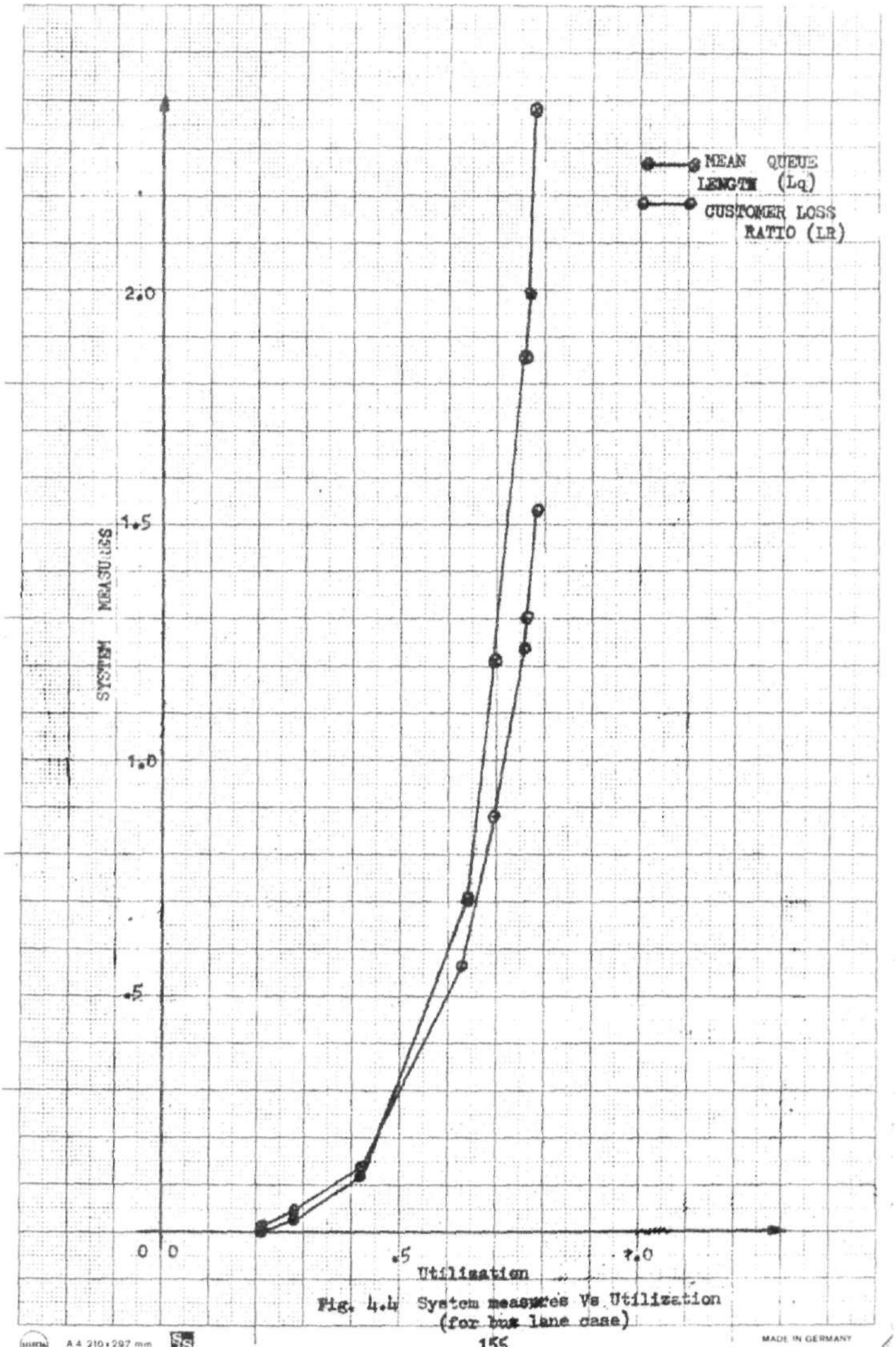


Fig. 4.4 System measures Vs Utilization (for bus lane case)

A 4 210 x 297 mm
MADE IN GERMANY

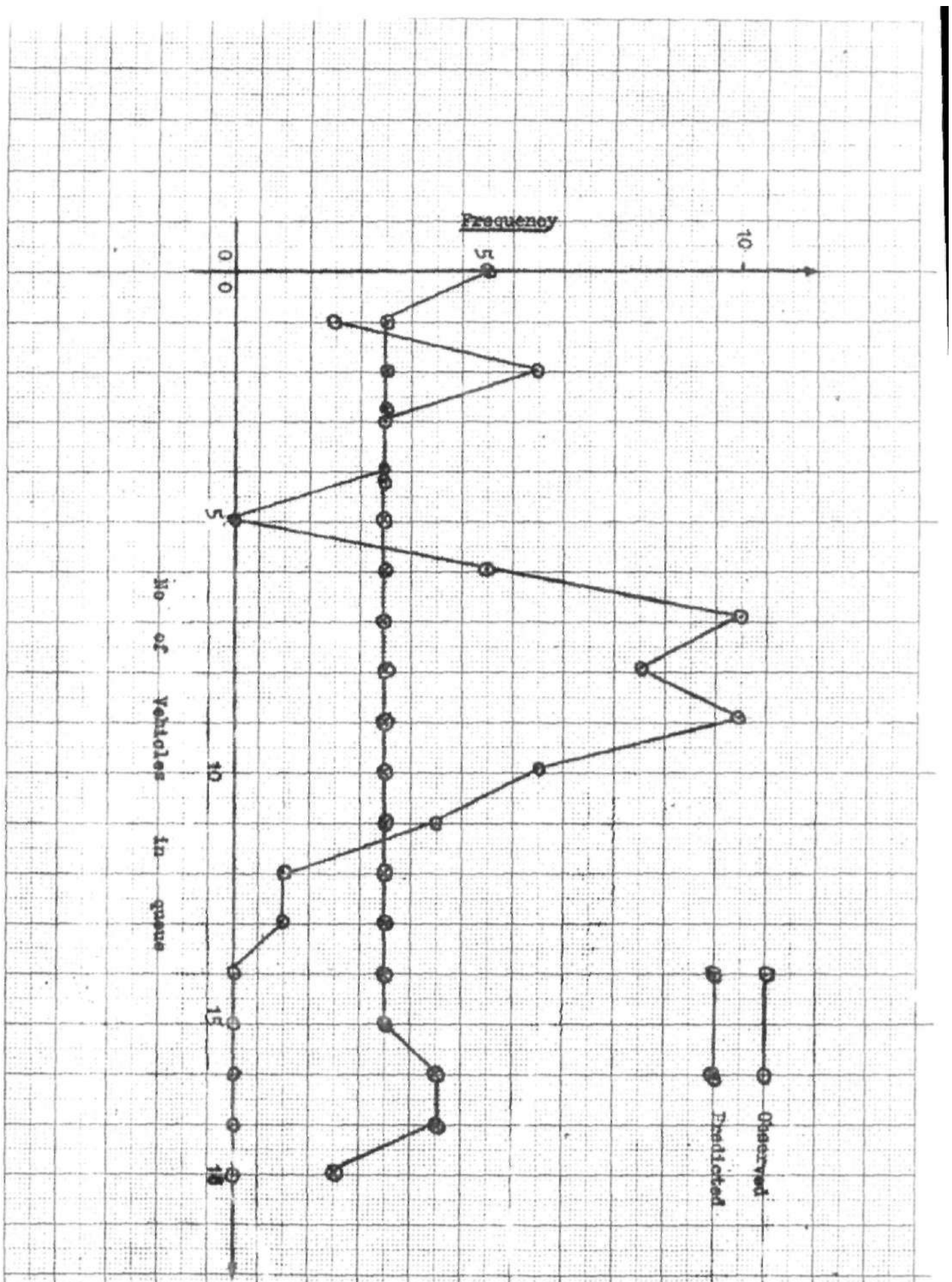


Fig. 4.5 Observed and Predicted frequency distribution of vehicle queue lengths (Truck lane lane_)

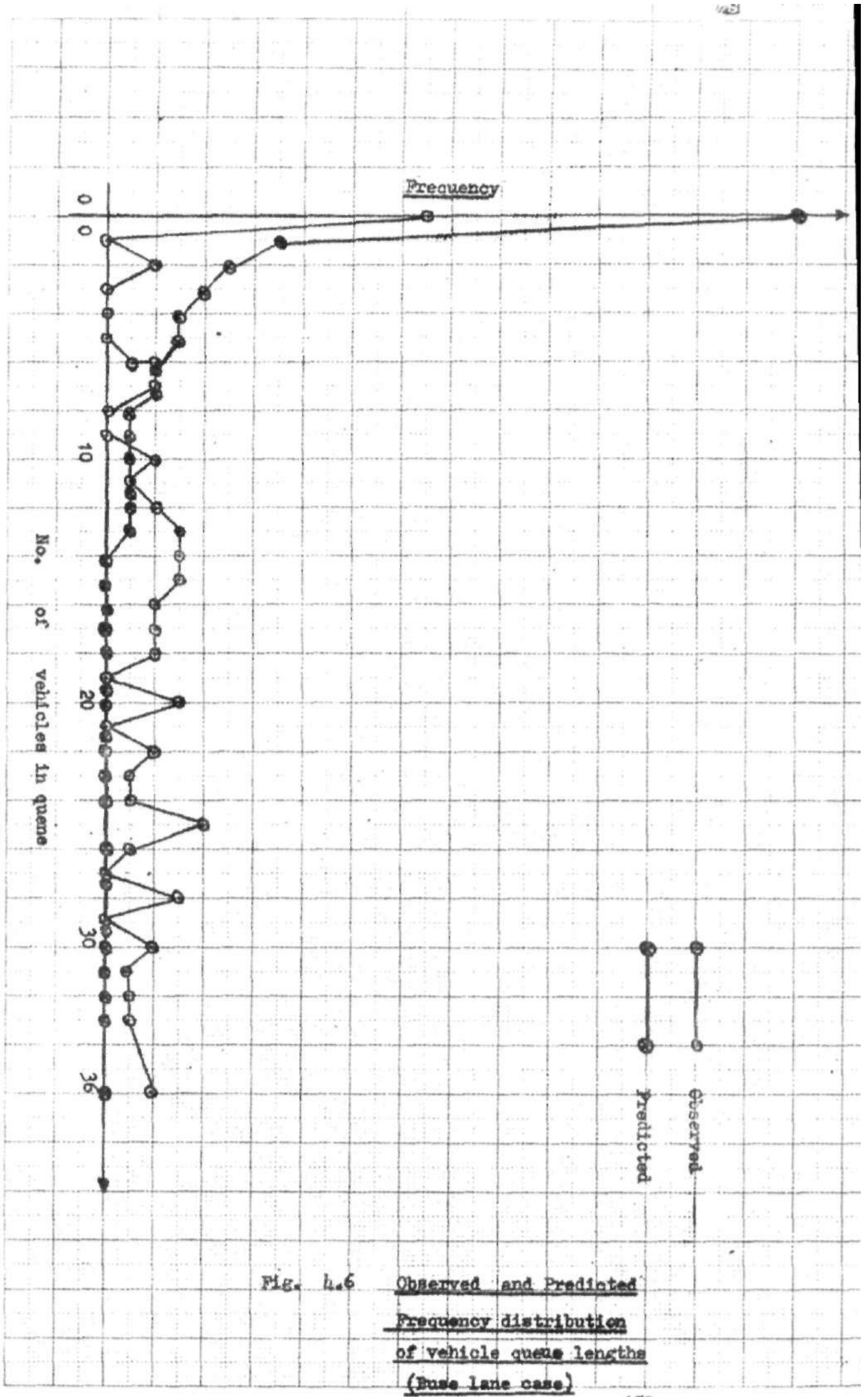


Fig. 4.6 Observed and Predicted Frequency distribution of vehicle queue lengths (Buse lane case)

4.2.4. Choice Of System Effectiveness Measures

All measures determined here are important in understanding the system but the customer loss ratio, the mean queue length and the mean waiting time could be considered as more important for this system. The customer loss ratio is normally very useful under situations of this type where customers are requiring only short service. Similarly, a motorist coming to queue will depend, for most of his decisions, on the mean queue length in terms of the number of vehicles he is actually observing. This can lead him to very many unpredictable behaviours which could even result in a breakdown of the queue. Also the mean waiting time is a very good decision parameter to a motorist who might be wanting to use the facility. At least he can appreciate off-hand the average time he might have to waste while queueing and if possible make decisions that could balance this effect.

4.3. MODEL VALIDATION

Validation is an essential part of model building and application of existing models to real life problems. Briefly it seeks to justify the model with regards to its prediction of the system behaviour. Often this is done either by comparing results obtained from the model in question with those of a known model that works well for the system or by using simulation results as a basis of comparison. Another popular method of achieving this is

to compare the system effectiveness measures predicted by the model with those observed and measured.

In this research, the last option is adopted and the comparison with the observed values is carried out by means of Tables 4.2, 4.3 and 4.4. Tables 4.2 and 4.3 show the queue length probabilities as observed and as predicted while the Table 4.5 shows a direct comparison of some measures of effectiveness predicted by the model and those actually observed. Exploratory graphical techniques have also been used resulting in the frequency diagrams of Figures 4.5 and 4.6 for the observed and predicted frequencies.

Statistical validation through the χ^2-test or other tests is also a useful tool for situations where a relatively high accuracy is required. For the kind of solution sought, however, the tables and graphs collectively justify that the model at least gives approximate answers to the problems of the system. Further statistical tests were therefore not considered necessary.

The major results with comments, are further summarised in the next and final chapter of this thesis.

TABLE 4.5.: VALUES OF UTILIZATION AGAINST SOME SYSTEM MEASURES

VEHICLE TYPE	MEAN WAITING TIME IN QUEUE W_q (Sec.)	CUSTOMER LOSS RATIO LR	MEAN NO. IN QUEUE L_q	BLOCKING PROBABILITY P_{BLS}	UTILIZATION ρ'
Cars	.000	.016	.014	.000	.295
"	.000	.030	.031	.000	.349
"	.360	.084	.116	.000	.458
"	.720	.094	.133	.000	.471
"	1.080	.110	.162	.000	.491
"	3.240	.142	.223	.000	.524
Cars	46.080	4.170	16.170	.293	1.293
Buses	.000	.017	.007	.000	.210
"	1.080	.041	.023	.000	.279
"	1.800	.141	.118	.000	.419
"	5.040	.563	.707	.000	.629
"	10.440	.865	1.210	.000	.698
"	22.320	1.235	1.863	.000	.754
"	21.240	1.298	1.978	.000	.762
Buses	17.640	1.527	2.378	.0001	.779
Trucks	10.800	.449	.184	.000	.409
"	5.040	.712	.349	.000	.490
"	8.640	.858	.451	.000	.525
Trucks	123.840	7.460	7.384	.0316	.990

TABLE 4.6.: SUGGESTED OPTIONS AND SOME OF THEIR PREDICTED SYSTEM MEASURES

OPTION NO.	COMMENTS	C	S	MEAN QUEUE LENGTH L_q	MEAN WAITING TIME IN QUEUE W_q (Sec)	LOSS RATIO LR	BLOCKING PROBABILITIES P_{BLS}	PROBABILITY OF EMPTY SYSTEM P_{00}
1	For these first 7 options all variables are fixed while different combinations of C & S are tried	1	1	15.479	171.000	15.479	.635	.000
2		1	2	29.407	162.360	14.704	.270	.000
3		1	3	8.152	32.760	2.978	.001	.022
4		2	1	2.408	15.840	1.424	.691	.000
5		2	2	8.276	31.320	2.824	.465	.000
6	Existing arrangement	2	3	16.179	46.080	4.172	.293	.000
7	1 more channel added to Existing No. at each stage	3	4	7.578	12.240	1.122	.179	.000
		WAITING ROOM N						
8	Every variable is fixed for these next four options while N is varied	80		30.504	86.760	7.866	.283	.000
9		65		24.697	70.200	6.369	.293	.000
10		25		9.211	26.280	2.375	.294	.000
11		10		3.405	9.720	.8782	.296	.000
		FIXED PROBABILITY β						
12	For these last four options all variables are fixed except the fixed probability of leaving Stage 1 and getting to Stage 2	.950		9.912	39.240	3.554	.493	.000
13		.800		9.796	25.200	2.291	.426	.000
14		.300		7.305	18.000	1.631	.001	.017
15		.100		.0342	0.360	.037	.000	.391

TABLE 4.6b: BRIEF DESCRIPTION OF THE SUGGESTED OPTIONS

OPTION NO.	DESCRIPTION
1.	One policeman in series with one toll collector. This implies collapsing the services at each of the channels into one super service facility at a single channel for both stages of the queueing system
2.	One policeman in series with two toll collectors. Here, while the two police channels are replaced by one super service channel, the three toll booth normally used for cars are reduced to just two channels
3.	One policeman in series with three toll collectors. This is the case of one super service channel replacing the two police channels while the three car lanes are retained.
4.	The case of two policemen in series with one toll collector. Here the three toll lanes constitute one super channel while the police lanes remain unchanged.
5.	Two policemen in series with two toll collectors.
6.	The existing arrangement of two policemen in series with three toll collectors.
7.	Three policemen in series with four toll collectors, implying an addition of one extra channel at each stage.
All other conditions being fixed:	
8.	No. of cars allowed in waiting room is 80
9.	No. of cars allowed in waiting room is 65
10.	No. of cars allowed in waiting room is 25
11.	No. of cars allowed in waiting room is 10
12.	Probability of getting to Stage 2 is .950
13.	Probability of getting to Stage 2 is .800
14.	Probability of getting to Stage 2 is .300
15.	Probability of getting to Stage 2 is .100

TABLE 4.10.: COMPARATIVE TABLE FOR THE DIFFERENT VEHICLE CLASSES USING SOME EFFECTIVENESS MEASURES PREDICTED FOR THE EXISTING FACILITY ORGANIZATION

VEHICLE TYPE	PROBABILITY OF AN EMPTY SYSTEM (P_{00})	MEAN QUEUE LENGTH (L_q)	FACILITY UTILIZATION (ρ')	MEAN WAITING TIME IN QUEUE (W_q) (Sec)	BLOCKING PROBABILITY (P_{BLS})	CUSTOMER LOSS RATIO (LR)
Trucks	.0414	7.384	.9903	128.8400	.0316	7.456
Buses	.1246	2.377	.7785	17.6400	.0001	1.527
Cars	.0000	16.179	1.2926	46.0800	.2926	4.172

CHAPTER 5

RESULTS, COMMENTS, RECOMMENDATIONS AND CONCLUSION

5.1 DISCUSSIONS AND COMMENTS

The major results of this study are presented in tables and graphs of previous chapters and a discussion of these results with comments and recommendations follows in this final chapter of the thesis.

Thus from the traffic flow pattern of Figures 2.1 and 2.2, it can be observed that the number of vehicles arriving at both stages of the queue does not remain constant throughout the observation period. Two distinct peaks appear to occur about the same time for both Fridays and Saturdays although the intensity is much higher on Saturdays and the 'slack' period rates are not statistically too different from the peak values. The traffic flow is characterized by two distinct types of variation: (i) on a macroscopic level, the average rate of flow, and (ii) on a microscopic level, even when the average level of flow remains constant, there are still the random variations of the minute to minute arrivals. For the purpose of this study, the microscopic level of variation has been considered secondary to the macroscopic level of variation in all analysis.

From the coefficients of variation, the histograms and the K-S statistical tests performed on the samples, it is clear that the negative exponential distribution is quite adequate to describe the arrival and service mechanisms in the system.

There is no significant difference between Friday and Saturday data collected for both lanes.

Even the week to week and month to month variations are found to be insignificantly different. This implies that data can be aggregated. Furthermore, stationarity conditions for both the arrivals and service mechanisms are found to be a reasonable assumption for all the time intervals of the day, within the peak periods of traffic, and this is the case irrespective of the week, month or day of week-end.

(ii) The arguments above hold good for the toll booth queues when the lanes appropriate to each vehicle class are subjected to the One-way Anova test. The only defection here is in the car lanes where it is found that the arrivals are different for the different lanes even though the same service can be assumed for them. Aggregation of data and stationarity of arrival and service patterns are therefore reasonable assumptions in most of the other cases.

From Table 4.4 and Figures 4.5 and 4.6, the predicted values appear not to be in a perfect agreement with the observed effectiveness measures but the agreement is good enough to enable one conclude that the Makino model is at least fairly adequate for the system. The mean waiting times (above 60 seconds) are about equal for cars and buses and so also are the customer loss ratios which are close to a value of 6 for both vehicle classes.

The mean waiting time for trucks is the greatest (above 100 seconds) although its associated customer loss ratio is smallest (about 5). Overall the prediction of these effectiveness measures appears to be better for Cars and trucks than for the buses where the predicted values are wider apart from those observed.

Table 4.5 and Figures 4.3 and 4.4 together suggest that blocking in the system does not set in until a certain critical value of the utilization factor is reached at the second stage channels. This critical value for the range of utilization factors used in this experiment, is at least greater than .524 for cars, equal to or greater than .779 for buses and at least greater than .525 for trucks. These asymptotic properties are better appreciated from the graphs under discussion.

The Table 4.6 of available options when considering improvements, as illustrated with the case of the cars shows that depending on the particular measure of effectiveness of interest, differnt options can be substituted for the present system. Thus if it is the general overall aim to minimize mean waiting time and loss ratio, then option 1 is definitely not okay. Rather one would consider option number 4 very seriously for an improved performance of the system. This is because, excluding option 7 which would involve building extra facilities, it gives the lowest values of the mean waiting time and loss ratio. Also for a wide range of values of the maximum waiting room, it can be seen

from options 8 to 11 that while the effectiveness measures appear to increase with increasing size of waiting room, the blocking probability decreases rather gradually with increasing waiting room.

The options 12 to 15 which represent the effects of either reducing or increasing the intensity of turning vehicles show that as the intensity of turning increases, the blocking effect and other measures reduce steadily to zero while the probability of the system being empty increases. This implies that the effect of removing the U-turning point (at least from its present location) is to further increase all other system measures including the blocking probability while the probability of the empty state becomes zero.

The present state of the system indicates that over 75% of all vehicles using the expressway either for the purpose of travelling out of Lagos or for U-turning back to Lagos are cars. The proportion of the vehicles thatare trucks is rather small while that of two-wheeled vehicles is so small to be considered negligible. On the average, the proportion of all vehicles turning is .3800 for Saturdays while that of Fridays is .5600. Similarly the proportion of buses that will get to the toll booths is .8400 while in the case of trucks and cars, this proportion is above .5000 for both cases.

From the comparative results Table 4.10 showing some values of the effectiveness measures for the system as currently organized, one can summarize as follows:

(a) For the car lanes, the probability of being in an empty state (i.e idle servers)is just not there. In fact, from the output sheet, it can be seen that the probability of the number of cars in the system being greater than 33 is about 99.9% which implies that almost for 100% of the time the number of cars present in the system is 33 and more. There is a very low probability of having a small number of cars in the system and thus the system is very likely to continue to block. The mean length in the car queue is 16 which implies about 5 to each lane resulting in a wait of 66 seconds and an over-utilization of the toll collectors. The blocking probability is high and cannot be neglected in analysis while the loss ratio of 6 could be considered unacceptable to a motorist requiring only a small service.

(b) For the bus lanes, the probability of being in the empty state is high enough to warrant concern since this is also an unhealthy queueing situation where servers are idle for most of the time. There is nothing much to worry about on the customers side since the queue length and waiting time are low enough to be considered reasonable. The toll collectors are operating above 70% utilization, while the loss ratio is acceptable.

The blocking probability is for this case negligible for practical purposes and the alternative Jackson model proposed for independent serial queues can be applied to the bus queues.

(c) For the truck lanes, the probability of an empty state is at an aceptable level but the queue length, waiting time and loss ratio of about 7.4 are high enough to warrant concern. For this case, the blocking probability is not negligible for a simpler analytical approach but the toll collectors are being used at near 100% utilization. Referring to the output, it can be seen that the probability of having a large number of trucks is more than that of having small number of trucks in the system. However, unlike the car lane case, at least, there exists a definite probability that small queue lengths and even the empty state will occur. This is also different from the bus lane case where from the output sheet, it is noticed that there is a higher probability of smaller number of buses occuring than do large numbers of buses.

What all these mean is that, from the customer point of view the bus lane arrangements are currently okay but the waste that is associated with idle facilities has to be evaluated against this customer satisfaction. If the customer point of view remains the criterion for evaluation, it is also evident that some improvements must be effected to reduce the fairly long periods of queue and thus the loss ratio.

These improvements have to be balanced against the need to use the facilities effectively as is being done now.

From the above considerations therefore, it could be further argued that the Makino model, with its basic limitations in terms of the simplifying assumptions made in this research, has sufficient merits and is quite adequate for this study. Surely blocking of facilities occurs in parts of the system and except for the case of buses where blocking is negligible, the alternative Jackson model of two independent serial queues cannot be applied with enough justification.

5.2 RECOMMENDATIONS

Although the situation is not as hopeless as patron complaints might paint it, it nevertheless will serve some useful purpose to recommend possible approaches to improve the system performance in such a way that facility costs can be balanced against customer satisfaction.

Generally, modifications can be made to arrivals, service mechanism or the queue discipline. However, for this system, it appears more practicable to modify the service mechanism and this can generally be done as follows: (a) decrease the mean service time; (b)reduce the coefficient of variation of service time (c) arrange that the service times are reduced during periods of more than average congestion;

(d) change the capacity of the system by providing more servers; (e) arrange that the capacity can be increased temporarily either when congestion is observed to be high or when more than the average number of customers are expected to arrive; (f) increase the server availability, either on the average or by arranging that service is more likely to be available when customers are present.

It is recommended that some or all of these be achieved through the following measures:

(i) introduction of incentives and better spaced relief for toll collectors.

(ii) reduction in shift hours

(iii) encouraging motorists to change to faster lanes always or whenever a particular service channel runs into problems.

(iv) intensifying the campaign for the need for correct-fare - only service as this reduces randomness implying a tendency towards constant service.

(v) better management appreciation and approach to the problem before it becomes critical.

(vi) the number of odd numbered cars is always greater than even numbered ones. Hence it might be a good idea to designate one lane to the even numbered cars while the odd numbered cars should have one channel and still share one lane with the cars with either change or other problems.

(vii) In view of the lower traffic on bus lanes, it might also be possible to divert cars to these lanes during critical periods. For this strategy, there must be good organization and experienced toll collectors must be used.

(viii) Services normally improve under the watchful eyes of the queueing manager and it is recommended that either himself or another top official should maintain more regular hours of work at the facility.

Normally, a decrease in mean service time and in the coefficient of variation is always desirable as the decrease in mean service time means more efficient use of the facility while a decrease in coefficient of variation results in more constant service characteristics. The constant rate of service has been shown to yield lower waiting times and improved services to customers [Gaver, (1953)].

However, for this system, what is important is the reduced coefficient of variation and the ratio of the rates of service at both stages and it is therefore more realistic to think in terms of increasing server availability for both stages rather than to think of providing more servers and more facilities which means more capital and operating costs.

Service capacity can also be elected to be increased or decreased temporarily depending on the traffic situation.

Operation at the critical periods therefore calls for innovative action in the form of temporary measures some of which are as suggested above. However any long term measure must consider the use of automation at the service channels and the choice of other system variables such that facility cost and customer satisfaction are balanced. For this purpose it is important to refer again to the list of options shown in Table 4.6 which uses the car case as an example. Of all the possible options into which the existing system can be organized, it is clear that although option 4 provides the best result in terms of customer service (as can be seen from the values of mean waiting time and loss ratio) the use of option 3 seems to provide a more balanced design (in terms of customer satisfaction and facility) than 4 and should be adopted if ever the present arrangement for cars is to be re-organized. Changing the positon of the U-turning point to probably somewhere behind the Police might be more beneficial to the motorists travelling out of Lagos as evidenced by the lower values of mean number in queue, mean waiting time in queue and the customer loss ratio. These can also be achieved by decreasing the waiting room and the choice of any of these options must be backed up by a thorough cost appraisal.

Another recommendation is in the area of the Police services being rendered. The imposition of Policemen on the system clearly means that the rate of arrival at the toll points sometimes depends on the rate at which the Policemen release the vehicles. Thus the poor performance of the queueing system facilities can be due sometimes to the Police holding back motorists even when there are no vehicles in the space between the Police point and the toll booths. This accounts for the sometimes observed high frequency of zero back-up on the individual lanes even though motorists are delayed unnecessarily too long behind the Police. The Police post therefore constitute a very big delay factor for motorists approaching the toll booths and motorists suffer terrible hold-ups before getting into the toll booth queues. It is therefore necessary in the overall interest of the motorists to give serious thought to the question of Policemen being present in the system by studying properly the measures of congestion at the Police queues. Their complete removal from the system or a change in their location may then be further justified.

The most important recommendations in connection with data collection is that which has to do with the method used for the date gathering. It is maintained that, for such systems as the one studied here where there are minute to minute variations and the hazard of collection is so much, the use of human collectors and timing devices is grossly inadequate without being

supplemented with mechanical devices. It would be best however, to collect such data by means of a one-second time lapse film taken from a vantage point on the study site but the financial aspects of such ventures are definitely beyond the financial resources available for this project.

5.3 SUGGESTIONS FOR FURTHER RESEARCH

Throughout the period of this research the following issues are given varying degrees of attention but because this study does not centre on such problems, they have either been completely left out or conveyed via over-simplifying assumptions. Such issues are mentioned here for further research pursuits in this area:

(a) It is appreciated that when a statistical confirmatory test gives evidence that an assumed theoretical distribution is suspect, a favourable option is to use the observed distribution for the service or inter-arrival time. This option and a simulation procedure (because of the mathematical inherent complexity in not using the exponential assumptions for a serial queueing process) is suggested in order to study the effect of Police sampling and their forced idle time on their own service time distribution.

(b) The issue of correlated arrivals and service, and state dependence were not looked into in this study.

They should be further investigated in order to establish the reasonableness of independence assumptions.

(c) An attempt should be made to analyse the system using one of the proposed alternative models as a means of comparison.

(d) The issue of optimization and scheduling of facilities and personnel should be more fully looked into in order to fully solve the operational problems.

(e) It is clear that the study carried out here has only given mean values for the effectiveness measures. For complete solution of a congestion problem it often becomes necessary to know not only the average delay but also the worst delay suffered by an individual unit. It is therefore suggested that another study be done to determine the waiting time distribution for the system.

(f) It must be mentioned quite clearly that the aim of this research is essentially to study the toll booth operations and not just a serial queue. For example, if the Police were not there, it should have been a straightforward case. But now that they are there, it would be enough just to evaluate their effect on the toll booth operations. This police effect is of course accommodated in the model and so specifically, only levels of service at this facility and not behind the Police is covered.

Further research is therefore encouraged in order to obtain operational values of the measures of effectiveness for the Police queues.

5.4 CONCLUSIONS

It is often said that delay at traffic controls such as Police, signal or toll booths is a matter of prejudiced comment by road u ers. It was therefore interesting to analyze objectively the average delays occuring at the Lagos - Ibadan expressway toll booth facility and still come out with such a feeling. Thus although the results show a clear case of customer dissatisfaction in the analysis carried out for cars and trucks, the levels of such indices of dissatisfaction are low enough to be considerd a little bit normal. The bus lanes do not appear to do otherwise about customer satisfaction for they in fact provide the best indices of satisfaction for the customer. There does not seem to be any justification, therefore, for the widely publicised patron complaint about this facility. It is however, understood that the motorists get to the toll booth queue already fuming with frustration and exhaustion as a result of the hold-up already suffered behind the Police check point and are therefore not willing to do another round of such a near 'ordeal'. People are apt to blame the lost time and the plight of the motorist on the operating conditions of the toll facility because they sort of believe that nothing can be done to improve the police service or make them leave their check point since they have full government backing.

They forget however that heaping up blames at wrong quarters never solve a problem:

The ultimate goal is to move in the direction of automatic methods of keeping the toll collection force in correct balance with the traffic requirements at all times in order to achieve optimum conditions within the limitations of the non-controllable variables in the operation. This being so, then this study at least has indicated the way.

One of the remoter aims of this research is to highlight the high incidence of facility over-loading that so much characterizes such facilities almost from inception. Such sights make one feel that proper feasibility studies were not done at the planning, design and construction stages of these facilities or that proper principles of management are not being used in their operation. However, this study has shown another side to the problem - that externally imposed conditions can seriously limit the performance ability of a facility no matter how noble the ideas behind its provision. The discussions here will therefore provide or rather always remind the technical experts, managers, top government officials and any such persons connected with the provision of public facilities that people are becoming more and more aware of their environment and their rights.

Therefore any solution to any problem that touches on the environment must always be seen to becoming from a group that is not only technically competent, but also economically convincing, ecologically sensitive, environmentally alert and politically adept.

APPENDIX 1A

DELAY + INTER-ARRIVAL TIME DATA

DATE: 8/1/82 NAME: EMMA

STARTING TIME: 1230OOPM DURATION: 20 MINS.

POSITION: BEFORE POLICE LANE: B

DAY OF WEEK: FRIDAY INITIAL NO. OF VEHICLES

NO. OF TOLL LANES OPEN: 6 IN LANE: 15

DIRECTION OF TRAFFIC: LAGOS - IBADAN ~~OR IBADAN - LAGOS~~

VEHICLE NO.	REGISTRATION No.	VEHICLE TYPE C = Car B = Bus T = Truck M = Motor cycle	TIME OF ARRIVAL
1	OG 221 AE	C	12 30 19
2	LA 91 K	C	12 30 37
3	OY 266	C	12 30 38
4	LAA 9984 K	C	12 30 43
5	OD 446	C	12 31 04
6	OG 324 R	B	12 31 30
7	OY 9099 A	C	12 31 44
8	LA 7202 SC	C	12 31 47
9	KW 635 AD	C	12 31 55
10	OY 9907 AN	C	12 32 02
11	OG 441 EB	C	12 32 20
12	OC 2222 JA	C	12 32 27
13	OD 3434 A	C	12 32 30
14	LA 143 WS	C	12 32 44
15	LA 4058 WB	C	12 32 55
16	OG 214 R	C	12 33 17
17	OY 9916 A	C	12 33 20
18	KW 1016 A	C	12 33 22
19	OY 8725 AB	C	12 33 24
20	BD 7080 BD	C	12 33 30
21	OY 101 JA	C	12 33 43
22	LZ 772	C	12 33 54
23	OY 2327 LA	B	12 34 03
24	OY 7374 HA	C	12 34 12
25	LA 3252 AH	C	12 34 26
26	OY 8721 YA	C	12 34 32
27	LA 5157 B	C	12 34 38
28	OD 7022 E	C	12 34 42
29	LA 528 WC	C	12 34 56
30	LA 3739 SC	C	12 35 12
31	OY 8223 LA	C	12 35 27

APPENDIX 1A

Vehicle No.	REGISTRATION NO	VEHICLE TYPE C = Car B = Bus T = Truck M = Motor cycle	TIME OF ARRIVAL
32	KW 3464 AA	C	12 35 31
33	Kw 5007 AA	C	12 35 41
34	OG 7849 J	C	12 36 03
35	LA 3367 ME	C	12 36 11
36	OY 905 FA	C	12 36 20
37	LA 871 A	C	12 36 25
38	OG 6090 Y	C	12 36 28
39	AEP 7825 S	C	12 37 05
40	OY 840 AE	C	12 37 12
41	OD 609 W	C	12 37 18
42	OD 217 C	C	12 37 24
43	LX 101	C	12 37 27
44	OD 888 H	C	12 37 29
45	OG 723 B	C	12 37 47
46	LA 6409 A	C	12 37 56
47	LA 166E	C	12 38 07
48	LA 759 J	C	12 38 20
49	OD 4932 R	C	12 38 27
50	OG 7996	C	12 38 29
51	LA 111 S	C	12 38 30
52	LSC 973	C	12 38 46
53	OY 672 A	C	12 38 50
54	OY 2573	C	12 39 09
55	BD 378	C	12 39 22
56	KW 701 A	C	12 39 34
57	LA 244 B	C	12 39 49
58	KW 59 AC	C	12 39 54
59	LAB 52	C	12 40 04
60	FGN 273	C	12 40 13
61	LA 66 MD	C	12 40 18
62	OY 8 B	C	12 40 25
63	OD 73 E	C	12 40 30
64	OG 287	C	12 40 36
65	LAA 199	C	12 40 41
66	OD 859 K	C	12 40 45
67	OG 88 A	C	12 41 23
68	LA 498 W	C	12 41 39
69	WP 768	C	12 41 50
70	RV 998	C	12 41 53
71	BDSG 159	C	12 42 03
72	LA 973 K	C	12 42 33
73	KD 961 A	C	12 42 54
74	KN 496	C	12 43 03
75	LA 446 F	C	12 43 08
76	LAB 591	C	12 43 12
77	BD 886 U	C	12 43 17
78	KD 996 D	C	12 43 19
79	BD 2478 F	C	12 43 20

APPENDIX 1B

DELAY : SERVICE TIME DATA

DATE: 23/1/82 NAME: GBED

TIME: 14000 PM DURATION: 20 MINS

POSITION: TOLL BOOTH LANE: CAR LANE 4

DAY OF WEEK: SATURDAY INITIAL NO. OF VEHICLES IN LANE: 7

NO OF TOLL LANES OPEN 6

DIRECTION OF TRAFFIC: LAGOS – IBADAN

VEHCILE NO.	VEHICLE REGISTRATION NO.	TIME OF ARRIVAL	TIME OF DEPARTURE	CONDITION OF SERVICE i.e. C FOR CUSTOM PROBLEM
1	LA 3006 ME	14 00 11	14 02 17	C
2	LA 288 W	14 03 35	14 04 06	C
3	OG 963 R	14 04 07	14 04 21	
4	LY 975	14 04 24	14 04 43	C
5	BD 998 U	14 04 46	14 04 56	
6	BD 4019	14 04 59	14 05 11	
7	LA 944 IM	14 05 15	14 05 28	
8	LAF 55 K	14 05 32	14 05 36	
9	LAB 596	14 05 41	14 05 50	
10	OY 6064	14 05 54	14 06 07	
11	LZ 9981	14 06 11	14 06 20	
12	OY 449 A	14 06 24	14 06 44	
13	LA 4353	14 06 48	14 06 58	
14	LA 9172	14 07 25	14 07 47	
15	WE 4833	14 08 05	14 08 10	
16	OY 772	14 08 15	14 08 45	
17	KW 974	14 08 49	14 09 00	
18	OD 1860 K	14 09 06	14 09 17	
19	OY 9381 L	14 09 23	14 09 40	
20	OD 969 Y	14 09 41	14 09 54	
21	LAC 545	14 09 57	14 10 07	
22	LY 6978	14 10 14	14 10 20	
23	LA 177 KD	14 10 25	14 10 27	
24	BD 777	14 10 33	14 10 46	
25	BDSG 785	14 10 56	14 11 05	
26	OG 303 R	14 11 17	14 11 36	
27	LA 607 M	14 11 38	14 11 50	
28	LAD 9719	14 12 11	14 12 15	
29	OG 14678	14 12 35	14 12 38	
30	OY 629	14 12 45	14 12 51	
31	LA 8865	14 12 56	14 13 08	
32	OY 4080 B	14 13 20	14 13 45	
33	KD 977	14 13 55	14 14 06	
34	OD 1090 M	14 14 08	14 14 27	
35	KN 1607 F	14 14 31	14 14 41	
36	KD 9763	14 14 44	14 14 51	
37	LAA 714 K	14 14 55	14 15 01	
38	LAA 2070	14 15 06	14 15 23	
39	FGN 8435	14 17 05	14 17 06	

APPENDIX 1B

vehicle No	VEHICLE REGISTRATION NO.	TIME OF ARRIVAL	TIME OF DEPARTURE	CONDITION OF SERVICE i.e. G FOR G AND P FOR PROBLEM
41	LAC 474	14 17 45	14 17 51	
42	LY 498 A	14 17 56	14 18 11	
43	LAB 471 D	14 19 55	14 20 08	
44	OG 7114 R	14 20 12	14 20 14	
45	LA 774 MC	14 20 28	14 20 41	
46	KV 191 F	14 20 45	14 20 56	
47	OYA 828 A	14 21 00	14 21 15	
48	LA 2008 E	14 21 20	14 21 22	
49	OY 441 Y	14 21 26	14 21 38	
50	KD 1182 D	14 21 42	14 21 44	
51	OY 424 Y	14 21 48	14 21 52	
52	OY 973 A	14 21 56	14 21 57	

APPENDIX 1C

DELAY + SERVICE TIME DATA

DATE: 23/1/82 NAME: EMMAN

STARTING TIME: 143000 P M DURATION: 20 MINUTES

POSITION: AT POLICE POINT LANE: A

DAY OF WEEK: SATURDAY INITIAL NO. OF VEHICLES: -

NO OF TOLL LANES OPEN: 6

DIRECTION OF TRAFFIC: LAGOS-IBDAN ~~OR IBADAN - LAGOS~~

VEHICLE NO.	TIME OF ARRIVAL	TIME OF DEPARTURE	TYPE OF VEHICLE C = Car B = Bus T = Truck M = Motorcycle
1	14 30 07	14 30 08	C
2	14 30 13	14 30 14	C
3	14 30 19	14 30 22	C
4	14 30 24	14 30 26	C
5 5	14 30 29	14 30 30	C
6	14 30 32	14 30 33	C
7	14 30 34	14 30 36	C
8	14 30 40	14 30 42	C
9	14 30 41	14 30 43	C
10	14 30 44	14 30 45	C
11	14 30 59	14 31 00	C
12	14 31 10	14 31 11	C
13	14 31 34	14 31 36	C
14	14 31 40	14 31 42	C
15	14 31 45	14 31 46	C
16	14 32 01	14 32 03	C
17	14 32 30	14 32 31	C
18	14 32 43	14 32 44	C
19	14 33 13	14 33 16	C
20	14 33 38	14 33 45	C
21	14 33 59	14 34 07	C
22	14 34 20	14 34 22	C
23	14 34 25	14 34 28	C
24	14 34 40	14 34 44	C
25	14 34 47	14 34 49	C
26	14 34 52	14 35 01	C
27	14 35 11	14 35 20	B
28	14 35 25	14 35 26	C
29	14 36 02	14 36 08	C
30	14 36 10	14 36 15	C
31	14 36 19	14 36 21	C
32	14 36 27	14 36 32	B
33	14 36 42	14 36 47	B
34	14 37 00	14 37 08	C
35	14 37 10	14 37 11	C
36	14 37 14	14 37 16	C
37	14 37 22	14 37 23	C

APPENDIX 1C

VEHICLE NO.	TIME OF ARRIVAL	TIME OF DEPARTURE	TYPE OF VEHICLE C = Car B = Bus T = Truck M = Motorcycle
38	14 37 25	14 37 28	C
39	14 37 31	14 37 35	C
40	14 37 40	14 37 46	C
41	14 38 02	14 38 06	C
42	14 38 14	14 38 16	C
43	14 38 19	14 38 21	C
44	14 38 22	14 38 24	B
45	14 38 26	14 38 30	B
46	14 39 05	14 39 07	T
47	14 39 15	14 39 20	T
48	14 39 24	14 39 27	T
49	14 39 40	14 39 42	C
50	14 39 48	14 39 50	C
51	14 39 57	14 39 59	B
52	14 40 07	14 40 10	B
53	14 40 40	14 40 44	C
54	14 40 55	14 40 57	C
55	14 41 00	14 41 05	C
56	14 41 11	14 41 12	C
57	14 41 16	14 41 18	C
58	14 42 00	14 42 01	C
59	14 42 10	14 42 12	C
60	14 42 20	14 42 28	C
61	14 42 30	14 42 33	C
62	14 42 35	14 42 37	C
63	14 42 42	14 42 44	C
64	14 42 54	14 42 58	C
65	14 43 05	14 43 10	C
66	14 43 16	14 43 19	C
67	14 43 21	14 43 22	C
68	14 43 23	14 43 26	C
69	14 43 29	14 43 30	C
70	14 43 34	14 43 36	C
71	14 43 39	14 43 42	C
72	14 43 43	14 43 47	C
73	14 43 54	14 43 57	C
74	14 44 03	14 44 05	C
75	14 44 17	14 44 18	C
76	14 44 20	14 44 24	C
77	14 44 30	14 44 40	C
78	14 44 45	14 44 50	C
79	14 44 55	14 44 57	C
80	14 45 02	14 45 05	C
81	14 45 30	14 45 38	C
82	14 45 40	14 45 43	C
83	14 45 50	14 45 53	C
84	14 46 01	14 46 06	B
85	14 46 10	14 46 11	C
86	14 46 15	14 46 19	C
87	14 46 22	14 46 24	C
88	14 46 29	14 46 30	C
89	14 46 36	14 46 38	B

APPENDIX 1C

VEHICLE NO.	TIME OF ARRIVAL	TIME OF DEPARTURE	TYPE OF VEHICLE C = Car B = Bus T = Truck M = Motorcycle
90	14 46 44	14 46 50	B
91	14 47 01	14 47 02	C
92	14 47 06	14 47 10	C
93	14 47 16	14 47 19	C
94	14 47 22	14 47 24	C
95	14 47 31	14 47 34	C
96	14 47 38	14 47 39	B
97	14 47 43	14 47 46	B
98	14 47 56	14 48 00	C
99	14 48 06	14 48 08	C
100	14 48 20	14 48 22	C
101	14 48 30	14 48 33	C
102	14 48 43	14 48 45	C
103	14 48 46	14 48 49	C
104	14 48 53	14 48 55	C
105	14 49 07	14 49 08	C
106	14 49 18	14 49 24	C
107	14 49 28	14 49 29	T
108	14 49 40	14 49 43	C
109	14 49 49	14 49 50	C
110	14 50 01	14 50 04	C
111	14 50 13	14 50 16	C
112	14 50 30	14 50 32	C
113	14 50 39	14 50 40	C
114	14 50 50	14 50 52	C
115	14 50 55	14 50 56	C
116	14 50 59	14 51 00	C
117	14 51 02	14 51 04	C

APPENDIX 1D

DELAY & SERVICE TIME DATA

DATE: 23/1/82 — NAME: ABUA

STRATING TIME: 143000 PM — DURATION: 20 MINUTES

POSITION: AT POLICE POINT — LANE: B

DAY OF WEEK: SATURDAY — TOTAL NO. OF VEHICLES -

NO OF TOLL LANES OPEN: 6

DIRECTION OF TRAFFIC: LAGOS - IBADAN OR ~~IBADAN - LAGOS~~

VEHICLE NO.	TIME OF ARRIVAL	TIME OF DEPARTURE	TYPE OF VEHICLE C = Car B = Bus T = Truck M = Motorcycle
1	14 30 04	14 30 19	C
2	14 30 23	-	C
3	14 30 27	-	C
4	14 30 33	-	T
5	14 30 38	-	C
6	14 30 40	-	C
7	14 30 46	-	C
8	14 30 49	-	C
9	14 30 53	-	C
10	14 30 57	-	C
11	14 31 01	-	C
12	14 31 08	-	C
13	14 31 10	-	C
14	14 31 11	-	C
15	14 31 19	-	C
16	14 31 26	-	C
17	14 31 32	-	C
18	14 31 35	-	C
19	14 31 38	-	C
20	14 31 43	-	C
21	14 31 49	14 32 03	B
22	14 32 06	14 32 09	C
23	14 32 16	-	T
24	14 32 25	-	C
25	14 32 29	-	C
26	14 32 32	-	T
27	14 32 39	-	C
28	14 32 49	-	C
29	14 32 52	-	C
30	14 33 09	-	C
31	14 33 11	-	C
32	14 33 19	-	C
33	14 33 20	-	C
34	14 33 22	-	C
35	14 33 30	-	C
36	14 33 34	-	C
37	14 33 40	-	C
38	14 33 43	-	C
39	14 33 48	-	C
40	14 33 54	-	C

APPENDIX 1D

VEHICLE NO	TIME OF ARRIVAL	TIME OF DEPARTURE	TYPE OF VEHICLE C = Car B = Bus T = Truck M = Motorcycle
41	14 34 02	-	C
42	14 34 05	-	C
43	14 34 07	-	C
44	14 34 13	-	B
45	14 34 16	-	C
46	14 34 19	-	C
47	14 34 25	-	C
48	14 34 31	-	C
49	14 34 35	-	C
50	14 34 39	-	C
51	14 34 40	-	C
52	14 34 44	-	C
53	14 34 51	-	C
54	14 34 57	-	C
55	14 34 59	-	C
56	14 35 09	-	T
57	14 35 23	-	C
58	14 35 28	-	C
59	14 35 31	-	B
60	14 35 56	-	C
61	14 35 59	-	C
62	14 36 02	-	C
63	14 36 09	-	C
64	14 36 13	-	C
65	14 36 16	-	C
66	14 36 31	14 36 45	B
67	14 36 49	-	C
68	14 36 54	-	C
69	14 36 55	-	C
70	14 37 00	-	C
71	14 37 05	-	C
72	14 37 10	-	C
73	14 37 14	-	B
74	14 37 17	-	C
75	14 37 23	-	C
76	14 37 26	-	C
77	14 37 31	-	C
78	14 37 42	-	C
79	14 37 45	-	C
80	14 37 46	-	C
81	14 38 26	-	C
82	14 38 33	-	C
83	14 38 39	-	C
84	14 38 46	-	C
85	14 38 55	-	C
86	14 39 01	-	C
87	14 39 10	-	C
88	14 39 14	-	C
89	14 39 20	-	C
90	14 39 26	-	C
91	14 39 30	14 39 34	C
92	14 39 39	-	C
93	14 39 43	-	C

APPENDIX 1D

VEHICLE NO.	TIME OF ARRIVAL	TIME OF DEPARTURE	TYPE OF VEHCILE C = Car B = Bus T = Truck M = Motorcycle
94	14 39 48	-	C
95	14 39 52	-	C
96	14 39 59	-	C
97	14 40 05	-	C
98	14 40 10	-	C
99	14 40 13	-	C
100	14 40 18	-	C
101	14 40 22	14 40 24	C
102	14 40 31	14 40 34	B
103	14 40 41	-	C
104	14 40 48	-	C
105	14 40 54	-	C
106	14 40 59	-	C
107	14 41 05	-	C
108	14 41 10	-	C
109	14 41 16	-	C
110	14 41 20	-	C
111	14 41 26	-	C
112	14 41 36	-	C
113	14 41 38	-	C
114	14 41 43	-	C
115	14 41 48	-	C
116	14 41 59	-	C
117	14 42 03	-	T
118	14 42 12	-	C
119	14 42 16	14 42 35	C
120	14 42 38	-	C
121	14 42 41	-	C
122	14 42 47	14 42 55	B
123	14 43 04	-	C
124	14 43 09	14 43 16	C
125	14 43 21	-	C
126	14 43 25	-	C
127	14 43 28	-	C
128	14 43 34	-	C
129	14 43 39	-	C
130	14 43 42	-	T
131	14 43 52	-	C
132	14 43 54	-	C
133	14 43 58	-	C
134	14 44 02	-	C
135	14 44 06	-	C
136	14 44 13	-	C
137	14 44 18	-	C
138	14 44 22	-	C
139	14 44 25	14 44 42	B
140	14 44 50	-	C
141	14 44 52	-	C
142	14 44 58	-	C
143	14 45 03	-	C
144	14 45 13	14 45 17	C

APPENDIX 1D

VEHICLE NO.	TIME OF ARRIVAL	TIME OF DEPARTURE	TYPE OF VEHICLE C = Car B = Bus T = Truck M = Motorcycle
145	14 45 19	-	C
146	14 45 27	14 45 54	C
147	14 45 58	-	C
148	14 46 01	-	C
149	14 46 08	-	C
150	14 46 13	-	C
151	14 46 19	-	C
152	14 46 25	-	C
153	14 46 30	-	C
154	14 46 36	-	C
155	14 46 47	-	C
156	14 47 02	14 47 43	T
157	14 47 51	-	C
158	14 47 58	-	C
159	14 48 03	-	C
160	14 48 12	-	B
161	14 48 17	-	C
162	14 48 27	-	C
163	14 48 38	-	C
164	14 48 52	-	C
165	14 49 04	-	C
166	14 49 18	-	C
167	14 49 19	-	C
168	14 49 26	-	C
169	14 49 56	-	T

APPENDIX 1 E

INTER-ARRIVAL TIME ANALYSIS DATA

DATE: 24/10/81 LANE: B

STARTING TIME: 105500 DURATION: 30 MINUTES

DAY OF WEEK: SATURDAY INITIAL NO. OF VEHICLES IN LANE: 20

POSITION: BEFORE POLICE

DIRECTION OF TRAFFIC: LAGOS - IBADAN ~~OR IBADAN-LAGOS~~

Vehicle No.	Vehicle Type C=Cars B=Bus T=Truck M=Motor cycle	Time of arrival	Inter-arrival Time for all Vehicles T_a	Inter-arrival Time for Cars T_c	Inter-arrival Time for Buses T_b	Inter-arrival Time for Trucks T_t	Inter-arrival Time for Motor-cycles T_m
1	B	105510	-	-	-	-	-
2	C	105514	4	-	-	-	-
3	C	105515	1	1			
4	B	105525	10	-	15	-	-
5	B	105539	14	-	14	-	-
6	B	105542	3	-	3	-	-
7	C	105546	4	21	-	-	-
8	C	105610	24	24	-	-	-
9	C	105629	19	19	-	-	-
10	T	105706	37	-	-	-	-
11	C	105712	6	43	-	-	-
12	B	105717	5	-	95	-	-
13	C	105724	7	12	-	-	-
14	B	105733	9	-	16	-	-
15	T	105743	10	-	-	37	-
16	B	105853	70	-	80	-	-
17	C	105901	8	97	-	-	-
18	T	105915	14	-	-	22	-
19	C	105924	9	23	-	-	-
20	C	105926	2	2	-	-	-
21	M	105930	4	-	-	-	-
22	C	105938	8	12	-	-	-
23	C	105949	11	11	-	-	-
24	C	105952	3	3	-	-	-
25	T	105957	5	-	-	37	-
26	C	110012	15	20	-	-	-
27	C	110034	22	22	-	-	-
28	C	110038	4	4	-	-	-
29	C	110040	2	2	-	-	-
30	T	111006	26	-	-	69	-
31	C	111040	34	40	-	-	-
32	C	111053	13	13	-	-	-
33	B	111042	9	-	169	-	-
34	B	111048	6	-	6	-	-
35	T	111059	11	-	-	53	-
36	B	111107	8	-	19	-	-
37	C	111110	3	37	-	-	-
38	C	111121	11	11	-	-	-
39	C	111125	4	4	-	-	-

Vehicle No.	Vehicle type C=Car B=Bus T=Truck M=Motor cycle	Time of Arrival	Inter-arrival Time for all Vehicles T_a	Inter-arrival Time for Cars T_c	Inter-arrival Time for Bus T_b	Inter-arrival Time for Trucks T_t	Inter-arrival Time for Motor cycles T_m
40	C	111142	17	17	-	-	-
41	C	111146	4	4	-	-	-
42	C	111150	4	4	-	-	-
43	C	111157	7	7	-	-	-
44	C	111206	9	9	-	-	-
45	C	111211	5	5	-	78	-
46	T	111217	6	-	-	-	-
47	T	111224	7	-	-	7	-
48	B	111231	-	-	81	-	-
49	B	111241	10	-	10	-	-
50	B	111244	3	-	3	-	-
51	B	111253	9	-	9	-	-
52	C	111256	3	105	-	-	-
53	B	111317	21	-	24	-	-
54	C	111320	3	24	-	-	-
55	C	111327	7	7	-	-	-
56	C	111330	3	3	-	-	-
57	C	111336	6	6	-	-	-
58	C	111340	4	4	-	-	-
59	C	111349	9	9	-	-	-
60	T	111351	2	-	-	147	-
61	C	111429	38	40	-	-	-
62	B	111438	9	-	81	-	-
63	T	111442	4	-	-	51	-
64	B	111445	3	-	-	-	-
65	C	111450	5	21	7	-	-
66	C	111505	15	15	-	-	-
67	C	111511	6	6	-	-	-
68	C	111517	6	6	-	-	-
69	C	111520	3	3	-	-	-
70	C	111527	7	7	-	-	-
71	C	111528	11	1	-	-	-
72	C	111529	1	1	-	-	-
73	C	111533	4	4	-	-	-
74	C	111538	5	5	-	-	-
75	T	111557	19	-	-	71	-
76	C	111608	11	30	-	-	-
77	C	111609	1	1	-	-	-
78	T	111617	8	-	-	20	-
79	C	111637	20	28	-	-	-
80	C	111642	5	5	-	-	-
81	C	111644	2	2	-	-	-
82	C	111651	7	7	-	-	-
83	C	111656	5	5	-	-	-
84	C	111704	8	8	-	-	-
85	C	111707	3	3	-	-	-
86	C	111709	2	2	-	-	-
87	C	111711	2	2	-	-	-
88	C	111718	7	7	-	-	-

Vehicle No	Vehicle Type C=Car B=Bus T=Truck M=Motor cycle	Time of arrival	Inter-arrival Time for all vehicles T_a	Inter-arrival Time for Cars T_c	Inter-arrival Time for Buses T_b	Inter-arrival Time for Truck T_t	Inter-arrival Time for Motor-cycles T_m
89	C	111722	4	4	-	-	-
90	C	111729	7	7	-	-	-
91	C	111738	9	9	-	-	-
92	C	111748	10	10	-	-	-
93	C	111750	2	2	-	-	-
94	C	111754	4	4	-	-	-
95	C	111757	3	3	-	-	-
96	C	111801	4	4	-	-	-
97	C	111805	4	4	-	-	-
98	C	111808	3	3	-	-	-
99	C	111818	10	10	-	-	-
100	C	111825	7	7	-	-	-
101	C	111831	6	6	-	-	-
102	C	111840	9	9	-	-	-
103	C	111843	3	3	-	-	-
104	C	111848	5	5	-	-	-
105	C	111903	5	5	-	-	-
106	C	111914	11	11	-	-	-
107	C	111919	5	5	-	-	-
108	C	111926	7	7	-	-	-
109	C	111932	6	6	-	-	-
110	C	111939	7	7	-	-	-
111	C	111945	6	6	-	-	-
112	C	111952	7	7	-	-	-
113	C	111957	5	5	-	-	-
114	C	112003	6	6	-	-	-
115	C	112006	3	3	-	-	-
116	C	112010	4	4	-	-	-
117	T	112016	6	-	-	239	-
118	C	112020	4	10	-	-	-
119	C	112025	5	5	-	-	-
120	C	112039	4	4	-	-	-
121	C	112034	5	5	-	-	-
122	C	112038	4	4	-	-	-
123	C	112042	4	4	-	-	-
124	C	112108	26	26	-	-	-
125	C	111213	5	5	-	-	-
126	C	112118	5	5	-	-	-
127	C	112123	5	5	-	-	-
128	B	112129	6	-	398	-	-
129	T	112202	33	-	-	106	-
130	C	112209	7	46	-	-	-
131	C	112212	3	3	-	-	-
132	C	112216	4	4	-	-	-
133	C	112219	3	3	-	-	-
134	B	112223	4	-	54	-	-
135	C	112225	2	6	-	-	-
136	C	112239	4	4	-	-	-
137	C	112242	3	3	-	-	-
138	B	112248	6	-	25	-	-

Vehicle No.	Vehicle Type C=Car B=Bus T=Truck M=Motor cycle	Time of arrival	Inter-arrival Time for all vehicles T_a	Inter-arrival Time for Cars T_c	Inter-arrival Time for Buses T_b	Inter-arrival Time for Trucks T_t	Inter-arrival Time for Motor-cycles T_m
139	C	112253	5	11	-	-	-
140	C	112259	6	6	-	-	-
141	C	112303	4	4	-	-	-
142	C	112307	4	4	-	-	-
143	C	112311	4	4	-	-	-
144	C	112312	1	1	-	-	-
145	C	112314	2	2	-	-	-
146	C	112318	4	4	-	-	-
147	B	112323	5	-	35	-	-
148	C	112325	2	7	-	-	-
149	C	112339	14	14	-	-	-
150	C	112342	3	3	-	-	-
151	B	112348	6	-	25	-	-
152	C	112353	5	11		-	-
153	C	112359	6	6	-	-	-
154	C	112403	4	4	-	-	-
155	C	112407	4	4	-	-	-
156	C	112411	4	4	-	-	-
157	C	112412	1	1	-	-	-
158	C	112414	2	2	-	-	-
159	C	112418	4	4	-	-	-
160	B	112423	5	-	25	-	-
161	C	112431	8	13	-	-	-
162	C	112435	4	4	-	-	-
163	C	112437	2	2	-	-	-
164	B	112441	4	-	18	-	-
165	C	112446	5	9	-	-	-
166	T	112509	23	-	-	127	-
167	B	112514	5	-	33	-	-
168	C	112520	6	34	-	-	-
169	C	112523	3	3	-	-	-
170	C	112526	3	3	-	-	-
171	C	112528	2	2	-	-	-
172	C	112534	6	6	-	-	-
173	C	112539	5	5	-	-	-
174	C	112543	4	4	-	-	-
175	B	112545	2		31	-	-
176	B	112551	6	-	6	-	-
177	C	112558	7	75		-	-
178	C	112604	6	6	-	-	-
179	B	112611	7	-	20	-	-
180	C	112619	8	15	-	-	-
181	C	112624	5	5	-	-	-
182	C	112627	3	3	-	-	-
183	C	112629	2	2	-	-	-
184	C	112630	1	1	-	-	-
185	C	112633	5	5	-	-	-
186	C	112636	1	1	-	-	-
187	C	112642	6	6	-	-	-
188	C	112651	9	9	-	-	-
189	C	112655	4	4	-	-	-
190	B	112659	4		41	-	-

APPENDIX 1E

Vehicle No.	Vehicle type C=Car, B=Bus T=Truck M=Motor cycle	Time of arrival	Inter-arrival Time for all vehicle T_a	Inter-arrival Time for Cars T_c	Inter-arrival Time for Buses T_b	Inter-arrival Time for Trucks T_t	Inter-arrival Time for Motor-cycles T_m
191	C	112717	18	22	-	-	-
192	C	112719	2	2	-	-	-
193	C	112737	18	18	-	-	-
194	C	112742	5	5	-	-	-
195	B	112750	8	-	51	-	-
196	C	112807	17	25	-	-	-
197	C	112809	2	2	-	-	-
198	C	112813	4	4	-	-	-
199	C	112826	13	13	-	-	-
200	C	112828	2	2	-	-	-
201	C	112830	2	2	-	-	-
202	C	112841	11	11	-	-	-
203	C	112845	4	4	-	-	-
204	C	112849	4	4	-	-	-
205	T	112852	3	-	-	223	-
206	C	112910	18	22	-	-	-
207	C	112914	4	4	-	-	-
208	C	112915	1	1	-	-	-
209	C	112918	3	3	-	-	-
210	C	112924	6	6	-	-	-
211	C	112926	2	2	-	-	-
212	C	112930	4	4	-	-	-
213	C	112932	2	2	-	-	-
214	C	112935	3	3	-	-	-
215	C	112951	16	16	-	-	-
216	C	112956	5	5	-	-	-
217	C	112957	1	1	-	-	-

APPENDIX 1 F

TOLL BOOTH DELAY DATA

NAME: ABUA

STARTING TIME: 14 2500PM

DATE: 27/11/82

DURATION: 20 MINUTES

DAY OF WEEK: FRIDAY

LANE: 2

VEHICLE TYPE - BUSES

DIRECTION OF TRAFFIC: LAGOS-IBADAN ~~OR IBADAN-LAGOS~~

SERIAL NO	VEHICLE REGISTRATION NO.	TIME OF ARRIVAL AT TOLL PLAZA	TIME OF ARRIVAL AT TOLL BOOTH
1	LA 707 B	14 25 09	14 25 38
2	KW 1538 BC	14 25 40	14 25 51
3	LAE 1582	14 26 02	14 26 32
4	LA 14 KD	14 26 50	14 27 10
5	LA 918 WC	14 26 54	14 27 50
6	LAE 9180	14 27 40	14 28 40
7	LSB 215	14 28 16	14 29 12
8	KN 8 KG	14 29 17	14 30 21
9	LA 193 SC	14 29 54	14 30 32
10	LA 732 AH	14 30 32	14 31 43
11	KN 9866	14 31 33	14 32 08
12	BD 137 A	14 31 49	14 32 41
13	LR 7073	14 31 56	14 32 52
14	OY 215 H	14 32 30	14 33 12
15	OY 8997 AE	14 34 01	14 35 28
16	LA 8494 SB	14 35 28	14 36 14
17	OYB 1942 A	14 35 55	14 39 48
18	OY 2437 B	14 42 50	14 42 34
19	OD 2554 A	14 43 23	14 44 03

APPENDIX 1G

TRAFFIC BEHAVIOUR DATA

DATE: 23/1/82 NAME: OBED, EMA, ABUA.

STARTING TIME: 15 2500 P.M. DURATION: 20 MINUTES

POSITION: AT TOLL BOOTHS LANE: ALL TOLL BOOTH LANES

DAY OF WEEK: SATURDAY INITIAL NO. VEHICLES

NO ON TOLL LANES OPEN 6 IN LANE: -

DIRECTION OF TRAFFIC: LAGOS - IBADAN ~~OR IBADAN - LAGOS~~

LANE NO.	FRAUDULENT CASES	JOCKEYING	BALKING
1	2	-	-
2	2	15	2
3	1	11	1
4	1	18	2
5	-	69	7
6	-	22	4

APPENDIX 1 H

COUNTING (POISSON) DISTRIBUTION DATA

DATE: 18/7/81 NAME: OBED

SAMPLING TIME: 115500 AM DURATION: 20 MINUTES

POSITION: TOLL PLAZA LANE: 1 2 3 4 5 6

DAY OF WEEK: SATURDAY INITIAL NO. OF VEHICLES: 1 0 0 3 4 2

NO OF TOOL LANES OPEN: 6

DIRECTION OF TRAFFIC: LAGOS - IBADAN ~~OR IBADAN - LAGOS~~

TIME OF ARRIVAL IN 30 SECOND INTERVALS	LANE 1	LANE 2	LANE 3	LANE 4	LANE 5	LANE 6
11 55 00	4	2	0	3	5	5
	0	0	0	3	1	2
11 56 00	0	1	0	3	2	1
	0	3	0	1	3	4
11 57 00	1	0	1	4	4	1
	3	0	0	2	3	3
11 58 00	2	0	1	1	3	2
	1	1	0	1	2	3
11 59 00	1	0	1	2	2	1
	0	2	0	1	3	2
12 00 00	1	1	0	4	3	2
	2	0	0	1	3	2
12 01 00	1	1	0	2	3	1
	1	2	0	3	3	3
12 02 00	1	1	1	1	2	1
	0	0	1	2	2	0
12 03 00	0	0	3	0	2	1
	0	1	1	0	1	2
12 04 00	0	0	2	1	1	2
	0	0	1	1	0	2
12 05 00	0	1	2	0	3	4
	2	3	0	3	3	2
12 06 00	0	1	1	3	2	3
	2	1	2	2	3	1
12 07 00	0	0	3	2	1	2
	1	1	0	1	4	5
12 08 00	1	1	0	1	3	4
	1	0	0	0	1	1
12 09 00	0	2	0	0	2	1
	2	3	0	0	2	1
12 10 00	3	4	0	0	3	1
	0	0	1	5	7	4
12 11 00	3	4	0	1	5	6
	1	0	1	5	4	5
12 12 00	0	0	0	1	1	1
	1	2	0	2	2	2
12 13 00	1	1	0	2	4	3
	1	2	0	1	2	3
12 14 00	1	2	3	2	1	3
12 15 00	0	1	0	1	1	2
	1	1	0	1	2	4
	1	1	0	1	3	2

APPENDIX 2

SPECIMEN CALCULATIONS ON RAW DATA

From standard texts on statistics,

The mean inter-arrival/service time $(\bar{T}) = \frac{\sum T}{n}$ ---- (i)

The standard deviation $(\delta) =$

$$\begin{cases} \sqrt{\dfrac{\sum T^2 - \frac{(\sum T)^2}{n}}{n}} & \text{for large samples.} \\ \sqrt{\dfrac{\sum T^2 - \frac{(\sum T)^2}{n}}{n-1}} & \text{for small samples.} \end{cases}$$

---- (ii)

and

Coefficient of variation (C.V) = $\frac{\delta}{\bar{T}}$ ----(iii)

where

n = sample size

T = observed inter-arrival or service time.

Using equations (i), (ii), (iii) and values from table of appendix IE

we have the following:

(a) All vehicles taken together

$\bar{T}a = 7.08$ sec.

$\delta_a = 7.35$ sec.

C.V = 1.04.

(b) For cars only.

$\delta_c = 14.16$ sec.

$\bar{T}c = 9.65$ sec.

C.V = 1.47

(c) For Buses only

$\bar{T}b = 48.14$ sec.

$\delta_b = 76.08$ sec.

C.V = 1.58

(d) For trucks only

$\bar{T}t = 85.80$ sec.

$\delta_t = 70.90$ sec.

C.V = 0.83

APPENDIX 3a

PROPORTION OF EACH VEHICLE CLASS IN TOLL BOOTH QUEUE BASED ON COUNTING DISTRIBUTION

DAY OF WEEK	PERIOD	LANE						TOTAL	PROPORTION OF TRUCKS (α_1)	PROPORTION OF BUSES (α_2)	PROPORTION OF CARS (α_3)
		1	2	3	4	5	6				
SAT.	1153-1213	67	87	123	195	234	261	967	.0693	.2172	.7135
SAT.	1250-1310	40	27	66	95	136	139	493	.0811	.1886	.7505
SAT.	1350-1410	37	36	40	52	107	131	403	.0918	.1886	.7196
SAT.	1435-1455	41	32	72	126	150	142	563	.0728	.1847	.7425
SAT.	1535-1555	56	51	38	69	138	98	450	.1244	.1977	.6777
SAT.	1155-1215	38	46	25	68	106	141	424	.0896	.1675	.7429
SAT.	1420-1440	33	35	27	28	88	76	287	.1150	.2160	.6689
SAT.	1243-1303	33	36	45	59	93	73	339	.0973	.2389	.6637
SAT.	1332-1352	21	17	32	49	85	73	277	.0758	.1768	.7472
SAT.	1402-1422	24	21	39	58	92	81	315	.0762	.1905	.7333
SAT.	1133-1153	33	37	38	114	109	118	449	.0735	.1670	.7595
SAT.	1445-1505	26	28	22	50	77	53	256	.1016	.1953	.7031
							Mean		0.0890	.1940	.7185

APPENDIX 3B

APPROXIMATE CALCULATION FOR THE PROPORTION OF U-TURNING VEHICLES THAT ARE OF CLASS i

Define:

α_1 = Proportion of all vehicles that are trucks.

α_2 = Proportion of all vehicles that are buses.

α_3 = Proportion of all vehicles that are cars.

α'_1 = Proportion of trucks in the toll booth queues.

α'_2 = Proportion of buses in the toll booth queues.

α'_3 = Proportion of cars in the toll booth queues.

α = Proportion of all vehicles that get to the toll booth queues.

α_U = Proportion of all vehicles that make a U-turn back to Lagos.

K = Total number of all vehicle classes arriving into the police queue.

K' = Total number of all vehicle classes arriving into the toll booth queues.

α''_1 = Proportion of U-turning vehicles that are trucks.

α''_2 = Proportion of U-turning vehicles that are buses.

α''_3 = Proportion of U-turning vehicles that are cars.

Using the case of the trucks, the following approximate relationship appears to hold for the particular condition of this study:

$$\alpha_1 K - \alpha'_1 K' = \alpha''_1 (K - K') = \alpha''_1 K(1 - \alpha) \quad \text{-----(3.1)}$$

On further simplification this yields

$$\alpha_1 K - \alpha'_1 \alpha K = \alpha''_1 K(1 - \alpha) \Rightarrow \alpha''_1 = \frac{\alpha_1 - \alpha'_1 \alpha}{1 - \alpha} \quad \text{--(3.2)}$$

Similarly for the buses

$$\alpha''_2 = \frac{\alpha_2 - \alpha''_2 \alpha}{1 - \alpha} \quad \text{----------(3.3)}$$

and for the cars

$$\alpha''_3 = \frac{\alpha_3 - \alpha''_3 \alpha}{1 - \alpha} \qquad \text{---- (3.4)}$$

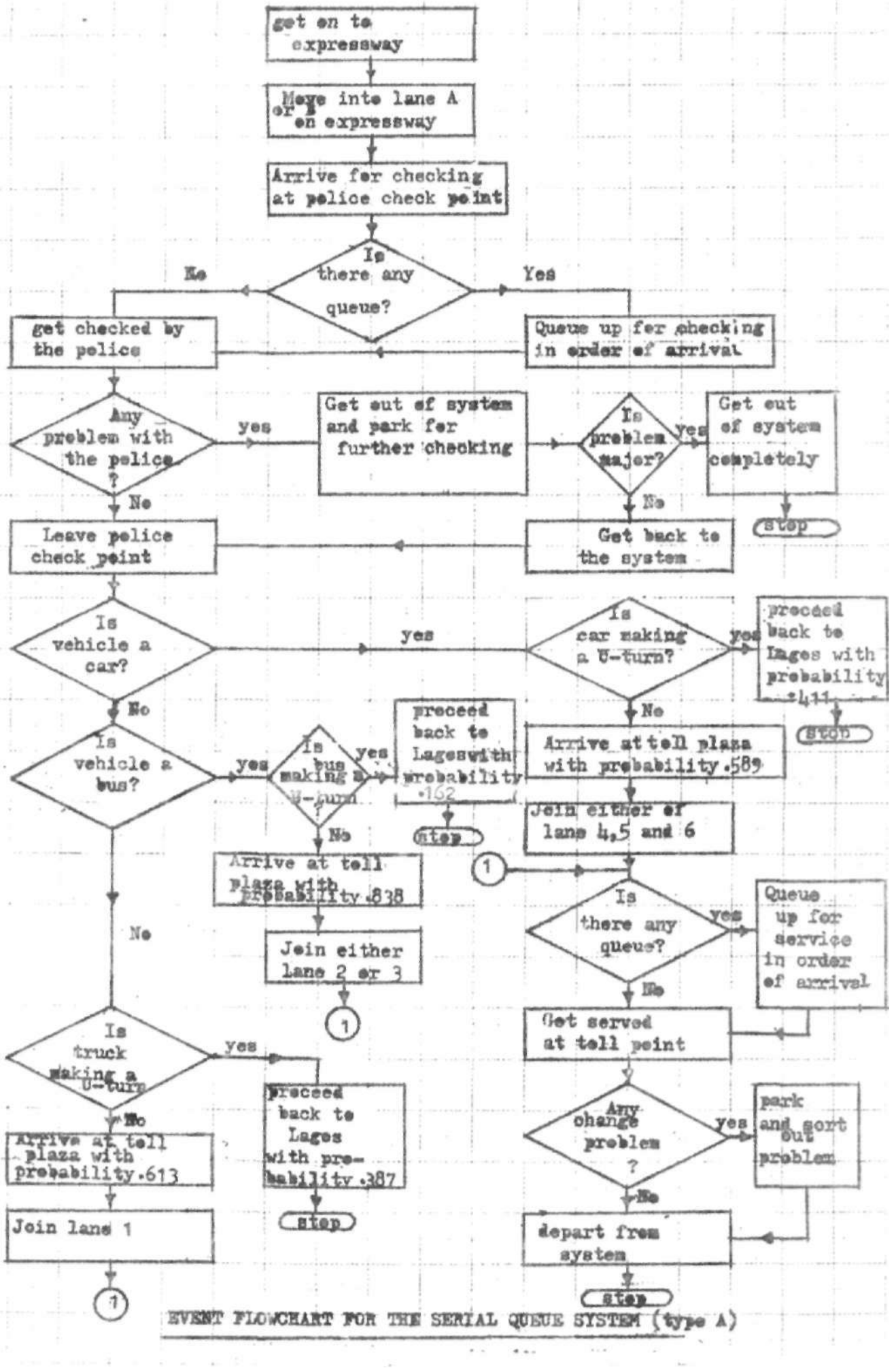
Now using values from Tables 4.7, 4.8 and Appendix 3a, these equations give the following approximate results:

$$\alpha''_1 = .0925$$
$$\alpha''_2 = .0619$$

and

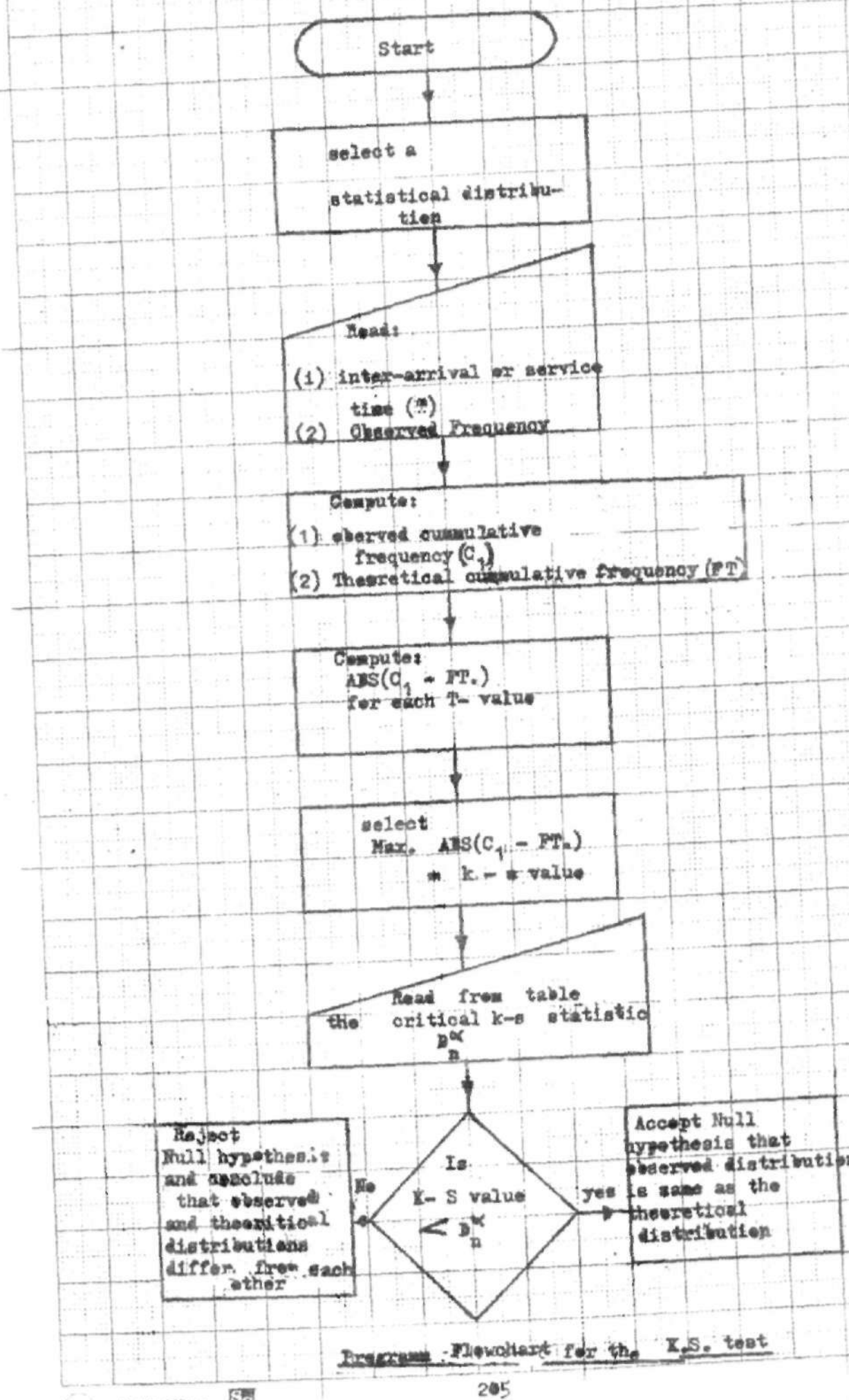
$$\alpha''_3 = .8249$$

APPENDIX 4a



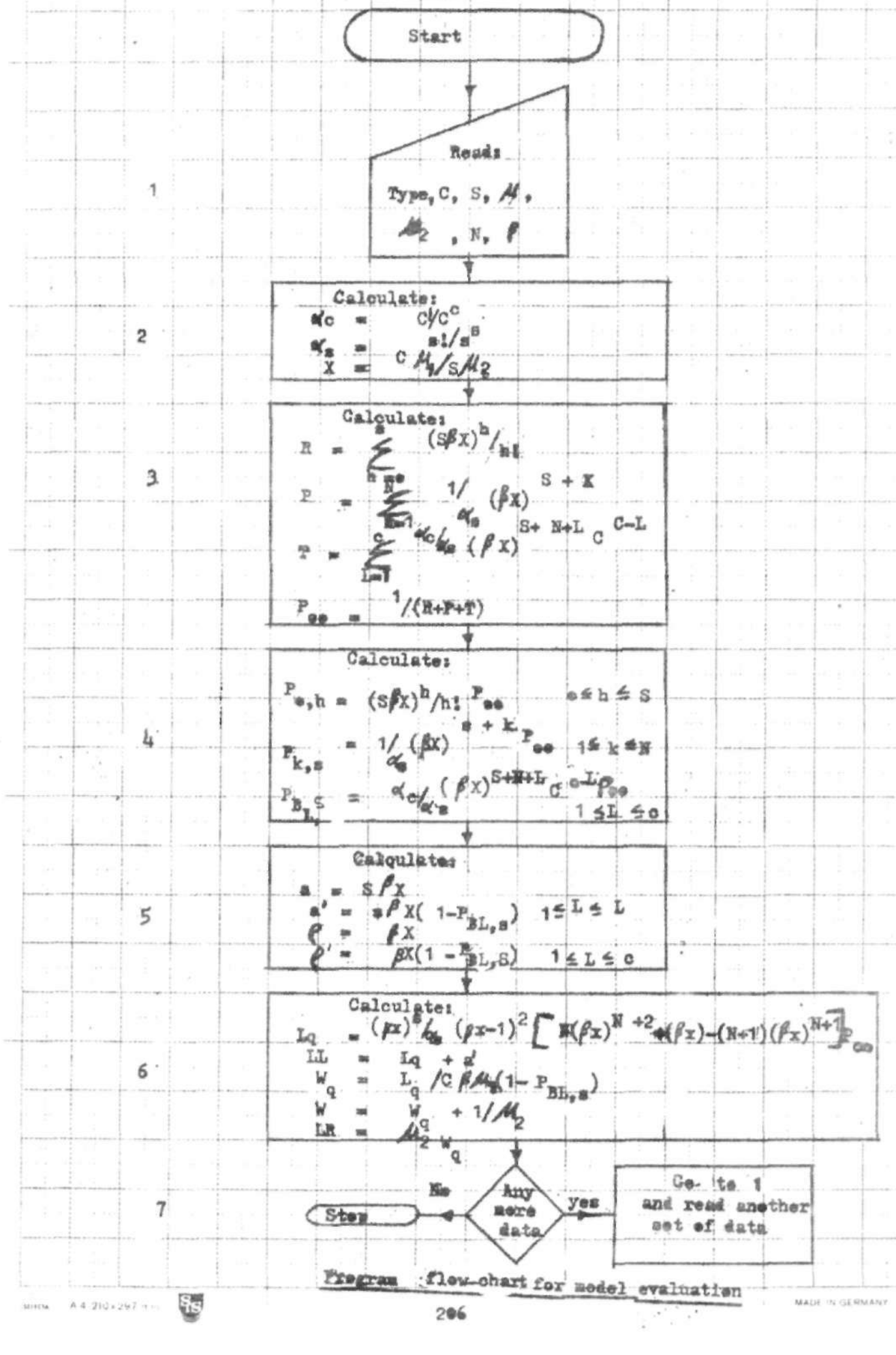
EVENT FLOWCHART FOR THE SERIAL QUEUE SYSTEM (type A)

APPENDIX 4b



Program -Flowchart for the K.S. test

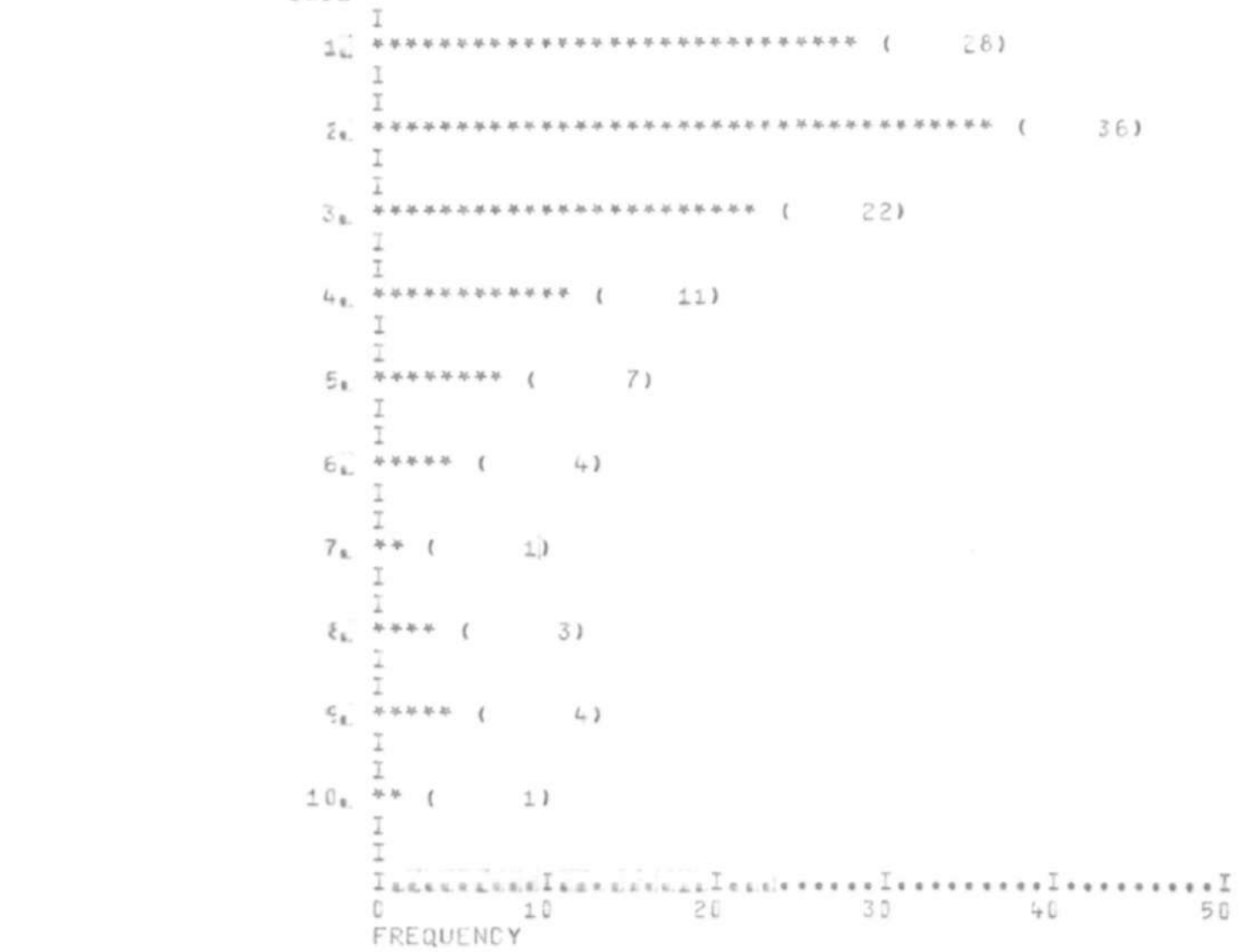
APPENDIX 4 C



Program :flow-chart for model evaluation

FILE FREQA (CREATION DATE = 82/05/21.)
SUBFILE SUB3

CAT



MEAN	2.957	STD ERR	.193	MEDIAN	2.347
MODE	2.000	STD DEV	2.090	VARIANCE	4.369
KURTOSIS	1.974	SKEWNESS	1.542	RANGE	9.000
MINIMUM	1.000	MAXIMUM	10.000	SUM	346.000
C.V. PCT	70.679	.95 C.I.	2.575	TO	3.340

VALID CASES 117 MISSING CASES 0

APPENDIX 4e

RESULTS OF TRUNCATED TOLL BOOTH QUEUING SYSTEM
FOR CARS

C= 2 S= 3 MU1= 1517 MU2= 326 N= 43 BETA= .5830

RHO	=	1.8272								
PLL	=	.1116								
LQ	=	16.1794								
A	=	5.4817								
ADASH	=	3.7261	3.8778							
RODASH	=	1.2420	1.2926							
WQ	=	.0133	.0128							
W	=	.0164	.0159							
LL	=	19.9055	20.0572							
LR	=	4.3422	4.1723							
POH	=	.0000	.0000	.0000	.0000					
PBLS	=	.3203	.2926							
PKS	=	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
PKS	=	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
PKS	=	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
PKS	=	.0001	.0002	.0004	.0008	.0014	.0026	.0047	.0086	.0157	.0287
PKS	=	.0525	.0959	.1753							

GOODNESS-OF-FIT TEST

LAM30A/MEU= .0755VEH.PER SECOND

TIME	OBS. FREQ.	OBS. CUM. FREQ.	EXPTD. CUM. FREQ.	K-S	
2.0	5	.1064	.1411	.0361	
3.0	4	.1888	.2026	.0146	
4.0	3	.2500	.2606	.0106	
5.0	4	.3330	.3143	.0187	
6.0	2	.3750	.3642	.0108	
7.0	4	.4580	.4104	.0476	
8.0	1	.4790	.4533	.0257	
9.0	2	.5210	.4930	.0280	
10.0	3	.5830	.5299	.0531	
11.0	1	.6040	.5640	.0400	
12.0	1	.6250	.5957	.0293	
13.0	1	.6460	.6251	.0209	
14.0	1	.6670	.6524	.0146	
15.0	4	.7500	.6776	.0724	MAX. K-S=.0724
17.0	1	.7710	.7228	.0482	CRITICAL K-S=.1900 AT 5%
18.0	1	.7920	.7430	.0490	ACCEPT NULL HYPOTHESIS
21.0	2	.8330	.7950	.0380	
23.0	1	.8540	.8237	.0303	
25.0	2	.8960	.8484	.0476	
28.0	1	.9170	.8792	.0378	
29.0	1	.9380	.8879	.0501	
37.0	2	.9790	.9367	.0423	
95.0	1	1.0000	.9992	.0008	

DATA FOLLOW NEGATIVE EXPONENTIAL DISTRIBUTION

BIBLIOGRAPHY

The following is part of the literature either cited in this thesis or referenced during the course of its preparation.

The journal references are quoted before the book references and the bibliography presented by Saaty formed an excellent guideline in the preparation of this list.

1. BALD, W.G. and UNDERWOOD, J.R. (1977), 'An Investigation Into The Effect Of Toll Booths On Morning Peak Traffic Flow', The Chartered Municipal Engineer, Vol. 104, pp. 205 - 208.

2. BURKE, P.J. (1956), 'The Output Of A Queueing System', Operations Research, Vol. 4, pp. 699 - 704.

3. DEBAVN, R.M. and KATZ, S. (1959), 'An Approximation To Distributions Of Summed Waiting Times', Operations Research, Vol. 7, No. 6.

4. EVERETT, J.L. (1953), 'State Probabilities in Congestion Problems Characterised by Constant Holding Times', Journal of Operations Research Society of America, Vol. 1, pp. 279 - 285.

5. GAVER, D.P. (1953), 'The Influence of Servicing Times in Queueing Processes', Journal of Operations Research, Vol. 2, pp 139 - 149.

14. LABETOULLE, J., PUJOLLE, G. SOULA, C. (1981), 'Stationary Distributions of Flows in Jackson Networks', Mathematics of Operations Research, Vol. 6, No. 2, pp. 173 - 185.

15. LESLIE, E. C. (1954), 'Traffic Delays at Tool Booths', Journal of Operations Research, Vol. 2, No. 2, pp. 107-138

16. MAKINO, T. (1964), 'On The Mean Passage Time Concerning Some Queueing Problems Of The Tandem Type', Journal Of The Operations Research Society Of Japan, Vol. 7, No. 1, pp. 17 - 47.

17. MORRISON, J.A. (1980), 'An Over-flow System, In which Queueing Takes Precedence', The Bell System Technical Journal, Vol. 60, No. 1, pp. 1 - 12.

18. NELSON, R.T. (1958), 'Waiting Time Distributions For Application To a Series Of Service Centres', Operations Research Vol. 6, pp. 856 - 862.

19. REICH, E. (1957), 'Waiting Times When Queues Are In Tandem, 'Ann. Math. Statist., Vol. 28, No. 3, pp. 768.

20. SHARMA, O.P. (1973), 'A Model For Queues in series', Journal of Applied Probability, Vol. 10 pp. 691 - 701.

21. SMITH, D.R. and WHITT, W. (1980), 'Resource Sharing For Efficiency In Traffic Systems', The Bell System Technical Journal, Vol 60, No. 1, pp. 39-55.

22. ANG, A.H-S. and TANG, W. H. (1975), Probability Concepts in Engineering Planning and Design. John Wiley and Sons, Inc., New York.

23. CHURCHMAN, C.W., ACKOFF, R.L. and ARNOFF, E.L. (1957), Introduction to Operations Research. John Wiley and Sons Inc., New York.

24. CONOVER, W.J. (1971), Practical Parametric Statistics. John Wiley and Sons Inc., New York.

25. COX, D.R. and SMITH, W.L. (1965), Queues. Methenn and Company Limited, London.

26. FREUND, J.E. and WALPOLE, R.E.(1980), Mathematical Statistics. Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

27. GIFFIN, W.C. (1978), Queueing - Basic Theory and Application. Grid Inc., Columbus, Ohio.

28. GROSS, D. and HARRIS, C.M. (1974), Fundamentals of Queueing Theory. John Wiley and Sons Inc., New York.

29. HANN, G.J. and SHAPIRO, S.S. (1967), Statistical Models in Engineering. John Wiley and Sons Inc., New York.

30. LEE, A.M. (1966), Applied Queueing Theory. Macmillan and Company Limited, London.

31. LEROY, S.F. and SHOUP, T.E. (1978) Introduction to Engineering including Fortran Programming. Prentice-Hall, Inc., Englewood Cliffs, New York.